

[7] J. W. Nillson, *Electric Circuits*, 3rd ed., Reading, MA: Addison-Wesley, 1990.

[8] D. L. Schilling and C. Belove, *Electronic Circuits*, 3rd ed., New York: McGraw-Hill, 1989.

## Further Information

More detailed developments of linear transformer circuit models can be found in most circuit analysis texts. Two example texts are [1] and [3].

Power transformers and their models are presented in considerable detail in electrical machinery texts such as [6]. These presentations also extend to multiple-coil, three-phase transformers, and tapped transformers. Included in most electrical machinery texts are in-depth discussions of hysteresis and eddy currents and of methods for reducing losses due to these effects. Also included are procedures for measuring the parameters for transformer circuit models.

The use of transformers in electronic circuits is considered in electronic circuit design texts and reference books such as [8] and [4]. Amplifier circuit models that include transformer circuit models are developed in some texts for tuned transformer coupled amplifier stages and transformer coupled load impedances. It is shown how these models can be used to determine amplifier frequency response and power transfer to the load. Other uses indicated for transformers in electronic circuits occur in power supplies and isolation amplifiers.

## 9.5 Semiconductor Diode

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Semiconductor diodes are made out of p-n semiconductor junctions. Nonlinear current-voltage characteristics of such junctions are used to rectify and shape electrical signals. Exponential current-voltage characteristics are sometimes used to build logarithmic amplifiers. The variations of junction capacitances with applied voltages are used to tune high-frequency electronic circuits. The semiconductor p-n junction illuminated by light will generate a voltage on its terminals. Such a diode is known as a solar battery. Also, the reverse diode current is proportional to the light intensity at the junction. This phenomenon is used in photodiodes. If a diode is biased in the forward direction, it can generate a light. In order to obtain high emission efficiency the light emitting diode (LED) should be made out of a semiconductor material with a direct energy band structure. This way electrons and holes can recombine directly between valence and conduction bands. Typically, LED's are fabricated using various compositions of  $\text{Ga}_y\text{Al}_{1-y}\text{As}_x\text{P}_{1-x}$ . The wavelength of generated light is inversely proportional to the potential gap of a junction material. When a light intensity is enhanced by additional micromirrors, then laser action occurs. The silicon diodes are not emitting light because the silicon has an *indirect* band structure and the probability of direct band-to-band recombination is very small.

When both sides of the junction are very heavily doped, then for small forward-biasing voltages (0.1–0.3 V) a large tunneling current may occur. For larger forward voltages (0.4–0.5 V) this tunneling current vanishes. This way the current-voltage characteristic has a negative resistance region somewhere between from 0.2 to 0.4 V [Fig. 9.68(d)]. Germanium and other than silicon semiconductors are used to fabricate tunnel diodes. The backward diode has slightly lower impurity concentrations than the tunnel diode and the tunneling current in the forward direction does not occur [Fig. 9.68(e)]. The backward diode is characterized by very sharp knee near zero voltage, and it is used for detection (rectifications) of signals with very small magnitude.

Diodes with high breakdown voltage have a p-i-n structure with an impurity profile shown in Fig. 9.67(d). Similar p-i-n structure is also used in microwave circuits as a switch or as

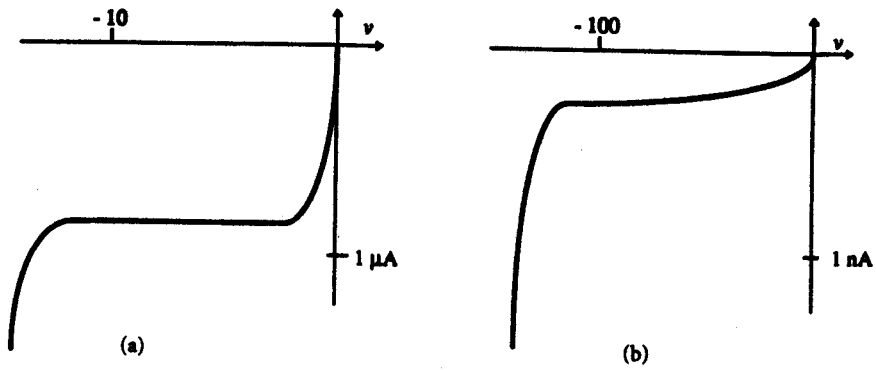


FIGURE 9.69 Reverse current-voltage characteristics: (a) germanium diode and (b) silicon diode.

