Communication Systems Modelling

with

tims

Volume A2
Further & Advanced Analog Experiments

Tim Hooper

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WHAT IS TIMS ?

TIMS is a **T**elecommunications **I**nstructional **M**odelling **S**ystem. It models telecommunication systems.

Text books on telecommunications abound with block diagrams. These diagrams illustrate the subject being discussed by the author. Generally they are small sub-systems of a larger system. Their behaviour is described by the author with the help of mathematical equations, and with drawings or photographs of the signal waveforms expected to be present.

TIMS brings alive the block diagram of the text book with a working model, recreating the waveforms on an oscilloscope.

How can TIMS be expected to accommodate such a large number of models ?

There may be hundreds of block diagrams in a text book, but only a relatively few individual block *types*. These block diagrams achieve their individuality because of the many ways a relatively few element *types* can be connected in different *combinations*.

TIMS contains a collection of these block types, or *modules*, and there are very few block diagrams which it cannot model.

PURPOSE OF TIMS

TIMS can support courses in Telecommunications at all levels - from Technical Colleges through to graduate degree courses at Universities.

This text is directed towards using TIMS as support for a course given at any level of teaching.

Most early experiments are concerned with illustrating a small part of a larger system. Two or more of these sub-systems can be combined to build up a larger system.

The list of possible experiments is limitless. Each instructor will have his or her own favourite collection - some of them are sure to be found herein.

Naturally, for a full appreciation of the phenomena being investigated, there is no limit to the depth of mathematical analysis that can be undertaken. But most experiments can be performed successfully with little or no mathematical support. It is up to the instructor to decide the level of understanding that is required.

EXPERIMENT AIMS

The experiments in this Volume build on those of Volume A1. It is advantageous to have completed as many of those as possible.

As before, the experiments have been written with the idea that each model examined could eventually become part of a larger telecommunications system, the aim of this large system being to transmit a *message* from input to output. The origin of this message, for the analog experiments in Volumes A1 and A2, would ultimately be speech. But for test and measurement purposes a sine wave, or perhaps two sinewaves (as in the two-tone test signal) are generally substituted. For the digital experiments (Volumes D1 and D2) the typical message is a pseudo random binary sequence.

The experiments are designed to be completed in about two hours, with say one hour of preparation prior to the laboratory session.

The four Volumes of ***Communication Systems Modelling with TIMS*** are:

A1 - Fundamental Analog Experiments

A2 - Further & Advanced Analog Experiments

D1 - Fundamental Digital Experiments

D2 - Further & Advanced Digital Experiments

Contents

AMPLITUDE MODULATION - METHOD 2

AMPLITUDE MODULATION - METHOD 2

ACHIEVEMENTS: *another method of modelling an amplitude modulated (AM) signal (see the experiment entitled* ***Amplitude modulation*** *in Volume A1); indirect method of phase measurement.*

PREREQUISITES: *completion of the experiment entitled* ***Amplitude modulation*** *in Volume A1 would be an advantage.*

PREPARATION

AM as DSBSC + carrier

There are many ways of generating an amplitude modulated (AM) signal. Refer to a text book for some of them. The class C modulated amplifier is a popular circuit. If the prime requirement of a transmitter is to generate AM, then this (or variations of it) is probably the best choice. But sometimes it is a secondary requirement. For example, a transmitter might primarily be designed for DSBSC generation. An ability to synthesise AM might be a secondary requirement; a suitable method is examined in this experiment.

In the experiment entitled *Amplitude modulation* (in Volume A1), you have already modelled the AM equation in the form of:

$$AM = E.(1 + m.\cos\mu t).\cos\omega t \qquad \text{........ 1}$$

There are other methods of writing this equation; for example, by expansion, it becomes:

$$AM = E.m.\cos\mu t.\cos\omega t + E.\cos\omega t \qquad \text{........ 2}$$

$$= DSBSC + carrier \qquad \text{........ 3}$$

The depth of modulation 'm' is determined by the *ratio* of the DSBSC and carrier amplitudes, since, from eqns.(2) and (3):

$$\text{ratio (DSBSC/carrier)} = (E.m) / E = m \qquad \text{........ 4}$$

phase requirement

The important practical detail here is the need to adjust the relative phase between the DSBSC and the carrier. This is not shown explicitly in eqn. (2), but is made clear by rewriting this as:

$$AM = E.m.\cos\mu t.\cos\omega t + E.\cos(\omega t + \alpha) \qquad \text{........ 5}$$

Here α is the above mentioned phase, which, for AM, must be set to:

$$\alpha = 0^o \qquad \text{........ 6}$$

Any attempt to model eqn. (2) by adding a DSBSC to a carrier cannot assume the correct relative phases will be achieved automatically.

It is eqn. (5) which will be achieved in the first instance, with the need for adjustment of the phase angle α to zero.

the equation model

A block diagram of the arrangement for modelling eqn. (2) is shown in Figure 1.

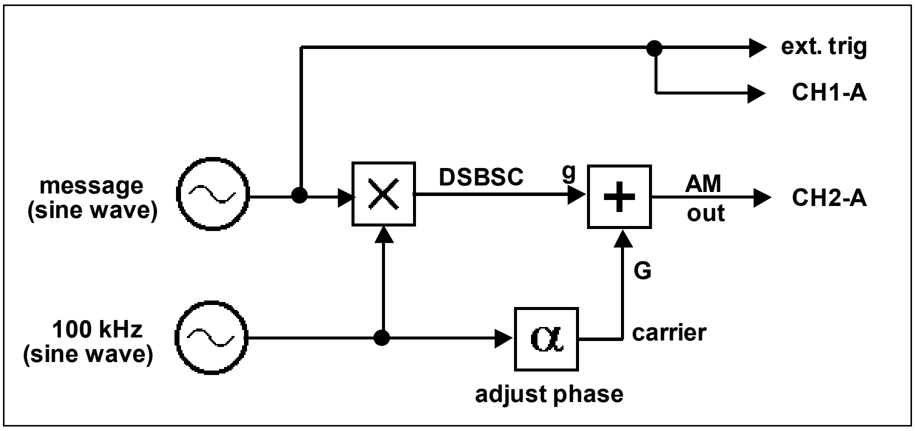

Figure 1: block diagram of AM generator

EXPERIMENT

the TIMS model

The block diagram of Figure 1 can be modelled by the arrangement of Figure 2.

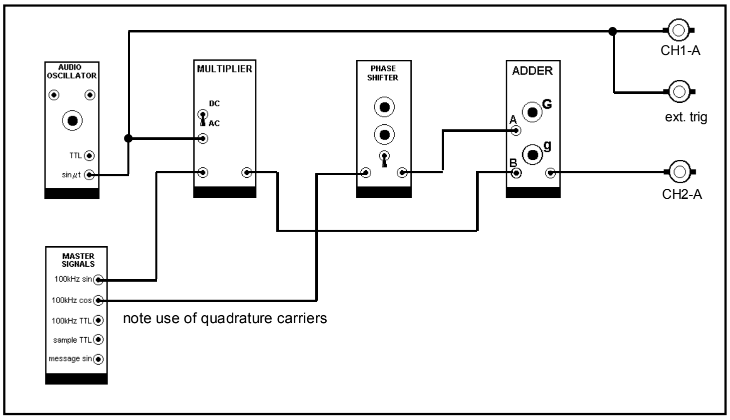

Figure 2: the AM generator model

adjusting the model

You will now set up a 100% amplitude modulated signal using the model of Figure 2.

The DSBSC and the carrier from which it is derived are required to be in phase for the generation of AM.

Examination of Figure 1 might suggest that no phase adjustment is required, since the DSBSC and carrier should be already in phase.

But the DSBSC and carrier are taken via separate paths to the ADDER, and these paths, and the ADDER, can introduce small and unknown (but stable) phase shifts. Thus at the adding point the two may not be in exact phase. A correction may be necessary. It is the purpose of the PHASE SHIFTER to make this *small* correction. The PHASE SHIFTER itself cannot introduce a *small* phase shift, even if the front panel control is fully anti-clockwise. This may, in any case, be in the *wrong* direction. To overcome this possible problem the carriers to the PHASE SHIFTER and to the MULTIPLIER (for the DSBSC generation) have *already* been shifted by 90^0 by using the quadrature outputs from the MASTER SIGNALS module. Thus the PHASE SHIFTER is required to introduce a further small adjustment either side of $\pm 90^0$, which is in its mid-range of control, rather than at one end.

T1 *patch up the model of Figure 2. Remember to set the on-board switch of the PHASE SHIFTER to the 'HI' (100 kHz) range before plugging it in.*

T2 *set the gain controls* ***g*** *and* ***G*** *of the ADDER fully anti-clockwise.*

T3 *set the oscilloscope to accept the 'ext. trig' signal, and adjust sweep and gain controls so that the message is displayed on CH1-A, filling about the top half of the screen.*

T4 *set the gain control of the oscilloscope CH2 to the same as that of CH1.*

T5 *without touching the sweep speed of the oscilloscope adjust the amplitude of the carrier signal, using the* ***G*** *control on the ADDER, so it is* ***about*** *2 volts peak-to-peak, but* ***exactly*** *aligned between two horizontal lines of the oscilloscope graticule.*

T6 *temporarily remove the carrier signal from the* ***G*** *input of the ADDER*

T7 *advance the ADDER gain control* ***g*** *until the DSBSC is displayed on CH2-A of the oscilloscope between the same two graticule lines as was the carrier.*

T8 *replace the carrier term into the* ***G*** *input of the ADDER.*

You have now set up the signal of eqn. (4), except for the phase angle α.

The output of the ADDER is expected to be a 100% amplitude modulated AM signal, but only if the relative phase is correct; that is, $\alpha = 0$.

It is an easy matter to adjust the phase:

T9 *vary the front panel control of the PHASE SHIFTER until, watching the display for CH2-A of the oscilloscope, a 100% AM is achieved.*

Q *although no instruction was given about how you might recognise the required phase condition, did you find this self evident ? Explain.*

phase adjustment

The above instruction offers no advice as to how the correct phase adjustment is to be achieved. But if you try it, there will be little doubt as to what to do.

You will notice that there is only one position of the PHASE SHIFTER control when the troughs of the signal will exactly 'kiss', *as is expected for 100% AM*.

The 'kiss' occurs when $\alpha = 0$. This is clearly illustrated in Figure 3.

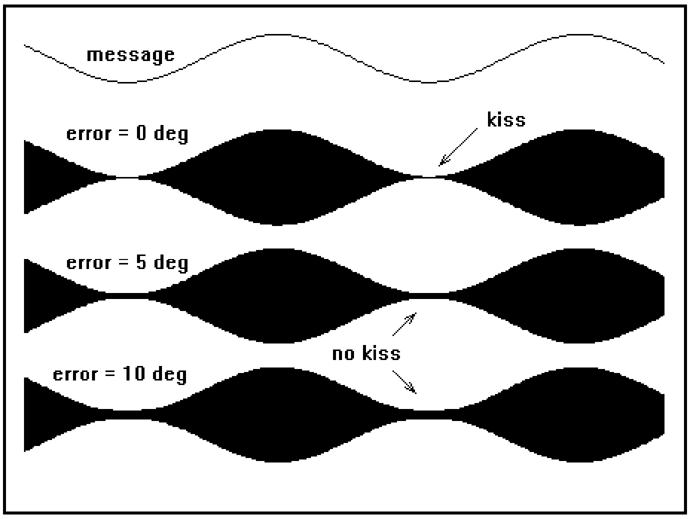

Figure 3: showing envelope for small phase errors

From a practical point of view the 'kiss' test is adequate as a method of phase adjustment. But remember it must be made under conditions for m = 1 - that is, with equal amplitude DSBSC and carrier signals at the ADDER output, and a sinusoidal message.

For an error of 5 degrees Figure 3 shows clearly that the troughs will not kiss; this is even more obvious for an error of 10 degrees.

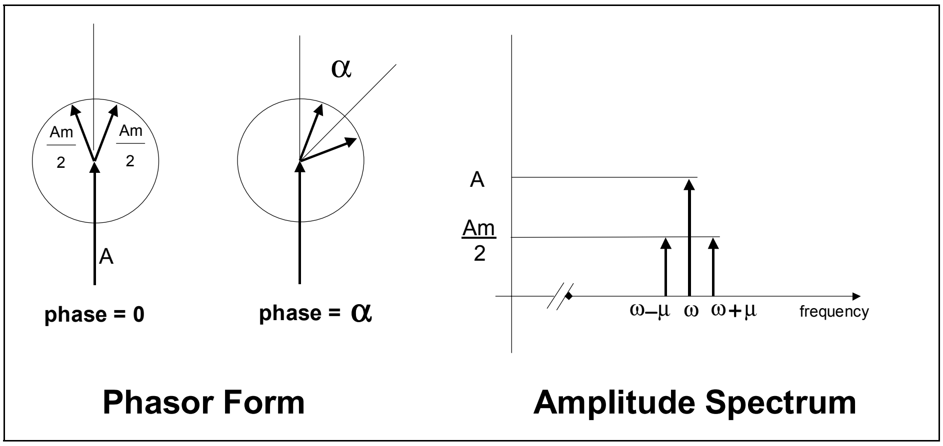

Figure 4: DSBSC + carrier, with m = 1

Representation of the DSBSC and carrier in phasor form is shown in Figure 4. This is another way of looking at the signal. It is clear that, when the phase angle is other than zero, no matter what the sideband amplitude, they could never add with the carrier to produce a resultant of zero amplitude, which is required for the 'kiss'. When the sidebands are in phase with the carrier, this can clearly only happen when m = 1 (as it is in the diagram).

Figure 4 also shows the amplitude spectrum. This is not affected as the phase changes.

There are other methods of phase adjustment. One would be to recover the envelope in an envelope detector (see earlier experiments) and adjust the phase until the distortion of the recovered envelope is a minimum. This is a practical method which achieves directly what is desired - without ever having to measure relative phase. In this way there may be some compensation for the inevitable distortion introduced both by the transmitter, at high depths of modulation, and the receiver.

TUTORIAL QUESTIONS

Q1 *show, with the aid of a phasor diagram, that, when the DSBSC and the carrier are of the same amplitude (the condition for 100% AM), the only way for them to periodically sum to zero is for their relative phase to be zero. This is the condition for the troughs to 'kiss'. As an extension of this, show that, for m < 1, they could never kiss.*

Q2 *how could you use a commercial phase meter to measure the relative phase between a carrier and a DSBSC ?*

Q3 *there was no need to make an explicit phase adjustment when modelling eqn. (1), whereas this was necessary when modelling eqn. (3). These two equations model the same signal. Comment.*

Q4 *if an AM signal was connected to the FREQUENCY COUNTER, would it read:*

a) the carrier frequency ?

b) the envelope frequency ?

c) what ?

Explain !

WEAVER`S SSB GENERATOR

WEAVER`S SSB GENERATOR

ACHIEVEMENTS: *exposure to Weaver`s SSB generator, and its alignment procedure.*

PREREQUISITES: *completion of some previous experiments involving linear modulated signal generation, especially* ***SSB generation - the phasing method****.*

EXTRA MODULES: *a total of four MULTIPLIERS, two PHASE SHIFTERS, and two TUNEABLE LPF modules is required. This is twice as many of these modules as are in the TIMS Basic Set.*

PREPARATION

principles

You should refer to a Text book for more detail on Weaver`s method of SSB generation [1]. This experiment will introduce you to some of its properties, and methods of alignment.

You are well advised to try the Tutorial Questions (especially Q1) before attempting the experiment, but after reading these preparatory notes..

Weaver`s method of SSB generation, like the phasing method, depends for its operation upon phase cancellation of two DSBSC-like signals. But:

1. it does not require wideband phasing networks, like the phasing method of SSB generation.
2. it does not require sharp cut-off filters, operating away from baseband, as does the filter method of SSB generation.
3. its unwanted components - those which are not fully removed (by phasing, as in the phasing method, or by imperfect filtering, as in the filter method) - do not cause interference to adjacent channels, since they fall *inside* the SSB channel itself.

[1] Weaver, D.K., "A third method of generation and detection of single sideband signals", *Proc. IRE*, Dec. 1956, pp. 1703-1705

Figure 1 shows a block diagram of the method.

There are two pairs of multipliers. These are referred to as quadrature multipliers, since they use 'carriers' phased relatively at 90^0. This configuration is found in many communications circuits.

The message bandwidth is defined as B Hz. It is shown as extending down to DC, but in practice (speech messages) this is not necessary - even undesirable. The DC requirement would introduce some complications, including the need for the first pair of quadrature multipliers to be DC coupled.

Two filters are required, but they are at baseband, where design and realization is simplified. They must, however, be matched (amplitude and phase responses) as closely as possible.

There are two phase shifters, but they are required to produce a 90^0 phase shift at a single frequency only.

Note that the *second* pair of quadrature multipliers should, ideally, be DC coupled. Think about it ! In practice DC coupling is not often provided, since it introduces DC-offset problems. As a result there can be a small gap in the message, as received, in the vicinity of (B/2) Hz. See Tutorial Question 6.

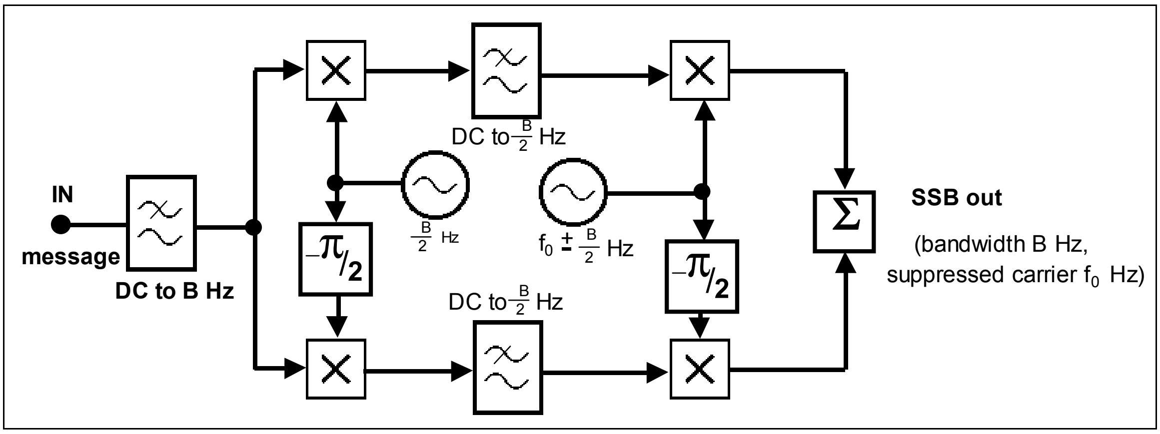

Figure 1: block diagram of Weaver`s SSB generator

Note that the input lowpass filter shown in the block diagram is not included in the patching diagram to follow (Figure 2). Its presence is recognised by not allowing the message source to be tuned above B Hz.

Tracing the message through either the upper arm or the lower arm alone is insufficient to deduce unambiguously what the frequency of the output signal will be. This is because of the cancellations which will take place in the summing block. Until the actual signals and their phases are known it is not possible to deduce which will cancel and which will add.

The analysis can be performed trigonometrically, using a single tone message.

Note that, what ever the output, the carrier frequency is *not* that of the second oscillator.

EXPERIMENT

the model

A suggested model of Weavers SSB generator is shown in Figure 2 below. It is best patched up stage-by-stage, from the input, checking operation until the output is reached.

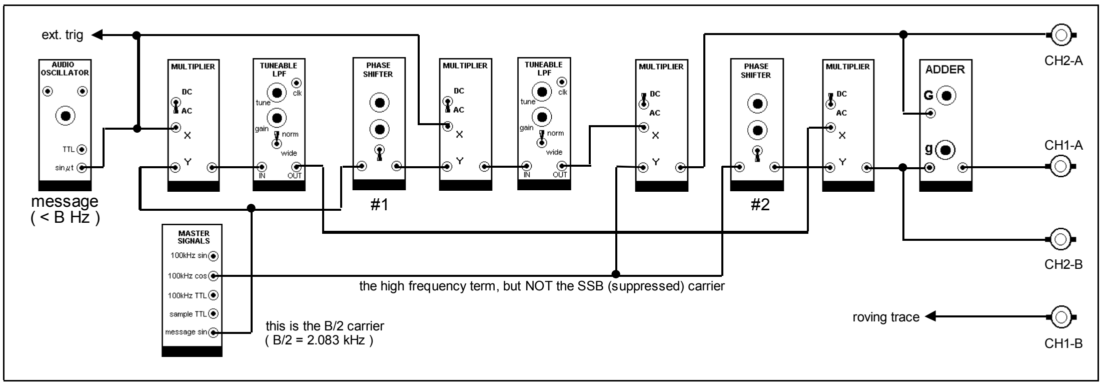

Figure 2: model of Weaver`s SSB generator

Please note: the '2 kHz message' from the MASTER SIGNALS module is not *Weaver`s* message, but is being used as the source of a stable, *low frequency carrier*, on B/2 Hz (refer the block diagram of Figure 1). Thus the message bandwidth must not exceed B Hz. A bandlimiting filter for the message is not included in the model (Figure 2), so the AUDIO OSCILLATOR message source is kept below 4 kHz for correct performance. But you should investigate what happens if this restriction is not adhered to - as it would not be if the message was not strictly bandlimited.

alignment

The alignment of Weaver`s SSB generator is quite straightforward, and will not be given in any great detail.

T1 *before plugging in the PHASE SHIFTER modules, set their on-board range switches; #1 module to 'LO', and #2 module to 'HI' (refer Figure 2).*

T2 *use the '2 kHz message' from the MASTER SIGNALS module to set the PHASE SHIFTER #1 to about* 90^0. *This is achieved to sufficient accuracy by displaying the input and output to the PHASE SHIFTER on two oscilloscope traces. Adjust the PHASE SHIFTER front panel control until one sinewave is delayed 1/4 period with respect to the other. Fine trimming cannot be carried out until the generator is near completion.*

T3 *set the TUNEABLE LPF modules to the same bandwidth, 'B/2', namely 2.083 kHz. With the front panel switch set to 'NORM', this makes the 'CLK' frequency 2.083 x 880 = 1833 kHz.*

T4 *set the message AUDIO OSCILLATOR to say 1 kHz. Call this* $\boldsymbol{f_m}$ ***Hz****, and make a record of it.*

T5 *now patch up according to Figure 2.*

T6 *trigger the oscilloscope externally to the message on* f_m *Hz. Use the roving trace CH1-B to confirm that the waveforms at the output of each of the low frequency MULTIPLIER modules is a DSBSC. There are no adjustments you can make if this is not so - check patching !*

T7 *trigger the oscilloscope to the 'roving trace' (CH1-B), and use it to confirm that the waveform at the output of each of the TUNEABLE LPF modules is a sine wave. There are no adjustments you can make if this is not so - check patching ! Its frequency will be* $(B/2 - f_m)$ *Hz - confirm this.*

T8 *adjust the gain of each of the TUNEABLE LPF modules filter to make the amplitude of each output sinewave about 4 volt peak-to-peak (TIMS ANALOG REFERENCE LEVEL).*

T9 *confirm with the 'roving trace' that the waveform at the output of each of the high frequency MULTIPLIER modules is a DSBSC. There are no adjustments you can make if this is not so - check patching ! Its 'message' will be the sinewave from the TUNEABLE LPF; use this for external oscilloscope triggering.*

When you reach this point all is ready for the final amplitude and phase adjustments, which will achieve the required result at the output of the ADDER.

T10 *switch the oscilloscope to CH1-A. Remove the patch lead from the* ***upper*** *input of the ADDER. Adjust the* ***lower*** *gain control until the output is a DSBSC of* ***about*** *4 volt peak-to-peak amplitude. Replace the upper patch lead.*

T11 *remove the patch lead from the* ***lower*** *input of the ADDER. Adjust the* ***upper*** *gain control until the output is a DSBSC of* ***about*** *4 volt peak-to-peak amplitude. Replace the lower patch lead.*

> *You are aiming for an ADDER output of a single sinewave. Although you have adjusted the amplitudes of the signals at the summing point to approximate equality, their relative phases have not been set.*

T12 *while watching the ADDER output, vary the phase of the phase shifter #2, aiming for a sinewave.*

T13 *fine trim one (only) of the ADDER gain controls for a better result.*

Remember: a perfect SSB (sinewave message) has a straight line envelope. Synchronise to its 'message' - in this case the output from either TUNEABLE LPF module. Think about it !

Repeat this, and the previous Task, until the best result is achieved.

The two high frequency MULTIPLIER modules, as shown in Figure 2, are set to accept DC. This is a requirement of the Weaver modulator. But in practice it is a problem, since DC offsets, preceding the two high-frequency multipliers, will degrade performance. You can check this by flipping the toggle switches to AC.

T14 *change the input coupling of the two high frequency MULTIPLIER modules to AC, and check if a superior performance can be obtained (a flatter envelope).*

T15 *with the two high frequency MULTIPLIER modules still AC coupled, vary the message frequency through the frequency B/2 (namely, the low frequency carrier frequency of 2.083 kHz) and demonstrate the gap in the response. Message frequencies near 2.083 kHz will be missing. But this imperfection is generally acceptable in practice, and AC coupling is used.*

T16 *is there any point to checking the adjustment of the PHASE SHIFTER #1 ?*

> *Alignment of the Weaver SSB generator is complete*

T17 *measure the frequency of the output. From the details of Figure 1 confirm that this is a possible outcome. What is the carrier frequency ? This means 'with what frequency sinewave would this SSB need to be multiplied to recover the correct message frequency in a convnetional demodulator ?' There will be two answers, only one of which is correct. Which one ? Why ?*

T18 *vary the message frequency over its allowed range of B Hz. Remember there is no message bandlimiting filter installed. Demonstrate that the generator performs satisfactorily over this range (except for the 'gap' previously identified).*

T19 *measure the sideband suppression ratio (SSR). Refer to the experiment entitled* ***SSB generation - the phasing method****, in Volume A1, for details.*

TUTORIAL QUESTIONS

Q1 *perform a trigonometrical analysis of Weaver`s method of SSB generation. Use a single tone for the message. Assume the 'B/2' filters are ideal (and matched). Assume the only impairments are small phase errors* α_1 *and* α_2 *in each of the* 90^0 *phase shifters. Obtain an expression for the sideband suppression ratio in decibels, as a function of these two phase errors.*

Q2 *from a knowledge of the properties of all of the modules you have used, predict from the previous calculation which sideband (upper or lower) your model will generate. Confirm by measurement.*

Q3 *use any method to determine the (suppressed) carrier frequency of the SSB generator, in terms of B and* f_0 *, both defined in Figure 1. Confirm from measurement.*

Q4 *use any method to determine the frequency of the SSB output from the generator, when:*

a) the 'B/2' oscillator is on 2.083 kHz
b) the message frequency is 500 Hz
c) the second oscillator frequency is 100 kHz
d) the phasing resulted in an UPPER sideband.

