ELEC 3400

Communication Systems

Chapter 4: Angle Modulation

Introduction

- Angle modulation
  - The angle of the carrier wave is varied according to the based band wave $\theta_i(t) = 2\pi f(t)t + \varphi(t)$
    - Frequency modulation: $f(t)=f(m(t))$, $\varphi(t)=constant$
    - Phase modulation: $f(t)= constant$, $\varphi(t)= \varphi(m(t))$
  - The carrier amplitude remains the same
- Performance
  - More robust to noise/interference
  - ... but at the cost of transmission bandwidth and complexity
  - Practical trade-off between the three

UNMODULATED CARRIER

$$s_c(t) = A_c \cos(\underbrace{2\pi f_c t + \theta}_{\psi(t)})$$

$\psi(t)$: Inst. Phase

FREQUENCY MODULATION

Inst. freq. $f_i(t) = f_c + k_f m(t)$

PHASE MODULATION

Inst. phase $\psi(t) = 2\pi f_c t + k_p m(t)$ radians

$$\Rightarrow s_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t)), \quad -\infty < t < \infty$$

FM : Inst. freq $f_i(t) \overset{def}{=} \frac{1}{2\pi} \frac{d\psi(t)}{dt}$ Hz

$$\Rightarrow \psi(t) = \int 2\pi f_i(t)\,dt$$

$$= \int_{-\infty}^{t} 2\pi f_i(\tau)\,d\tau$$

$$\Rightarrow S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

Recall

$$S_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

Ex $m(t) = \cos(2\pi f_0 t)$

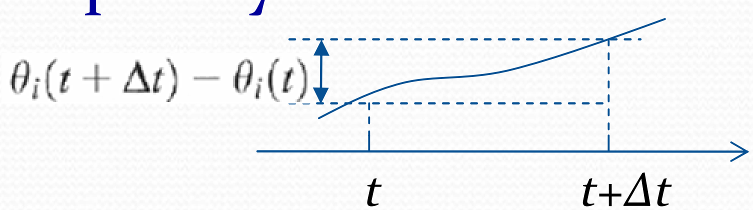
$$\Rightarrow S_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p \cos(2\pi f_0 t)\right)$$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + \left(\frac{k_f}{f_0}\right) \sin(2\pi f_0 t)\right)$$

$$\left[\int \cos(2\pi f_0 t)\, dt = \frac{\sin(2\pi f_0 t)}{2\pi f_0} \right]$$

Basic Definitions

- The angle $\theta_i(t)$ of the carrier is a function of the information wave $s(t) = A_c \cos[\theta_i(t)]$
- If $\theta_i(t)$ increases monotonically with time, the average frequency over an interval is



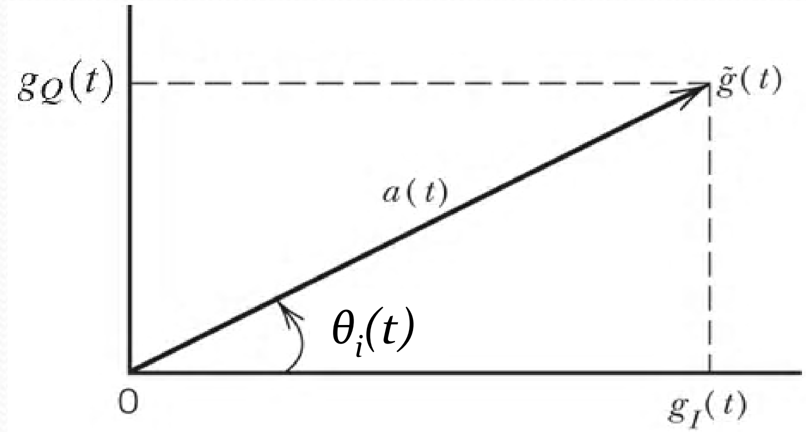
$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t}$$

- Let $\Delta t \rightarrow 0$, the instantaneous frequency of $s(t)$, measured in *radians per second*, is

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right] = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

Interpretation of *S(t)*

- As a rotating phasor
  - Length: A_c
  - Angle: $\theta_i(t)$
  - Angular velocity: $d\theta_i(t)/dt$
- Unmodulated signal
  - $\theta_i(t) = 2\pi f_c t + \varphi_c$
- Angle modulation
  - A_c constant
  - Making either f_c or φ_c a function of $m(t)$



Two Commonly Used Methods

- Phase modulation (PM)
  - The instantaneous angle is varied linearly with *m(t)*
  - k_p: phase sensitivity of the modulator, in *radians per*

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$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

- Frequency modulation (FM)
  - The instantaneous frequency is varied linearly with *m(t)*
  - per volt

$$f_i(t) = f_c + k_f m(t)$$

ψ

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

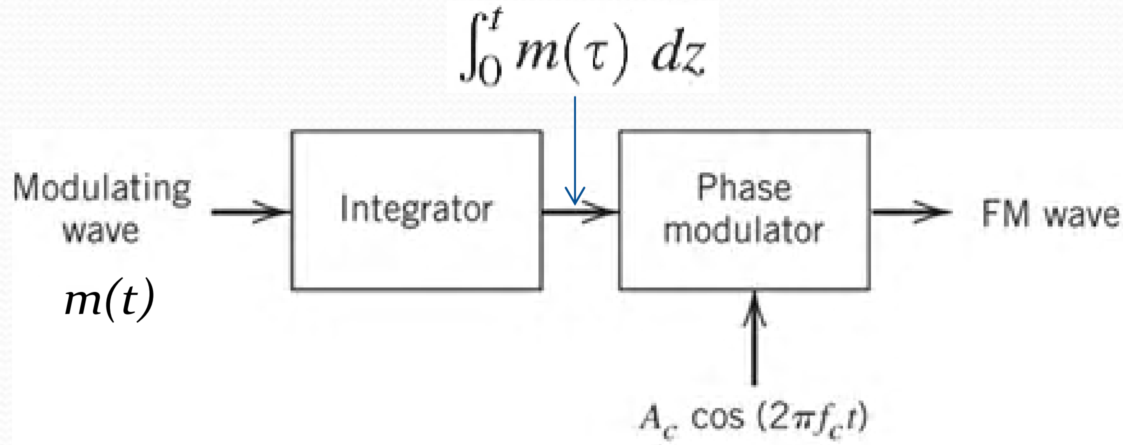
$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

Relationship Between PM and FM

- Equivalence of PM and FM
  - An FM signal with $m(t)$ can be regarded as a PM signal with $\int_0^t m(\tau)\, dz$
  - A PM signal with $m(t)$ can be regarded as an FM signal with $\frac{d[m(t)]}{dt}$
  - Due to the integral/differentiative relationship between phase and frequency
- All PM properties can be deduced from those of FM, and vice versa
  - Only need to study one of these two

Relationship Between PM and FM (contd.)

PM → FM:



FM → PM:

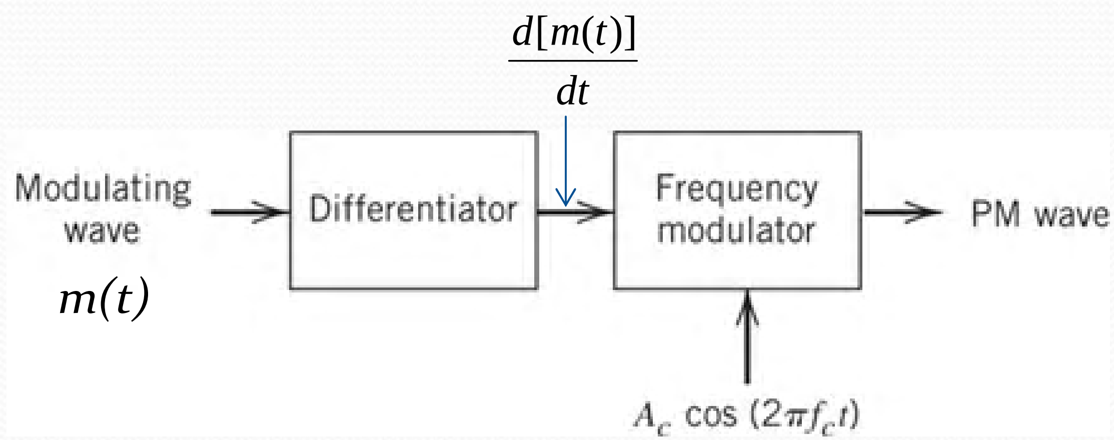


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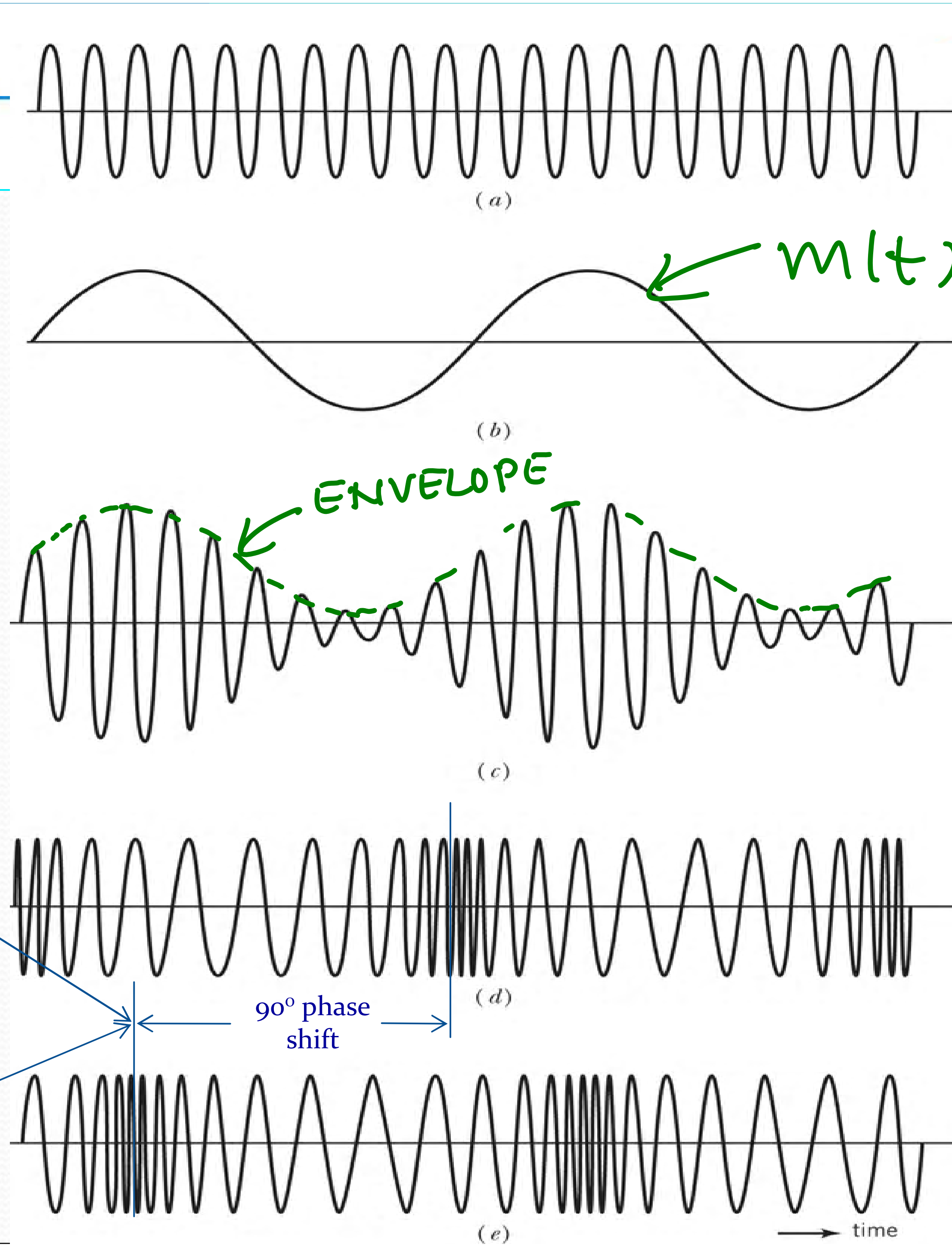
Illustration



AM:

$s(t)=m(t)\times\cos(2\pi f_c t)$

$+ k\cos(2\pi f_c t)$

PM:

$\phi_i(t) = k_p m(t)$

$= k_p \cos(2\pi f_m t)$

FM:

$\phi_i(t) = k_f 2\pi \int_0^t \cos(2\pi f_m \tau) d\tau$

$= \frac{k_f}{f_m} \sin(2\pi f_m t)$

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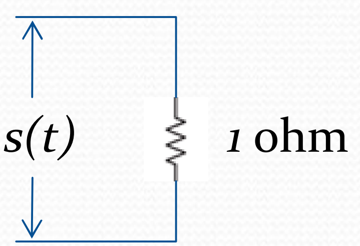
Property 1: Constancy of Transmitted Power

- Modulated signal has constant amplitude

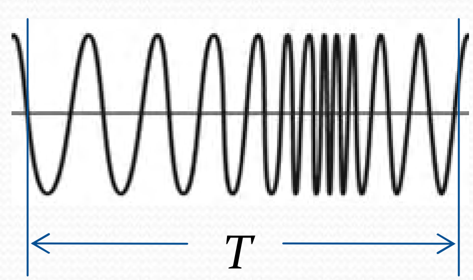
$$s(t) = A_c \cos[\theta_i(t)]$$

- Average transmitted power is



$$\begin{aligned} P_{av} &= \frac{1}{T}\int_0^T A_c^2 \cos^2[\theta(t)]dt \\ &= \frac{A_c^2}{2T}\int_0^T \{\cos[2\theta(t)]+1\}dt \\ &= \frac{A_c^2}{2T}T + \frac{A_c^2}{2}\int_0^T \cos[2\theta(t)]dt \\ &= \frac{A_c^2}{2} \end{aligned}$$

=0?



Property 2: Nonlinearity of the Modulation Process

- AM is linear: $s(m_1(t)+m_2(t))=s(m_1(t))+s(m_2(t))=s_1(t)+s_2(t)$
- PM and FM are not
  - e.g., for PM:

$$m(t) = m_1(t) + m_2(t)$$
$$s(t) = A_c \cos[2\pi f_c t + k_p(m_1(t) + m_2(t))]$$
$$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)]$$
$$s_2(t) = A_c \cos[2\pi f_c t + k_p m_2(t)]$$

  - The principle of superposition is violated, since

$$s(t) \neq s_1(t) + s_2(t)$$

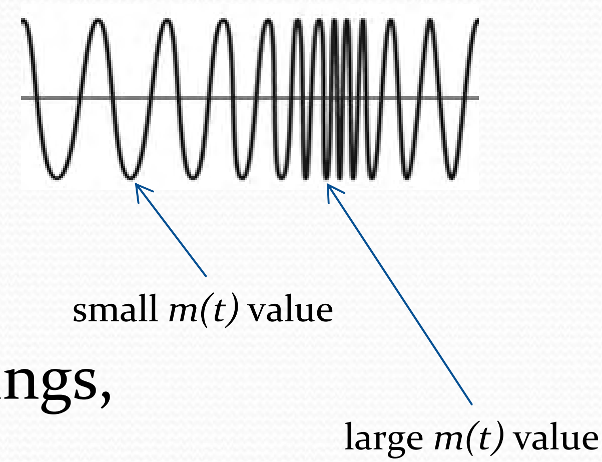
- Complicates the spectral analysis and noise analysis
- … but superior noise performance: due to nonlinearity

Property 3: Irregularity of Zero-Crossings

- Zero-crossing: when $s(t)$ crosses zero
- No longer have a perfect regularity in their spacing
- Also attributed to the nonlinear characteristics of the modulation process
- FM and PM: $m(t)$ resides in the zero-crossings of the modulated wave $s(t)$
  - Using zero-crossings to sample $m(t)$
- AM, $m(t)$ residues in the envelope of $s(t)$
  - Using peaks of $c(t)$ to sample $m(t)$

Property 4: Visualization Difficulty of Message Waveform

- AM
  - $m(t)$ ←→ envelope of $s(t)$
- PM and FM
  - $s(t)$ has constant amplitude
  - $m(t)$ is in the form of zero-crossings, which is hard to visualize
- Also attributed to the nonlinearity of the modulation process



Property 5: Trade-Off of Transmission Bandwidth and Noise Performance

- Advantage
  - Improved noise performance
    - Many noise and interference are additive
    - Angle of a sinusoidal wave is less sensitive to additive noise
  - AM: amplitude is sensitive to additive noise
- Attend at the expense of larger transmission bandwidth requirement
- Trade-off
  - Exchanging an increase in transmission bandwidth for an improvement in noise performance
  - Not for AM: since transmission bandwidth is fixed

Example 4.1: Zero-Crossings

- Linear modulating wave
  - a=1 volt/s, carrier f_c=1/4 Hz, k_p=π/2 radians/volt
- Phase modulation

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

For zero-crossings:

$$2\pi f_c t_n + k_p a t_n = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, \ldots$$

$$t_n = \frac{\frac{1}{2} + n}{2f_c + \frac{k_p}{\pi} a} \qquad t_n = \frac{1}{2} + n, \qquad n = 0, 1, 2, \ldots$$

Example 4.1: Zero-Crossings (contd.)