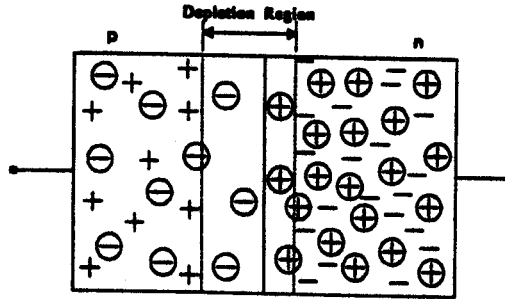


FIGURE 9.70 Illustration of the  $p$ - $n$  junction.

(electrons in  $p$ -type and holes in  $n$ -type) can be found using the equations

$$n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A} \quad p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D} \quad (9.100)$$

The intrinsic carrier concentration  $n_i$  is given by

$$n_i^2 = \xi T^3 \exp\left(-\frac{V_g}{V_T}\right); \quad V_T = \frac{kT}{q} \quad (9.101)$$

where  $V_T = kT/q$  is the thermal potential ( $V_T = 25.9$  mV at 300 K),  $T$  is absolute temperature in K,  $q = 1.6 \cdot 10^{-16}$  C is the electron charge,  $k = 8.62 \cdot 10^{-5}$  eV/K is the Boltzmann constant,  $V_g$  is potential gap ( $V_g = 1.124$  V for silicon), and  $\xi$  is a material constant. For silicon, intrinsic concentration  $n_i$  is given by

$$n_i = 7.98 \cdot 10^{15} T^{\frac{1}{2}} \exp\left(-\frac{6522}{T}\right) \quad (9.102)$$

For silicon at 300 K,  $n_i = 1.5 \cdot 10^{10} \text{ cm}^{-2}$ .

When a  $p$ - $n$  junction is formed, the fixed electrostatic lattice charges form an electrical field at the junction. Electrons are pushed by electrostatic forces deeper into the  $n$ -type region and holes into the  $p$ -type region, as illustrated in Fig. 9.70. Between  $n$ -type and  $p$ -type regions there is a depletion layer with a built-in potential which is a function of impurity doping level and

intrinsic concentration  $n_i$ :

$$V_{pn} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = V_T \ln\left(\frac{n_n p_p}{n_i^2}\right) = V_T \ln\left(\frac{n_n}{n_p}\right) = V_T \ln\left(\frac{p_p}{p_n}\right) \quad (9.103)$$

The junction current as a function of biasing voltage is described by the diode equation:

$$i = I_s \left[ \exp\left(\frac{v}{V_T}\right) - 1 \right] \quad (9.104)$$

where

$$I_s = A q n_i^2 V_T \left( \frac{\mu_p}{\int_0^{L_p} n_n dx} + \frac{\mu_n}{\int_0^{L_n} p_p dx} \right) \quad (9.105)$$

where  $n_n \approx N_D$ ,  $p_p \approx N_A$ ,  $\mu_n$  and  $\mu_p$  are mobility of electrons and holes,  $L_n$  and  $L_p$  are diffusion length for electrons and holes, and  $A$  is the device area.

In the case of diodes made of silicon or other semiconductor materials with a high energy gap, the reverse-biasing current cannot be calculated from the diode Eq. (9.104). This is due to the carrier generation-recombination phenomenon. Lattice imperfection and most impurities are acting as generation-recombination centers. Therefore, the more imperfections there are in the structure, the larger the deviation from ideal characteristics.