Confirm by measurement.

Q5 *when the generator was aligned you measured the output frequency. From this single measurement can you state:*

a) if you have an upper or lower sideband ?

This question cannot be answered without

i) knowing something about the internal arrangement of the generator

or

ii) increasing the message frequency, and observing if the sideband frequency increases or decreases.

b) what was the SSB suppressed carrier frequency ?

Q6 *the first pair of quadrature multipliers (Figure 1) need to be DC coupled only if the message contains a DC component. Circuit designers try to avoid DC coupling, as this requires care in minimizing DC offsets. AC coupling is preferred, and this is acceptable for speech. The second pair of quadrature multipliers must be DC coupled, unless one is prepared to accept a small gap in the message (as received) in the region of (B/2) Hz. Explain.*

Q7 *the amplitude of the individual outputs from the ADDER were set to about 4 volts peak-to-peak before final trimming. These were DSBSC-style signals. When they were present together, and the system aligned, about what amplitude would you expect for the final SSB signal ? Show your reasoning. Check by measurement.*

WEAVER`S SSB DEMODULATOR

WEAVER`S SSB DEMODULATOR

ACHIEVEMENTS: *alignment of Weaver`s SSB receiver*

PREREQUISITES: *completion of experiment entitled **Weaver`s SSB generator** in this Volume.*

EXTRA MODULES*: a total of four MULTIPLIERS, two PHASE SHIFTERS, and two TUNEABLE LPF modules is required. This is twice as many of these modules as are in the TIMS Basic Set.*

PREPARATION

It is assumed that you have completed the experiment entitled *Weaver`s SSB generator*, where the general principles of Weaver`s method were met.

The demodulator employs the same principles, although 'in reverse', as it were.

Its block diagram is shown in Figure 1.

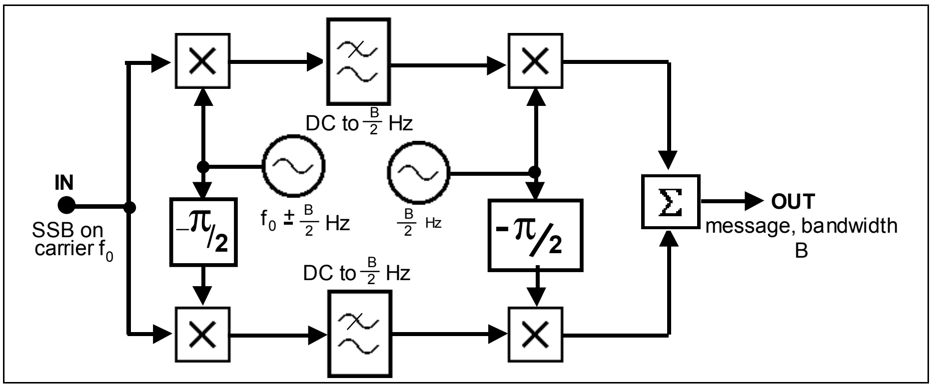

Figure 1: model of Weaver`s SSB demodulator

As in the case of the phasing method of SSB generation, two lowpass filters are shown at the *inputs* to the second pair of quadrature multipliers. In practice these two are replaced by a single lowpass filter, of bandwidth B Hz, at the *output* of the summing block. Similar comments apply here as were made when describing the phasing-type SSB demodulator.

EXPERIMENT

If you have completed the experiment entitled *Weaver`s SSB generator* you should have no difficulty in modelling Weaver`s demodulator !

Your low frequency carrier will be the nominal 2 kHz ‘message’ available from the MASTER SIGNALS module.

When aligned, you should demonstrate that the receiver looks out at the RF spectrum on one side only of the carrier to which it is tuned. You should confirm that this window is B Hz wide [1].

[Refer to the technique of testing used in the experiment entitled ***SSB demodulation – the phasing method*** in Volume A1. This used a VCO to simulate an SSB signal derived from a single tone message. Thus, with the VCO tuned to 102 kHz, it simulates the USB of a 100 kHz SSB transmitter, derived from a 2 kHz message.

When the receiver is tuned to receive the VCO on the high side (USB) of the 100 kHz carrier (say at 102 kHz) then there should be no output from the receiver when the VCO is tuned to 98 kHz (the LSB)].

TUTORIAL QUESTIONS

Q1 *analyse the system illustrated in Figure 1, and show that it has a window on the frequency spectrum of bandwidth B/2 Hz. Show exactly where this is located in the spectrum.*

Q2 *describe the practical problems associated with placing the* ***two*** *filters at the inputs to the second pair of multipliers, and the different set of problems encountered when they are replaced by a* ***single*** *filter at the summing block output.*

[1] where B is defined in Figure 1

CARRIER ACQUISITION AND THE PLL

CARRIER ACQUISITION AND THE PLL

ACHIEVEMENTS: *introduction to a method of carrier acquisition using the phase locked loop (PLL)*

PREREQUISITES: *familiarity with the generation and demodulation of DSBSC - completion of the experiments entitled* ***DSBSC generation*** *and* ***Product demodulation - synchronous & asynchronous*** *(both in Volume A1)*

ADVANCED MODULES: *100 kHz CHANNEL FILTERS*

EXTRA MODULES: *a third MULTIPLIER module would be an advantage*

PREPARATION

carrier acquisition methods

As you will know there is often a need, *at the receiver*, to have a copy of the carrier which was used *at the transmitter*. See, for example, the experiment entitled *Product demodulation -synchronous & asynchronous* in Volume A1.

This need is often satisfied, in a laboratory situation, by using a *stolen carrier*. This is easily done with TIMS. But in commercial practice, where the receiver is remote from the transmitter, this local carrier must be derived from the received signal itself.

The use of a stolen carrier in the TIMS environment is justified by the fact that it enables the investigator (you) to concentrate on the main aim of the experiment, and not be side-tracked by complications which might be introduced by the carrier acquisition scheme.

bandpass filter

There have been many schemes proposed for the purpose of deriving carrier information from the received signal. Many of these depend for their operation on the existence of a component, however small, at carrier frequency, in the transmitted signal itself. This is often called a ‘pilot carrier’. Commercial practice was to set the pilot carrier at a level of about 20 dB below the major sidebands [1]. Modern practice is to omit pilot carriers completely.

A signal with pilot carrier is illustrated in Figure 1. It is a DSBSC, derived from a single tone, with a small amount of carrier added. See Tutorial Question Q2.

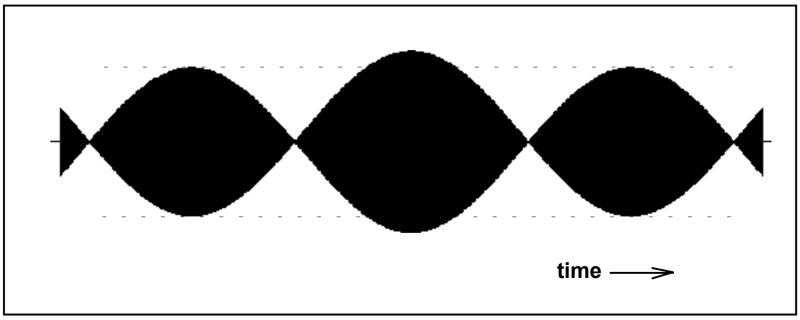

Figure 1: DSB with small carrier leak

To extract a term at carrier frequency from this signal one could use a narrowband bandpass filter. Unfortunately this scheme, as simple as it appears to be, has its problems. For example, in practice the frequency stability of the transmitted carrier may be such that the receiver filter would need either to track it, or be wide enough to encompass it under all conditions. In this latter case the filter may allow some sidebands to pass as well, thus impairing the purity of the recovered carrier. So generally something more sophisticated is required.

the phase locked loop (PLL).

The PLL configuration includes a non-linear feedback loop. See Figure 2. To analyse its performance to any degree of accuracy is a non-trivial exercise. To illustrate it in simplified block diagram form is a simple matter.

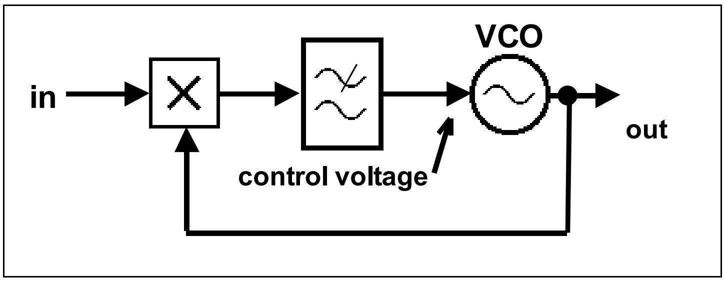

Figure 2: the basic PLL

To describe its behaviour in elementary terms is also a simple matter.

If there is a component at the desired frequency at the input, it will appear at the output in filtered and amplitude stabilised form. In addition, if the frequency of the input changes, the PLL output is capable of following it.

The PLL behaves like a narowband tracking filter.

[1] typically defined relative to the transmitter peak envelope power (PEP).

Of course, there are conditions upon this happening.

The principle of operation is simple - or so it would appear.

Consider the arrangement of Figure 2 in open loop form. That is, the connection between the filter output and VCO control voltage input is broken.

Suppose there is an unmodulated carrier at the input.

The arrangement is reminiscent of a product demodulator. If the VCO was tuned precisely to the frequency of the incoming carrier, ω_0 say, then the output would be a DC voltage, of magnitude depending on the phase difference between itself and the incoming carrier.

For two angles within the 360^0 range the output would be precisely zero volts DC.

Now suppose the VCO started to drift slowly off in frequency. Depending upon which way it drifted, the output voltage would be a slowly varying AC, which if slow enough looks like a varying amplitude DC. The sign of this DC voltage would depend upon the direction of drift.

Suppose now that the loop of Figure 2 is closed. If the sign of the slowly varying DC voltage, now a VCO *control voltage*, is so arranged that it is in the direction to urge the VCO back to the incoming carrier frequency ω_0, then the VCO would be encouraged to 'lock on' to the incoming carrier. The carrier has been 'acquired'.

> *Notice that, at lock, the phase difference between the VCO and the incoming carrier will be 90^0.*

Matters become a little more complicated if the incoming signal is now modulated. Suppose it was an AM signal. There is always a carrier, and the sidebands are always symmetrically displaced about it. Qualitatively you may tend to agree that, if the sidebands were not too large, the PLL would still lock on to the carrier, which is the largest component present; and so it does.

Being a non-linear arrangement, as analysis will show, it is not so much the *largest* component present as the *central* component to which the PLL will lock. In fact, the amplitude of the central component need not be large (under some conditions it can even be zero ! Non-linearities will generate energy at the carrier frequency).

Rather than attempt to justify this statement analytically (it is a non-trivial exercise) you will make a model of the PLL, and demonstrate that it is able to derive a carrier from a DSB signal which contains a pilot carrier.

squaring

What happens if the received signal has no pilot carrier ?

This is the general case in modern practice. The solution is to subject the signal to a non-linear operation. This will generate new spectral components. A popular non-linear characteristic is that of squaring.

Such a signal processing block is illustrated in block diagram form in Figure 3 below:

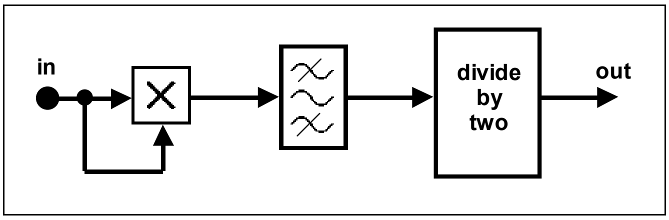

Figure 3: the squaring circuit

The divide-by-two block is shown with an analog input. An analog output is implied. Internally the circuitry may be digital.

This arrangement will generate a component at carrier frequency from a true DSBSC signal.

It is easy to show, in a simple case, that this is so. For example.

$$\text{DSBSC} = a(t).\cos\omega t \qquad \text{........ 1}$$

$$\text{DSBSC squared} = a^2(t)\ [\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega)t \qquad \text{........ 2}$$

$$= \tfrac{1}{2}.a^2(t) + \tfrac{1}{2}.a^2(t).\cos 2\omega t \qquad \text{........ 3}$$

$$= \text{low frequency term} + \text{DSB at } 2\omega \qquad \text{........ 4}$$

Here a(t) is the message. After squaring it *must* have a DC term, together with some other low frequency terms.

> *Since there is a large DC term in $a^2(t)$, then there **must** be a large term at 2ω in the product $a^2(t).\cos 2\omega t$.*

A bandpass filter will extract this. It may be amplitude limited (to stabilise the amplitude) and then halved in frequency. This may be sufficient processing for some applications.

The purpose of the BPF is to separate the terms in $a^2(t)$ from those around the frequency of 2ω. For the case where these are widely separated then an RC highpass filter would probably be adequate. It is assumed that the original DSBSC is reasonably free of noise and interference.

However, if improved properties are required (see Tutorial Question Q7.), a phase locked loop (PLL) may replace the bandpass filter [2].

squarer plus PLL

For the case where a component at carrier frequency is definitely not present, and the advantages of a dynamic tracking bandpass filter are desired, then the squarer plus PLL is recommended. This is illustrated in Figure 4 below.

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2 or at least ease the requirements of this BPF

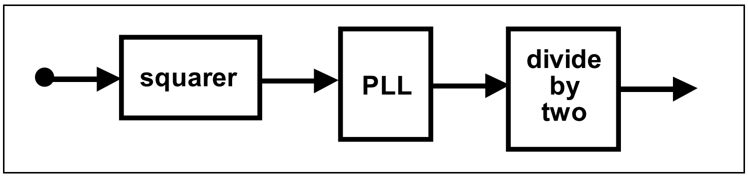


Figure 4: squarer-plus-PLL

The squaring arrangement ensures that a component at the desired (carrier) frequency will be present at the input to the PLL. The PLL operates at 2ω.

So the combination of a squarer and PLL, together with a third multiplier in a product demodulator arrangement, constitutes a popular, basic synchronous receiver.

An alternative arrangement of three multipliers and associated operational blocks constitutes the Costas loop.

the Costas loop

A Costas loop is another well known arrangement which is capable of extracting a carrier from a received signal.

This arrangement is examined in the experiment entitled *The Costas loop* (this Volume).

EXPERIMENT

During this experiment you will consider in turn:

1. a bandpass filter (BPF)
2. the PLL
3. the squarer
4. the squarer plus PLL
5. as time permits, a complete synchronous receiver

the pilot carrier and BPF

When a small but constant amplitude component at carrier frequency (a *pilot* carrier) accompanies the transmitted signal it can be extracted with a bandpass filter. This technique is reasonably self evident and will not be examined. See Tutorial Question Q1.

the PLL

You will now model the PLL of Figure 2, and use a DSB plus small carrier (Figure 1) as its input. The arrangement is shown modelled in Figure 5.

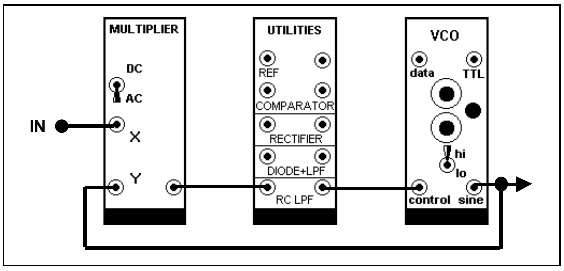


Figure 5: a model of the PLL of Figure 2

T1 *patch up the model of Figure 5 above. The VCO is in 'VCO mode' (check* SW2 *on the circuit board). The input signal, a DSB based on a 100 kHz carrier (locked to the TIMS 100 kHz MASTER), is available at TRUNKS (or you could model it yourself). Initially set the* GAIN *of the VCO fully anti-clockwise.*

T2 *tune the VCO close to 100 kHz. Observe the 100 kHz signal from MASTER SIGNALS on CH1-A, and the VCO output on CH2-A. Synchronize the oscilloscope to CH1-A. The VCO signal will not be stationary on the screen.*

T3 *slowly advance the* GAIN *of the VCO until lock is indicated by the VCO signal (CH2-A) becoming stationary on the screen. If this is not achieved then reduce the* GAIN *to near-zero (advanced say 5% to 10% of full travel) and tune the VCO closer to 100 kHz, while watching the oscilloscope. Then slowly increase the* GAIN *again until lock is achieved.*

T4 *while watching the phase between the two 100 kHz signals, tune the VCO from outside lock on the low frequency side, to outside lock on the high frequency side. Whilst in lock, note (and record) the phase between the two signals as the VCO is tuned through the lock condition. Theory suggests (?) they should be 90^0 apart in the centre of the in-lock tuning range. See Tutorial Question Q5.*

the squaring multiplier

Even without spectrum analysis facilities it is possible to give a convincing demonstration of the truth of eqn.(4) above, which revealed the generation of a DSB *with* carrier (at 2ω) from a DSB *without* carrier (at ω).

This is achieved by modelling the arrangement of Figure 3, as illustrated in Figure 6.

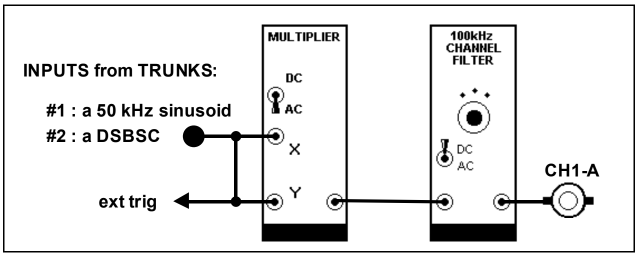


Figure 6: squaring. The model of Figure 3 without divide-by-2

First check the operation of squaring on a sinusoidal input.

T5 *patch up the model of Figure 6. Select the input #1 from TRUNKS (Check the waveform, and measure its frequency, to confirm it is the 50 kHz signal).*

T6 *select* CHANNEL *#1 on the 100 kHz CHANNEL FILTERS module. This is a straight through connection. Toggle to select* DC*.*

T7 *examine the output of the 100 kHz CHANNEL FILTERS module on CH1-A. Observe and record the waveform, and the envelope shape. This is the output from the MULTIPLIER as a squarer. Confirm the presence of a DC component.* ***note****: the MULTIPLIER is switched to 'AC'. This means any DC at either input will be blocked, but* ***not*** *any DC at the output.*

T8 *switch the 100 kHz CHANNEL FILTERS module to its* CHANNEL *#3. This is a 100 kHz bandpass filter (BPF).*

T9 *observe the change of waveform from the 100 kHz CHANNEL FILTERS module. Confirm it is twice the frequency of the first input, and is sinusoidal.*

Now replace the sinusoidal input with a DSBSC based on a 50 kHz carrier.

T10 *change the input from the nominal 50 kHz sinusoid to the 50 kHz DSBSC. Confirm, at least from all appearances, and expectations, that this* ***is*** *the DSBSC based on a 50 kHz carrier.*

T11 *select the straight through connection,* CHANNEL *#1, on the 100 kHz CHANNEL FILTERS module. Toggle to pass* DC*. Observe the output. Since there is no filtering this is the square of a DSBSC. You may not have anticipated what it would look like, but at least confirm that there is a significant DC component present. It is this component which will produce the desired double frequency carrier term - refer eqns.(3).*

T12 *whilst still observing the 100 kHz CHANNEL FILTERS output, select* CHANNEL *#3 - the bandpass filter. Confirm that the signal does indeed now look like a DSB* ***plus*** *carrier, as per eqn.(4). It must be in the vicinity of 100 kHz, since it passed through the bandpass filter. See Tutorial Question Q6.*

You have now confirmed that the squaring circuit has produced a significant component at twice the frequency of the suppressed carrier of a DSBSC signal.

A narrowband BPF filter could extract this from the other spectral components.

TIMS does not have a 100 kHz narrowband bandpass filter.

But a PLL can do the job. A PLL *behaves* like a *bandpass* filter, although it is built around a *lowpass* filter - the lowpass filter in the feedback loop. See Tutorial Question Q4.

the PLL + squarer

You will now combine squarer (modelled in Figure 6) and the PLL (modelled in Figure 5), using the output of the squarer as the input to the PLL.

The input to the squarer will be the DSBSC based on a 50 kHz carrier.

The full model is shown in Figure 7.

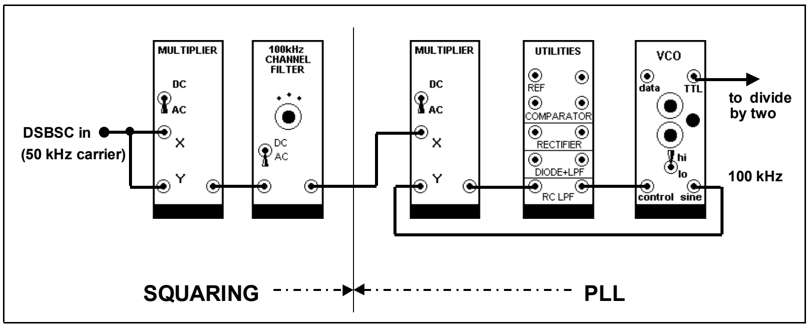


Figure 7. model of the squarer plus PLL

The divide-by-two would add nothing to the demonstration, so it has been omitted.

T13 *combine the models of Figures 5 and 6. Use as input to the squaring circuit the nominal 50 kHz DSBSC.*

T14 *go through the procedure to lock the PLL to the 100 kHz output from the squarer. Describe the setting up, and locking procedure, in your notes.*

T15 *refer to Tutorial Question Q8.*

So far you have locked the PLL to a signal of constant amplitude. But in practice it would be required to lock on to a modulated signal whose message was varying.

What would happen, for a speech message, during significant pauses ?

If you have a third MULTIPLIER module you can examine your carrier acquisition circuit - squarer plus PLL - as the source of local carrier for a product demodulator, and receiving such a signal.

The product demodulator was examined in the experiment entitled *Product demodulation - synchronous and asynchronous* in Volume A1.

A DSBSC, based on a 50 kHz carrier, and with a speech message, will be available at TRUNKS during the latter part of the experiment.

For further details see your Laboratory Manager.

TUTORIAL QUESTIONS

Q1 *suppose a signal has a pilot carrier. This can be used to produce a local carrier by bandpass filtering, or with a PLL. Compare the two methods.*

Q2 *draw an approximate amplitude spectrum of the signal of Figure 1, knowing that it is a DSBSC plus small carrier term, and explain how this was done. How would you then define the level of the pilot carrier ?*

Q3 *compare the advantages of a bandpass filter based on a lowpass filter - the PLL - with a 'conventional' bandpass filter.*

Q4 *explain how you are able to confirm that the VCO of a PLL has locked on to the input signal, whose exact carrier frequency is unknown, when the signal is:*

a) an unmodulated carrier

b) an envelope modulated signal

Q5 *in Task T4 the two signals may not have been close to 90^0 apart at the centre of lock. How could this be, when theory suggests otherwise - or does it ?*

Q6 *from the observed DSB plus carrier from the 100 kHz CHANNEL FILTERS module of Task T12, and knowing the model configuration, draw an amplitude/frequency spectrum of this signal. Confirm, by trigonometrical analysis, the relative amplitudes of the spectral components.*

Q7 *name some of the improved features of the squarer-plus-PLL compared with the squarer alone.*

Q8 *there are many parameters associated with a phase locked loop which are of interest, and their measurement could form the basis of another experiment (or an extension of this one). Two properties of interest are* CAPTURE RANGE *and* LOCK RANGE. *You should find out about these. Of what importance was the setting of the VCO* GAIN *(sensitivity) control, and of the VCO frequency control ?*

SPECTRUM ANALYSIS - THE WAVE ANALYSER

SPECTRUM ANALYSIS - THE WAVE ANALYSER

ACHIEVEMENTS: *to examine the basic spectrum analyser model; modelling a WAVE ANALYSER; to consider the effects of non-linearities upon performance.*

PREREQUISITES: *completion of the experiment entitled* ***Product demodulation - synchronous and asynchronous*** *in Volume A1.*

EXTRA MODULES: *the SPECTRUM UTILITIES module (not in the TIMS Basic Set of modules).*

PREPARATION

the SPECTRUM ANALYSER

Spectrum analysers are found in most homes. They are the domestic radio receiver and TV set. These devices are capable of examining parts of the radio spectrum, and they report what they find either aurally or visually. From their front panel controls we can deduce the frequency from the channel to which they are tuned, and the nature of the spectrum within that channel from the sound and/or picture.

A professional spectrum analyser is an instrument for identifying the amplitude spectrum of an electrical signal (typically one whose spectrum does not vary with time). The majority of commercially available instruments cover a very wide frequency spectrum (Hz to GHz), are extremely accurate, and expensive. They generally provide a visual display of the amplitude-frequency spectrum.

principle of operation

The principle of a spectrum analyser is represented by a tuneable filter, as shown in Figure 1.

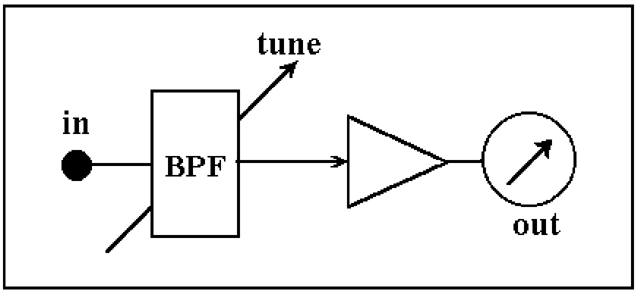


Figure 1: principle of the spectrum analyser.

The arrow through the bandpass filter (BPF) shown in Figure 1 implies that the centre frequency to which it is tuned may be changed. The filter bandwidth will determine the frequency *resolution* of the instrument. The internal noise generated in the circuitry, and the gain of the amplifier, will set a limit to its *sensitivity*.

The symbol of circle-plus-central-arrow represents a voltage indicator of some sort. For the moment we will disregard its response characteristic (RMS, peak, average ?), but agree that it will indicate in some way the presence of an output from the filter.

The frequency of the signal to which the analyser responds is that of the centre frequency of the BPF.

Instruments which require the user to make a manual search, one component at a time, are generally called *wave analysers*; those which perform the frequency sweep automatically and show the complete amplitude-frequency response on some sort of visual display are called *spectrum analysers*.

practical variation

Tuneable bandpass filters are difficult to manufacture. Thus the arrangement of Figure 1 is not used in an instrument covering a wide frequency range. Figure 2 shows a practical compromise. Although this circuit behaves as a *tuneable bandpass* filter, it uses a *fixed lowpass* filter.