- Frequency modulation

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

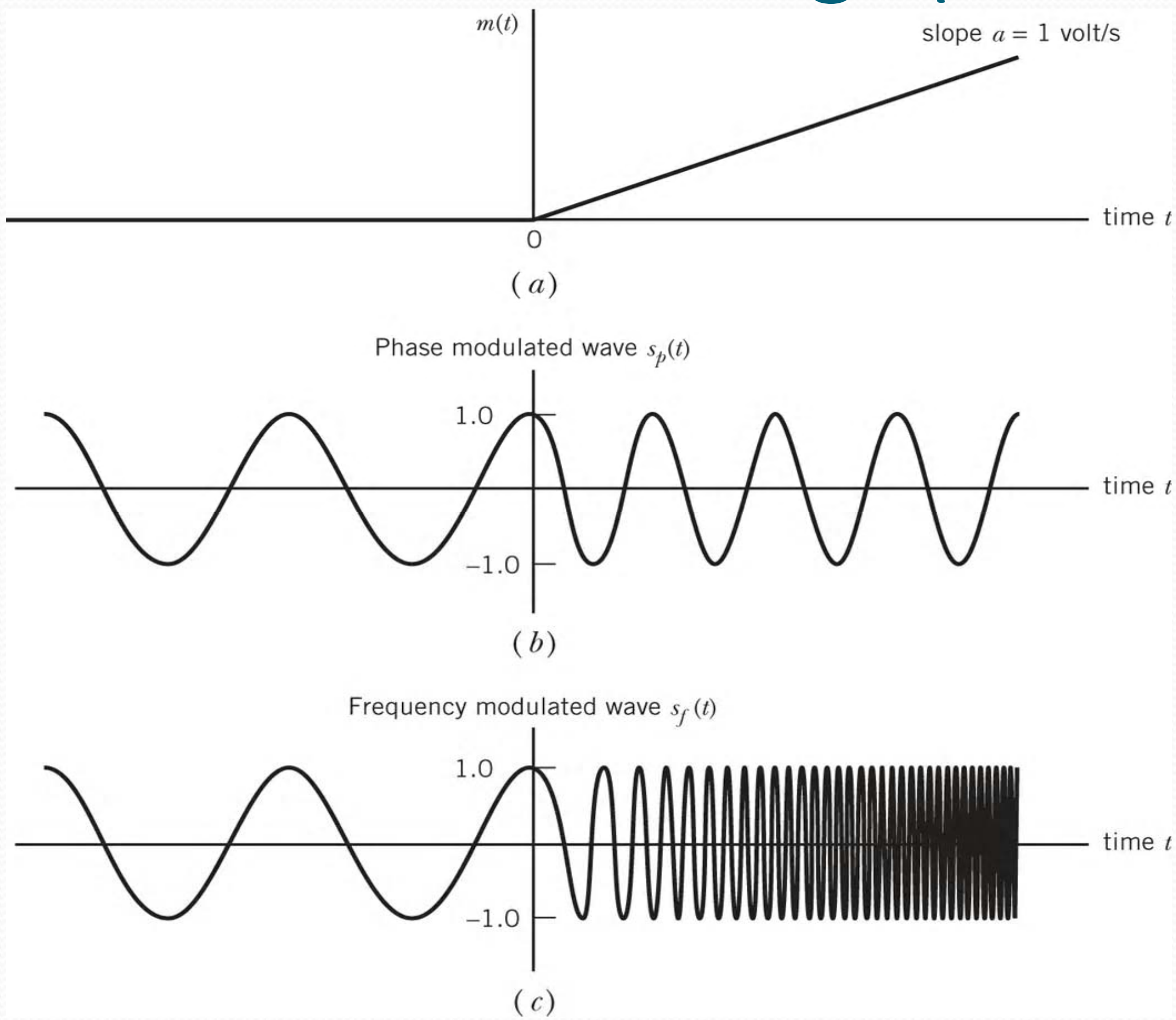
For zero-crossings:

$$2\pi f_c t_n + \pi k_f a t_n^2 = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, \ldots$$

$$t_n = \frac{1}{a k_f}\left(-f_c + \sqrt{f_c^2 + a k_f\left(\frac{1}{2} + n\right)}\right),$$

$$t_n = \frac{1}{4}(-1 + \sqrt{9 + 16n}), \qquad n = 0, 1, 2, \ldots$$

Example 4.1: Zero-Crossings (contd.)



Frequency Modulation

- Nonlinear modulation:
  - $s(t)$ is a nonlinear function of $m(t)$
  - Hard for spectral analysis
- Analyze the simplest case: single-tone modulation
  - Information wave: $m(t) = A_m \cos(2\pi f_m t)$
  - Instantaneous frequency:
    $$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t)$$
  - Frequency deviation: $\Delta f = k_f A_m$
  - The angle $\theta_i(t)$ is:
    $$\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

    Modulation index: β

Frequency Modulation (contd.)

- Modulation index: $\beta = \Delta f / f_m$
- The FM signal is:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

- Depending on the value of β, two cases of FM
  - Narrow-band FM, for which β *is* small compared to one radian
  - Wide-band FM, for which β is large compared to one radian

Narrow-Band FM

- FM signal with single tone *m(t)* is

$$s(t) = A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)]$$

- For small β, we have

If $|x| << 1$
- $\cos(x) \approx 1$
- $\sin(x) \approx x$

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

$$s(t) \simeq A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c\{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\}$$

- Similar to an AM signal with the same *m(t)*

$$s_{\mathrm{AM}}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c\{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\}$$

- Requires the same transmission bandwidth as AM

difference

Wide-Band FM

- For arbitrary $\beta=\Delta f/f_m$, use the complex representation of band-pass signals

$m(t) \propto \cos(2\pi f_m t)$

- Assuming f_c is large enough:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

$$s(t) = \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] = \mathrm{Re}[\tilde{s}(t) \exp(j2\pi f_c t)]$$

  - The complex envelope of $s(t)$ is

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)]$$

  - A periodic signal with period $1/f_m$ and fundamental frequency f_m:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t)$$

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Wide-Band FM (contd.)

- The coefficient is

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t)\exp(-j2\pi n f_m t)\, dt$$
$$= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt$$
$$= \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx = A_c J_n(\beta)$$

$x = 2\pi f_m t$

- $J_n(\beta)$: the n-th order Bassel function of the first kind and argument β

Wide-Band FM (contd.)

- We have

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t)$$

$$s(t) = A_c \cdot \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t] \right]$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$$