### Forward $I$ - $V$ Diode Characteristics

The diode Eq. (9.104) was derived with an assumption that injected carriers are recombining on the other side of the junction. The recombination within the depletion layer was neglected. In real forward-biased diodes, electrons and holes are injected through the depletion region and they may recombine there. The recombination component of the forward-biased diode is given by

$$i_{\text{rec}} = q w A \frac{n_i}{2\tau_0} \exp\left(\frac{v}{2V_T}\right) = I_{r0} \exp\left(\frac{v}{2V_T}\right) \quad (9.106)$$

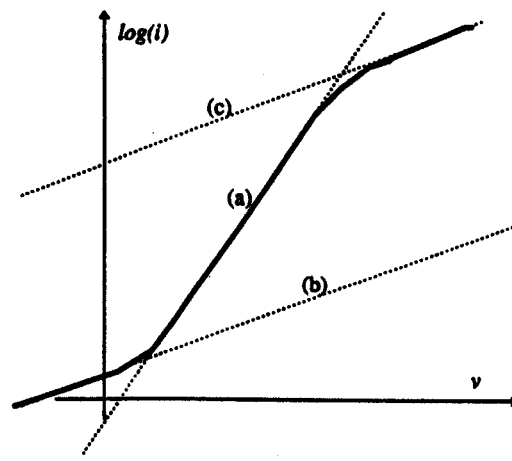
where  $w$  is the depletion layer thickness and  $\tau_0$  is the carrier lifetime in the depletion region. The total diode current  $i_T = i + i_{\text{rec}}$  where  $i$  and  $i_{\text{rec}}$  are defined by (9.104) and (9.106). The recombination component dominates at low current levels, as Fig. 9.71 illustrates.

Also in very high current levels, the diode Eq. (9.104) is not valid. Two phenomena cause this deviation. First, there is always an ohmic resistance that plays an important role for large current values. The second deviation is due to high concentration of injected minority carriers. For very high current levels, the injected minority carrier concentrations may approach, or even become larger than the impurity concentration. An assumption of the quasi-charge neutrality leads to an increase of the majority carrier concentration. Therefore, the effective diode current is lower, as can be seen from (9.105). The high current level in the diode follows the relation

$$i_h = I_{h0} \exp\left(\frac{v}{2V_T}\right) \quad (9.107)$$

Figure 9.71 shows the diode  $I$ - $V$  characteristics, which include generation-recombination, diffusion, and high current phenomena. For modeling purposes, the forward diode current can be approximated by

$$i_D = I_0 \exp\left(\frac{v}{\eta V_T}\right) \quad (9.108)$$



**FIGURE 9.71** Current-voltage characteristics of the p-n junction in the forward direction: (a) diffusion current, (b) recombination current, and (c) high-level injection current.

where  $\eta$  has a value between 1.0 and 2.0. Note, that the  $\eta$  coefficient is a function of current, as can be seen in Fig. 9.71. It has a larger value for small and large current regions and it is close to unity in the medium current region.

### Reverse $I$ - $V$ Characteristics

The reverse leakage current in silicon diodes is mainly caused by the electron-hole generation in the depletion layer. This current is proportional to the number of generation-recombination centers. These centers are formed either by a crystal imperfection or deep impurities, which create energy states near the center of the energy gap. Once the reverse voltage is applied, the size of the depletion region and the number of generation-recombination centers increase. Thus, the leakage current is proportional to the thickness of the depletion layer  $w(v)$ . For a step-abrupt junction

$$w = \sqrt{\frac{2\epsilon\epsilon_0(V_{pn} - v)}{qN_{\text{eff}}}} \quad (9.109)$$

For other impurity profiles,  $w$  can be approximated by

$$w = K(V_{pn} - v)^{\frac{1}{n}} \quad (9.110)$$

The reverse-diode current for small and medium voltages can therefore be approximated by

$$i_{\text{rev}} = Aw(v) \frac{qn_i}{2\tau_0} \quad (9.111)$$

where  $n_i$  is given by (9.101) and  $w$  by (9.109) or (9.110). The reverse current increases rapidly near the breakdown voltage. This is due to the avalanche multiplication phenomenon. The multiplication factor is often approximated by