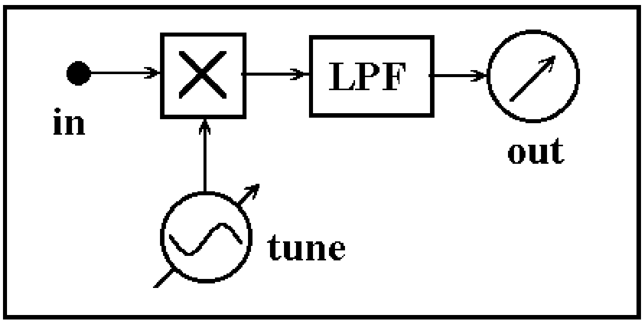


Figure 2: practical spectrum (wave) analyser

The arrangement of Figure 1 is a tuned radio frequency receiver (TRF), and that of Figure 2 is based on the principle of the superheterodyne receiver ('superhet'). Refer to the literature circa 1920 to learn about the historical development of these two configurations. The practical difficulties of the former, and the advantages of the latter, are discussed.

The frequency to which the analyser responds is that of the sinusoidal, tuneable, ‘local’ oscillator.

the WAVE ANALYSER

This experiment will be concerned with a *wave analyser,* which was defined above.

the model

We would like to model a wave analyser that would be of use for future experiments with TIMS. Its *tuning range* must cover the audio spectrum from say 300 Hz to 10 kHz, as well as say 10 kHz either side of our standard carrier frequency of 100 kHz.

Its *frequency resolution* requirements are modest, determined principally by the fact that we would like to examine the individual spectral components in the sidebands either side of 100 kHz modulated signals. In TIMS these are seldom closer than say 250 Hz. A resolution of 100 Hz would be adequate; this is a very modest requirement.

We cannot model the arrangement of Figure 1, since we do not have a tuneable BPF with a bandwidth of 100 Hz, covering such a wide range as 250 Hz to 110 kHz.

> *The problems associated with the realization of the scheme of Figure 1 are now apparent.*

The scheme of Figure 2, meeting the above specification, would require a LPF with a cut-off of around 50 Hz. In addition a tuneable oscillator is required; this will need to cover the audio as well as the 100 kHz range.

TIMS provides the tuneable oscillator in the form of the VCO module.

Although a 50 Hz lowpass filter is not difficult to design, there is no such electronic filter in the TIMS BASIC SET of modules. But a moving coil volt meter will serve as the output indicator. Due to the inertia of the mechanical movement it will only respond to DC and very low-frequency signals. It will therefore *also* serve as the *lowpass filter.*

The SPECTRUM UTILITIES module has been designed for the purpose.

spectrum measurement - single component input

To make a spectral component measurement it is necessary to understand the principle of the analyser. For a simple first example, suppose the signal v(t) appears at the input, where:

$$v(t) = V_1 \cos(2\pi f_1 t) \qquad \text{........ 1}$$

and that:

$$\text{VCO output} = V_{vco}.\cos(2\pi f_2 t) \qquad \text{........ 2}$$

Then:

$$\text{multiplier output} = \tfrac{1}{2}.k.V_1.V_{vco}\,[\cos 2\pi(f_1 - f_2)t + \cos 2\pi(f_1 + f_2)t] \quad \text{........ 3}$$

where ‘k’ is a constant of the multiplier.

The LPF filter built into the meter-module will remove the term at frequency $(f_1 + f_2)$, and the meter will respond to the term at frequency $|(f_1 - f_2)| = \delta f$. Let the amplitude of this signal be V_m.

$$V_m = (\tfrac{1}{2}.k.V_{vco})\,V_1 \quad \text{........ 4}$$

Since the amplitude of the VCO output is a constant, the magnitude of the meter reading V_m will be *proportional to the amplitude of the input component* V_1. We will call the constant of proportionality ***S***, the *conversion sensitivity* [1], so we have:

$$\boldsymbol{S} = 2/(k.V_{vco}.) \quad \text{........ 5}$$

$$V_1 = \boldsymbol{S}.V_m \quad \text{........ 6}$$

Since V_1 is the *amplitude* of the unknown signal, this last equation gives the scaling factor to be applied to the meter reading.

The *frequency* of the input component must lie within ± δf Hz of the VCO frequency f_2.

The inertia of the moving coil meter prevents it responding to signals of more than a few Hz. For this the VCO frequency must be set close to the frequency of the unknown component at the input. As the frequency difference δf is slowly reduced to zero, the meter will at first ‘quiver’ (say δf is 10 Hz or less); then start to oscillate with greater and greater swings as δf approaches zero.

The peak amplitude of the swing will be V_m, reached as δf approaches zero.

Despite the last statement, setting the frequency error to precisely zero is not desirable. Should δf = 0 then the term of interest becomes a constant DC voltage, and its amplitude would depend upon the phase angle between the unknown component at the input, and the VCO signal. This phase is unknown, and so would introduce an unnecessary complication.

So to measure the amplitude of the unknown component we set δf to one or two Hz, and make a note of the peak reading of the meter and the frequency of the VCO. From this, and the last equation, the unknown amplitude V_1 can be derived.

spectral measurement - two component input

Suppose there are two components at the input. Provided they are separated by at least the frequency resolution of the analyser, only one will produce an output from the filter at any time [2].

[1] *conversion*, because of the frequency change between input and output

[2] this assumes linear operation of all circuits

practical considerations

precautions

A moving coil volt meter will not respond to signals of more than 10 Hz or so, due to its mechanical inertia. This does not prevent its moving coil from being burned out by other AC signals of excessive amplitude. So, as a practical precaution, the meter in the SPECTRUM UTILITIES module is protected by a low-order LPF. This will also remove the component(s) from the MULTIPLIER at the sum frequency ($f_1 + f_2$).

There is a sample-and-hold facility, for capturing the peak swing of the meter. This should be used with care, and its reading not mis-interpreted, since it bypasses the filtering effect of the mechanical inertia of the meter, and will capture all and any signals which reach the meter. Consequently you should have some idea of the relative amplitudes and location of components before using this facility, and an appreciation of the response of the built-in LPF. For further information refer to the ***TIMS User Manual***.

searching methods

Searching for spectral components takes a certain amount of practice. If the VCO frequency is changed at too great a rate the meter will not have time to respond, and components of significant amplitude will be missed. If and when the meter does respond, adjust the VCO frequency carefully until the meter is oscillating very slowly, and record the peak meter reading. Use the sample-and-hold facility if appropriate.

where ?

In practice one usually has a good idea of where the unknowns are going to be - what is sought is their relative amplitude. Thus the searching process is not as difficult as it might at first appear.

how large ?

No great significance is placed on the measurement of *absolute* amplitudes - relative amplitudes are what we really want. So pre-calibration is seldom necessary.

It is often convenient to tune to the largest component of interest, and then to adjust the meter to full scale deflection (FSD) using the on-board variable resistor RV1, labelled GAIN. This reading becomes the reference.

EXPERIMENT

SPECTRUM UTILITIES module

The SPECTRUM UTILITIES module contains a centre-reading moving coil meter, with lowpass filtering, and a sample-and-hold facility. Read about it in the ***TIMS User Manual***. Pay particular attention to the precautions necessary if you use the sample-and-hold facility.

VCO fine tuning

Read about the VCO module in the ***TIMS User Manual***. In the present application it is important to know the techniques of coarse and fine tuning.

- ***coarse tuning*** is done with the front panel f_o control (typically with no input connected to V_{in}).
- for ***fine tuning*** it is convenient to set the GAIN control of the VCO to some small value. Then fine tuning is then done by varying the DC voltage, from the VARIABLE DC module, which is conected to the V_{in} input. The smaller the GAIN setting the finer is the tuning.

The WAVE ANALYSER model

A wave analyser is shown modelled in Figure 3 below.

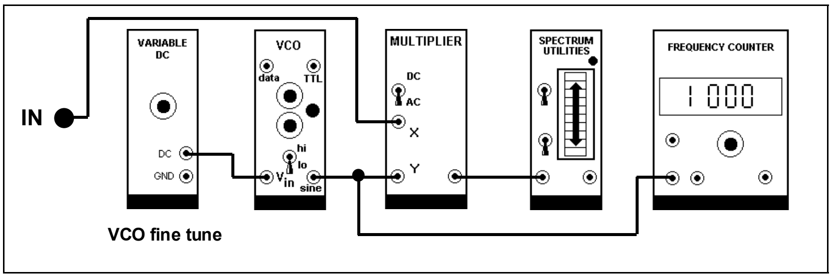


Figure 3: the WAVE ANALYSER model

T1 *patch up a model of the block diagram of Figure 2. A suggested scheme is illustrated in Figure 3. Before plugging in the VCO set the on-board switch SW2 to select the VCO mode. Remember the LPF is simulated by the inertia of the moving coil meter movement in the SPECTRUM UTILITIES module.*

test and calibration

Before using the WAVE ANALYSER to measure some unknown signals it needs to be tested and calibrated in both the audio and 100 kHz regions. You can use the output from an AUDIO OSCILLATOR as a source of test signal for the former (say at 1 kHz and 10 kHz)), and the 100 kHz sinewave from the MASTER SIGNALS module for the latter.

T2 *select a 1 kHz sinewave from an AUDIO OSCILLATOR as your source of test signal. Measure its amplitude V_1 at the input to the WAVE ANALYSER, using the oscilloscope.*

T3 *connect the VCO output to the FREQUENCY COUNTER, and tune the VCO to the expected vicinity of the test signal, until the volt meter reading oscillates slowly. Record the peak reading V_m of the meter.*

There is a variable SCALING resistor RV1 on the circuit board of the SPECTRUM UTILITIES module. You may find it convenient, when measuring spectra, to adjust this so the meter reads full scale deflection (FSD) on a reference component - typically the largest to be encountered.

T4 *check the operation of the on-board SCALING adjustment of the SPECTRUM UTILITIES module.*

T5 *calculate the conversion sensitivity V_1 / V_m of your WAVE ANALYSER.*

T6 *repeat the last three tasks for a 100 kHz test input.*

You will now have three determinations of the sensitivity **S** of the WAVE ANALYSER. Ideally they should all be the same. This assumes:

- the VCO output amplitude is constant over the full LO and HI frequency ranges
- the k factor of the MULTIPLIER is independent of frequency
- probably the k factor of the MULTIPLIER *will* vary slightly between the LO and HI frequency ranges, and so you may need both an S_{LO} and an S_{HI}.

T7 *derive an expression for the '1 kHz conversion sensitivity' of the WAVE ANALYSER in terms of its circuit constants (including the VCO output voltage, multiplier k factor, meter sensitivity). If you do not know the value of 'k', then set up a temporary arrangement, and measure it. Compare this conversion sensitivity with the direct measurement.*

spectrum analysis

It is now time to use the WAVE ANALYSER to examine the spectrum of a DSBSC signal - this was promised in the experiment entitled *DSBSC generation* in Volume A1.

DSBSC spectrum

T8 *set up a DSBSC signal using an AUDIO OSCILLATOR, MULTIPLIER, and the 100 kHz sine wave from the MASTER SIGNALS module.*

T9 *use the oscilloscope to measure the amplitude of the DSBSC (in the time domain). From this, and a knowledge of the frequency of the AUDIO OSCILLATOR, sketch the amplitude spectrum of the DSBSC (in the frequency domain). Show clearly the amplitude and frequency scales.*

T10 *connect the DSBSC to your WAVE ANALYSER, and search for spectral components in the range 90 kHz to 110 kHz. Sketch the measured amplitude spectrum. Show clearly the amplitude and frequency scales.*

T11 *compare the last two spectra, and account for any discrepancies.*

spectra of unknown signals

When happy with the results of the DSBSC spectrum measurement, have a look at the signals at TRUNKS. These have been sent to you for analysis.

practical hints

In practice it is *relative amplitudes* which are of interest. Thus one seldom needs to carry out an amplitude calibration. This saves time in setting up the model. Also, one usually knows *where* the components are, so searching is simplified. It is convenient to find the largest component, and then to set the sensitivity of the meter (with the on-board SCALING adjustment) to indicate full scale on this component.

T12 *use your WAVE ANALYSER to determine the amplitude/frequency spectra of the signals at TRUNKS. Note that the sensitivity of the SPECTRUM UTILITIES meter can be adjusted with an on-board control.*

future spectral measurements

In future experiments you will find this inexpensive WAVE ANALYSER is a useful measurement tool. The model just examined will be found adequate for your purposes. But remember: its circuits must never be overloaded. In practice this means that the signal at the analyser input must not overload the input MULTIPLIER.

Overload - meaning *peak amplitude overload* - will mean spurious readings. Use the oscilloscope to ensure that the peak amplitude of the input signal *never exceeds* the TIMS ANALOG REFERENCE LEVEL. See the Tutorial Question below.

TUTORIAL QUESTIONS

Q1 *the resolution of a wave analyser relates to the width of the 'window' through which it looks at the input spectrum. If the BPF of Figure 1 is 'brick wall' with a passband 2 Hz wide, how would you describe the frequency resolution of the instrument ? (note: a filter response even approaching this would be difficult to realize in analog form).*

Q2 *if the LPF of Figure 2 is 'brick wall', and passes frequencies from DC to 1 Hz, how would you describe the frequency resolution of the instrument ? (note: a reasonable approximation to this filter response, in analog form, would not be impossible to implement).*

In answering the above two questions it was assumed the circuits were linear. Any non-linearity in the circuitry can degrade performance, as is illustrated by answering the next question.

Q3 *what happens if the amplitude of the input signal is 'too high' ? Suppose that there is an amplifier between the input terminal and the BPF of Figure 1, which is 'brick wall', with a 2 Hz bandwidth. Suppose the amplifier has a non-linear gain characteristic given by:*

$$e_{out} = a_1 e_{in} + a_3 e_{in}^3$$

where $a_1 = 1$ and $a_3 = -0.1$ and the input signal e_{in} is a DSBSC.

Derive an expression for the spectrum reported by the output meter, in the vicinity of 100 kHz, when the input is a DSBSC on a 100 kHz carrier, and derived from a 1 kHz sinusoidal message.

The above question is an exercise in trigonometry. It will illustrate one of the phenomena of intermodulation distortion.

hint: *give the DSBSC an amplitude V, and look for sum and/or difference ('intermodulation') components in the vicinity of the sidebands. Show that these are related in a non-linear way to V, and discuss the consequences.*

Q4 *explain the reason for the precautions necessary when using the sample-and-hold facility of the SPECTRUM UTILITY module.*

AMPLIFIER OVERLOAD

AMPLIFIER OVERLOAD

ACHIEVEMENTS: *an introduction to the definition and measurement of distortion in wideband and narrowband systems.*

PREREQUISITES: *completion of the experiment entitled* ***Spectrum analysis - the WAVE ANALYSER*** *in this Volume.*

EXTRA MODULES: *SPECTRUM UTILITIES.*

PREPARATION

not too little - not too much

One of the aims of an analog transmission system, such as an audio amplifier or a long-distance telephone circuit, is to present at the output a faithful reproduction of the signal at the input. Analog systems will always introduce some signal degradation, however defined, but it is the aim of the analog design engineer to keep the amount of degradation to a minimum.

As you are probably already aware, if signal levels within a system rise ‘too high’, then the circuitry will overload; it is no longer operating in a linear manner. As will be seen later in this experiment, extra, unwanted, distortion components will be generated. These distortion, or noise [1], components, are signal-level dependent. In this case the noise components arise due to the presence of the signal itself.

Conversely, if signal levels within a system fall ‘too low’, then the internal circuit noise, which is independent of signal level, will eventually swamp the small, wanted, output. The background noise of the TIMS system is held below about 10 mV peak - this is a fairly loose statement, since this level is dependent upon the bandwidth over which the noise is measured, and the model being examined at the time. A general statement would be to say that TIMS endeavours to maintain a SNR of better than 40 dB for all models.

Thus analog circuit design includes the need to maintain signal levels at a level 'not to high' and 'not too low', to avoid these two extremes.

The TIMS working level, or ANALOG REFERENCE LEVEL has been set at 4 volts peak-to-peak. Modules will generally overload if this level is exceeded by say a factor of two.

1 noise is here considered to be anything that is not wanted.

It is the purpose of this experiment to introduce you to the phenomenon of circuit overload, and to offer some means of defining and measuring its effects.

amplifier 'gain'

ideal amplifier 'gain'.

Consider an amplifier which is said to have a gain of '**g**'. This is understood to mean that, if v_i is the input signal voltage, then the output v_o is given by:

$$v_o = g \cdot v_i \qquad \text{........ 1}$$

This would be described as an ideal amplifier. Its input/output (i/o) characteristic would be a straight line, with a slope of g volts/volt.

In simple terms, **g** is a dimensionless constant. More generally it can be complex, and frequency dependent; but such complications will be ignored in the work to follow.

real amplifiers

Unfortunately, a real amplifier does not have an ideal straight-line input/output characteristic. It is more likely to look like that of Figure 1.

It is obvious to the eye that the characteristic shown in Figure 1 could not be considered straight, or *linear*, except perhaps for input signal amplitudes in the range say ± ½ volt.

In this range the slope of the characteristic is 10 volts/volt.

The amplifier is said to have a *small signal gain* of 10. Input signal amplitudes in the range say ± ½ volt would be considered *small* signals for *this* amplifier.

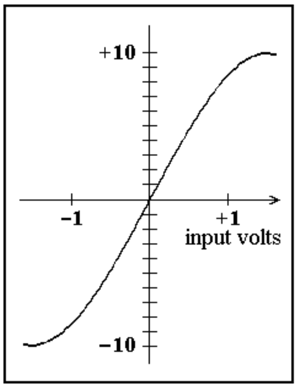


Fig 1: typical characteristic

A typical amplifier characteristic is likely to flatten off and become parallel to the horizontal axis, whereas this characteristic, as defined by eqn. (2) below, will approach and finally cross the input-axis for larger input amplitudes.

So this approximation to an amplifier characteristic should be used for input amplitudes restricted to the range 0 to say ±1½ volts.

Its actual input/output relationship is given by:

$$v_o = g_1.v_i + g_3.v_i^3 \qquad \text{........ 2}$$

where v_i and v_o are the input and output voltages, respectively, and

$$g_1 = 10 \qquad \text{........ 3}$$

$$g_3 = -1.5 \qquad \text{........ 4}$$

Note that the range of the so-called linear part of the characteristic is not obvious from a cursory examination of eqn. (2) alone. We shall later obtain a method of defining an acceptable operating input signal range.

harmonic distortion

calculation of harmonic distortion components

Intuition tells us that an amplifier with the input-output characteristic of Figure 1 will introduce distortion, but what sort of distortion ? This can be checked analytically by nominating a test input signal, and then determining the corresponding output.

Let the test signal be a single tone, v_i, where:

$$v_i = V.\cos\mu t \qquad \text{........ } 5$$

Substituting this into eqn. (2), and expanding, gives:

$$v_o = g_1 V \cos\mu t + g_3 V^3 (3/4 \cos\mu t + 1/4 \cos3\mu t) \qquad \text{........ } 6$$

Notice that the cubic term has given rise to two new components; one on the same frequency as the input signal, and the other on its third harmonic.

After combining like harmonic terms the last equation can be rewritten as:

$$v_o = [g_1 V + (3/4) g_3 V^3] \cos\mu t + (1/4) g_3 V^3 \cos3\mu t \qquad \text{........ } 7$$

Notice that, at the output:

1) the amplitude of the *wanted* term $\cos\mu t$ is no longer simply g_1 times the input amplitude (as suggested by eqn. (1)).

2) there is an extra, *unwanted* term, on the third harmonic of the input.

The original signal has been distorted.

This can be observed in the time domain, using an oscilloscope. For the example under discussion, the output, for an input of amplitude V = 1 volt, is shown in Figure 2.

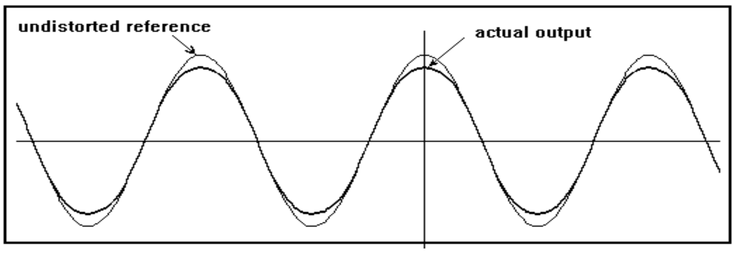


Figure 2: the output voltage waveform of eqn. (7), for an input amplitude of V = 1 volt

definition of harmonic distortion - THD

Notice that the analysis has been performed so as to describe the output in terms of harmonic components of the input. The fundamental, or first harmonic, is the *wanted* term, and all higher harmonic terms (in this example there is only one) are *unwanted*.

The wanted and unwanted harmonic terms can be compared, and some measure defined, to describe the *amount* of *harmonic* distortion.

The comparison is usually made on a power basis, described as the 'total harmonic distortion', or THD, and defined as:

$$THD = 10\log_{10}\frac{\sum H_1^2}{\sum_{j=2}^{n} H_j^2}\,dB \qquad \text{........ 8}$$

where H_1 is the amplitude of the output signal on the same frequency as the input test signal, and the H_j are the amplitudes of the 2nd and higher harmonics, or unwanted, terms.

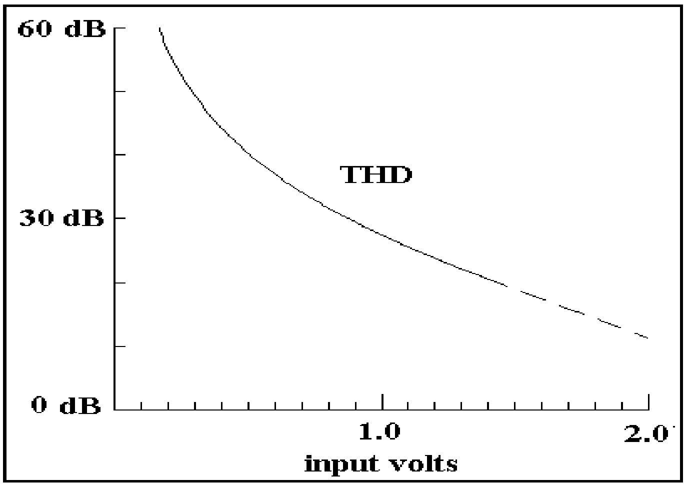


Figure 3: THD versus input amplitude for the response of eqn. (2)

For the example above, where the input amplitude V was 1 volt peak, this evaluates to a THD of 27.48 dB.

Figure 3 shows the THD plotted for input amplitudes in the range 0 to 1½ volts.

Having decided on an acceptable THD for this amplifier (say 40 dB), the user can now specify a maximum input level (½ volt) from Figure 3.

measurement of THD

There are instruments available which measure THD directly. They supply their own test input signal to the device under test, measure the total AC power output, subtract the power due to the wanted signal, and present the THD expressed in decibels.

You will make your own measurements with TIMS by modelling a WAVE ANALYSER [2], measuring the amplitudes of the individual output components, and then applying the THD formula.

Note that this THD is specific to the actual input signal amplitude used for the measurement. There is no subsequent simple step to enable:

1. prediction of the THD for another input signal amplitude.
2. determination of the non-linear characteristic of the amplifier.

This is because of the non-linear relationship between the input signal amplitude and the corresponding output THD.

narrow band systems

The previous discussion requires some modification if the system being examined is narrow band [3].

There are many circuits in an analog communications system which are narrowband, since many communications signals themselves are narrow band. A narrowband system is one which has had its frequency response intentionally restricted. This generally simplifies the circuit design, and eliminates out-of-band noise.

To simplify the discussion, suppose there is a bandpass filter at the system output, so that only frequencies over a narrow range either side of the measurement frequency, μ rad/s, will pass to the output. Let the non-linearities be in the circuitry preceding the filter.

If this were the case for the example already discussed, the output waveform would show no sign of distortion, but instead be a pure sinewave ! No distortion would be visible on an oscilloscope connected to the output, because the distorting third harmonic signal would not reach the output. An instrument for measuring THD with a single tone input would register no distortion at all !

Is the system linear or not ? It is definitely non-linear. This can be demonstrated by observing that the relationship between *input amplitude* and *output amplitude* is not linear.

Using the methods so far employed, this does not show up as waveform distortion, and would not be revealed by a spectrum analysis of the output

measurement of a narrow band system

The single-tone test signal we have been using so far is inappropriate for the measurement of THD in a narrow-band system. What is needed is a more demanding test signal, which will reveal the non-linearity, and which is more representative of the signals to be found in most systems. The non-linearity is

2 see the experiment entitled ***Spectrum Analysis - the Wave Analyser***, or model a spectrum analyser using the TIMS320 module.

3 'wideband' and 'narrowband' signals are defined in the chapter entitled *Introduction to Modelling with* TIMS.

revealed by the transmission of *two* signals simultaneously, namely the *two tone test signal*.

the two-tone test signal

The two-tone test signal consists of two equal amplitude sinusoids, of comparable frequency. Thus:

$$v(t) = V(\cos\mu_1 t + \cos\mu_2 t) \quad \text{where say } \mu_1 < \mu_2 \qquad \text{........ } 9$$

Let us use this as the input to the non-linear amplifier previously examined with a single tone input.

The amplitudes of the various output terms, from trigonometrical expansion of v(t) when substituted into eqn. (2), are shown below:

$$\cos(\mu_1).t \implies g_1.V + (3/4).g_3.V^3 + (3/2).g_3.V^3 \qquad \text{........ } 10$$

$$\cos(\mu_2).t \implies g_1.V + (3/4).g_3.V^3 + (3/2).g_3.V^3 \qquad \text{........ } 11$$

$$\cos(3\mu_1).t \implies (1/4).g_3.V^3 \qquad \text{........ } 12$$

$$\cos(3\mu_2).t \implies (1/4).g_3.V^3 \qquad \text{........ } 13$$

$$\cos(2\mu_1 - \mu_2)\,t \implies (3/4).g_3.V^3 \qquad \text{........ } 14$$

$$\cos(\mu_1 - 2\mu_2)\,t \implies (3/4).g_3.V^3 \qquad \text{........ } 15$$

$$\cos(2\mu_1 + \mu_2)\,t \implies (3/4).g_3.V^3 \qquad \text{........ } 16$$

$$\cos(\mu_1 + 2\mu_2)\,t \implies (3/4).g_3.V^3 \qquad \text{........ } 17$$

short cuts

The calculation of these amplitude coefficients can be very tedious. But one soon observes certain phenomena involved, and with a narrowband system applies short cuts to avoid unnecessary work. Remember that the two frequencies μ_1 and μ_2 are close together.

Some observations are:

- The trigonometrical expansions can only generate terms on harmonics of the original frequencies, and on sum and difference frequencies of the form $(n.\mu_1 + m.\mu_2)$ and $(n.\mu_1 - m.\mu_2)$, where n and m are positive integers.
- Components with frequencies μ_1 and μ_2 will pass through the system, but none of their higher harmonics.
- Of the sum and difference frequencies $(n.\mu_1 + m.\mu_2)$ and $(n.\mu_1 - m.\mu_2)$, it is agreed that only those of the difference group, where n and m differ by unity, will pass through the system.
- When n is *odd*, the expansion of $(\cos\mu t)^n$ gives rise to all odd harmonics counting down from the n^{th}.