Exact analysis, no approximation

- Spectrum

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

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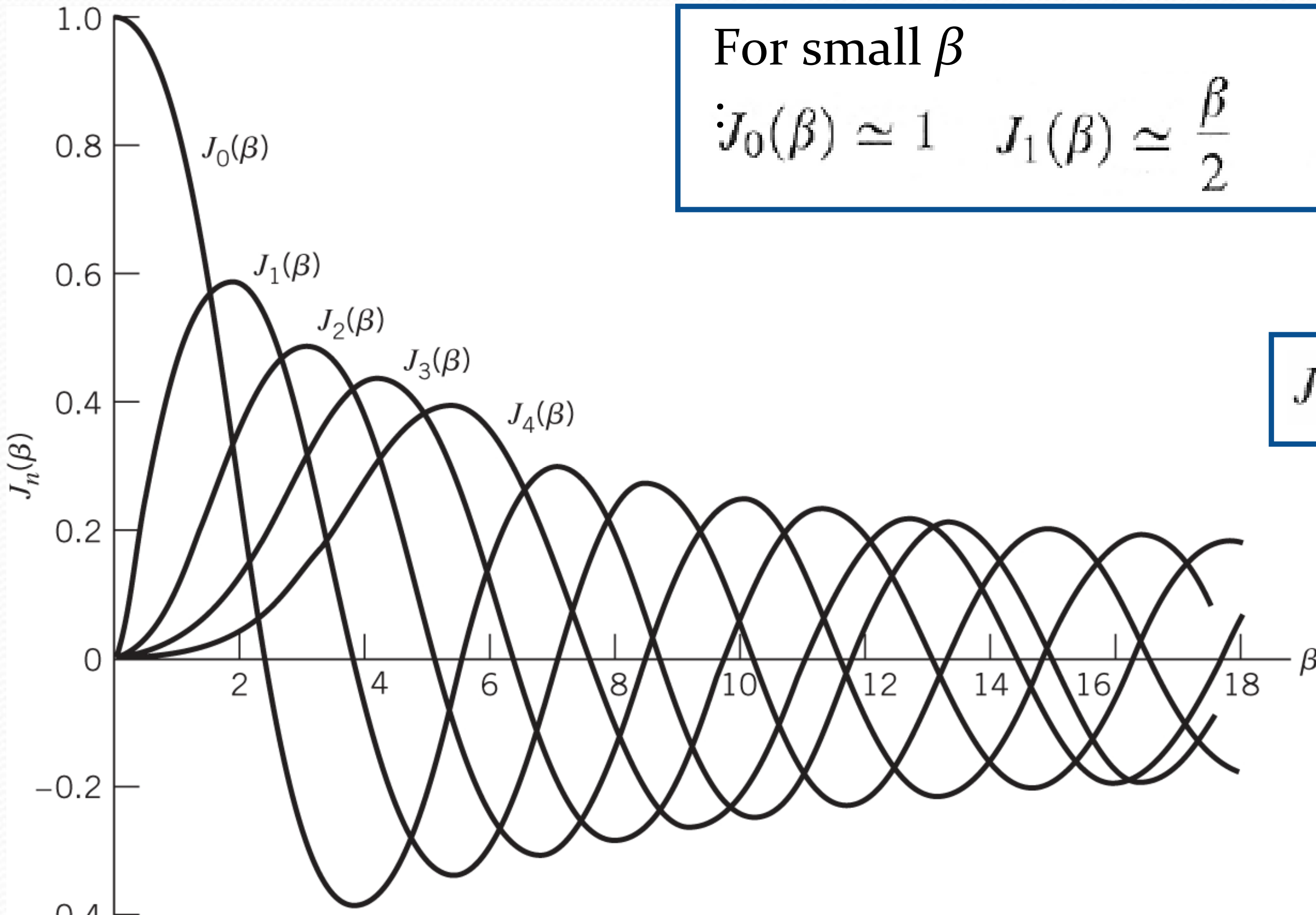
Bessel Function



For small β

$J_0(\beta) \simeq 1 \quad J_1(\beta) \simeq \frac{\beta}{2} \quad J_n(\beta) \simeq 0, \quad n > 2$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Observations

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

- The spectrum of an FM signal
  - n=0: a carrier component
    - Amplitude of the carrier is $J_0(\beta)$
  - n >0 and n < 0: an infinite set of side frequencies located symmetrically on both sides of the carrier at frequency separations of f_m, $2f_m$, $3f_m$, ...
  - Unlike AM that is band limited, infinite bandwidth
- When $\beta \ll 1$, $J_n(\beta) \approx 0$, for n>2: single pair of side frequencies → narrowband FM
- Total transmit power: $P = \frac{1}{2}A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{1}{2}A_c^2$

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Example 4.3

- Modulating signal

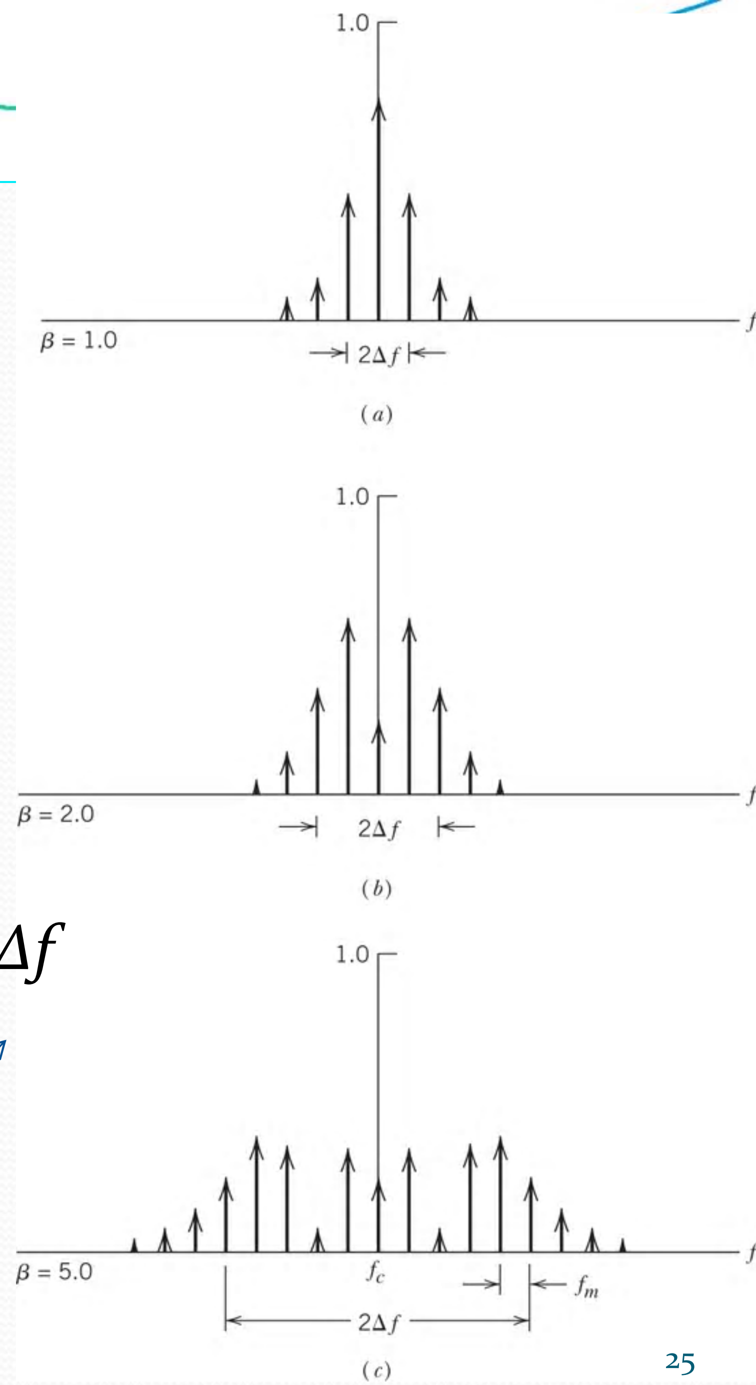
$$m(t) = A_m \cos(2\pi f_m t)$$

  - Modulation index

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

  - f_m determines the spacing
  - βf_m or A_m determines interval $2\Delta f$
- Case I:
  - Fixed f_m
  - Increased β

The spectrum on the right hand side of the plane. There is a symmetric part on the left hand side.