$$m = \frac{1}{1 - \left(\frac{v}{BV}\right)^m} \quad (9.112)$$

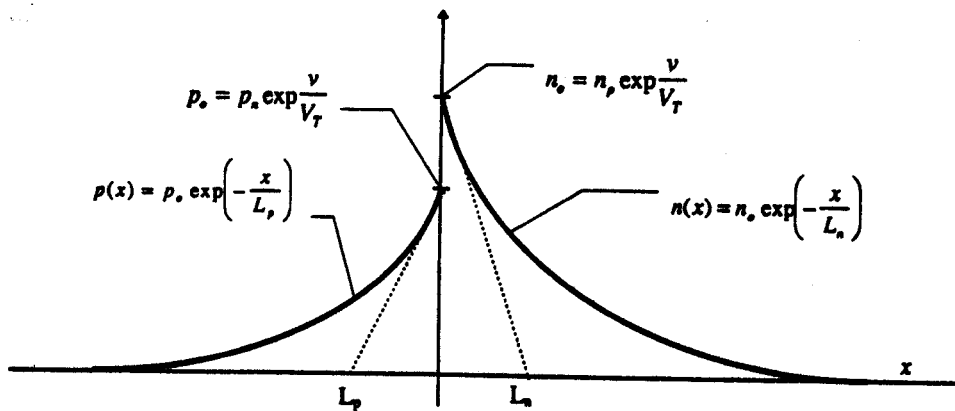


FIGURE 9.72 Minority carrier distribution in the vicinity of the p-n junction biased in the forward direction.

where  $BV$  stands for the breakdown voltage and  $m$  is an exponent chosen experimentally. Note, that for the reverse biasing both  $v$  and  $BV$  have negative values and the multiplication factor  $M$  reaches an infinite value for  $v = BV$ .

## Diode Capacitances

Two types of capacitances are associated with a diode junction. One capacitance, known as diffusion capacitance, is proportional to the diode current. This capacitance exists only for the forward-biased condition and has the dominant effect there. The second capacitance, known as the depletion capacitance, is a weak function of the applied voltage.

### Diffusion Capacitance

In a forward-biased diode, minority carriers are injected into opposite sides of the junction. Those minority carriers diffuse from the junction and recombine with the majority carriers. Fig. 9.72 shows the distribution of minority carriers in the vicinity of the junction of uniformly doped n-type and p-type regions. The electron charge stored in the p-region corresponds to the area under the curve, and it is equal to  $Q_n = qn_0L_n$ . Similarly, the charge of stored holes  $Q_p = qp_0L_p$ . The storage charge can also be expressed as  $Q_n = I_n\tau_n$  and  $Q_p = I_p\tau_p$ , where  $I_n$  and  $I_p$  are electron and hole currents at the junction and  $\tau_n$  and  $\tau_p$  are the lifetimes for minority carriers. Assuming  $\tau = \tau_n = \tau_p$  and knowing that  $I = I_p + I_n$  the total storage charge at the junction is  $Q = I\tau$ . The diffusion capacitance can be then computed as

$$C_{\text{dif}} = \frac{dQ}{dv} = \frac{d}{dv} \left[ \tau I_0 \exp\left(\frac{v}{\eta V_T}\right) \right] = \frac{\tau I_B}{\eta V_T} \quad (9.113)$$

As one can see, the diffusion capacitance  $C_{\text{dif}}$  is proportional to the storage time  $\tau$  and to the diode biasing current  $I_B$ . Note that the diffusion capacitance does not depend on the junction area, only on the diode current. The diffusion capacitances may have very large values. For example, for 100 mA current and  $\tau = 1 \mu\text{s}$  the junction diffusion capacitance is about 4  $\mu\text{F}$ . Fortunately, this diffusion capacitance is connected in parallel to the small-signal junction resistance  $r = \eta V_T/I_B$ , and the time constant  $rC_{\text{dif}}$  is equal to the storage time  $\tau$ .

### Depletion Capacitance

The reversed-biased diode looks like a capacitor with two "plates" formed of p-type and n-type regions and a dielectric layer (depletion region) between them. The capacitance of a

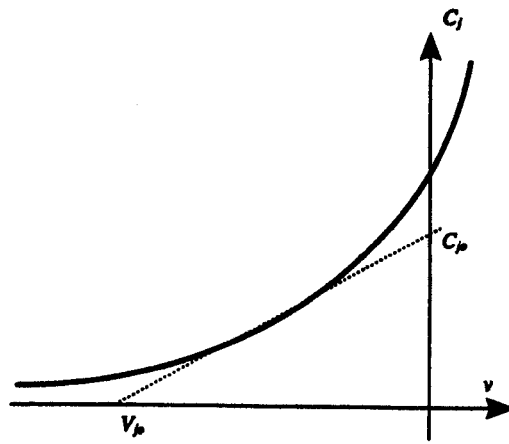


FIGURE 9.73 Capacitance-voltage characteristics for reverse-biased junction.