- When n is *even*, the expansion of $(\cos\mu t)^n$ gives rise to all even harmonics counting down from the n^{th}. Note that the count goes down to the zeroeth term, which is DC.

Taking these observations into account when dealing with a narrowband system, the number of necessary calculations can be reduced.

signal-to-distortion ratio - SDR

We have seen that when the test input is a single tone, the distortion components are restricted to being harmonics of this signal. But with a more complex test signal other distortion products are possible. As has just been seen, a two-tone test input gives rise to harmonics of each of these signals, as well as *intermodulation products*. Intermodulation products (IPs) arise from the *products* of two or more signals, and fall on frequencies which are the sums and differences of multiples of their harmonics.

The measure of distortion can no longer be called THD, since the sum and difference frequencies are present in addition to the harmonic terms, so the term *signal-to-distortion ratio*, or SDR, is used. It is evaluated using the same principle as THD, namely:

$$SDR = 10\log_{10}\frac{\sum_{i=1}^{n} W_i^2}{\sum_{j=1}^{m} U_j^2}\,dB \qquad \text{........ } 18$$

where the amplitude of the *n* wanted terms is W_i and of the *m* unwanted terms is U_j. These are the terms which actually reach the output. In a narrow band system many others will be generated which will not reach the output.

two-tone example

We will now apply eqn. (18) to calculate the SDR for the characteristic of Figure 1. The input signal is defined by eqn. (9), with V = 0.5 volts. This makes the peak amplitude of the two-tone signal equal to 1 volt, which is comparable with the amplitude used earlier for the single tone testing. First we will include all unwanted components in the calculation, making this a wideband result.

wideband SDR = 27.01 dB 19

For the narrowband case the terms to be neglected, in this example, are those on the third harmonics of μ_1 and μ_2 and the intermodulation products on the sum frequencies. The result is then:

narrowband SDR = 30.25 dB 20

Remember that, had a single tone been used for this narrowband system, there would have been no signal-dependent distortion products found at the output, and the amplifier would have appeared ideal.

The noise output would not, in fact, have been zero. We have been dealing with large-signal operation, and so have ignored the existence of random noise.

noise

In the work above we have divided the output signal into wanted and unwanted components. All the unwanted components so far had magnitudes which were directly dependent upon the amplitude of the input signal. Such unwanted components are referred to as *signal dependent* noise.

Also to be considered in any system is random noise, or system noise. This arises naturally in all circuitry, and its magnitude is independent of any input signal [4].

In the work above we have made no mention of such noise. This is because it has been assumed insignificant with respect to signal dependent noise. This was a reasonable assumption, since, in a well designed system, signal dependent noise only occurs for large input signal magnitudes.

two-tone test signal generation

The two-tone signal can be made from any two signals of suitable frequencies; a convenient pair of signals for TIMS is the nominal 2 kHz message at the MASTER SIGNALS module and an AUDIO OSCILLATOR. Refer to the block diagram of Figure 4 below.

A lot can be learned about the two tone signal if it can be displayed in a convenient manner. Recognising it as a form of DSBSC makes this an easy matter.

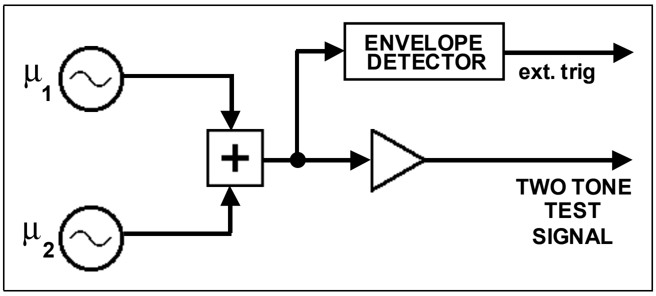


Figure 4: two-tone test signal generation

the two-tone seen as a DSBSC

Recall that a two-tone test signal has been defined earlier as in eqn. (9). The spectrum of this signal is identical with that of a DSBSC, defined as:

$$\text{DSBSC} = 2.V.\cos\mu t.\cos\omega t \quad \text{........ 21}$$

$$= V\,[\cos(\omega - \mu)t + \cos(\omega + \mu)t\,] \quad \text{........ 22}$$

To force the signal of eqn. (21) to match that of eqn. (9), it is necessary that:

$$\omega - \mu = \mu_1 \quad \text{........ 23}$$

4 although it *can* be dependent on the presence of an input generator, the output impedance of which can influence the system noise.

$$\omega + \mu = \mu_2 \qquad \text{........ 24}$$

From these two equations the DSBSC frequencies are:

$$\mu = (| \mu_2 - \mu_1 |) / 2 \quad \text{rad/s} \qquad \text{........ 25}$$

$$\omega = (\mu_1 + \mu_2) / 2 \quad \text{rad/s} \qquad \text{........ 26}$$

To display a DSBSC stationary on the screen a triggering signal is required that is related to its *envelope*. The envelope of the DSBSC of eqn. (21) is a full wave rectified version of $\cos\mu t$, but there is no signal at this frequency [eqn. (25)] if the two-tone signal is made by the addition of two tones as per eqn. (9).

The appropriate triggering signal can be generated with an envelope detector acting on the two-tone signal. Remember this is a rectifier followed by lowpass filter which will pass a few harmonics (ideally the first only) of the difference frequency of eqn. (25), but not the sum frequency eqn. (26).

EXPERIMENT

experimental set-up

For this experiment you will need a WAVE ANALYSER. We suggest you use the model examined in the experiment entitled *Spectrum analysis - the WAVE ANALYSER*.

For the non-linear amplifier you will use the COMPARATOR within the UTILITIES module. Please refer to the ***TIMS User Manual***. The COMPARATOR has an analog (YELLOW) output socket, which you will use, and in this application it is called a CLIPPER. The CLIPPER has a non-linear input/output characteristic. That is, it will overload with input signal amplitudes comparable with the TIMS ANALOG REFERENCE LEVEL of 4 volts peak-to-peak. The overload characteristic can be varied by means of two on-board DIP switches. For the present application you will use the 'soft clipping' characteristic; this is set with SW1 switched to 'ON/ON', and SW2 switched to 'OFF/OFF'. The REF input socket, used for the COMPARATOR, is not used for CLIPPER applications.

From now on the CLIPPER will be referred to as the 'DUT', namely the 'device-under-test'.

For the purpose of the experiment you could consider it to be an amplifier in an audio system, where it is required to amplify speech signals. Thus it is a wideband device.

You should base your experimental set up on the block diagram of Figure 5.

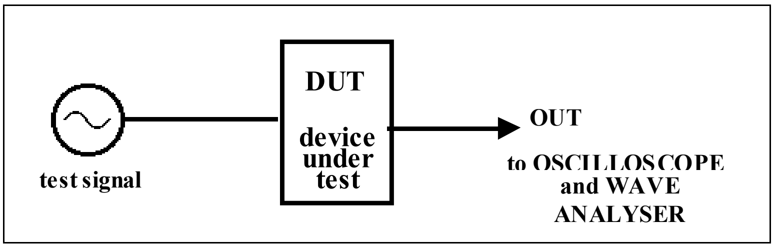


Figure 5: test setup

The WAVE ANALYSER model is shown in Figure 6

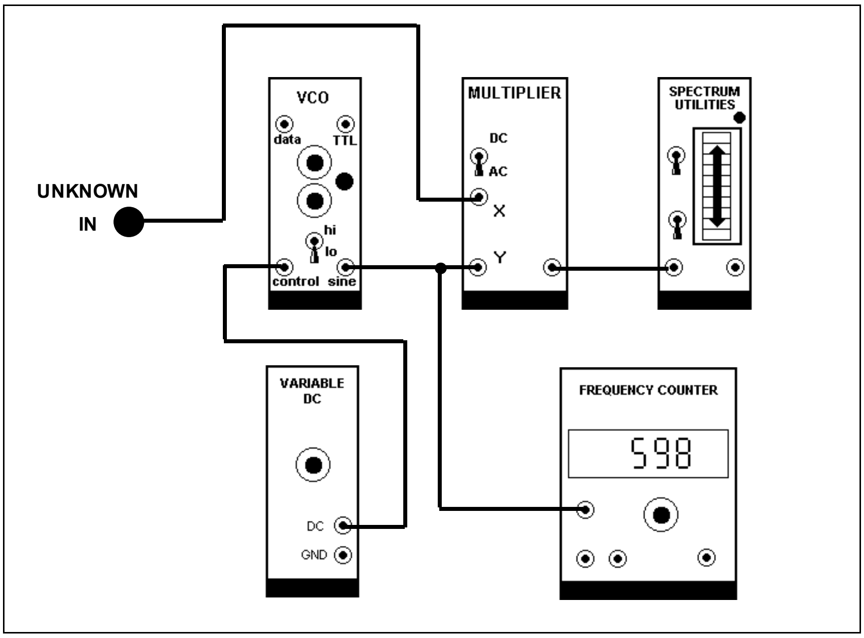


Figure 6: the WAVE ANALYSER model.

single tone testing

measurement of THD

T1 *set the DIP switches on the DUT to the low gain (soft limiting) position, before inserting the module into the TIMS SYSTEM UNIT. This is the device under test ('DUT'). Include the patchings to the SCOPE SELECTOR inputs.*

T2 *patch up the model as in Figure 7 below.*

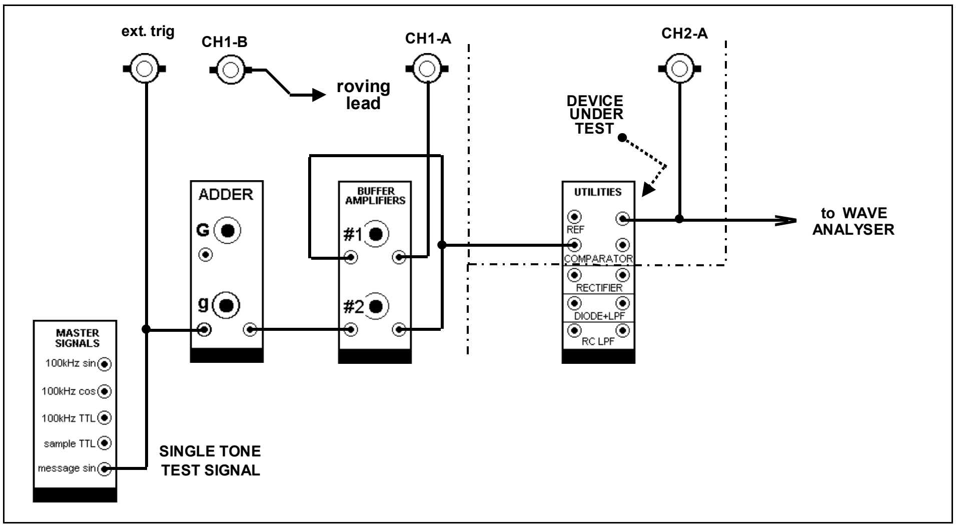


Figure 7: the single tone test model

The ADDER, in cascade with BUFFER #2, will be used later in the two-tone test set up. BUFFER #2 will be used for test signal amplitude control. The other BUFFER, #1, is used for polarity reversal and level adjustment of the oscilloscope display of the input test signal.

T3 *set the gain of the ADDER to about ½, and that of BUFFER #2 to about unity.*

T4 *use the 'ext. trig' from a constant amplitude version of the test source, as shown, to trigger the oscilloscope. Set the sweep speed to show one or two periods of the test signal.*

T5 *switch to CH1-A and CH2-A. Set both channels to the same gain, say ½ volt/cm. You are looking at both the input and output signals of the DUT.*

T6 *use the oscilloscope shift controls to superimpose the two traces. Use BUFFER #1 to equalize their amplitudes.*

T7 *notice that BUFFER #2 varies the amplitude of* ***both*** *traces, so they stay superimposed at low input amplitudes, before distortion sets in. Adjust the gain of BUFFER #2 until the output signal indicates the onset of moderate distortion; that is, when its shape is obviously different from the input waveform, which is also being displayed on the oscilloscope. A ratio of about 5:6 for the distorted and undistorted peak-to-peak amplitudes gives a measurable amount of distortion.*

You have duplicated Figure 2.

Record the amplitude of the signal at the input to the DUT.

You are now set up moderate overload of the DUT. You are ready to measure the distortion components.

T8 *without disturbing the arrangement already patched up, model a WAVE ANALYSER. For example, use the one examined in the experiment entitled* ***Spectrum analysis - the WAVE ANALYSER****. This is shown in Figure 6.*

T9 *use the WAVE ANALYSER to search for, and record, the presence of all significant components at the output of the DUT for the conditions of the previous Task. Theory suggests that these will be at 2.084 kHz (set by the nominal 2 kHz test signal from the MASTER SIGNALS module) and its* ***odd*** *multiples (odd, because of the approximate cubic shape of the transfer function of the DUT).*

T10 *from your measurements of the previous Task calculate the amount of harmonic distortion (THD) under the above conditions.*

T11 *now reduce the level of the input signal to the DUT by (say) 50% (using the gain control of BUFFER #2).*

T12 *measure the amplitude of the largest unwanted component - the third harmonic of the input.*

You should have observed that:

> *whereas the amplitude of the input was reduced by 50%, that of the largest unwanted component fell by more than this.*

This is a phenomenon of non-linear distortion.

Now try a two-tone test, looking for intermodulation products as well as harmonics. You will work on the same amplifier (DUT) as before.

two-tone testing

measurement of SDR

If you built the model of Figure 7, then you are almost ready. The new test set-up is illustrated in Figure 8 below.

The ADDER combines the two signals in equal proportions. The tones should be of 'comparable frequency'; say within 10% or less of each other. BUFFER #2 at the ADDER output is used as a joint level control. As before, BUFFER #1 enables the 'before' and 'after' signals, displayed on CH1-A and CH2-A respectively, to be matched in amplitude.

T13 *patch up the model of Figure 8 below. Include the patchings to the SCOPE SELECTOR inputs.*

T14 *the two tones are the nominal 2 kHz message from the MASTER SIGNALS module, and a second from an AUDIO OSCILLATOR. Set the second tone close to the first, say 1.8 kHz.*

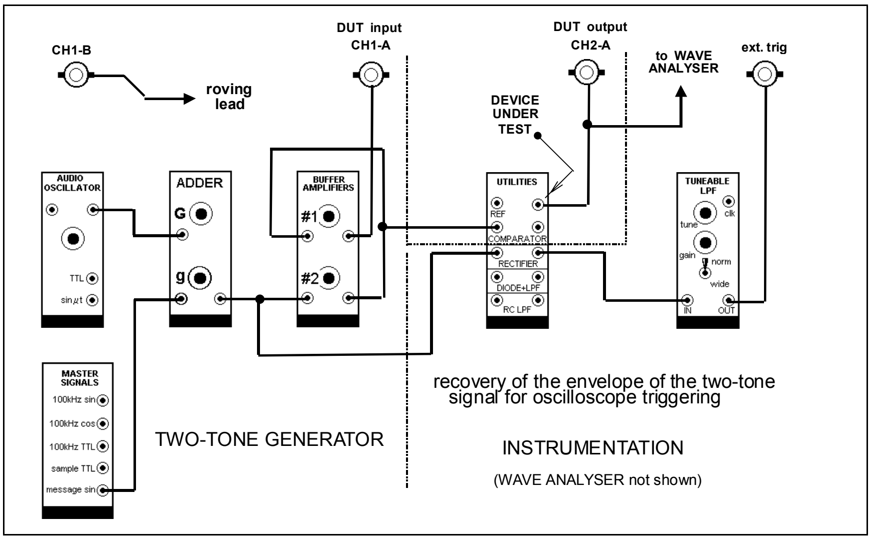


Figure 8: two-tone test setup

The two-tone signal necessitates new oscilloscope triggering arrangements. As already explained, an envelope recovery circuit is needed, and this is shown modelled with a RECTIFIER in the UTILITIES module, and a TUNEABLE LPF.

Three of the four SCOPE SELECTOR positions are shown permanently connected. The fourth, CH1-B, can be used as a roving lead, for various waveform inspections, including the next task.

T15 *adjust the two tones at the ADDER output to equal amplitudes (say 2 volt peak-to-peak each). Adjust the gain of BUFFER #2 to about unity.*

T16 *check the envelope detector output, using CH1-B. What is wanted is a periodic signal at envelope frequency, suitable for oscilloscope triggering. Its shape is not critical. Tune the filter to its lowest bandwidth; set the front panel passband* GAIN *control to its mid range.*

T17 *when satisfied with the previous Task, use the output of the envelope detector to trigger the oscilloscope, and check on CH1-A (with BUFFER #1 set to mid-gain) that the envelope of the two-tone signal is stationary on the screen (showing one or two periods of the envelope). Note that this display will show up any imperfection in the equality of the amplitudes of the two tones (or could have been used to set them equal in the first place).*

T18 *switch to CH1-A and CH2-A. Set both these oscilloscope channels to the same gain. You are now observing the input and output of the DUT. Adjust the gain of BUFFER #2 so that the output (CH2-A) is not distorted - that is, the same waveform as CH1-A.*

T19 *use the oscilloscope shift controls to overlay the two waveforms. Adjust the gain of BUFFER #1 until they are of exactly the same amplitude, and so appear as a single trace on the screen.*

T20 *now slowly increase the gain of BUFFER #2. Both traces will get larger,* ***but remain overlaid****, until the signal level into the DUT exceeds its linear operating range, and the output begins to show distortion. The two waveforms will no longer be identical, and the difference should be clearly visible.*

T21 *familiarize yourself with the overloading process by covering the full gain range of BUFFER #2. Then set the gain to the point where distortion of the DUT output waveform is 'moderate'* [5]*. A ratio of output to input signal amplitudes of about 5:6 is suggested as a start. This is the condition under which you will be making some distortion measurements. Record the input amplitude to the DUT; sketch the input and output waveforms.*

Remember the DUT has a cubic non-linearity. This suggests it will generate odd order harmonic and intermodulation products. If the two test signals are f_1 and f_2, then the largest unwanted components are likely to be on frequencies $3f_1$, $3f_2$, $(2f_1 \pm f_2)$ and $(2f_2 \pm f_1)$.

T22 *use the WAVE ANALYSER to search for, and record, the presence of all significant components in the output of the DUT for the conditions of the previous Task. Record clearly the frequency of each component found, and relate it to the frequencies of the two-tone signal.*

T23 *from your measurements of the previous Task calculate the SDR, using eqn. (18).*

[5] if you make it too slight the distortion components will be hard to find !

In the above calculation you might have included all components which you measured.

But suppose the output signal DUT had then been transmitted via a channel with a bandpass characteristic. Then many of the distortion components would have been removed. But distortion would still have been measured, since those intermodulation products close to the two wanted tones would have been passed.

T24 *re-calculate the SDR, assuming transmission was via a bandpass filter.*

conclusions

During the experiment you might have taken the opportunity to listen to the signals with and without distortion, to gain a qualitative idea and appreciation of what level of distortion - with these types of signals - is detectable by ear.

This experiment has served as an introduction to the methods of measuring and describing the non-linear performance of an analog circuit. A problem associated with the measurement of a narrowband system has been demonstrated, together with a method of overcoming it.

TUTORIAL QUESTIONS

Q1 *why were the two tones, in the two-tone test signal, set relatively close in frequency to each other ?*

Q2 *equation (19) suggests an alternative method of making a two-tone test signal. It has the particular advantage of providing a synchronizing signal (from the low frequency source), the two tones are automatically of equal amplitude, and the whole signal can be swept across the spectrum with one control - that of the high frequency oscillator. What are some disadvantages of this method of generation ?*

Q3 *explain qualitatively how the display of the two added tones, as a DSBSC signal, can be used to equalize the amplitudes of the two tones. Use a phasor diagram or other method to explain the process quantitatively.*

Q4 *two businesses advertise the same amplifier, one saying it is a 50 watt amplifier, and the other a 60 watt amplifier. There is no dishonesty. How could this be ?*

APPENDIX

some useful expansions.

In analysing a non-linear system in terms of sinusoidal signals as in the above work, the aim is to convert expressions in terms of *powers* of sinusoidal signals to expressions in terms of *harmonics* of the fundamental frequencies involved.

Some useful expansions are:

$\cos^2 A = 1/2 + 1/2.\cos 2A$

$\cos^3 A = 3/4.\cos A + 1/4.\cos 3A$

$\cos^4 A = 3/8 + 1/2.\cos 2A + 1/8.\cos 4A$

$\cos^5 A = 5/8.\cos A + 5/16.\cos 3A + 1/16.\cos 5A$

$\cos^6 A = 5/16 + 15/32.\cos 2A + 3/16.\cos 4A + 1/32.\cos 6A$

Perhaps you can see the pattern developing ? It is clear that:

- when n is *odd*, the expansion of $(\cos mt)^n$ gives rise to all odd harmonics counting down from the n^{th}.
- when n is *even*, the expansion of $(\cos \mu t)^n$ gives rise to all even harmonics counting down from the n^{th}. Note that the count goes down to the zeroeth term, which is DC.

After an expression has been reduced to the sum of harmonic terms, those of similar frequency must be combined, taking into account their relative phases. Thus:

$$V_1.\cos\mu_1 t + V_2.\cos\mu_1 t = (V_1 + V_2).\cos\mu_1 t$$

but

$$V_1.\cos\mu_1 t + V_2.\sin\mu_1 t = V.\cos(\mu_1 t + \alpha)$$

where

$$V = \sqrt{(V_1^2 + V_2^2)}$$

and

$$\alpha = \tan^{-1}(\frac{V_2}{V_1})$$

As an exercise, develop the above expansions in terms of sin functions. There must be some obvious similarities, but just as importantly there must be differences. Explain !

Further useful expansions may be found in Appendix B to this Text.

FREQUENCY DIVISION MULTIPLEX

FREQUENCY DIVISION MULTIPLEX

ACHIEVEMENTS: *principle of FDM; appreciation that more than one channel can be recorded on a single track of a domestic tape recorder; demodulation of FDM.*

PREREQUISITES: *completion of experiments on DSBSC and SSB demodulation eg,* ***Product demodulation - synchronous and asynchronous****.*

PREPARATION

The *principle* of frequency division multiplexing - FDM - should be familiar to everyone, since every domestic radio or TV receiver is a frequency de-multiplexer.

You are aware that there are many local radio stations operating without mutual interference. They occupy their own allocated frequency channels without overlap. They have been frequency multiplexed into the radio spectrum.

A de-multiplexer is merely a device which can select one station (channel) from all others. It is a frequency selective device.

the application

Although the above description was indeed of FDM, it is not a typical example of an FDM system. The principle is generally applied when it is required to transmit, simultaneously, more than one message via a particular communications channel, such as a transmission line (say twisted pair or coaxial), and all channels emanate from the same location. Such a medium is thought of as being basically a single channel, but FDM techniques enable it to carry multiple channels simultaneously.

The motivation comes from the observation that a particular transmission medium, say twisted pair, or coaxial cable, has a bandwidth much wider than that of a single voice channel, and so it seems wasteful to use it for one voice channel only.

multi-channel tape recorder

In this experiment the principle of FDM has been used to place several channels on a single track of an audio tape recorder.

The bandwidth of a good quality domestic audio tape recorder is at least 16 kHz, and so is adequate for the purpose of accepting four 3 kHz audio channels side by side, with spaces between to act as 'guard bands'.

Two schemes are suggested, namely scheme 1 and scheme 2.

scheme 1

A possible spectrum is shown in Figure 1.

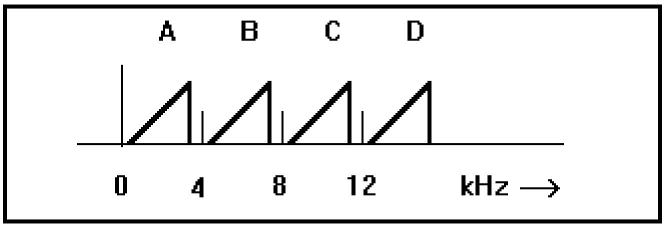


Figure 1: FDM scheme 1

There are four channels in the FDM spectrum of Figure 1, each based on independent messages of 3 kHz bandwidth.

Channel A is located in its normal position in the frequency spectrum.

Channels B, C and D have been translated in frequency by individual USSB generators, based on carriers of 4, 8, and 12 kHz respectively.

To recover channel A, a 3 Hz lowpass filter would be sufficient. For the other three channels, SSB receivers would be required. A de-multiplexing scheme is illustrated in Figure 2.

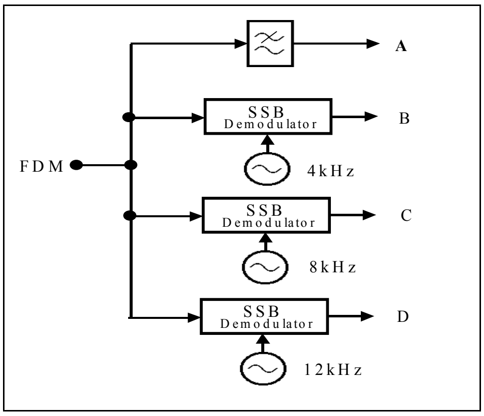


Figure 2: de-multiplexer for scheme 1

Notice that the *three* carrier sources might, in practice, be derived, by division, on a master clock of, say, 24 kHz. This is typical of FDM systems.

If only one channel is required at a time, then only one SSB demodulator need be supplied.

scheme 2

A less ambitious scheme, but which none-the-less illustrates the principle, is shown in Figure 3.

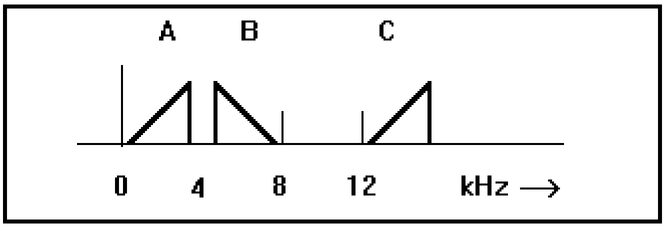


Figure 3: FDM scheme 2

This system has only three channels, but requires a less complex de-multiplexer. Instead of SSB demodulators, DSBSC demodulators are adequate.

A suitable de-multiplexer is illustrated in Figure 4.

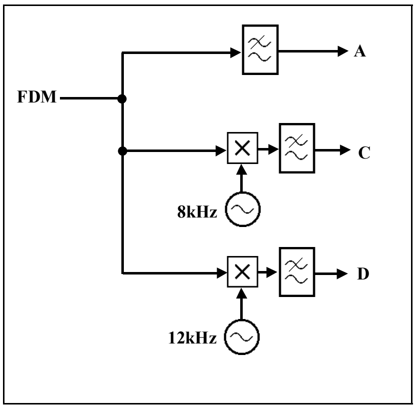


Figure 4: demodulation scheme for Figure 3

Unless announced otherwise, it will be assumed below that Scheme 2 is in operation.

avoid crosstalk

During the experiment it is important to avoid crosstalk.