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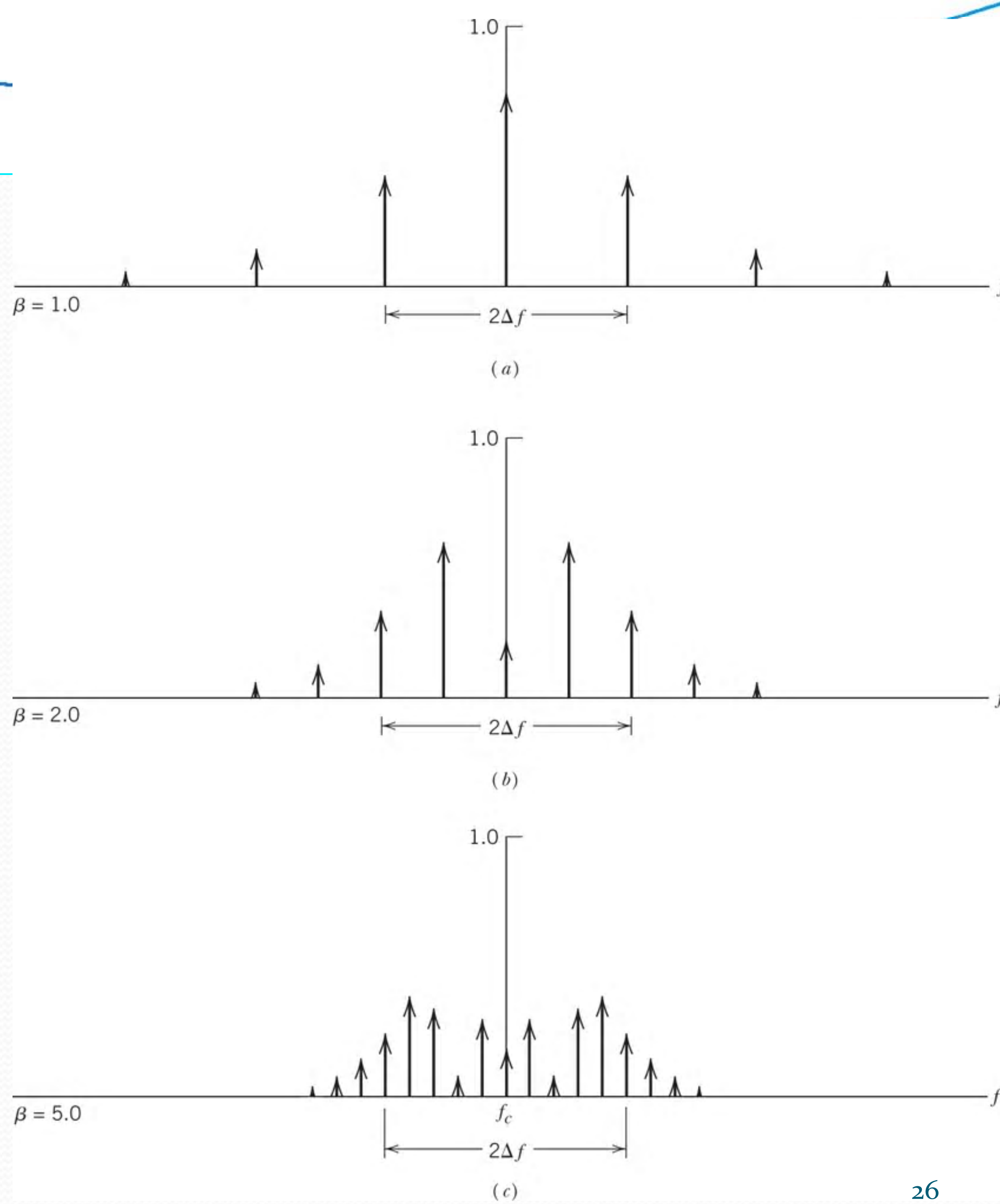
Example 4.3 (contd.)

- Case II:
  - Decreased f_m
  - Fixed A_m
- An increasing number of spectral lines crowding into the fixed interval $f_c - \Delta f < |f| < f_c + \Delta f$
- When $\beta \rightarrow \infty$, the FM bandwidth $\rightarrow$ $2\Delta f$



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Transmission Bandwidth of FM

- An FM signal contains an infinite number of side frequencies
  - Lossless transmission: infinite bandwidth
  - For practical systems:
    - Where to cut: trade-off between distortion and bandwidth efficiency
- **First approach** – Carson's Rule
  - Observing two extremes:
    - Large β : $B_T \rightarrow 2\Delta f$
    - Small β : $B_T \rightarrow 2(\Delta f + f_m)$
  - An empirical estimate: $B_T \simeq 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$

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Transmission Bandwidth of FM (contd.)

- **Second approach**: universal curve
  - An estimate based on retaining the maximum number of significant side frequencies
  - Significant: whose amplitudes are all greater than some threshold
    - e.g.: 1% of the unmodulated carrier amplitude
- The separation between two frequencies beyond which none of the side frequency is greater than 1% of the unmodulated carrier
  - $B_T = 2 \times n_{max} \times f_m$
  - n_{max} : the largest one with $|J_n(\beta)| > 0.01$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

Finding n_{max}

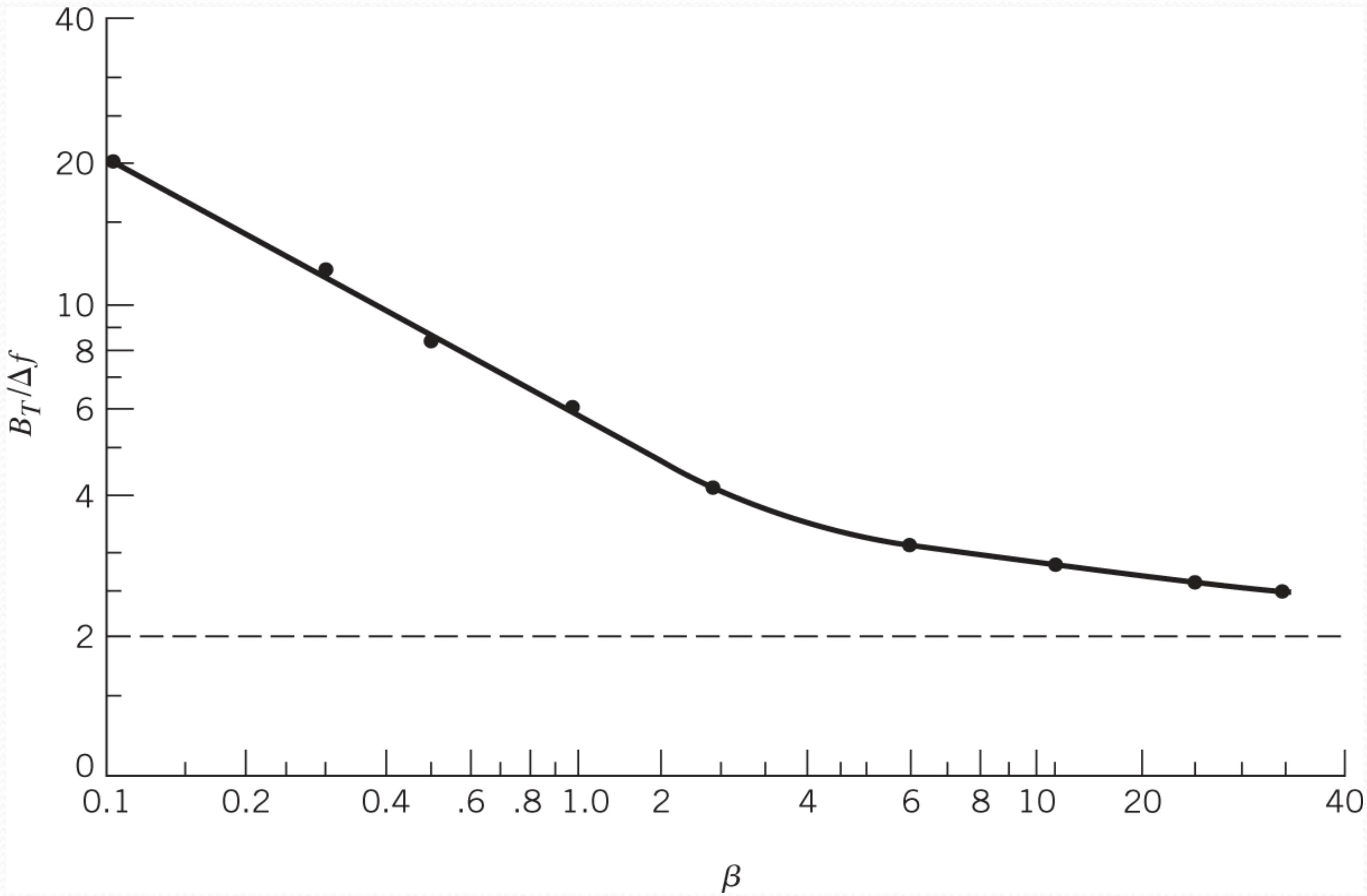
- Table 4.1

TABLE 4.1 Number of significant side frequencies of a wide-band FM signal for varying modulation index

Modulation Index β	Number of Significant Side Frequencies $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

Finding n_{max} (contd.)