reversed-biased junction can then be written as

$$C_{\text{dep}} = A \frac{\epsilon}{w} \quad (9.114)$$

where  $A$  is a junction area,  $\epsilon$  is the dielectric permittivity of semiconductor material, and  $w$  is the thickness of the depletion layer. The depletion layer thickness  $w$  is a weak function of the applied reverse-biasing voltage. In the simplest case, with a step-abrupt junction, the depletion capacitance is

$$C_j = \sqrt{\frac{qN_{\text{eff}}\epsilon\epsilon_0}{2(V_{pn} - v)}}; \quad \frac{1}{N_{\text{eff}}} = \frac{1}{N_D} + \frac{1}{N_A} \quad (9.115)$$

The steepest capacitance-voltage characteristics are in  $p^+ - i - p - n^+$  diodes with the impurity profiles shown in Fig. 9.67(f). In general, for various impurity profiles at the junction, the depletion capacitance  $C_j$  can be approximated by

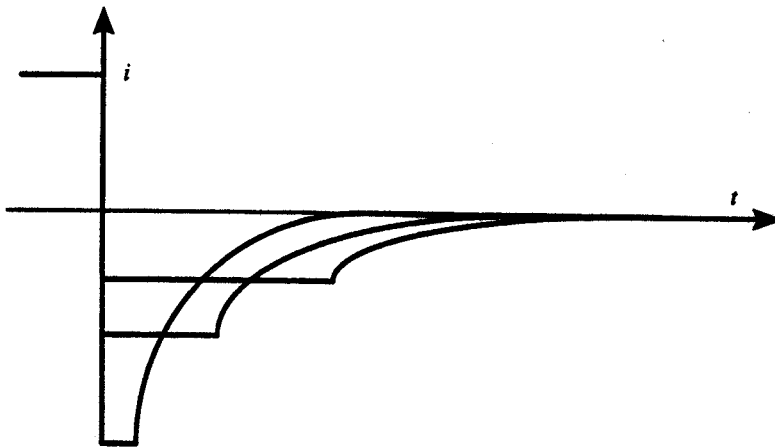
$$C_j = C_{j0} \left( 1 - \frac{v}{V_{pn}} \right)^{\frac{1}{n}} \quad (9.116)$$

or using linear approximation as shown in Fig. 9.73:

$$C_j = C_{j0} \left( 1 - \frac{v}{V_{j0}} \right) \quad (9.117)$$

## Diode as a Switch

The switching time of the p-n junction is limited mainly by the storage charge of injected minority carriers into the vicinity of the junction (electrons injected in p-type region and holes injected in n-type region). When a diode is switched from the forward to the reverse direction these carriers may move freely through the junction. Some of the minority carriers recombine with time. Others are moved away to the other side of the junction. The diode cannot recover



**FIGURE 9.74** Currents in diode with large minority carrier lifetimes after switching from the forward to the reverse direction.

its blocking capability as long as a large number of the minority carriers exist and can flow through the junction. An example of the current–time characteristics of a diode switching from the forward to the reverse direction is shown in Fig. 9.74. A few characteristics that are shown in the figure are for the same forward current and different reverse currents. Just after switching, these reverse currents are limited only by external circuitry. In this example, shown in Fig. 9.74, most of the minority carriers are moved to the other side of the junction by the reverse current, and the recombination mechanism is negligible. Note that the larger the reverse current flowing after switching, the shorter the time required to recover the blocking capability. This type of behavior is typical for commonly used high-voltage diodes.

In order to shorten the switching time, diodes sometimes are doped with gold or other deep-level impurities to create more generation centers and to increase the carrier recombination. This way, the minority carrier lifetimes of such switching diodes are significantly reduced. The switching time is significantly shorter, but it is almost independent of the reverse-diode current after switching, as Fig. 9.75 shows. This method of artificially increasing recombination rates has some severe disadvantages. Such switching diodes are characterized by very large reverse leakage currents and small breakdown voltages.