In an analog system it is essential to operate in a linear manner to avoid the generation of harmonic and intermodulation distortion. This is especially important in a multi-channel system if interference between channels - *crosstalk* - is to be avoided.

If the crosstalk was introduced at the transmitter there is nothing you can do about it at the receiver [1]. It is your responsibility to avoid circuit overload at the receiver.

If at any time you observe intrusion of the message from another channel into the channel to which you are tuned, check that it is not your circuitry overloading.

[1] do you agree ?

The most likely place for intermodulation to occur is in the MULTIPLIER. Check this by reducing the amplitude of the FDM from TRUNKS. This can be done by using a BUFFER AMPLIFIER.

It is the *ratio* of unwanted-to-wanted signal power that you are trying to reduce.

> *In a typical overloaded system it can be shown that a 6 dB reduction of signal level should result in a reduction of the unwanted signal power by* ***more*** *than 6 dB*

This phenomenon is demonstrated in the experiment entitled *Amplifier overload* in this Volume.

EXPERIMENT

An FDM signal is available at your TRUNKS panel.

You will note that the demodulators of Figure 4 are not true SSB demodulators. This is not necessary, since, although each channel is in fact an SSB signal, there is no signal on the opposite sideband. Secondly, not being DSBSC there is no need to include a phase changer in the carrier paths.

The two carrier signals can be obtained from an AUDIO OSCILLATOR and VCO modules. Note that, in both schemes, the receiver for channel A is *in principle* the same as that for the other channels, with a local oscillator on *zero kHz*. This can be simulated by replacing the oscillator signal with a 2 volt DC signal from the VARIABLE DC module. Alternatively, for channel A, dispense with the MULTIPLIER altogether, and use the LOWPASS FILTER only (as in Figures 2 and 4).

Since you have only one HEADPHONE AMPLIFIER you will only be able to listen to one channel at a time, by connecting the input of the HEADPHONE AMPLIFIER to the appropriate MULTIPLIER output (for channel B and C), or the FDM itself (for channel A).

T1 *patch up the de-multiplexer scheme of Figure 4. For the two carriers use an AUDIO OSCILLATOR and a VCO. You will have to share the HEADPHONE AMPLIFIER (with its 3 kHz LPF) between channels.*

T2 *locate the FDM signal at the TRUNKS panel.*

T3 *initially set the peak amplitude of the FDM signal into your demultiplexer at the TIMS ANALOG REFERENCE LEVEL.*

T4 *show that it is possible to separate the individual channels with your demultiplexer. Record their carrier frequencies (nominally 0, 8, and 12 kHz in the scheme of Figure 3, but the signal at TRUNKS could differ from this). Remember that, if there is significant crosstalk, check the effect of a reduction of the amplitude of the FDM signal from TRUNKS by using a BUFFER AMPLIFIER.*

T5 *make sure you investigate both methods of recovering channel A. That is, without the MULTIPLIER, or with the MULTIPLIER and a zero frequency local oscillator signal.*

T6 *from your results draw the spectrum, in the format of Figure 3, of the FDM signal you have been sent.*

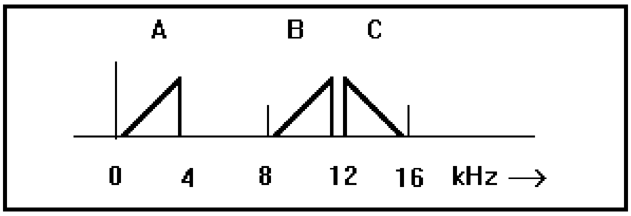
T7 *introduce crosstalk. This can be done by connecting, if not already done so, a BUFFER AMPLIFIER between TRUNKS and the input to your demultiplexer. Initially set the gain of the BUFFER AMPLIFIER to unity, and re-confirm that crosstalk is low (or non-existent). Then increase the BUFFER AMPLIFIER gain, by say a factor of two, and check the crosstalk. This crosstalk 'measurement' can only be an estimate whilst all channels are carrying speech. You should make some comments on how this estimate was made and recorded.*

Record the peak amplitude of the FDM signal into your demultiplexer when it is set just below the onset of crosstalk.

TUTORIAL QUESTIONS

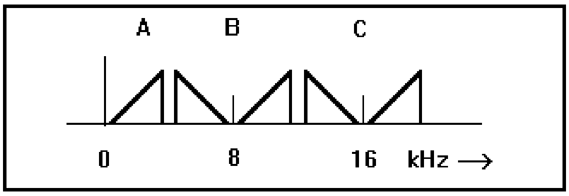
Q1 *draw a block diagram and describe its use to make a tape recording of the FDM signal you have been sent.*

Q2 *assuming you have a DSBSC demodulator only (not an SSB demodulator) could you separate the channels of the scheme illustrated below ?*



Q3 *you have shown that a true SSB demodulator is not necessary in order to recover each channel from the scheme of Figure 3. In principle, however, there is an advantage in using a true SSB demodulator. Explain.*

Q4 *to avoid the need for SSB generators to make the FDM signals, an alternative scheme could generate the spectrum shown below. Could a DSBSC demodulator be used in principle to recover the three channels independently ? Can you suggest a possible practical problem(s) with this scheme ?*



Q5 *discuss the meaning and significance of the term 'guard band' in the present context.*

Q6 *in the last Task, was the peak amplitude of the FDM signal above or below the TIMS ANALOG REFERENCE LEVEL when the crosstalk became unacceptable (this will have to be a qualitative judgement). Comment.*

PHASE DIVISION MULTIPLEX

PHASE DIVISION MULTIPLEX

ACHIEVEMENTS: *phase division multiplex (PDM) demodulation; significance of the exact quadrature condition; cross-talk estimation*

PREREQUISITES: *DSBSC generation; synchronous demodulation*

PREPARATION

Phase division multiplex [1], PDM, is a modulation technique which allows two DSBSC channels, sharing a common, suppressed carrier, to occupy the same spectrum space. It is possible to separate the channels, upon reception, by phase discrimination. Apart from communications applications, especially in digital communications, the technique is also used for colour difference signals in some TV systems.

the transmitter

Figure 1 shows a block diagram of the arrangements at the transmitter.

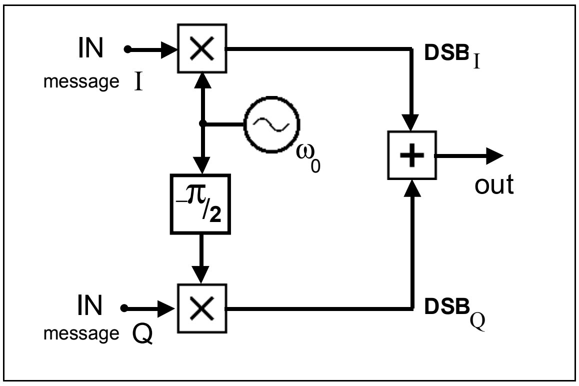


Figure 1: The PDM Generator

[1] also known as *quadrature phase division multiplexing*, or *quadrature-carrier multiplexing*, or *quadrature amplitude modulation (QAM),* or *orthogonal multiplexing*

There are two message channels, I (in-phase) and Q (quadrature) and these are converted to DSBSC signals - DSB_I and DSB_Q. The messages should be bandlimited (not shown) to the same bandwidth, say 3 kHz if they are speech. Each DSBSC will therefore occupy a 6 kHz bandwidth. The two DSBSC signals are *added* together. They will *overlap* in frequency, since they share a common carrier of ω rad/s. So the bandwidth of the PDM will also be 6 kHz.

The key to the system lies in the fact that there is a 90 degree - *quadrature* - phase difference between the carriers supplied to the two DSBSC generators.

the receiver

Consider a single DSBSC demodulator as studied in an earlier experiment. It was learned there that, when receiving a DSBSC signal, it was possible to adjust the phasing of the local carrier such that the received message amplitude was reduced to zero.

Suppose now a second DSBSC was added at the transmitter, as has been done in Figure 1. Since both the transmitter and receiver are operating in a linear manner, this should make no difference, at the receiver, to the null, already achieved (of channel I, say). Consequently, if the second DSBSC, channel Q, is of a different relative phase, it will NOT be nulled, and will appear at the demodulator output.

To listen to the message from channel I, it is merely a matter of changing the receiver phasing to null channel Q.

In principle the two channels at the transmitter need not be in exact phase quadrature. So long as there is a finite phase difference, no matter how small, one of the channels at the receiver can be nulled, leaving the other.

The disadvantage of a phase difference other than 90 degrees is that this results in a degradation of signal to noise ratio, as observed at the demodulator output. Whereas the output noise level is *not* sensitive to the phase of the local carrier, the amplitude of the recovered message is. You can show that, for a null of one channel, the output amplitude from the other is a maximum when the two channels are in phase quadrature. However, this maximum is fairly broad. An error of 45 degrees from quadrature at the transmitter will result in a 3 dB degradation from the maximum possible amplitude at the receiver.

What is important is not so much the *accuracy* of the channel phase difference, but its *stability*. It is also assumed that, what ever the phase difference at the transmitter may be, the receiver will be adjusted appropriately.

In practice, to simplify carrier acquisition by the receiver, a small amount of 'pilot' carrier, typically about 20 dB below the peak DSB level, may be inserted at the transmitter.

EXPERIMENT

At one of the TRUNKS outputs there is PDM signal. It carries two channels, which will be arbitrarily named as 'I' and 'Q'. They carry independent messages, one speech (I), the other a single tone (Q).

Locate the PDM signal with your oscilloscope.

With two independent messages, as there are, there is no 'text book' type of stationary display which can be reproduced on your oscilloscope. However, knowing the message on one channel is a single tone, the PDM will take on the appearance of a text book [2] DSBSC during speech pauses on the other channel. The envelope of this DSBSC will not remain stationary, but it may remain so for periods long enough to verify this statement.

A two-channel demodulator, capable of selecting channels from this PDM signal, is illustrated in Figure 2.

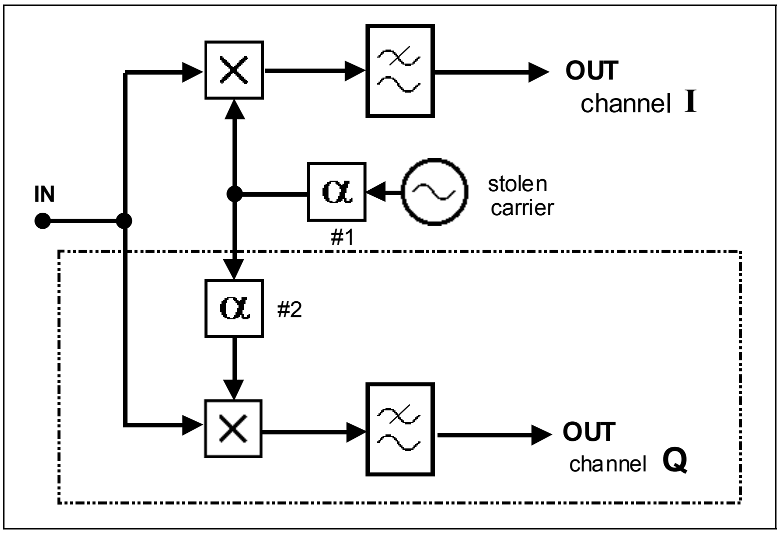


Figure 2: a demodulator for PDM.

a single-channel receiver

You may decide to omit those modules in the dotted box required for channel Q. In this case you will be able to receive either channel, but only one at a time. To do this:

T1 *patch up a TIMS model of the block diagram of Figure 2, omitting that part in the dotted box. Steal the carrier from the MASTER SIGNALS module. Use the 3 kHz LPF in the HEADPHONE AMPLIFIER.*

T2 *use the oscilloscope to select the PDM signal from TRUNKS.*

[2] a DSBSC derived from a single tone

T3 *connect the oscilloscope to monitor the output from channel I of the demodulator. Switch the 'trig' to 'channel A', in auto mode (if available). Start with a sweep speed of say 1 ms/cm. There is no appropriate signal for oscilloscope synchronization.*

The oscilloscope will now be displaying the output of Channel I of the demodulator. This will most likely show contributions from *both* transmitter channels. You can listen with HEADPHONES, as well as view on the oscilloscope. The display will not be stationary during the nulling process, but when the tone channel is isolated you can synchronize the oscilloscope to it.

The nulling procedure is best performed by concentrating the ear, if using headphones, or the eye, if using the oscilloscope, on the message from the *unwanted* channel. It is up to you to decide which this unwanted channel should be, but you may find it easier to null the channel carrying the tone rather than the speech.

T4 *adjust the phase (front panel control on the PHASE SHIFTER #1) until the unwanted channel is nulled out.*

If both channels are carrying speech the visual method of nulling would be very difficult. When the channels are carrying quite different types of messages, as they are here, it is less difficult. Could you automate either of these procedures ?

a two-channel receiver

To model a receiver which is capable of demodulating both transmitter channels simultaneously it is necessary to add the modules within the dotted box of Figure 2.

T5 *add the modules for channel Q recovery.*

T6 *listen to the output of Channel I of your receiver, and null out one channel from the transmitter with PHASE SHIFTER #1*

T7 *listen to the output of Channel Q of your receiver, and null out the other channel from the transmitter with PHASE SHIFTER #2*

TUTORIAL QUESTIONS

Q1 *obtain an expression for the degradation of SNR at the receiver, as a result of a phase difference from the ideal 90 at the transmitter, and confirm that it is 3 dB for a 45 degree error. It is assumed that the receiver has been (re)adjusted to accommodate what ever phasing is in use at the transmitter*

Q2 *since it is not necessary that the two DSBSC of a two-channel PDM signal be phased exactly 90 degrees apart, why not use three channels, and put them at 60 degrees apart ? or four channels at 45 degrees apart ? Discuss the possibilities of such a system.*

Q3 *if there is a small phase error α in a PDM receiver a listener to one channel will hear a low-level copy of the message on the other channel; this is called 'cross-talk'. Obtain an expression for the level of cross-talk as a function of phase error.*

Q4 *there are two PHASE SHIFTER modules in Figure 2. An alternative connection would be to place PHASE SHIFTER #1 between the carrier source and the top MULTIPLIER, and PHASE SHIFTER #2 between the carrier source and the lower MULTIPLIER. Can you see any advantages in this ?*

ANALYSIS OF THE FM SPECTRUM

ANALYSIS OF THE FM SPECTRUM

introduction

To understand the next few experiments it is necessary to have a basic understanding of the nature of phase modulated (PM) and frequency modulated (FM) signals. These notes define the *angle modulated* signal, of which PM and FM are special cases.

FM, under well defined conditions, offers certain features, including a method of trading bandwidth for signal-to-noise ratio. These notes will not discuss signal-to-noise properties, but instead will concentrate on an analysis of the spectral properties of the signal.

definition of modulation

Consider the signal:

$$y(t) = E.\cos(\omega t + \phi) \qquad \text{........ 1}$$

This signal possesses, by definition:

- an amplitude 'E'
- a total phase $(\omega t + \phi)$
- an instantaneous frequency defined as the time rate of change of total phase.

Any one of these three parameters may be modulated by a message $A.\cos\mu t$. Which ever parameter is chosen, then, by definition:

- the *rate* of variation of the chosen parameter should be directly proportional to the rate of variation of the message alone (μ)
- the *amount* of variation of the chosen parameter should be directly proportional to the amplitude of the message alone (A)

Other parameters may vary at the same time, as will be seen in what follows, but these variations will not be strictly in accordance with the above definitions.

phase modulation (PM) - definition

According to the above requirements a signal will be phase modulated, by the message A.cosμt, if:

$$\text{total phase} = \omega t + k_1.\cos\mu t \qquad \text{........ 2}$$

and provided k_1 is linearly proportional to A, the message amplitude.

Hence:

$$PM = E\cos(\omega t + k_1 \cos\mu t) \qquad \text{........ 3}$$

is a phase modulated signal.

Note that, for PM:

$$\text{instantaneous frequency} = \omega - k_1.\mu.\cos\mu t \qquad \text{........ 4}$$

Although the frequency is also varying with the message, the variation is not directly proportional to the message amplitude alone. Hence, by definition, this is not frequency modulation.

frequency modulation (FM) definition

According to the above requirements a signal will be frequency modulated, by the message A.cosμt, if:

$$\text{instantaneous frequency} = \omega + k_2.\cos\mu t \qquad \text{........ 5}$$

and provided k_2 is linearly proportional to A, the message amplitude.

The total phase is obtained by integration of the instantaneous frequency, and thus the signal itself must be:

$$FM = E.\cos(\omega t + \frac{k_2}{\mu}\sin\mu t) \qquad \text{........ 6}$$

Although the phase is also varying with the message, the variation is not directly proportional to the message amplitude alone. Hence, by definition, this is not phase modulation.

angle modulation - general form

The defining equation, for both PM and FM, can be written in the form:

$$y(t) = E.\cos(\omega t + \beta.\sin\mu t) \qquad \text{........ 7}$$

One can choose β to represent either PM or FM as the case may be, and according to the definitions above. Thus:

for PM $\beta = \Delta\phi$, the peak phase deviation 8

and for FM $\beta = \Delta\phi / \mu$ 9

The parameter β is often called the *deviation*.

Both PM and FM fall into a class known as *angle modulated* signals.

receivers

There are demodulators for these signals.

The demodulator in a PM receiver responds in a linear manner to the variations in phase of the PM signal, and the receiver output is a copy of the original message. Likewise the demodulator in an FM receiver responds in a linear manner to the instantaneous frequency variations of the FM signal.

There is an output from a PM receiver if the input is an FM signal, and from an FM receiver if the input is a PM signal. But these outputs will not be related to the message in a linear manner.

In the experiment entitled *Envelopes* (in volume A1) a general expression is defined for a modulated signal, namely:

$$y(t) = a(t).\cos[\omega t + \phi(t)] \qquad \text{........ } 10$$

where a(t) and $\phi(t)$ were defined as involving components at or near the message frequency only.

The envelope was defined as being |a(t)|.

Phase variations are described by $\phi(t)$.

> *By definition, the output from both a PM and an FM receiver will be $\phi(t)$, provided a(t) is a constant.*

spectrum analysis

The spectrum of the angle modulated signal y(t) of eqn.(7) above can be obtained by trigonometrical expansion.

This is an interesting exercise, and you should do it yourself, since it is instructive to see where the various terms in the expansion come from.

Firstly you will need to know that:

$$\cos(\beta.\sin\phi) = J_0(\beta) + 2\ [\ J_2(\beta).\cos 2\phi + J_4(\beta).\cos 4\phi + ...] \qquad \text{........ } 11$$

$$\sin(\beta.\sin\phi) = 2\ [\ J_1(\beta).\sin\phi + J_3(\beta).\sin 3\phi + J_5(\beta).\sin 5\phi + ...] \qquad \text{........ } 12$$

$$\cos(\beta.\cos\phi) = J_0(\beta) - 2\ [\ J_2(\beta).\cos 2\phi - J_4(\beta).\cos 4\phi + ...] \qquad \text{........ } 13$$

$$\sin(\beta.\cos\phi) = 2\ [\ J_1(\beta).\cos\phi - J_3(\beta).\cos 3\phi + J_5(\beta).\cos 5\phi - ....] \qquad \text{........ } 14$$

Here $J_n(\beta)$ is a Bessel function of the first kind, argument β, and order n.

You will also need to know that:

$$J_{-n}(\beta) = (-1)^n J_n(\beta) \qquad \text{........ } 15$$

Using the above formulae, y(t) of eqn.(7) can be expanded into two infinite series. These can be combined, and then condensed, into the compact form:

$$y(t) = E.\sum_{n=-\infty}^{n=\infty} J_n(\beta).\cos(\omega_0 + n\mu)t \qquad \text{........ 16}$$

Notice that the choice of +cos() and +sin() in the defining equation (7) leads to +cos() in the compact version of eqn.(16). A different combination of cos and sin in the defining equation will produce a different *looking* compact version. Whilst the frequency and amplitudes in each must be the same (why ?) the phases will inevitably be different. It is difficult and unnecessary to memorise the exact form of any of these expansions of the defining equation.

spectral properties

Despite the apparent complexity of eqn.(16), which describes the spectrum of the signal, it is a surprisingly easy matter to make *and remember* many general observations about the nature of this spectrum.

What you should commit to memory is the general trends of the Bessel functions, which are illustrated in Figure 1 below.

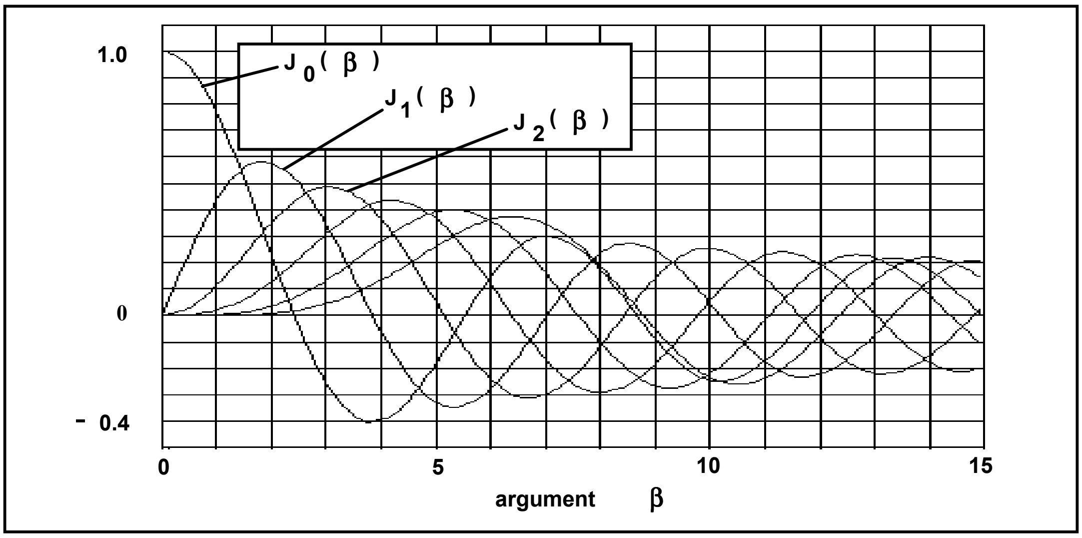


Figure 1: Plots of Bessel Functions

Notice that, as a function of β, all the curves are damped oscillatory. Except for $J_0(\beta)$, they start from zero, rise to a positive maximum, then oscillate about zero with ever decreasing amplitudes (damped). The exception, $J_0(\beta)$, starts from an amplitude of unity. Their zero crossings are *not* uniform, so the functions are not periodic. If you remember these general trends you will be able to make many qualitative observations about the behaviour of the FM spectrum.

By a careful examination of eqn.(16), and the plots of the Bessel functions in Figure 1, the following properties can be deduced. It is useful to retain an understanding of them all:

- spectral components are numbered left and right (±) counting from the central component at ω (number n = 0).
- the spectral lines are spaced μ/(2π) Hz apart
- amplitude of the ±nth component from the centre is $E.J_n(\beta)$. Because of eqn.(15) these two components are of equal amplitudes. Thus the spectrum is symmetrical about the central component at ω.
- the bandwidth is mathematically infinite, but in engineering terms the signal is considered confined within limits which contain all 'significant components'.
- you should have an understanding of the term 'significant sidebands' (or spectral components). This is discussed below.
- as β increases, the bandwidth, however defined, increases
- as β increases, individual spectral lines do *not* increase in amplitude monotonically. Their amplitudes are determined by $J_n(\beta)$, plots of which appear in Figure 1.
- for particular values of β the amplitude of particular sidefrequency pairs becomes zero (these are the 'Bessel zeros').
- the total power in the spectrum is constant, and independent of β. This can be deduced in more than one way. It is easy if you know that:

$$\sum_{n=-\infty}^{n=\infty} J_n^2(\beta) = 1$$

- the largest ever component is the one at ω rad/s (often called the 'carrier'), for the special case when β = 0
- sidefrequency pairs are alternately in phase-quadrature, and in-phase, with the term at ω (perhaps this is less obvious)

bandwidth

As engineers we should always know the bandwidth of the signals with which we are dealing. In many cases bandwidth estimation is not difficult. But for an angle modulated signal there is no single, closed form formula.

Mathematically the bandwidth of the angle modulated signal of eqn.(7) is infinite, since the amplitude of the component nμ rad/s from the central component is $E.J_n(\beta)$, and this only approaches zero as n approaches infinity.

significant sideband criterion

Fortunately, as engineers, practical concerns enable us to overcome this apparent problem.

Firstly, we only require a component to be relatively small for us to be able to ignore it. This requires some sort of reference. For this we use the amplitude of the unmodulated carrier, which is $E.J_0(0)$ (which is equal to E, since $J_0(0) = 1$). Insignificance is then defined as some fraction of the reference. Thus we have the '1% significant sideband criterion', which declares a component insignificant if it is 1% or less of the reference.

Secondly, if we can define a minimum width window in the frequency spectrum outside of which there are no components of significance, then this window will define its bandwidth. There is no simple formula for evaluating the width of this window, although there are several well known 'rules of thumb'.

example

Let us examine the spectrum of the angle modulated signal, defined by eqn.(7), with:

$$\beta = 5,$$

and

$$\mu / (2\pi) = 3 \text{ kHz}$$

Reference to the Bessel tables will give the *relative* amplitudes of the spectral components. It is usual to draw the *normalized* amplitude spectrum. This is for the case E = 1.

Go to the column in the tables for $\beta = 5$. Using the 1% significant sideband criterion search *upwards* from the *bottom* [1] of the column until the first entry is found whose magnitude exceeds 0.01 (1% of 1). The order of the Bessel function is given in the left-most column; this is 'n', and this gives the number of significant sidebands *either side* of the carrier. The Tables give n = 8. Thus the bandwidth is 2x8x3 = 48 kHz.

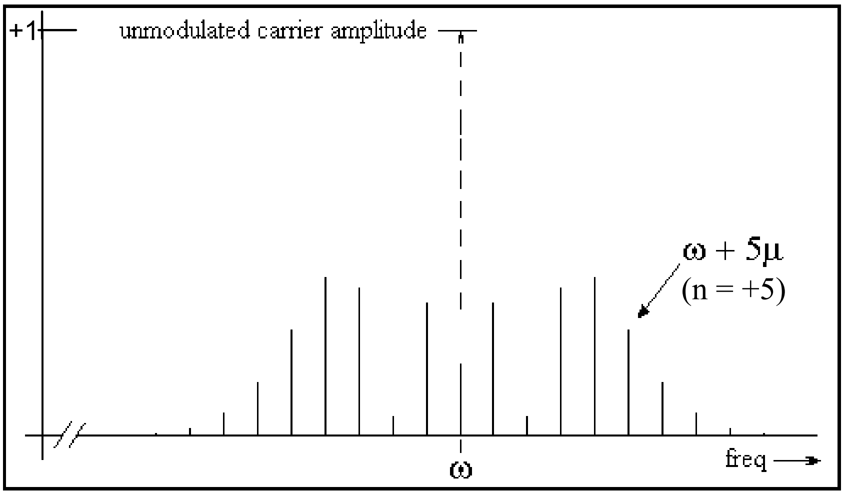


Figure 2: Amplitude Spectrum for β = 5

bandwidth on a power basis

Another method of defining the bandwidth is on a power basis - it is that window which contains x% of the total power in the signal. The power of successive components is added, *counting outwards* from the carrier, until the required proportion is accumulated. The reference is the power in the *unmodulated* carrier.