- The universal curve



Discussions

- For a general *m(t)* with highest frequency *W*
  - Estimate using a worst-case tone-modulation analysis
  - Deviation ratio *D*: $$D = \frac{\Delta f}{W} = \frac{k_f \,|\, m(t)\,|_{\max}}{W}$$
    - D is equivalent to β in the case of single-tone modulation
    - W if equivalent to f_m in the case of single-tone modulation
  - Carson's Rule for transmission bandwidth:

$$B_T \approx 2(\Delta f + W) = 2\Delta f\left(1 + \frac{W}{\Delta f}\right) = 2\Delta f\left(1 + \frac{1}{D}\right)$$

Same form as single tone modulation, D ←→ β

- Carson's rule: underestimate → Lower bound
- Universal curve: overestimate → Upper bound

CARSON'S RULE :

FM Bandwidth $\approx 2(W + \Delta f)$

where W = BW of message $m(t)$

Δf = (peak) frequency deviation

Ex: Given modulated signal

$$s(t) = 100 \cos\left[\underbrace{2\pi \times 10^6}_{f_c} t + 10 \sin(2000\pi t) + 5 \sin(4000\pi t)\right]$$

Find the bandwidth of $s(t)$

Solution:

Instantaneous Phase $\psi(t)$

Carson's rule $BW \simeq 2(\Delta f + W)$

Find Δf: Inst. freq $f_i(t) \overset{\text{def}}{=} \frac{1}{2\pi} \frac{d\psi(t)}{dt}$

$$\Rightarrow f_i(t) = \frac{1}{2\pi}\left[2\pi \times 10^6 + 10(2000\pi)\cos(2000\pi t) + 5(4000\pi)\cos(4000\pi t)\right]$$

$$= \underbrace{10^6}_{f_c} + 10^4 \cos(2000\pi t) + 10^4 \cos(4000\pi t) \ \text{Hz}$$

$$\Delta f \stackrel{\text{def}}{=} \max_{-\infty < t < \infty} \left| f_i(t) - f_c \right|$$

$$= \max_{-\infty < t < \infty} 10^4 \left| \cos(2000\pi t) + \cos(4000\pi t) \right|$$

$$= 2 \times 10^4 \ \text{Hz} \quad (\text{Try } t = 0)$$

$$= 20 \ \text{kHz}$$

Find W:

(a) PM signal? $\quad s(t) = A_c \cos(\underbrace{2\pi f_c t + k_p m(t)}_{\psi(t)})$

$$\Rightarrow k_p m(t) = 10 \sin(\underbrace{2000\pi t}_{1\text{kHz}}) + 5 \sin(\underbrace{4000\pi t}_{2\text{kHz}})$$

$\Rightarrow$ $W = BW$ of $m(t)$

$= 2\,kHz$

$\Rightarrow$ Carson's rule $B_T \approx 2(20+2)\,kHz = 44\,kHz$

(b) FM signal? $f_i(t) = f_c + k_f m(t)$

$$\Rightarrow k_f m(t) = 10^4 [\cos(\underbrace{2000\pi t}_{1\,kHz}) + \cos(\underbrace{4000\pi t}_{2\,kHz})]$$

$\Rightarrow$ $W = 2\,kHz$ & $B_T \approx 44\,kHz$

Example 4.4

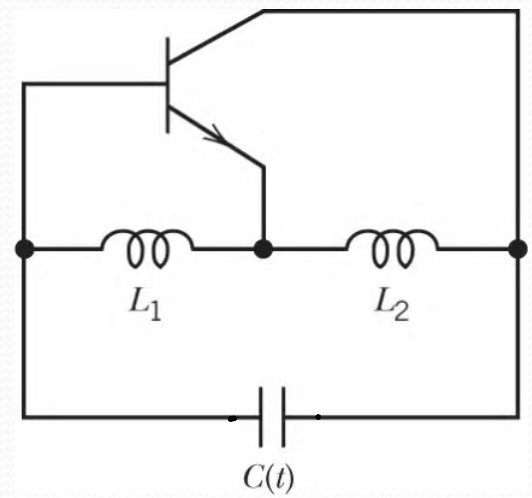
- North America, commercial FM broadcast radio
  - Δf =75 kHz
  - Maximum audio frequency W=15 kHz
  - $D = \Delta f / W = 75/15 = 5$
- Carson's rule:
  - $B_{T_1} = 2(\Delta f + W) = 2(75+15) = 180$ kHz
- Universal curve:
  - From Figure 4.9, $B_{T_2} = 3.2\Delta f = 240$ kHz
- An FM radio channel is B_T =200 kHz
  - $B_{T_1} < B_T < B_{T_2}$

Allocated BW = 200 kHz

FM Modulator Implementation

(diode)

- Voltage-controlled oscillator
  - A Hartley oscillator
  - Varactor: capacitance depends on the voltage applied to its electrodes



$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

- Assuming a sinusoidal modulating wave with f_m

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t) \qquad f_0 = \frac{1}{2\pi\sqrt{C_0(L_1 + L_2)}}$$

$$f_i(t) \simeq f_0\left[1 - \frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right] \qquad f_i(t) \simeq f_0 + \Delta f \cos(2\pi f_m t)$$

FM Demodulator

- A direct method
  - A frequency discriminator: instantaneous output amplitude is directly proportional to the instantaneous frequency of the input signal

$$\psi(t) \Rightarrow f_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = f_c + k_f m(t)$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int m(t)dt\right]$$

$$A_c \frac{d\cos(\psi)}{d\psi} \times \frac{d\psi}{dt}$$

$$\frac{d}{dt}\left[s_{FM}(t)\right] = -A_c\left[2\pi f_c + k_f m(t)\right]\sin\left[2\pi f_c t + k_f \int m(t)dt\right]$$

With an envelope detector, we get:

$$v_1(t) = A_c\left|2\pi f_c + k_f m(t)\right|$$

Need to choose k_f to avoid phase reversal

With a DC blocker, we get: $v_2(t) = A_c k_f m(t)$

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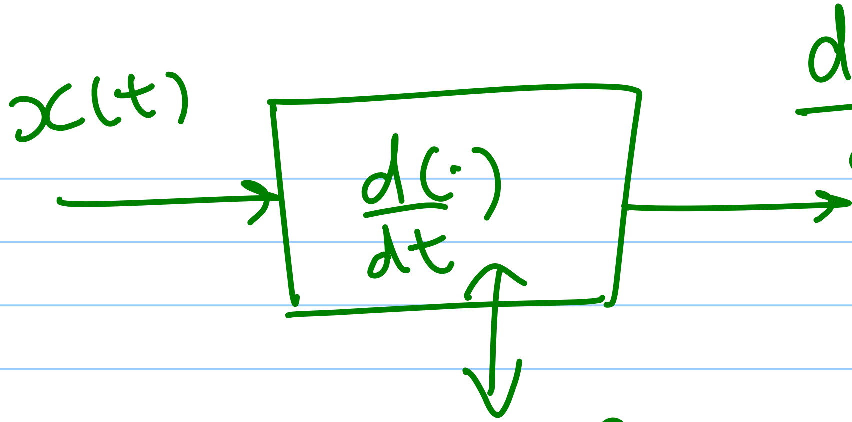
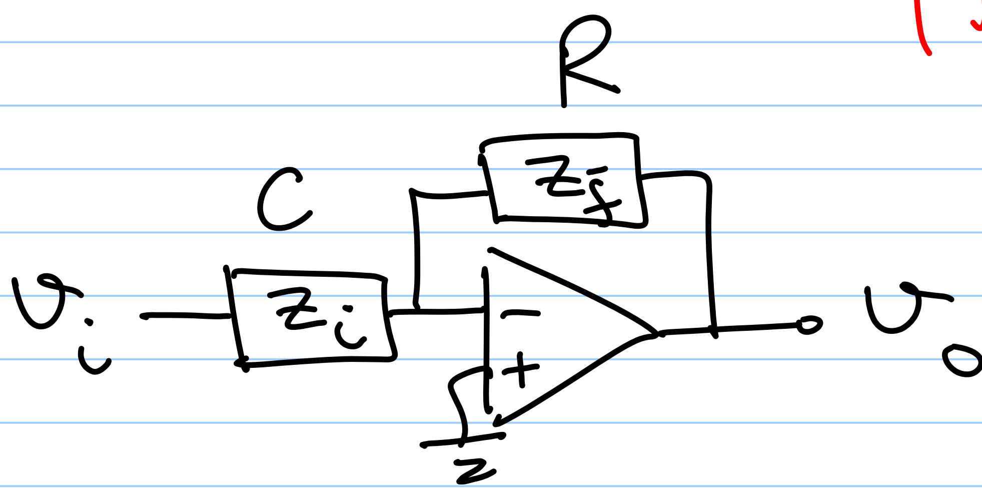