The best switching diodes utilize metal–semiconductor contacts. They are known as Schottky diodes. In such diodes there is no minority carrier injection phenomenon; therefore, these diodes recover the blocking capability instantaneously. Schottky diodes are also characterized by a relatively small (0.2–0.3 V) voltage drop in the forward direction. However, their reverse leakage current is larger, and the breakdown voltage rarely exceeds 20–30 V. Lowering the impurity concentration in the semiconductor material leads to slightly larger breakdown voltages, but at the same time, the series diode resistances increase significantly.

## Temperature Properties

Both forward and reverse diode characteristics are temperature dependent. These temperature properties are very important for correct circuit design. The temperature properties of the diode can be used to compensate for the thermal effects of electronic circuits. Diodes can be used also as accurate temperature sensors. The major temperature effect in a diode is caused by the strong temperature dependence of the intrinsic concentration  $n_i$  [(9.101) and (9.102)] and by the exponential temperature relationship of the diode Eq (9.104). By combining (9.101) and (9.104) and assuming the temperature dependence of carrier mobilities, the voltage drop on the

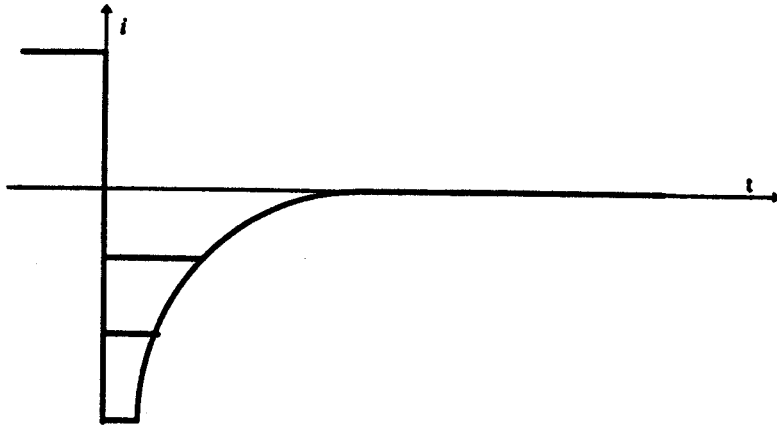


FIGURE 9.75 Currents in diode with small minority carrier lifetimes after switching from the forward to the reverse direction.

forward-biased diode can be written as

$$v = \eta \left[ V_T \ln \left( \frac{i}{\xi T^\alpha} \right) + V_g \right] \quad (9.118)$$

or diode current

$$i = I_0 \left( \frac{T}{T_0} \right)^\alpha \exp \left( \frac{(\eta/\eta) - V_g T_0}{V_{T_0} T} \right) \quad (9.119)$$

where  $V_g$  is the potential gap in semiconductor material,  $V_g = 1.124$  V for silicon and  $V_g = 1.424$  V for GaAs, and  $\alpha$  is a material coefficient ranging between 2.5 and 4.0.

The temperature dependence of the diode voltage drop  $dv/dT$  can be obtained by calculating the derivative of (9.118)

$$\frac{dv}{dT} = \frac{v - \eta(V_g + \alpha V_T)}{T} \quad (9.120)$$

For example, in the case of the silicon diode with a 0.6 voltage drop and assuming  $\eta = 1.1$ ,  $\alpha = 3.0$ , and  $T = 300$  K, the  $dV/dT = 1.87$  mV/°C.

The reverse-diode current is a very strong function of the temperature. For diodes made of semiconductor material with a small potential gap, such as germanium, the diffusion component dominates. In this case, the reverse current is proportional to

$$i_{\text{rev}} \propto T^\alpha \exp \left( -\frac{qV_g}{kT} \right) \quad (9.121)$$

For diodes made of silicon and semiconductors with a higher energy gap, the recombination is the dominant mechanism. In this case, reverse leakage current is proportional to

$$i_{\text{rev}} \propto T^\alpha \exp \left( -\frac{qV_g}{2kT} \right) \quad (9.122)$$

Using (9.122) one may calculate that for silicon diodes at room temperatures, the reverse leakage current doubles for about every 10°C.