The two methods converge to the same estimate as the window widens. Depending on instrumentation available, or mathematical techniques being used, one may be more convenient than the other.

[1] you have to make an engineering judgement that there are no entries in the column *below* this point !

bandwidth restriction

If an FM signal is passed through a bandpass filter of insufficient bandwidth to pass all sidebands of significance, then the output from an ideal receiver will be distorted. The search for an exact analysis of this distortion has occupied mathematicians since well before the days of Armstrong and the introduction of FM commercially.

It is common practice to refer to tables of measured results for particular situations.

amplitude limiting

Amplitude limiters are used extensively in angle modulated systems.

It is an easy matter to describe the function of an amplitude limiter:

an amplitude limiter removes variations in the envelope of a signal.

The input in the present context is a narrow band modulated signal defined by

$$y(t) = a(t).\cos[\omega t + \phi(t)] \qquad \text{........ 17}$$

where both a(t) and $\phi(t)$ contain components at or near the message frequency only. Some properties of this signal were discussed in the experiment entitled *Envelopes*.

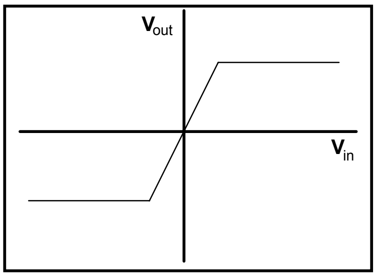


Figure 3: limiter characteristic

An amplitude limiter can be imagined as an amplifier with a characteristic such as that of Figure 3.

Being a non-linear device it is not an easy matter to analyse its operation on a general signal. It will be sufficient for our purposes to declare that:

the amplitude limiter will convert a(t) of eqn.(10) to a constant.

This requires qualification of the nature of a(t). If y(t) is an amplitude modulated signal, the amplitude limiter can remove moderate amounts of amplitude modulation; but it would require infinite gain to remove the envelope of a 100% amplitude modulated signal. Practical considerations need to be taken into account when assessing the capabilities of the amplitude limiter in each application.

Further, it should be obvious that there will be groups of components (bands) located around the harmonics of ω.

These bands have useful properties. Thus the amplitude limiter is followed by a bandpass filter to select one of these wanted bands.

properties of the harmonic terms

The harmonic terms have properties which are useful in angle modulation systems. These can be examined by a study of the limiter characteristic. Its transfer function can be described in many ways, one being:

$$\text{output} = a_1.(e_{in}) + a_3.(e_{in})^3 + a_5.(e_{in})^5 + ... \qquad .... \; 18$$

where e_{in} is the input signal.

For the case:

$$e_{in} = E.\cos\varphi(t) \qquad .... \; 19$$

then eqn.(18) can be re-written as:

$$\text{output} = E_1.\cos\varphi(t) + E_3.\cos3\varphi(t) + E_5.\cos5\varphi(t) + ... \qquad .... \; 20$$

Mathematicians may object to the next claim without proof, but as an engineer you can check experimentally that, if:

$$\varphi(t) = [\omega t + \beta.\cos\mu t] \qquad .... \; 21$$

then substitution into eqn.(20) will lead to:

$$\begin{aligned}\text{output} = \; & E_1.\cos(\omega t + \beta.\cos\mu t) + E_3.\cos(3\omega t + 3\beta.\cos\mu t) \\ & + E_5.\cos(5\omega t + 5\beta.\cos\mu t) + \; ....\end{aligned} \qquad .... \; 22$$

Note carefully what has and has *not* happened:

- the limiter output has bands of spectral components centred on odd harmonics of the original carrier at ω rad/s
- each of these bands of components is an angle modulated signal
- the degree of angle modulation of the input signal was β. The degree of angle modulation of the n^{th} harmonic has been increased to $n\beta$.
- the rate of angle modulation of the input signal was μ rad/s. The rate of angle modulation in the harmonics is *still* μ rad/s, which is as wanted. There has been *no multiplication of the rate.*
- typically $E_{n+1} < E_n$.

frequency deviation multiplication

In the context of angle modulation the combination of *amplitude limiter* and *bandpass filter* is called a *frequency multiplier.*

Suppose there is an angle modulated signal, carrier ω rad/s, at the input to the amplitude limiter. If the output of the amplitude limiter is passed to the bandpass filter, centred on $n\omega$, then this output is an angle modulated signal with n times the frequency deviation of the input. The fact that this new signal is on a carrier frequency of n times the input frequency is of secondary importance.

The frequency multiplier would have been better called a *deviation multiplier*, since this is its application in the context of angle modulation.

You will meet the amplitude-limiter-plus-bandpass-filter combination in the experiment entitled *FM and deviation multiplication* (this Volume).

tutorial questions

Q1 *suppose you are given the amplitudes of three adjacent side frequencies in the spectrum of a PM signal. Knowing 'n', can you use the relationship:*

$$\beta = \frac{2.n.J_n(\beta)}{J_{n+1}(\beta) + J_{n-1}(\beta)}$$

to determine the value of β at the transmitter output ?

Q2 *the signal of eqn.(7) appears across a 50 ohm resistor. What power is dissipated in the resistor for the case E = 5 volts and $\beta = 5$?*

Q3 *calculate the number of sideband pairs required in the signal*

$$y(t) = E.\cos(\omega t + \beta.\cos\mu t)$$

so that it contains 95% of the power in the unmodulated carrier. Consider the cases:

$\beta = 1$, and
$\beta = 5$

Q4 *a 100 MHz FM signal appears across a 50 ohm load resistor. The amplitude spectrum, based on a 100 MHz carrier, is recorded as in the table:*

a. what was the message frequency ?

b. what was the power dissipated in the 50 ohm load ?

c. what would be the bandwidth on the 5% significant sideband criterion ?

d. what was the peak phase deviation of the transmitted signal ?

e. what was the peak frequency deviation of the transmitted signal ?

f. compare the peak frequency deviation and the signal bandwidth. Any comments ?

freq MHz	***amplitude volts***
100.0000	1.7461
100.0020	0.4101
100.0040	1.6330
100.0060	1.3111
100.0080	0.5480
100.0100	1.9157
100.0120	2.0944
100.0140	1.5509
100.0160	0.9004
100.0180	0.4362
100.0200	0.1826
100.0220	0.0676
100.0240	0.0225
100.0260	0.0068
100.0340	0.0000
100.0360	0.0000

Q5 *in the Tables of Bessel Coefficients you will see some entries are negative. Of what significance is this with relation to the spectrum of an FM signal ?*

Q6 *a phase modulated transmitter radiates a signal at 100 MHz, derived from a 2.5 kHz single tone message. The transmitter output peak phase deviation is $\Delta\phi$. Draw the amplitude spectrum for the cases:*

$\Delta\phi = 1$

$\Delta\phi = 5$

$\Delta\phi = 10$

Q7 *what is the bandwidth of each of the signals of the previous Question, based on the 2% significant sideband criterion ? Are these bandwidths in the ratio 1:5:10 ? Comment.*

Q8 *a frequency modulated transmitter radiates a signal at 100 MHz, derived from a 2.5 kHz single tone message. The transmitter output peak frequency deviation is $\Delta\phi$ = 10 kHz. What is the bandwidth of the FM signal on the 1% significant sideband criterion ?*

If the message frequency is changed to 10 kHz, what now is the bandwidth on the same criterion ?

Q9 *an FM signal with a peak frequency deviation of 20 kHz on a 160 MHz carrier is multiplied with a 100 MHz sinewave, and the products at 260 MHz selected with a bandpass filter. What is the frequency deviation of the 260 MHz signal ?*

INTRODUCTION TO FM USING A VCO

INTRODUCTION TO FM USING A VCO

ACHIEVEMENTS: *FM generation using a VCO. Confirmation of selected aspects of the FM spectrum. Demodulation using a zero crossing counter demodulator.*

PREREQUISITES: *familiarity with the contents of the chapter entitled* ***Analysis of the FM spectrum****; completion of the experiment entitled* ***Spectrum analysis - the WAVE ANALYSER****, both in this Volume.*

EXTRA MODULES: *SPECTRUM UTILITIES. A second VCO is required for spectral measurements.*

INTRODUCTION

This experiment has been written to satisfy those who are familiar with the existence of the ubiquitous VCO integrated circuit, and wish to explore it as a source of FM.

The VCO - *voltage controlled oscillator* - is available as a low-cost integrated circuit (IC), and its performance is remarkable. The VCO IC is generally based on a bi-stable 'flip-flop', or 'multi-vibrator' type of circuit. Thus its output waveform is a rectangular wave. However, ICs are available with this converted to a sinusoid. The mean frequency of these oscillators is determined by an RC circuit.

The *controllable* part of the VCO is its frequency, which may be varied about a mean by an external control voltage.

The variation of frequency is remarkably linear, with respect to the control voltage, over a large percentage range of the mean frequency. This then suggests that it would be ideal as an FM generator for communications purposes.

Unfortunately such is not the case.

The relative instability of the centre frequency of these VCOs renders them unacceptable for modern day communication purposes. The uncertainty of the centre frequency does not give rise to problems at the receiver, which may be taught to track the drifting carrier (see this Volume for the experiment entitled *Carrier acquisition and the PLL*, which introduces the phase locked loop - PLL). The problem is that spectrum regulatory authorities insist, and with good reason, that communication transmitters maintain their (mean) carrier frequencies within close limits [1].

It is possible to stabilise the frequency of an oscillator, relative to some fixed reference, with automatic frequency control circuitry. But in the case of a VCO which is being frequency modulated there is a conflict, with the result that the control circuitry is complex, and consequently expensive.

For applications where close frequency control is not mandatory, the VCO is used to good effect [2].

This experiment is an introduction to the FM signal. The wideband FM signal is very convenient for studying some of the properties of the FM spectrum.

spectrum

Examination of the spectrum will be carried out by modelling a WAVE ANALYSER. This instrumentation was introduced in the experiment entitled *Spectrum analysis - the WAVE ANALYSER* (in this Volume).

Specifically, two properties of an FM signal will be examined:

first Bessel zero

Check your Bessel tables and confirm that $J_0(\beta) = 0$ when $\beta = 2.45$

This Bessel coefficient controls the amplitude of the spectral component at carrier frequency, so with $\beta = 2.45$ there should be a carrier null.

You will be able to set $\beta = 2.45$ and so check this result.

special case - β = 1.45

For $\beta = 1.45$ the amplitude of the first pair of sidebands is equal to that of the carrier; and this will be $J_0(1.45)$ times the amplitude of the unmodulated carrier (always available as a reference).

You should confirm this result from Tables of Bessel functions (see, for example, the Appendix C to this volume).

This is one of the many special cases one can examine, to further verify the predictions of theory.

the zero-crossing-counter demodulator

A simple yet effective FM demodulator is one which takes a time average of the zero crossings of the FM signal. Figure 1 suggests the principle.

1 typically within a few parts per million (ppm) or less, or a few Hertz, which ever is the smaller.

2 suggest such an application

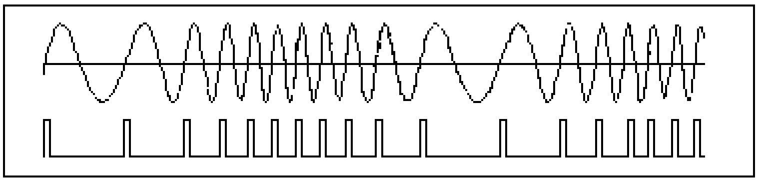


Figure 1: an FM signal, and a train of zero-crossing pulses

Each pulse in the pulse train is of *fixed width*, and is located at a zero crossing of the FM signal [3]. This is a pulse-repetition-rate modulated signal. If the pulse train is passed through a lowpass filter, the filter will perform an averaging operation. The rate of change of this average value is related to the message frequency, and the magnitude of the change to the depth of modulation at the generator.

This zero-crossing-counter demodulator [4] will be modelled in the latter part of the experiment. The phase locked loop (PLL) as a demodulator is studied in the experiment entitled *FM demodulation and the PLL* in this Volume.

EXPERIMENT

A suitable set-up for measuring some properties of a VCO is illustrated in Figure 2.

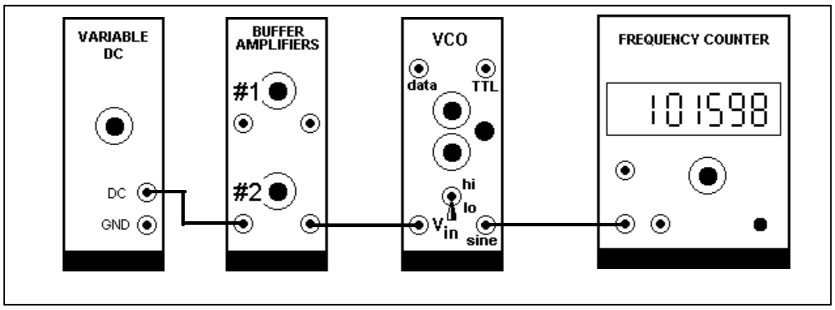


Figure 2: the FM generator

For this experiment you will need to measure the sensitivity of the frequency of the VCO to an external control voltage, so that the frequency deviation can be set as desired.

The mean frequency of the VCO is set with the front panel control labelled f_o.

The mean frequency can as well be varied by a DC control voltage connected to the V_{in} socket. Internally this control voltage can be amplified by an amount determined by the setting of the front panel GAIN control. Thus the frequency sensitivity to the external control voltage is determined by the GAIN setting of the VCO.

A convenient way to set the sensitivity (and thus the GAIN control, which is not calibrated), to a definable value, is described below.

[3] only positive going zero crossings have been selected

[4] also called the *zero crosssing detector*

T1 *before plugging in the VCO, set the mode of operation to 'VCO' with the on-board switch SW2. Set the front panel switch to 'LO'. Set the front panel* GAIN *control fully anti-clockwise.*

T2 *patch up the model of Figure 2.*

T3 *use the FREQUENCY COUNTER to monitor the VCO frequency. Use the front panel control* f_0 *to set the frequency to 10 kHz.*

deviation sensitivity

T4 *set the VARIABLE DC module output to* <u>*about*</u> *+2 volt. Connect this DC voltage, via* BUFFER *#2, to the* V_{in} *socket of the VCO.*

T5 *with the BUFFER #2 gain control, set the DC at the VCO* V_{in} *socket to* <u>*exactly*</u> *-1.0 volt. With the VCO* GAIN *set fully anti-clockwise, this will have no effect on* f_0*.*

T6 *increase the VCO* GAIN *control from zero until the frequency changes by 1 kHz. Note that the direction of change will depend upon the polarity of the DC voltage.*

> *The* GAIN *control of the VCO* ***is now set*** *to give a 1 kHz peak frequency deviation for a modulating signal at* V_{in} *of 1 volt peak.*

The gain control setting will now remain unchanged.

For this setting you have calibrated the sensitivity, ***S***, of the VCO for the purposes of the work to follow.

Here:

$$S = 1000 \text{ Hz/volt} \qquad \text{........ } 1$$

deviation linearity

The linearity of the modulation characteristic can be measured by continuing the above measurement over a range of input DC voltages. If a curve is plotted of DC volts versus frequency deviation the linear region can be easily identified.

A second, dynamic, method would be to use a demodulator, using an audio frequency message. This will be done later. In the meantime:

T7 take a range of readings of frequency versus DC voltage at V_{in} of the VCO, sufficient to reveal the onset of non-linearity of the characteristic. This is best done by producing a plot as the readings are taken.

the FM spectrum

So far you have a *theoretical* knowledge of the spectrum of the signal from the VCO, but have made no *measurement* to confirm this.

The instrumentation to be modelled is the WAVE ANALYSER, introduced in the experiment entitled *Spectrum analysis - the WAVE ANALYSER*. It is assumed you have completed that experiment.

the WAVE ANALYSER

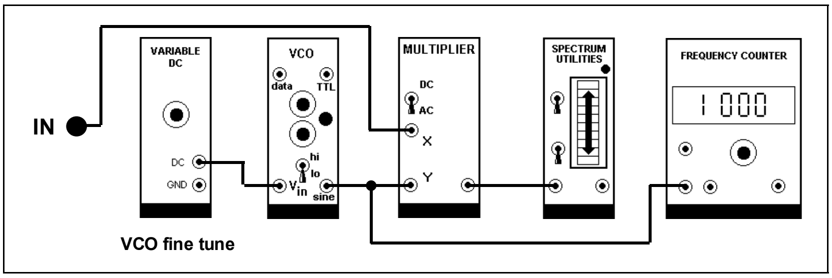


Figure 3: the WAVE ANALYSER model.

The VARIABLE DC voltage, with a moderate setting of the VCO GAIN control, is used as a fine tuning control.

Remember that one is generally not interested in absolute amplitudes - what is sought are *relative* amplitudes of spectral components.

Two such spectra will now be studied.

1. the first Bessel zero of the carrier term will be set up.
2. the amplitude of the carrier will be made equal to that of each of the first pair of sidebands.

The VCO will be set up with a sinusoidal message of 500 Hz.

T8 *in the FM generator model of Figure 2 replace the variable DC module with an AUDIO OSCILLATOR, tuned to 500 Hz.*

T9 *patch up the WAVE ANALYSER of Figure 3.*

T10 *revise your skills with the WAVE ANALYSER. Set the frequency deviation of the FM VCO to zero, with the BUFFER amplifier #2, and search for the carrier component. Set the on-board* SCALING *resistor of the SPECTRUM UTILITIES module to obtain a full scale deflection (FSD) on the carrier.*

first Bessel zero

Refer to the work, under this heading, which was done preparatory to starting the experiment.

Since

$\beta = \Delta f / f_m$

and

$\Delta f = V_{in}\, \boldsymbol{S}$

then set

$V_{in} = \beta . f_m / \boldsymbol{S} \cong 1.22$ peak volt

T11 *adjust, with the AUDIO OSCILLATOR supplying the message, via the BUFFER amplifier, for* $\beta = 2.45$

T12 *use the WAVE ANALYSER to confirm the amplitude of the carrier has fallen very low. Fine trim of the BUFFER amplifier gain control should find the null. Check that* β *is still close to 2.45.*

T13 *find one of the adjacent sidebands. Its amplitude should be* $J_1(\beta)$ *times the amplitude of the unmodulated carrier (measured previously).*

since $J_1(2.54) \cong 0.5$

then each of the first pair of sideband should be of amplitude half that of the unmodulated carrier.

special case - β = 1.45

Refer to the work, under this heading, which was done preparatory to starting the experiment.

T14 *set* $\beta = 1.45$ *and confirm that the carrier component, and either or both of the first pair of sideband, are of similar amplitude.*

T15 *check that each of the components just measured is* $J_1(1.45)$ *times the amplitude of the unmodulated carrier.*

There are many more special cases, of a similar nature, which you could investigate. The most obvious would be the Bessel zeros of some of the side-frequencies.

You will now use this generator to provide an input to a demodulator.

FM demodulation

A simple FM demodulator, if it reproduces the message without distortion, will provide further confirmation that the VCO output is indeed an FM signal.

A scheme for achieving this result was introduced earlier - the zero-crossing-counter demodulator - and is shown modelled in Figure 4.

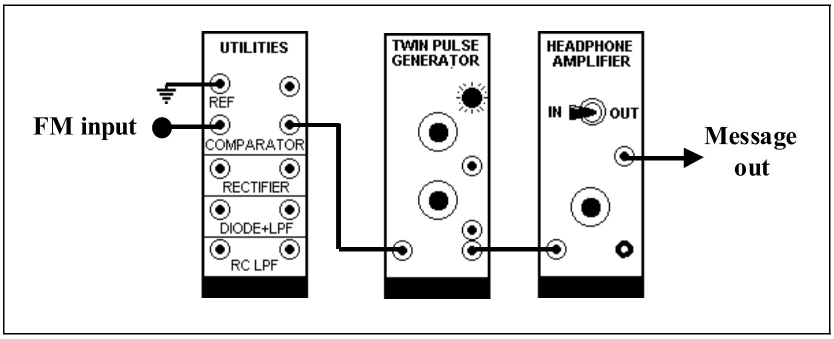


Figure 4: an FM demodulator using a zero-crossing demodulator

The TWIN PULSE GENERATOR is required to produce a pulse at each positive going zero crossing of the FM signal. To achieve this the FM signal is converted to a TTL signal by the COMPARATOR, and this drives the TWIN PULSE GENERATOR.

note: the input signal to the HEADPHONE AMPLIFIER filter is at *TTL* level. It is TIMS practice, in order to avoid overload, not to connect a TTL signal to an analog input. Check for overload. If you prefer you can use the yellow analog output from the TWIN PULSE GENERATOR. This is an AC coupled version of the TTL signal.

T16 *before plugging in the TWIN PULSE GENERATOR set the on-board MODE switch SW1 to SINGLE. Patch up the demodulator of Figure 4.*

T17 *set the frequency deviation of the FM generator to zero, and connect the VCO output to the demodulator input.*

T18 *using the WIDTH control of the TWIN PULSE GENERATOR adjust the output pulses to say a mark/space ratio of 1:1.*

T19 *observe the demodulator output. If you have chosen to take the TTL output from the TWIN PULSE GENERATOR there should be a DC voltage present. Why ? Notice that it is proportional to the width of the pulses into the LPF of the HEADPHONE AMPLIFIER.*

T20 *introduce some modulation at the VCO with the BUFFER amplifier gain control. Observe the output from the LPF of the HEADPHONE AMPLIFIER using the oscilloscope. Measure its frequency (and compare with the message source at the transmitter).*

T21 *show that the amplitude of the message output from the demodulator:*

a) varies with the message amplitude into the VCO. Is this a linear variation ?

b) varies with the pulse width from the TWIN PULSE GENERATOR. Is this a linear variation ?

c) is constant with the frequency of the message to the VCO. Does this confirm the VCO is producing FM, and not PM ?

T22 *increase the message amplitude into the VCO until distortion is observed at the receiver output. Can you identify the source of this distortion ? Record the amplitude of the message at the VCO. You may need to increase the GAIN of the VCO.*

conclusions

There are many other observations you could make.

- have you checked (by calculation) the bandwidth of the FM signal under all conditions ? Could it ever extend 'below DC' and cause wrap-around (fold-back) problems ?
- do you think there is any conflict with the nearness of the message frequency to the carrier frequency ? Why not increase the carrier frequency to the limit of the VCO on the 'LO' range - about 15 kHz
- why not avoid possible problems caused by the relatively large ratio μ/ω and change to the 100 kHz region ?
- try a more demanding distortion test with a two-tone message.

TUTORIAL QUESTIONS

Q1 *name some applications where moderate carrier instability of an FM system is acceptable.*

Q2 *draw the amplitude/frequency spectrum of the signal generated in task* ***T14***.

Q3 *how would you define the bandwidth of the signal you generated in Task* ***T14*** *?*

Q4 *what will the FREQUENCY COUNTER indicate when connected to the FM signal from the VCO ? Discuss possibilities.*

Q5 *derive an expression for the sensitivity of the demodulator of Figure 4, and compare with measurements.*

$$sensitivity = \frac{output\ (message)\ amplitude}{input\ (FM)\ \ frequency\ deviation}$$

Q6 *what is a magnitude for* β, *in* $J_n(\beta)$, *for a Bessel zero, if:*

a) $n = 1$
b) $n = 3$

FM AND THE SYNCHRONOUS DEMODULATOR

FM AND THE SYNCHRONOUS DEMODULATOR

ACHIEVEMENTS: *confirmation of some properties of the spectrum of an angle modulated (FM) signal; action of a synchronous demodulator on this signal; appreciation of the relative phases between the sidefrequency pairs.*

PREREQUISITES: *familiarity with the contents of the Chapter entitled* ***Analysis of the FM spectrum*** *in this Volume****;*** *completion of the experiment entitled* ***Product demodulation - synchronous & asynchronous*** *in Volume A1.*

EXTRA MODULES: *SPECTRUM UTILITIES*

PREPARATION

You are going to look at the operation of the synchronous demodulator on the angle modulated signal:

$$y(t) = E.\cos[\omega t + \varphi(t)] \qquad \text{........ 1}$$

For brevity this angle modulated signal will be referred to as being 'FM'.

synchronous demodulation - linear modulation

In earlier experiments the term 'synchronous demodulator' was used to describe the arrangement of Figure 1.

This arrangement (also known as a synchronous product demodulator) has been used to recover the message from AM, DSBSC, and SSB. Since the message was recovered from the modulated signal input in each case, it was not unreasonable to call it a *demodulator*.

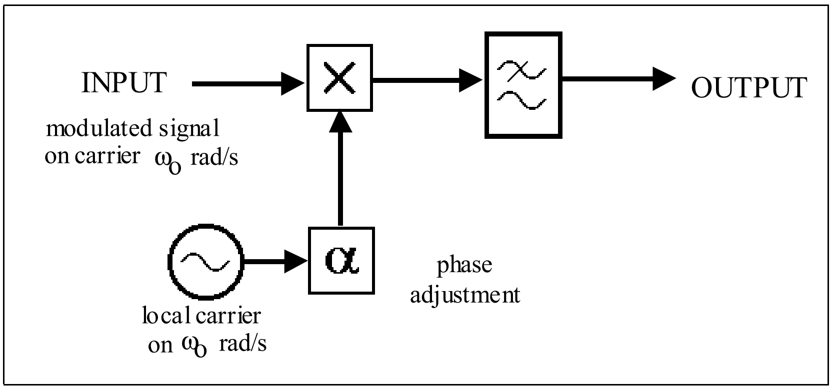


Figure 1: the synchronous demodulator

This was true because these signals were undergoing *linear modulation*. This class of signal was defined in the experiment entitled *DSBSC generation*.

Recall that, for a double sideband signal, as the phase angle α is adjusted, the output amplitude from a synchronous demodulator will vary, including reduction to zero. Zero output results when the local carrier, and the input DSB, are in phase-quadrature.

synchronous demod of non-linear modulation

An FM signal is a member of the class defined as undergoing *non-linear modulation*. These were defined in the chapter entitled *Analysis of the FM spectrum*.

What will happen when the input to the synchronous demodulator is an FM signal ? Will it recover the message $\varphi(t)$ of eqn.(1) ? The answer is a definite '*no*' ! None-the-less, what happens is of interest, and forms the subject of this experiment.