$$\frac{dx(t)}{dt} = y(t)$$

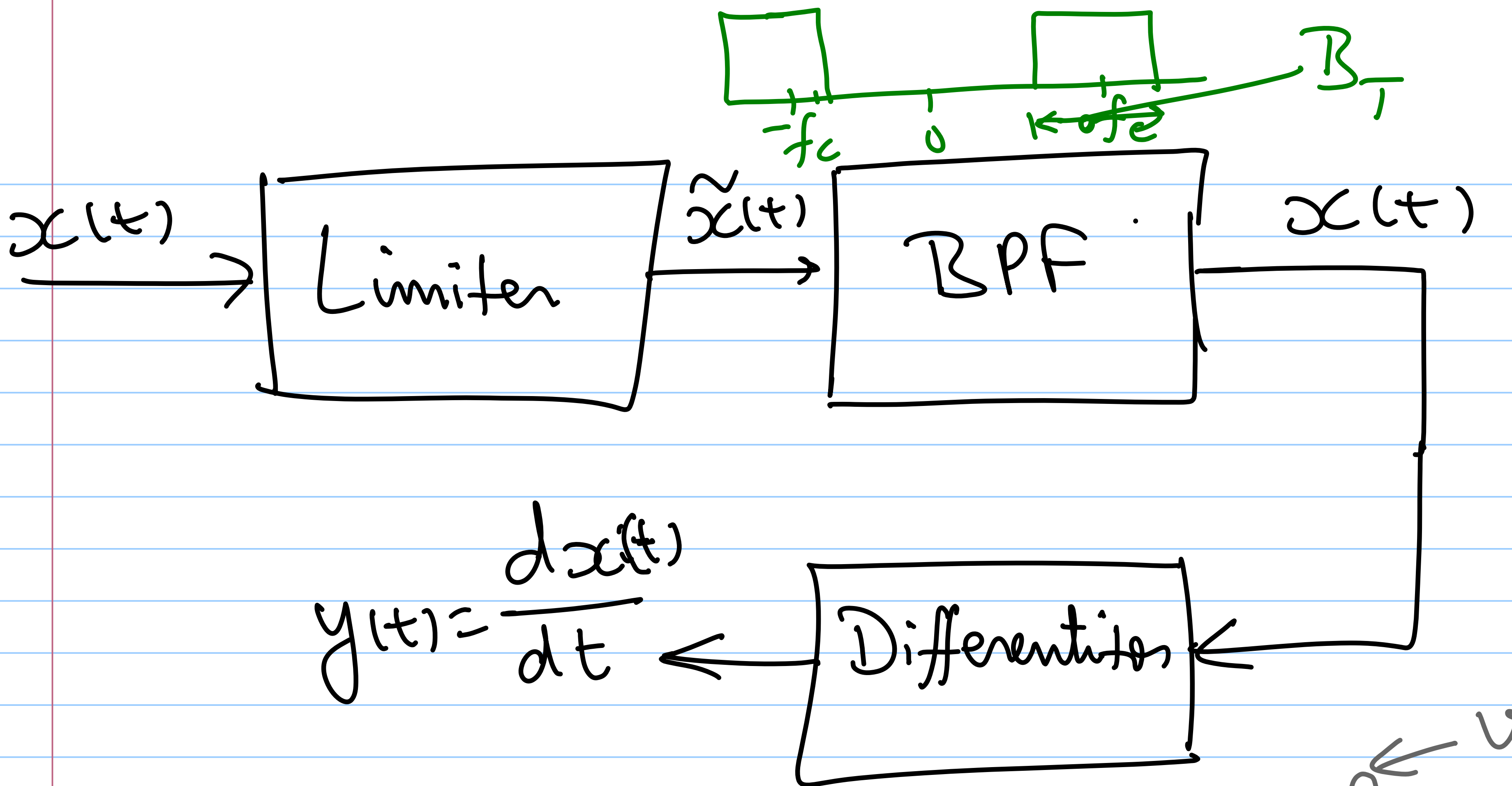
$$j2\pi f, \quad -\infty < f < \infty$$

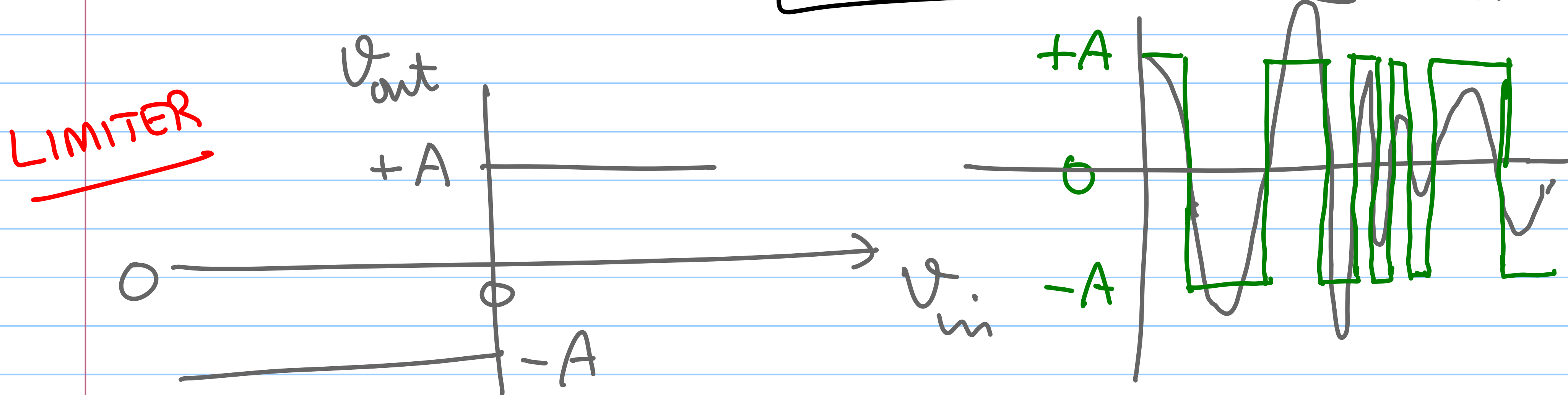
PRACTICAL

$$|f - f_c| \le (B_T/2)$$ ← DESIGN



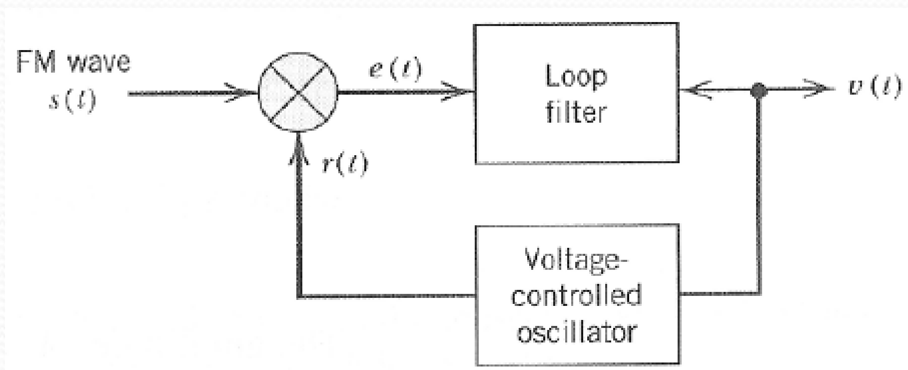
$$\frac{V_o(f)}{V_i(f)} = -\frac{Z_f(f)}{Z_i(f)}$$


x(t)
Limiter
x̃(t)
BPF
x(t)
-fc
0
fc
B_T
Differentiator
y(t) = dx(t)/dt


LIMITER
v_out
+A
0
-A
v_in
+A
0
-A
v_in

FM Demodulator (contd.)

- Indirect method
  - Phase-locked loop (PLL)
    - A negative feedback system
    - Used for synchronization, frequency division/multiplication, frequency modulation, and indirect frequency demodulation
  - To generate an $r(t)$ that tracks the phase angle of $s(t)$



Nonlinear Effects in FM Systems

- A nonlinear channel with

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t)$$

- FM signal: $v_i(t) = A_c \cos[2\pi f_c t + \phi(t)]$
- We have:

$$v_o(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)]$$

$$v_o(t) = \frac{1}{2} a_2 A_c^2 + \left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right) \cos[2\pi f_c t + \phi(t)] + \frac{1}{2} a_2 A_c^2 \cos[4\pi f_c t + 2\phi(t)] + \frac{1}{4} a_3 A_c^3 \cos[6\pi f_c t + 3\phi(t)]$$

Nonlinear Effects (contd.)

- Need to separate the desired signal with carrier f_c
- According to Carson's rule, the necessary condition for separating the desired FM signal with f_c from that with $2f_c$ is

$$2f_c - (2\Delta f + W) > f_c + \Delta f + W \quad \rightarrow \quad f_c > 3\Delta f + 2W$$

- With a band-pass filter centered at f_c and with bandwidth $2\Delta f + 2W$, we get

$$v_o'(t) = \left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right) \cos[2\pi f_c t + \phi(t)]$$

Nonlinear Effects (contd.)