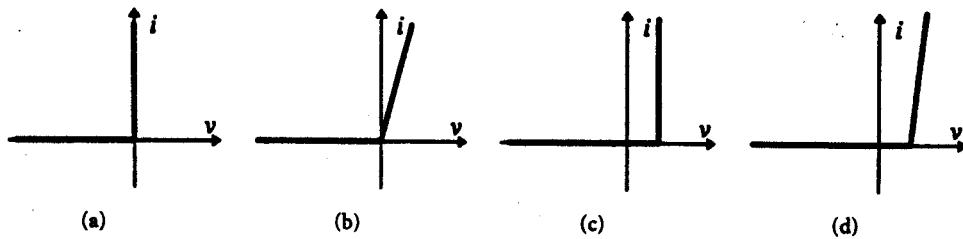


FIGURE 9.76 Various ways to linearize diode characteristics.

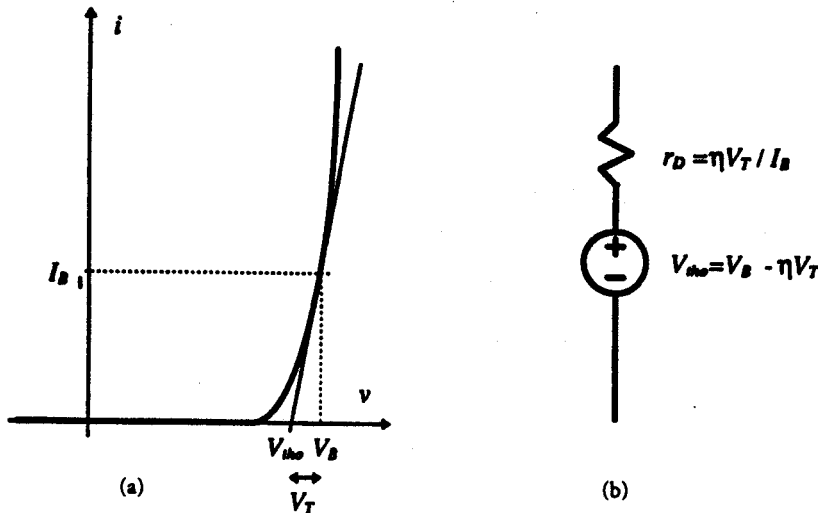


FIGURE 9.77 Linearization of the diode: (a) diode characteristics and (b) equivalent diagram.

The breakdown voltage is also temperature dependent. The tunneling effect dominates in diodes with small breakdown voltages. This effect is often known in literature as the Zener breakdown. In such diodes the breakdown voltage decreases with the temperature. The avalanche breakdown dominates in diodes with large breakdown voltages. When the avalanche mechanism prevails, then the breakdown voltage increases 0.06 percent to 0.1 percent /°C. For medium-range breakdown voltages, one phenomenon compensates the other, and temperature-independent breakdown voltage can be observed. This zero temperature coefficient exists for diodes with breakdown voltages equal to about  $5 V_T$ . In the case of the silicon diode, this breakdown voltage, with a zero temperature coefficient, is equal to about 5.6 V.

## Piecewise Linear Model

Nonlinear diode characteristics are often approximated by a piecewise linear model. There are a few possible approaches to linearize the diode characteristics, as Fig. 9.76 shows. The parameters of the most accurate linearized diode model are shown in Fig. 9.77(a), and the linearized diode equivalent circuit is shown in Fig. 9.77(b).

The modified diode Eq. (9.108) also can be written as

$$v = \eta V_T \ln \left( \frac{i}{I_0} \right) \quad (9.123)$$

For the biasing points  $V_B$  and  $I_B$ , the small-signal diode resistance  $dv/di$  can be computed from (9.123) as

$$r = \frac{dv}{di} = \frac{\eta V_T}{I_B}; \quad V_{th0} = V_B - \eta V_T \quad (9.124)$$

and it is only the function of the thermal potential  $V_T$  and the biasing current  $I_B$ . Note that the small-signal diode resistance is almost independent of the diode construction or semiconductor material used. If one requires that this linearized diode have  $I_B$  current for  $V_B$  voltage, then the piecewise diode characteristics should be as in Fig. 9.97(a). The equivalent Thevenin and Norton circuits are shown in Fig. 9.97(b). In the case of large-signal operation, the diode can be approximated by shifting the characteristics to the left by  $\Delta V$ . In this case, the threshold voltage becomes  $V_{th0} = V_B - 2V_T$  instead of  $V_{th0} = V_B - \eta V_T$ .

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