The arrangement of Figure 1 in this application is best thought of as a *frequency translator*, for the special case where the translation is back to baseband.

FM - spectral properties to be verified

The arrangement illustrated in the above Figure 1 is operating *synchronously* - this means that the incoming and local carriers are on exactly the same frequency. Their relative phases are not yet defined, but may be adjusted by varying the phase angle α .

Remember that the amplitude spectrum of an FM signal, derived from a single tone message, is symmetrical about the central (carrier) term on ω_o. The spectrum consists of pairs of sidefrequencies (DSBSC) alternately in phase-quadrature, and in-phase, with the carrier term. Spectral lines are spaced apart by the message frequency.

From the experiment entitled *Product demodulation - synchronousand asynchronous* you will recall that the upper and lower sidebands of the input signal will be frequency translated down to the baseband (audio) region where their respective contributions will now *overlap* in frequency. The amplitude of the *single* resultant from each *pair* of sidefrequencies will depend upon the relative phases of the two components being combined. These in turn will be determined by the phase angle α.

The 'demodulator' will give an output of many frequency components, one from each of the sidefrequency pairs. These will be on exact harmonics of the original message. The output will thus *not* be a copy of the message.

An FM demodulator would need to (re)combine all the sidefrequencies of an FM signal into a single *component at message frequency. So strictly the arrangement of Figure 1 is* ***not a demodulator*** *in this context.*

Since the pairs of DSBSC in the FM spectrum are alternately in-phase, and in phase-quadrature, their resultants will not maximise simultaneously as the phase angle α is rotated. Thus the amplitudes of odd harmonics of the message will be maximised when the even harmonics fall to zero, and vice versa.

All of the above properties will be verified in the experiment to follow.

You will not make measurements with a WAVE ANALYSER at 100 kHz. You will make baseband measurements to confirm this.

EXPERIMENT

At TRUNKS is an FM signal. It is based on a 100 kHz carrier (of which you have a copy at the MASTER SIGNALS module) and a single tone message near 1 kHz. The frequency deviation is sufficient to ensure several pairs of significant sidefrequencies.

T1 *look at the TRUNKS #1 signal with your oscilloscope. By choice of a sweep speed of, say 1 ms/cm, confirm that its envelope is of constant amplitude, so, if indeed it is modulated, it probably is an FM signal. Use a faster sweep speed and see if you can detect any non-uniformity of the zero-crossings (the compressed and expanded spring analog), thus further confirming the possibility of its being FM.*

FM spectrum determination at baseband

You will use the 'synchronous demodulator' to determine the nature of the FM spectrum. Note that this method makes the measurements at baseband frequencies.

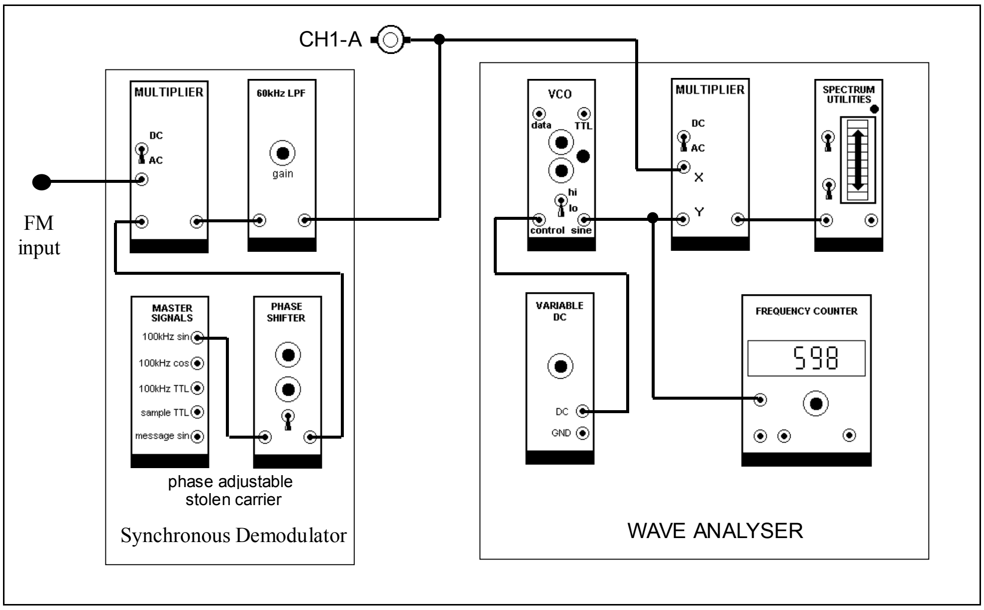


Figure 2: synchronous 'demodulation' of the FM signal.

T2 *patch up the arrangement of Figure 2, which is a 100 kHz SYNCHRONOUS DEMODULATOR and a baseband WAVE ANALYSER. Before plugging in the VCO set the on-board switch to 'VCO'. The front panel frequency range switch will be set to 'LO'.*

You will now have the opportunity to check the observation made in the chapter entitled *Analysis of the FM spectrum* that:

- sidefrequency pairs are alternately in quadrature and in phase with the carrier term at ω.

This means that the first pair of sidefrequencies are in *phase-quadrature* with the carrier.

If the local carrier is set *in-phase* with the received carrier by means of the PHASE SHIFTER then any carrier component present in the FM signal will show up as a DC component at the 'demodulator' filter output.

By maximising the DC output from the filter you are putting the receiver local oscillator in-phase with the received carrier.

Under this condition there will be no output from the odd sidefrequency pairs.

Conversely there will be no output from the even sidefrequency pairs when the carrier is in *phase-quadrature*.

*Thus, **with a little ingenuity**, you can use the WAVE ANALYSER to identify the nature of the spectrum **at 100 kHz**, by making measurements **at audio frequencies**.*

T3 *set the oscilloscope to respond to DC, and adjust the PHASE SHIFTER to maximise the amplitude of the DC output from the 60 kHz LPF. The local carrier is now **in-phase** with the received carrier.*

T4 *use the WAVE ANALYSER to measure the relative amplitudes of all significant components from the 60 kHz LPF. These will be present due to the even-order sidefrequency pairs in the 100 kHz spectrum. Call them V_2, V_4, etc. You cannot measure V_0 (the DC component) since the VCO does not tune down to DC. You could replace the VCO signal with a fixed voltage from the VARIABLE DC module, but if you do this you will need to be sure about what amplitude to use.*

It is now necessary to put the local oscillator into the **phase-quadrature** condition.

T5 *swap from the **sin** to **cos** output of the 100 kHz signal from the MASTER SIGNALS module.*

T6 *use the WAVE ANALYSER to measure the amplitude of all significant components from the filter. These will be present due to the odd-order sidefrequency pairs in the 100 kHz spectrum. Call them V_1, V_3, V_5 etc.*

Make sure you appreciate how you are now in a position to reconstruct the 100 kHz relative amplitude spectrum from the baseband measurements just completed. Do not forget that there is a ‘factor-of-two’ correction to be applied to the sidefrequencies, but *not* to the carrier [1].

[1] each baseband component came from a sidefrequency *pair*.

T7 *use your baseband measurements to construct an amplitude spectrum of the 100 kHz signal. Compare results with the direct measurement made earlier (or the amplitude spectrum supplied).*

alternative spectrum analysis

The assertion that the sidefrequencies are alternately in-phase and in phase-quadrature can be checked [2] without the aid of the WAVE ANALYSER. Instead use the TUNEABLE LPF at the output of the synchronous demodulator.

This filter can be narrowed to pass only the message frequency at μ rad/s (and DC). By adjusting the phase α the DC output can be maximised, while the AC output (the signal at message frequency) is simultaneously reduced to zero. If the filter is then widened to put the passband edge just above 2μ the component at twice the message frequency will observed at the output, since this component is maximised with the DC.

T8 *use the TUNEABLE LPF in place of the WAVE ANALYSER and isolate separately the DC, the component at μ rad/s, and the component at 2μ rad/s. Record their amplitudes.*

T9 *compare the (relative) amplitudes measured in the previous task with the corresponding results obtained using the baseband WAVE ANALYSER. Comment.*

[2] strictly the check is valid only for the first two pairs.

TUTORIAL QUESTIONS

Q1 *in Task 1, why pick a sweep speed of 1 ms/cm when looking for an envelope ?*

Q2 *would it be of serious consequence if the phase, when transferring from the* **cos** *to* **sin** *outputs of the MASTER SIGNALS module (Task* ***T5****), was a few degrees off a true 90°phase shift ?*

Q3 *for the final Task you were able to isolate the single component at μ rad/s. This is the message frequency. Why could not the SYNCHRONOUS DEMODULATOR, with the filter tuned as it is, then be called an FM demodulator ?*

Q4 *did you confirm that the sidefrequency pairs of the FM signal are alternately in-phase and in phase-quadrature ? Explain.*

Q5 *did you confirm that the sidefrequency pairs of the FM signal are spaced apart by the message frequency ? Explain.*

Q6 *show how the measurements made at baseband were used to determine the amplitude spectrum of the FM signal at 100 kHz. Draw the amplitude spectrum of the FM signal.*

Q7 *the FM signal at TRUNKS is represented by y(t), where:*

$$y(t) = E.\cos[\omega t + \beta \cos\mu t]$$

Can you determine, from the amplitude spectrum of the previous Question, the magnitude of 'β' ?

Q8 *what is the 'factor-of-two' correction which was referred to when mapping from the baseband amplitude spectrum (measured with the synchronous demodulator) to the 100 kHz spectrum ?*

ARMSTRONG'S PHASE MODULATOR

ARMSTRONG'S PHASE MODULATOR

ACHIEVEMENTS: *modelling Armstrong's modulator; quadrature phase adjustment; deviation calibration; introduction to the amplitude limiter.*

PREREQUISITES: *earlier modulation experiments; an understanding of the contents of the Chapter entitled* ***Analysis of the FM spectrum*** *in this Volume*.

EXTRA MODULES: *SPECTRUM UTILITIES; 100 kHz CHANNEL FILTERS (optional).*

PREPARATION

FM generation

As its name implies, an FM signal carries its information in its frequency variations. Thus the message must vary the frequency of the carrier.

Spectrum space being at a premium, radio communication channels need to be conserved, and users must keep to their allocated slots to avoid mutual interference.

There is a conflict with FM - the carrier must be maintained at its designated centre frequency with close tolerance, yet it must also be moved (modulated) by the message.

A well know source of FM signals is a voltage controlled oscillator (VCO). These are available cheaply as integrated circuits. It is a simple matter to vary their frequency over a wide frequency range; but their frequency stability is quite unsatisfactory for today`s communication systems. Refer to the experiment entitled *Introduction to FM using a VCO* in this Volume.

Armstrong`s modulation scheme [1] overcomes the problem [2]. It does not change the frequency of the source from which the carrier is derived, yet achieves the objective by an *indirect* method. It forms the subject of this experiment.

---

1 Armstrong`s system is well described by D.L. Jaffe ‘Armstrong`s Frequency Modulator’, Proc.IRE, Vol.26, No.4, April 1938, pp475-481.

2 but introduces another- it is not capable of wide frequency deviations

Armstrong`s modulator

Armstrong's modulator is basically a *phase modulator*; it can be given a frequency modulation characteristic by an integrator inserted between the message source and the modulator. For a single tone message, at one frequency, it is not possible to tell, by what ever measurement, if the integrator is present (so it is an FM signal) or not (a PM signal). Only with a change of message frequency can one then make the decision - by noting the change to the spectral components, for example.

theory

You are already familiar with amplitude modulation, defined as:

$$AM = E.(1 + m.\sin\mu t).\sin\omega t \qquad ...... 1$$

This expression can be expanded trigonometrically into the sum of two terms:

$$AM = E.\sin\omega t + E.m.\sin\mu t.\sin\omega t \qquad ...... 2$$

In eqn.(2) the two terms involved with 'ω' are in phase. Now this relation can easily be changed so that the two are at 90 degrees, or 'in quadrature'. This is done by changing one of the sinωt terms to cosωt. The signal then becomes what is sometimes called a *quadrature modulated* signal. It is Armstrong`s signal.

Thus:

$$\text{Armstrong`s signal} = E.\cos\omega t + E.m.\sin\mu t.\sin\omega t \qquad ...... 3$$

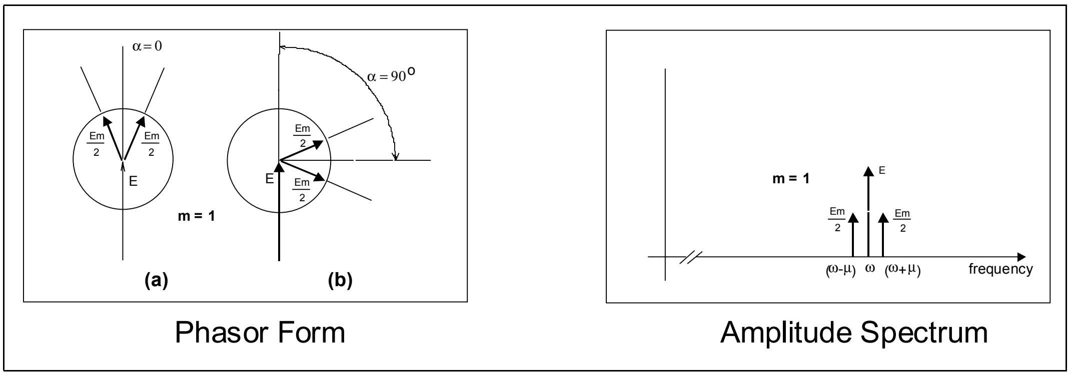


Figure 1: DSBSC + carrier

The signals described by both eqn. (2) and eqn. (3) are shown in phasor form in Figure 1 (a) and (b) above. The amplitude spectrum is also shown; it is the same for both cases (a) and (b).

Each diagram shows the signals for the case m = 1. That is to say, the amplitude of each side frequency component is half that of the carrier.

In the phasor diagram the side frequencies are rotating in *opposite directions*, so their resultant stays in the same direction - co-linear with the carrier for (a), and in phase-quadrature for (b).

Both eqn. (2) and eqn. (3) can be modelled by the arrangement of Figure 2 below.

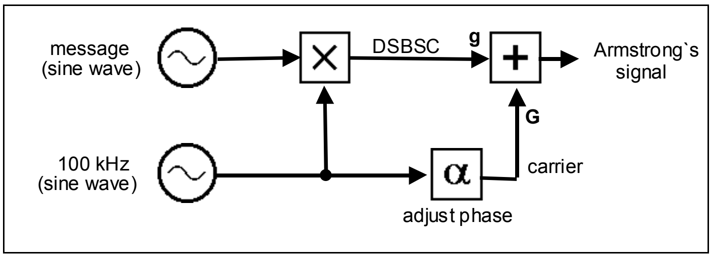


Figure 2: Armstrong`s phase modulator

phase deviation

Having defined Armstrong`s signal it is now time to examine its potential for producing PM.

But first: we have been using the symbol 'm' for the ratio of DSBSC to carrier amplitude, because our starting point was an amplitude modulated signal, and 'm' has been traditionally the symbol for depth of amplitude modulation.. The amplitude modulation was converted to quadrature modulation. We will acknowledge this in the work to follow by making the change from 'm' to '$\Delta\phi$'. Thus:

$$\Delta\phi = m \qquad \text{........ 4}$$

Analysis shows (see the Appendix to this experiment) that the carrier of Armstrong`s signal is undergoing phase modulation.

The peak phase deviation is proportional to the ratio of DSBSC to CARRIER peak amplitudes at the ADDER output; but it is not a linear relationship. The peak phase deviation, $\Delta\phi$, is given by:

$$\Delta\phi = \arctan\left[\frac{DSBSC}{CARRIER}\right] \textit{radians} \qquad \text{...... 5}$$

Remember that the amplitude of the DSBSC is directly proportional to that of the message, so the message amplitude will determine the amount of phase variation.

For small arguments, arctan(arg) ≈ arg

Thus to minimize distortion at the receiver the ratio of DSBSC to carrier must be kept small.

A receiver to demodulate a phase modulated signal is sensitive to these phase deviations.

To keep the *received* signal distortion to acceptable limits the peak phase deviation at Armstrong`s modulator should be restricted to a fraction of a radian, according to distortion requirements as per Figure 3. The analysis of distortion is discussed in the Appendix to this experiment.

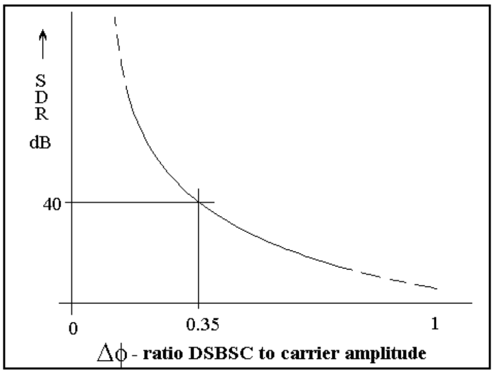


Figure 3: distortion from Armstrong`s modulator

practical realization of Armstrong`s modulator

The principle of Armstrong's method of phase modulation, or his frequency modulator (with the added integrator as described earlier), is used in commercial practice. But the circuitry employed to generate this signal is often not as straightforward as the arrangement of Figure 2. It is not always possible to isolate, and so measure separately, the amplitudes of the DSBSC and the CARRIER. So it is not possible to calculate the phase deviation, in such a simple, straight forward manner.

Amplitude limiters are also incorporated in the circuitry. These intentionally remove the envelope, which otherwise could be used as a basis for measurement.

In these cases other methods must be used to set up and calibrate the phase deviation of the modulator. These include, for example, the use of a calibrated demodulator. There is also the method of 'Bessel zeros'. This is an elegant and exact method, and is examined in the experiment entitled *FM and Bessel zeros* in this Volume.

EXPERIMENT

patching up

In this experiment you will learn how to set up Armstrong`s modulator for a specified phase deviation, and a unique method of phase adjustment.

Armstrong`s generator, in block diagram form in Figure 2, is shown modelled in Figure 4 below.

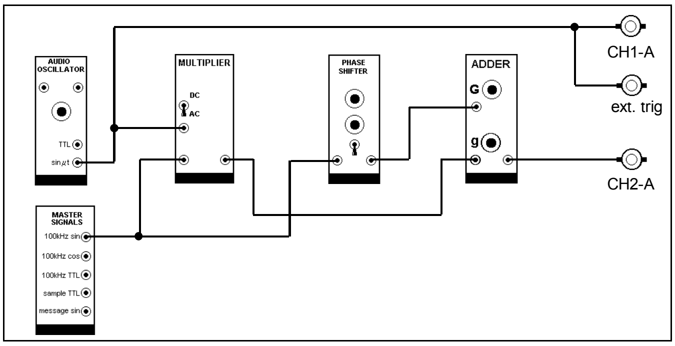


Figure 4: the model for Armstrong`s modulator

T1 *patch up the model in Figure 4. You will notice it is exactly the same arrangement as was used for modelling AM. The major difference for the present application will be the magnitude of the phase angle* α *- zero degrees for AM, but* 90^0 *for Armstrong.*

T2 *choose a message frequency of about 1 kHz from the AUDIO OSCILLATOR.*

model adjustment

T3 *check that the oscilloscope has triggered correctly, using the external trigger facility connected to the message source. Set the sweep speed so that it is displaying two or three periods of the message, on CH1-A, at the top of the screen.*

Now pay attention to the setting up of the modulator. The signal levels *into* the ADDER are at TIMS ANALOG REFERENCE LEVEL, but their relative magnitudes (and phases) will need to be adjusted at the ADDER *output*.

To do this:

T4 *rotate both **g** and **G** fully anti-clockwise.*

T5 *rotate **g** clockwise. Watch the trace on CH2-A. A DSBSC will appear. Increase its amplitude to about 3 volts peak-to-peak. Adjust the trace so its peaks just touch grid lines exactly a whole number of centimetres apart. This is for experimental convenience; it will be matched by a similar adjustment below.*

T6 *remove the patching cord from input **g** of the ADDER*

T7 *rotate **G** clockwise. The CARRIER will appear as a band across the screen. Increase its amplitude until its peaks touch the same grid lines as did the peaks of the DSBSC (the time base is too slow to give a hint of the fine detail of the CARRIER; in any case, the synchronization is not suitable).*

T8 *replace the patch cord to **g** of the ADDER.*

At the ADDER output there is now a DSBSC and a CARRIER, each of *exactly* the same peak-to-peak amplitude, but of unknown relative phase.

Observe the envelope of this signal (CH2-A), and compare its shape with that of the message (CH1-A), also being displayed.

T9 *vary the phasing with the front panel control on the PHASE SHIFTER until the almost sinusoidal envelope (CH2-A) is of **twice** the frequency as that of the message (CH1-A). The phase adjustment is complete when alternate envelope peaks are of the **same amplitude**.*

As a guide, Figure 5 shows three views of Armstrong's signal, all with equal amplitudes of DSBSC and carrier, but with different phase errors (ie, errors from the required 90^0 phase difference between DSBSC and carrier).

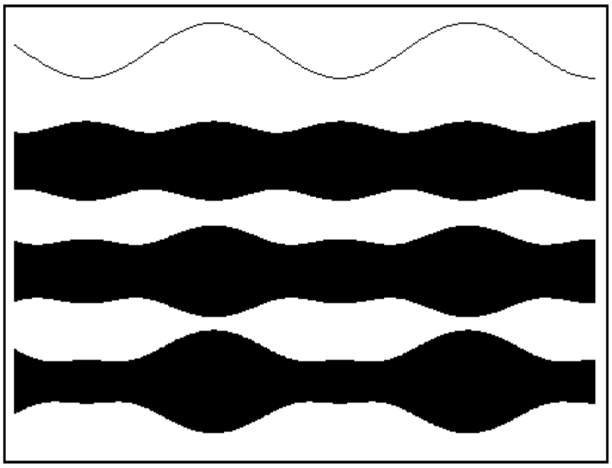


Figure 5: Armstrong's signal, with $\Delta\phi = 1$, and phase errors of 45 deg (lower), 20 deg,(centre) and zero (upper).

Armstrong's phase adjustment

An error from quadrature at the transmitter will show up as distortion of the recovered message at the receiver. This will be in 'addition' to the inherent distortion introduced by the approximation arctan(arg) ≈ (arg). The 'addition' would be anything but linear, and difficult to evaluate, but easy to measure for a particular case.

How can the phase of the DSBSC and the added carrier be adjusted to be in *exact* quadrature ?

An analysis of the envelope of Armstrong`s signal is given in the Appendix to this experiment. There it is shown that:

1. when in phase quadrature, the envelope is sinusoidal-like in shape (Figure 5 above) with adjacent peaks of *equal* amplitude.
2. the envelope waveform is periodic, with the fundamental frequency being *twice* that of the message from which the DSBSC was derived.

Each of these two findings suggests a different method of phase adjustment.

phase adjustment using the envelope

T10 *trim the front panel control of the PHASE SHIFTER until adjacent peaks of the envelope are of equal amplitude. To improve accuracy you can increase the sensitivity of the oscilloscope to display the peaks only.*

Equating heights of adjacent envelope peaks with the aid of an oscilloscope is an acceptable method of achieving the quadrature condition. For communication purposes the message distortion, as observed at the receiver, due to any such phase error, will be found to be negligible compared with the inherent distortion introduced by an ideal Armstrong modulator.

phase adjustment using 'psycho-acoustics'

There is another fascinating method of phase adjustment, first pointed out to the author by M.O. Felix.

The envelope of Armstrong`s signal is recovered, using an envelope detector, and is monitored with a pair of headphones. For the in-phase condition this would be a pure tone at message frequency. As the phase is rotated towards the wanted 90 degrees difference it is very easy to detect, by ear, when the fundamental component disappears (at μ rad/s, and initially of large amplitude), leaving the component at 2μ rad/s, initially small, but now large. This is the quadrature condition.

T11 *model an envelope detector, using the RECTIFIER in the UTILITIES module, and the 3 kHz LPF in the HEADPHONE AMPLIFIER module. Connect Armstrong`s signal to the input of the envelope detector. Listen to the filter output (the envelope) with headphones. Set the PHASE SHIFTER as far off the quadrature condition as possible, and concentrate your mind on the fundamental. Slowly vary the phase. You will hear the fundamental amplitude* ***reduce to zero****, while the second harmonic of the message appears. Notice how sensitive is the point at which the fundamental disappears ! This is the quadrature condition.*

Note that you have been able to detect the presence of a low (finally zero) amplitude tone in the presence of a much stronger one. This was only possible because the low amplitude term was a sub-harmonic of the higher amplitude term. The opposite is extremely difficult. This is a phenomenon of psycho-acoustics.

practical applications

Remember: Armstrong`s modulator generates phase, or frequency, modulation, by an *indirect* method. It does not disturb the frequency stability of the carrier source, as happens in the case of modulators using the direct method - eg, the VCO.

But, to keep the distortion to acceptable limits, Armstrong`s modulator is capable of *small* phase deviations only - see the Appendix to this experiment. This is insufficient for typical communications applications. The deviation can be increased by additional processing, namely by *frequency multipliers*.

The frequency multiplier has been discussed in this Volume entitled *Analysis of the FM spectrum*. You can learn about them in the experiment entitled *FM deviation multiplication* (this Volume). Refer also to the Appendix to this experiment.

spectral components

In later experiments you will be measuring the spectral components of wideband FM signals. In this experiment all we have is Armstrong`s signal, which, after amplitude limiting, has relatively few components of any significance. But they are there, and you can find them.

So now you will model the WAVE ANALYSER, which was introduced in the experiment entitled *Spectral analysis - the WAVE ANALYSER* (this Volume), and look for them.

Table A-1 in the appendix to this experiment shows you what to expect. Notice it will be possible to find only three, possibly five, components (including the carrier) with any confidence (the simple WAVE ANALYSER you will be using has its limitations), but confirming their amplitude ratios as predicted is a satisfying exercise.

Remember: there are only *three* components at the output of Armstrong`s modulator (as modelled by you already).

To create the FM sidebands Armstrong`s signal must first be:

1. *amplitude limited*, to produce a narrowband FM signal (NBFM)

and then

2. *frequency multiplied*, to generate a wideband FM signal (WBFM).

You will take step (1) in this experiment, and then step (2) in a later experiment.

The amplitude limiting is performed by the CLIPPER in the UTILITIES module [3] (with gain set to 'hard limit'; refer to the ***TIMS User Manual***).

Although, for the experiment, there is no need to add a filter following the amplitude limiter (it won`t change the spectral components in the region of 100 kHz), in practice this would be done, and so in the block diagram of Figure 6 below this is shown. If you have a 100 kHz CHANNEL FILTERS module you should use it in this position.