- Unlike AM, FM is *not affected by the channel induced amplitude nonlinearities*
  - Widely used for microwave radio and satellite systems
  - Permits the use of highly nonlinear amplifiers and power transmitters, to produce a max power for long distance
- But still extremely sensitive to phase nonlinearilities

The Superheterodyne Receiver

- Other functions of a receiver, in addition to demodulation
  - Carrier-frequency tuning: to select the desired signal (or, station)
  - Filtering: separate the desired signal from other modulated signals
  - Amplification: compensate for the channel attenuation
- The superheterodyne receiver
  - A special type of receiver that fulfils all three functions, particularly the first two, in an elegant and practical fashion
  - Practically all radio and TV receivers are of the superheterodyne type
  - Consists of: radio-frequency (RF) section, mixer and local oscillator, intermediate frequency (IF) section, demodulator, and power amplifier

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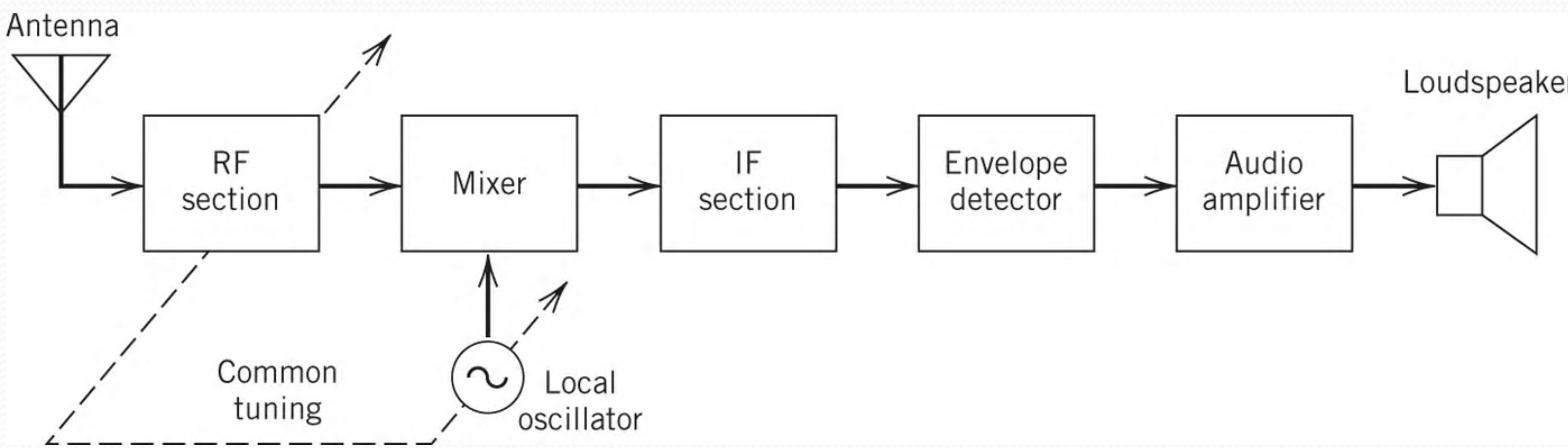
Basic Elements

- Basic difference between AM and FM superheteerodyne receivers
  - The use of an FM demodulator such as limiter-frequency discriminator



Theme Example: Analog and Digital FM Cellular Telephones

- Advanced Mobile Phone Service (AMPS), 1983
- The initial cellular telephone system in North America
- FDMA with 30 kHz channels
- A pair of channels are assigned to each user, 45 MHz apart
- Analog FM for voice transmission
- Frequency-shift Keying (FSK) for data transmission (will be discussed in chapter 9)
- FM design
- Voice is limited to 3K, the FM modulator is designed with peak deviation Δf=12 kHz
- Carson's rule: $B_T=2(\Delta f+W)=2(12+3)=30$ kHz

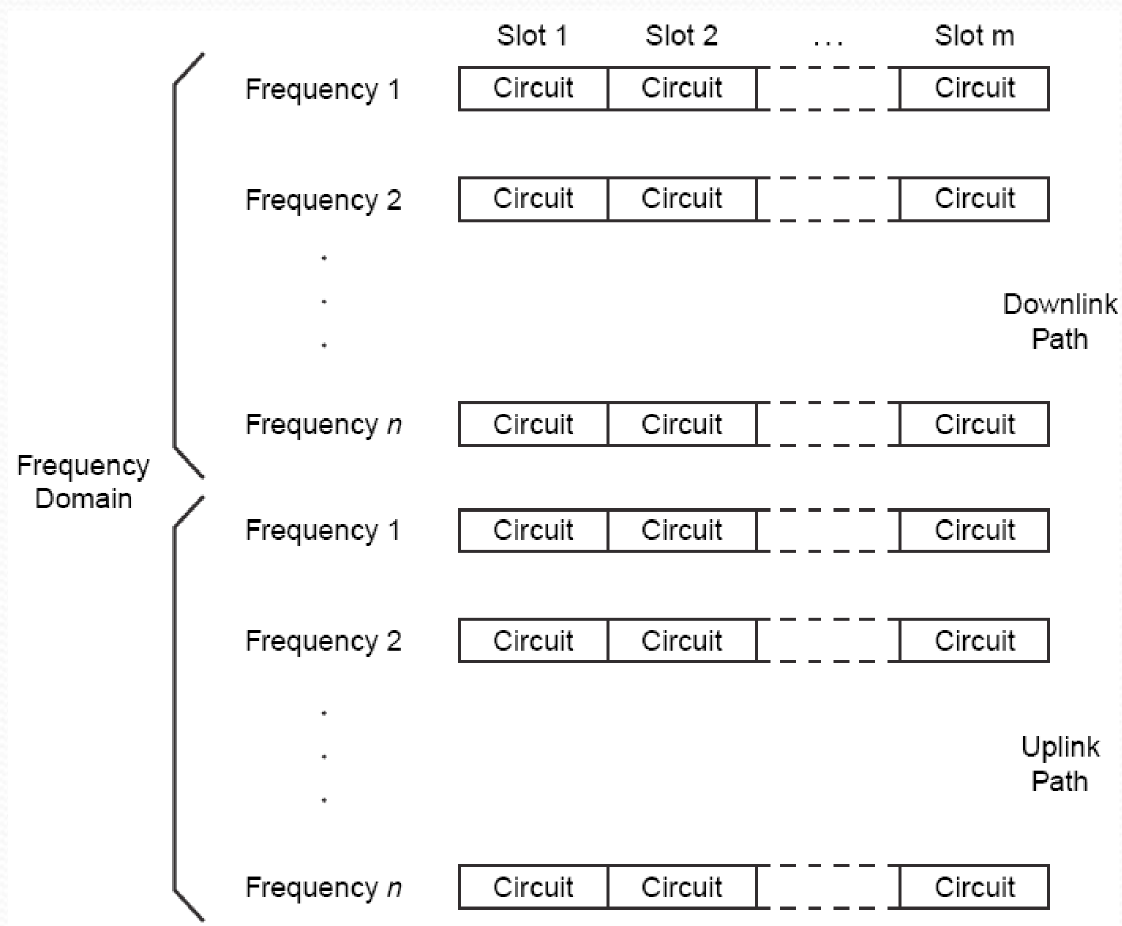
Theme Example: Analog and Digital FM Cellular Telephones (contd.)

- Pros
  - Can use high efficiency power amplifiers
  - Can operate in saturation without distorting the envelope, which is constant
  - Constant amplitude helps combating fading on mobile links
- Cons
  - No protection from eavesdropping
  - Not bandwidth efficient

Theme Example: Analog and Digital FM Cellular Telephones (contd.)

- Global System for Mobile Communications (GSM)
  - Successor of AMPS
  - Built on FM related advantages, but improves bandwidth efficiency by
    - A more complex multiplexing strategy
    - Digital representation of data
- A frequency band is 200 kHz, which is shared by 32 voice calls in one direction
  - Combination of FDMA and TDMA
  - Number of calls that an be supported per unit bandwidth
    - $(30/200)\times 32=4.8$ times improvement over AMPS

Theme Example: Analog and Digital FM Cellular Telephones (contd.)



Summary

- Angle modulation
  - FM (frequency) and PM (phase)
  - Equivalence of the two
  - Properties
- Hard for spectral analysis
- Study single tone FM for insights
  - An empirical rule: Carson's rule for approximate evaluation of the transmission bandwidth of FM B_T
  - Determined by the modulation index β or the deviation ratio D for nonsinusoidal FM
- Constant amplitude: robust to noise and interference
- Trade-off between transmission bandwidth and noise performance