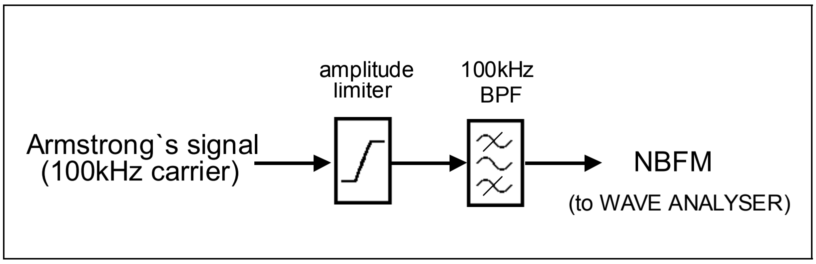


Figure 6: Armstrong`s NBFM signal

T12 *set up for equal amplitudes of DSBSC and carrier into the ADDER of the modulator (β = 1), and confirm you have the quadrature condition. A message frequency of about 1 kHz will be convenient for spectral measurements.*

Although a ratio of DSBSC to carrier of unity will result in significant distortion at the output from a demodulator (refer Figure 3) one can still predict the amplitude spectrum and confirm it by measurement.

[3] version V2 or later

T13 *at the output of your Armstrong modulator add the AMPLITUDE LIMITER (the CLIPPER in the UTILITIES module) and filter (in the 100 kHz CHANNEL FILTERS module) as shown in block diagram form in Figure 6.*

T14 *model a WAVE ANALYSER, and connect it to the filter output. There is no need to calibrate it; you are interested in* ***relative amplitudes****.*

T15 *set the phase deviation to zero (by removing the DSBSC from the ADDER of the modulator). Observe and sketch the waveform of the signals into and out of the channel filter. Find the 100 kHz carrier component with the WAVE ANALYSER. This, the* ***unmodulated carrier****, is your* ***reference****. For convenience adjust the sensitivity of the SPECTRUM UTILITIES module so the meter reads full scale.*

T16 *replace the DSBSC to the ADDER of the modulator. The carrier amplitude should drop to 84% of the previous reading (if you leave the meter switch on* ***HOLD*** *nothing will happen !). This amplitude change is displayed in Table A-1 of the appendix to this experiment.*

T17 *search for the first pair of sidebands. They should be at amplitudes of 38% of the unmodulated carrier.*

T18 *there are further sideband pairs, but they are rather small, and will take care to find.*

T19 *you could repeat the spectral measurements for $\beta = 0.5$ (which are also listed in Table A-1).*

T20 *you were advised to look at the signal from the filter when there was no modulation. Do this again. Synchronize to the signal itself, and display ten or twenty periods. Then add the modulation. You will see the right hand end of the now modulated sinewave move in and out (the 'oscillating spring' analogy), confirming the presence of frequency modulation (there is no change to the amplitude).*

TUTORIAL QUESTIONS

Q1 *by writing eqn.(3) in the general form of* $a(t).\cos[\omega t + \phi(t)]$, *obtain approximate expressions for both* $a(t)$ *and* $\phi(t)$ *as functions of 'm' (or the equivalent,* $\Delta\phi$*).*

Q2 *can a conventional phase meter be used to set the DSBSC and carrier in quadrature ? Explain.*

Q3 *a 4 volt peak-to-peak DSBSC is added to a 5 volt peak-to-peak carrier, in phase-quadrature. Calculate:*

a) the peak to peak and the trough-to-trough amplitudes of the resultant signal.

b) the phase deviation of the carrier, after amplitude limiting.

Q4 *suppose the phasing in an Armstrong modulator is adjusted by equating adjacent envelope maxima. Obtain an approximate expression for the phase error* α, *from the ideal quadrature, as a function of a small error in this amplitude adjustment.*

Q5 *if there is an error in the phasing of an Armstrong modulator, the output could be written as*

$$y(t) = E.\cos\omega t + E.\cos\mu t.\sin(\omega t + \alpha)$$

Obtain an approximate expression for the phase deviation, following amplitude limiting, for small α, *the phase error from quadrature.*

Q6 *the phasing in an Armstrong modulator is adjusted by listening for the null of the message in the envelope (the psycho-acoustic method). If during this adjustment the fundamental amplitude is reduced to 40 dB below the amplitude of the second harmonic of the message, what would be the resulting phase error ?*

APPENDIX

Analysis of Armstrong`s signal

If we define Armstrong`s signal as:

Armstrong`s signal = $\cos\omega t + \Delta\phi.\sin\mu t.\sin\omega t$ A.1

and then write this in the general form of a narrowband modulated signal we have:

Armstrong`s signal = $a(t).\cos[\omega t + \phi(t)]$ A.2

where:

$$a(t) = \sqrt{(1 + (\Delta\phi)^2 \sin^2 \mu t)} \qquad \text{...... A.3}$$

$$\phi(t) = \arctan(-\Delta\phi.\sin\mu t) \qquad \text{...... A.4}$$

$$\text{where} \quad \Delta\phi = (\text{DSBSC / carrier}) \qquad \text{........ A.5.}$$

The expressions for both a(t) and $\phi(t)$ can be expanded into infinite series. For small values of $\Delta\phi$, say ($\Delta\phi < 0.5$), they approximate to:

$$\text{approx. } a(t) = (1 + \frac{(\Delta\phi)^2}{4}) + (\frac{(\Delta\phi)^2}{4}\sin 2\mu t) \qquad \text{...... A.6}$$

$$\text{approx. } \phi(t) = (\Delta\phi - \frac{(\Delta\phi)^3}{4})\cos\mu t - \frac{(\Delta\phi)^3}{12}\cos 3\mu t \qquad \text{...... A.7}$$

Equation A.6 confirms that, to a first approximation, the Armstrong envelope is sinusoidal and of twice the message frequency. There will be higher order even harmonics of the message, but, as you will have observed in the psycho-acoustic test earlier, no component at message frequency.

Equation A.7 shows that the phase modulation is proportional to $\Delta\phi$, as wanted, but that there is odd harmonic distortion in the received message. The need to keep the distortion to an acceptable value puts an upper limit on the size of $\Delta\phi$. Figure 3, shown previously, graphs the expected signal-to-distortion ratio to a better approximation.

Remember that eqn.A.7 gives the distortion from an ideal demodulator - it gives no clue as to the spectrum of the Armstrong signal.

the amplitude limiter

From your work on angle modulated signals you will appreciate that the signal

$$y(t) = \cos[\omega t + \phi(t)] \qquad \text{...... A.8}$$

has the potential for many spectral components.

This is an angle modulated signal, which is what is expected from Armstrong`s modulator.

But Armstrong`s signal, as defined by eqn.(A.1), has only *three* components ! We know this since it is the linear sum of a DSBSC (two components) and a carrier (one component).

Then where are all the spectral components suggested by eqn.(A.7) ?

The signal of the form of eqn.(A.8) is what we want, but we have one of the form of eqn.(A.2). The difference is the multiplying term a(t).

We would like a(t) to become a constant.

This is the function of the *amplitude limiter*. It is required to remove envelope variations.

The amplitude limiter has been discussed in the Chapter entitled *Analysis of the FM spectrum*.

Armstrong`s spectrum after amplitude limiting

Obtaining the spectrum of the amplitude-limited Armstrong signal whilst taking into account the inevitable distortion involves an expansion of eqn.(A.8) with:

$$\phi(t) = \arctan(\beta.\cos\mu t) \qquad \text{...... A.9}$$

The phase function can be expanded into an odd harmonic series of μ.

$$\phi(t) = \beta_1.\cos\mu t + \beta_3.\cos 3\mu t + \beta_5.\cos 5\mu t + ... \qquad \text{........ A.10}$$

This expansion is then substituted in eqn.(A.8). Taking any more than two terms makes the expansion of eqn.(A.8) extremely tedious, and so this means that the approximation is only valid for say the range

$$0 < \beta < 1$$

Even with two terms in $\phi(t)$ the expansion of eqn.(A.8), to obtain the spectrum, is a tiresome exercise. But when finished one has an analytic expression for the spectrum for small β.

An alternative is to use a fast Fourier transform and evaluate the spectrum for specific values of β. This has been done, and Table A-1 below lists the amplitude spectrum for $\beta = 0.5$ and $\beta = 1$

Component	***amplitude $\beta = 0$***	***amplitude $\beta = 0.5$***	***amplitude $\beta = 1$***
carrier ω	1.0	0.945	0.835
$\omega \pm 1\ \mu$	0.000	0.23	0.381
$\omega \pm 2\ \mu$	0.000	0.026	0.072
$\omega \pm 3\ \mu$	0.000	0.006	0.031
$\omega \pm 4\ \mu$	0.000	0.001	0.009
$\omega \pm 5\ \mu$	0.000	0.000	0.004
$\omega \pm 6\ \mu$	0.000	0.000	0.001

Table A-1

What would happen if this signal, for $\beta = 1$, was processed by a frequency multiplier ? The deviation would be increased. What would be the new spectrum ? The analytic derivation of the new spectrum is decidedly non-trivial [4]. The easy way to find the answer is to generate it, and then measure it !

Although not specifically suggested, there will be an opportunity for this in the experiment entitled *FM deviation multiplication* in this Volume.

[4] remember, this is Armstrong`s signal, involving the arctan function. Derivation of the spectrum of a pure FM signal, with $\beta = 1$, is relatively straight forward (see ***Analysis of the FM Spectrum)***..

FM DEVIATION MULTIPLICATION

FM DEVIATION MULTIPLICATION

ACHIEVEMENTS: *introduction to the frequency multiplier; wide-band FM spectrum from Armstrong`s modulator; wide-band spectrum measurement.*

PREREQUISITES: *familiarity with the theory outlined in the Chapter entitled* ***Analysis of the FM spectrum*** *and completion of the experiment entitled* ***Armstrong`s phase modulator****, both in this Volume. For Tables of Bessel Coefficients see the* ***Appendix*** *to this Volume.*

EXTRA MODULES: *100 kHz CHANNEL FILTERS (V.2); FM UTILITIES; SPECTRUM UTILITIES.*

PREPARATION

the angle modulated signal

This experiment is about generating a wideband angle modulated signal, based on an Armstrong modulator, and examining its spectrum.

An angle modulated signal is one defined as:

$$y(t) = E.\cos(\omega t + \beta.\cos\mu t) \qquad \text{........ } 1$$

where the parameter β is the one which varies with the message in accordance with the definition given in the Chapter entitled *Analysis of the FM spectrum.*

Two examples of an angle modulated signal are those of phase modulation (PM) and frequency modulation (FM).

You will need an understanding of the theory if you are going to relate your measurements to expectations. This is especially so if the measurements are not in complete agreement, since you should be able to explain the discrepancy !

In a following experiment, entitled *FM and Bessel zeros*, some more insights into the behaviour of the spectrum with changing depths of modulation - deviation - will be examined.

FM or PM ?

If either a PM or an FM transmitter is modulated by a single tone message, of fixed amplitude and frequency, there is no way an observer, by examining the spectrum, can distinguish which transmitter is being used.

They are both described by eqn.(1) above.

The degree of modulation (the 'deviation') is determined by the magnitude of β.

An expansion of eqn. (1) shows that the *number* of sidebands, and their *amplitudes*, are governed completely by the magnitude of β. Their frequency spacing is determined by the message frequency of μ rad/s.

- the magnitude of β is directly proportional to the message amplitude.
- for a PM signal β is equal to the peak phase deviation $\Delta\phi$.
- for an FM signal β is equal to $\Delta\phi$ / μ

The difference between the two signals shows itself when the message frequency is changed. Suppose there is an increase of message frequency:

- for a PM transmitter β remains fixed. The amplitude of each spectral component remains the same, but their spacing increases. The bandwidth therefor increases in proportion to the message frequency change.
- for an FM transmitter the magnitude of β reduces, since it is inversely proportional to the message frequency. The spacing of spectral components increases. Some spectral component amplitudes will increase, some reduce, but the sum of the squares of their amplitudes remains fixed. The bandwidth will *not* increase in proportion to the message frequency change, since a smaller β requires less components. To a rough approximation, the bandwidth remains much the same.

In the experiment to follow you will be using a fixed message frequency, so it is immaterial whether it is called an FM or a PM signal. In principle it is going to be a PM transmitter, since there will not be an integrator (1 / μ characteristic) associated with the message source.

the need for frequency multipliers

To achieve the signal-to-noise ratio advantages of which FM is capable it is necessary to use a deviation large enough to ensure that the signal has at least two or more pairs of 'significant sidebands'.

A *frequency multiplier* is a device for increasing the deviation. It would have been better to have called it a *deviation multiplier*. It does indeed multiply the frequency, but this is of secondary importance in this application.

Frequency multipliers are used following PM and FM modulators which are themselves unable, usually because of linearity considerations, to provide sufficient deviation.

They are essential for most applications of Armstrong`s modulator, which was examined in the experiment entitled *Armstrong`s phase modulator* (this Volume). This modulator is restricted to phase deviations of less than one radian before the generated distortion becomes unacceptable.

Frequency multipliers consist of an *amplitude limiter* followed by a *bandpass filter*. They are discussed in the Chapter entitled *Analysis of the FM spectrum*. These devices are also called *harmonic multipliers*.

wideband FM with Armstrong`s modulator

The experiment will examine the operation of a FREQUENCY MULTIPLIER (in fact, two such operations in cascade) on the output of an ARMSTRONG MODULATOR. The spectral components will be identified with a WAVE ANALYSER. This instrumentation was introduced in the experiment entitled *Spectrum analysis - the WAVE ANALYSER* in this Volume.

The aim of this experiment is to generate a signal at 100 kHz with sufficient frequency deviation to enable several significant sidebands to be found in the spectrum. This requires a phase deviation of a radian or more. Since the Armstrong modulator is only capable of a deviation of a fraction of a radian, if distortion at the demodulator is to be kept low, then a FREQUENCY MULTIPLIER is required.

FM UTILITIES

The FM UTILITIES module contains two amplitude LIMITER sub-systems, a 33.333 kHz BPF, and a source of sinusoidal carrier at 11.111 kHz. These sub-systems will be combined as shown in Figure 1 below.

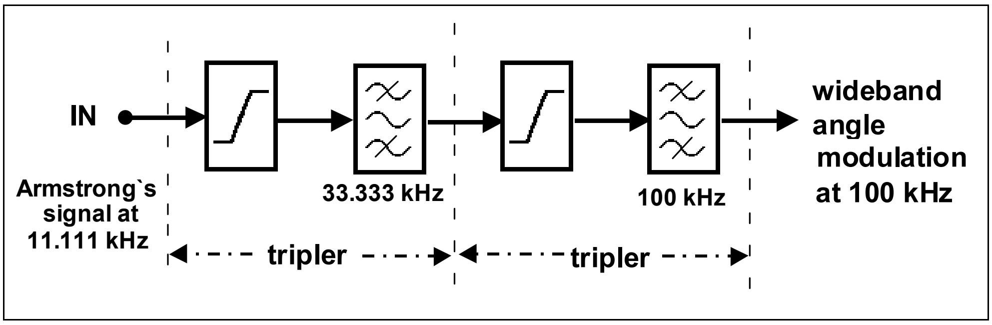


Figure 1: deviation multiplication times-9

The Armstrong modulator (not shown) uses the 11.111 kHz sinusoidal carrier.

The 100 kHz BPF is available in the 100 kHz CHANNEL FILTERS module.

EXPERIMENT

The modelling arrangement of the block diagram of Figure 1 is illustrated in Figure 2. This has been sub-divided into three distinct parts, as in Figure 1, and each is described below.

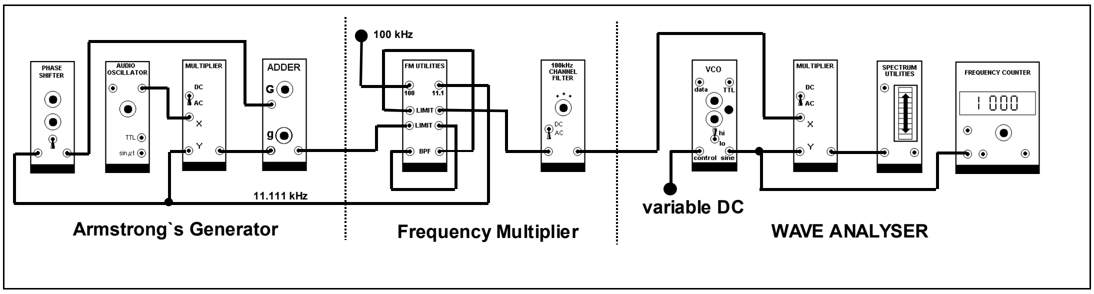


Figure 2: the models

the Armstrong modulator

For an FM signal at 100 kHz the Armstrong modulator must operate at a frequency several multiples *below* this. The frequency of 11.111 kHz has been chosen, providing a deviation multiplication, and an inevitable frequency multiplication, of times-9.

The 11.111 kHz sinusoidal carrier is obtained from the FM UTILITIES module. This contains a sub-system which produces a sinusoidal carrier at 11.111 kHz from a sinusoidal input at 100 kHz.

the frequency multiplier times-9

The FM UTILITIES module can model the two triplers of Figure 1. It has two LIMITER sub-systems, and a 33.3 kHz BPF. The 100 kHz BPF is obtained from a 100 kHz CHANNEL FILTERS module (channel #3). The patching is shown in Figure 2.

the WAVE ANALYSER

The spectrum of the FM signal is examined with a WAVE ANALYSER, modelled as shown in Figure 2. Of particular importance is the method of fine tuning the VCO, using a combination of the controls of the VARIABLE DC and the GAIN of the VCO.

The fine tuning technique was described in the experiment entitled *Spectrum analysis - the WAVE ANALYSER.*

choosing the message frequency

The design of the BPF in a frequency multiplier chain is a compromise between allowing sufficient bandwidth for the desired sidebands, while still attenuating the adjacent harmonics of the non-linear action of the preceding LIMITER.

This is generally not a problem in a commercial system for speech, since the frequency ratios (speech bandwidth, and so the modulated bandwidth, to the carrier frequency) are significantly lower. But in the modelling environment of TIMS, where carrier frequencies are relatively low, some design difficulties do arise.

You have been presented with existing 33.333 kHz and 100 kHz bandpass filters, so have no control over their bandwidths. They will set an upper limit to your message frequency. This you will need to determine.

First see Tutorial Questions Q2 and Q3.

Bandwidth measurement is best performed *before* patching up the complete system, since, using the suggested method for the 33 kHz BPF, it is necessary to have easy access to the VCO board for tuning purposes (see below).

Alternatively you might prefer to devise your own method of providing a 33 kHz variable frequency sinewave, *without* the need to use the VCO in FSK mode. See Tutorial Question Q4.

BPF bandwidth measurement

33 kHz BPF

T1 *set the on-board switch SW2 of the VCO to 'FSK'. Plug in the FM UTILITIES module and the VCO, leaving plenty of room for hand-access to the on-board control RV8 (FSK2) of the VCO. Connect a TTL HI to the DATA input of the VCO (this switches the output of the VCO to the FSK2 frequency). Select the HI frequency mode of the VCO with the front panel toggle switch.*

T2 *connect the VCO output to the input of the 33 kHz BPF of the FM UTILITIES module, and the output to the oscilloscope. Sweep the VCO frequency through the filter, and measure the frequency response.*

100 kHz BPF

T3 *using the VCO in 'VCO' mode make a sweep of the 100 kHz BPF of the 100 kHz CHANNEL FILTERS module, and measure its bandwidth.*

message frequency

T4 *from the two bandwidth measurements determine an upper limit for the message frequency. See Tutorial Question Q3.*

You will now realize that extending the message frequency to 3 kHz is not possible, considering the bandwidths involved. Something below 1 kHz would be preferable.

assembling the models

The system to be patched up is illustrated in Figure 2.

preparation

T5 *before plugging in the PHASE SHIFTER (of the Armstrong modulator) set the on-board switch to 'HI'. Set the on-board switch of the VCO (of the WAVE ANALYSER) to 'VCO', and the front toggle switch to 'HI'.*

T6 *plug in the modules. Put the ARMSTRONG MODULATOR to the left, the FREQUENCY MULTIPLIER in the centre, and the WAVE ANALYSER to the right. Do not yet do any patching. This will be done in easy stages, detailed below.*

Armstrong`s carrier

To test the frequency multiplier, first use only the 11.111 kHz unmodulated carrier from the Armstrong modulator.

The 11.111 kHz sinusoidal carrier is provided by the FM UTILITIES module, derived from a 100 kHz sinusoidal input. This carrier is 1/9 of the final output carrier frequency of 100 kHz.

T7 *patch the 11.111 kHz sine wave from the FM UTILITIES module via the phase shifter to the '**G**' input of the ADDER. Adjust the ADDER output to the TIMS ANALOG REFERENCE LEVEL. Leave the '**g**' input of the ADDER empty.*

The carrier is now ready for testing the FREQUENCY MULTIPLIER.

modelling the frequency multiplier

the first tripler

The first tripler is modelled with a LIMITER and 33.333 kHz BPF in the FM UTILITIES module.

T8 *patch the output of the Armstrong modulator (the unmodulated 11.111 kHz carrier) to the first tripler. Confirm that the output is a 33.333 kHz sinusoid at or about the TIMS ANALOG REFERENCE LEVEL. Its amplitude may be adjusted with an on-board trimmer. Precise level adjustment is not necessary.*

the second tripler

The second tripler is modelled with a LIMITER from the FM UTILITIES module, and a 100 kHz BPF from the 100 kHz CHANNEL FILTERS module.

T9 *patch the output of the 33.333 kHz BPF to the unused LIMITER of the FM UTILITIES module. Connect its output to the 100 kHz CHANNEL FILTERS module, switched to the 100 kHz BPF.*

T10 *confirm that the output of the second tripler is a 100 kHz sinewave. It should be at or about the TIMS ANALOG REFERENCE LEVEL. Its amplitude may be adjusted with an on-board trimmer. Precise level adjustment is not necessary.*

You have now modelled the FREQUENCY MULTIPLIER, although with an unmodulated carrier.

Armstrong`s modulator

It is now time to complete the model of Armstrong`s modulator by adding the DSBSC to the carrier in the ADDER.

We want the Armstrong modulator to have as large a deviation as possible, so as to have several significant sidebands on the 100 kHz carrier, yet not so large as to generate too much distortion (which would upset the predictable amplitude ratios of the sidebands).

After adjusting the quadrature phase of the DSBSC and carrier, a suggestion is to set the DSBSC to carrier amplitude ratio to a about 1:3.

T11 *complete the modelling of the ARMSTRONG MODULATOR. Set the AUDIO OSCILLATOR to, say, 500 Hz [1]. There is no DC involved, so switch the MULTIPLIER to AC coupling. Remember it is easier to set the phase [2], while watching the envelope, with the ratio of DSBSC to CARRIER approximately unity. The detailed procedure was described in the experiment entitled **Armstrong`s phase modulator**.*

T12 *having adjusted the phase, reduce the phase deviation to 0.33 radians, in preparation for the next part of the experiment. Remember to keep signal levels, where possible, at or near the TIMS ANALOG REFERENCE LEVEL.*

[1] do you agree that this is acceptable ? If not, or in any case, you are free to choose your own frequency.

[2] the on-board switch SW1 is probably best set to 'HI'.

measuring the FM spectrum

The ARMSTRONG MODULATOR now has a phase deviation of 0.33 radians. The FREQUENCY MULTIPLIER output will be a phase modulated signal with a peak phase deviation of nine times this, or 3.0 radians.

T13 *examine the output of the FREQUENCY MULTIPLIER with the oscilloscope. As seen in the time domain (the oscilloscope being triggered by the signal itself) it will have the appearance of a compressed and expanded spring. It should be at about the TIMS ANALOG REFERENCE LEVEL. Notice that the envelope is flat.*

Now use the WAVE ANALYSER to check the 100 kHz spectrum.

it is essential that you are familiar with the method of fine tuning the VCO

T14 *patch up the WAVE ANALYSER. Set the VCO to the 'HI' frequency range with the front panel switch.*

T15 *examine the output of the FREQUENCY MULTIPLIER with the WAVE ANALYSER. Record the frequency and amplitude of each spectral component of significance [3].*

T16 *use the Tables of Bessel Coefficients in Appendix C to this text to draw the amplitude spectrum of an angle modulated signal with $\beta = 3.0$. Compare with your measurements.*

special cases

It is always interesting to investigate special cases. The obvious cases are those of the Bessel zeros. With these special values of β the amplitude of particular components falls to zero. These and other cases are examined in the experiment entitled *FM and Bessel zeros*.

[3] that is, significantly above the noise level.

TUTORIAL QUESTIONS

Q1 *see first the tutorial questions in the Chapter entitled* ***Analysis of the FM spectrum****.*

Q2 *suppose a message frequency range of 300 to 3000 Hz was desired. Knowing the maximum phase deviation allowed by the limitations of the Armstrong modulator, specify the characteristics of the two BPF of the system. Remember they must pass the wanted sidebands, but stop the unwanted components from the preceding LIMITER.*

Q3 *from your measurements of the bandpass filter bandwidths, and knowing the limits upon the phase deviation set by the Armstrong modulator, how would you specify the upper message frequency ?*

Q4 *you may have found it inconvenient to use the suggested method of tuning the VCO around 33 kHz for the BPF measurement. Devise another method, which does not involve adjustment of the on-board control of the VCO; that is, all frequency changes are made using front panel controls of existing TIMS modules.* ***hint****: there is a 130 kHz sinusoid at TRUNKS.*

FM AND BESSEL ZEROS

FM AND BESSEL ZEROS

ACHIEVEMENTS: *calibration of the frequency deviation of an FM transmitter using the method of Bessel Zeros.*

PREREQUISITES: *completion of the experiments entitled **Armstrong`s phase modulator**, and **FM deviation multiplication** in this Volume; a knowledge of the relationships between the phase deviation and the spectrum of a PM signal. See **Appendix C** to this text for Tables of Bessel Coefficients.*

EXTRA MODULES: *100 kHz CHANNEL FILTERS (version 2); FM UTILITIES, SPECTRUM UTILITIES.*

PREPARATION

introduction

This experiment investigates methods of deviation calibration of a PM transmitter by observation of the spectrum. It includes the method of 'Bessel zeros'.

The outcome of the experiment could be a calibration curve, showing the position of the modulator deviation control versus β, the deviation of the modulator [1].

This curve could already have been obtained in the experiment entitled *Armstrong`s phase modulator*, by measuring the ratio of the DSBSC to carrier amplitudes out of the ADDER. From this ratio the magnitude of the spectral components could have been deduced by *calculation*. But in *this* experiment you will be examining the spectrum itself, and from this working backwards to determine the phase deviation.

Make sure you appreciate the difference between the two methods.

The model required to generate a PM signal is that used in the experiment entitled *FM deviation multiplication*. Refer to that experiment for setting up details.

[1] the *principle* of the method is what will be learned. You cannot actually plot the curve, since TIMS knobs are not graduated

Figure 1 shows the arrangement in simplified block diagram form, and Figure 2 shows patching details.

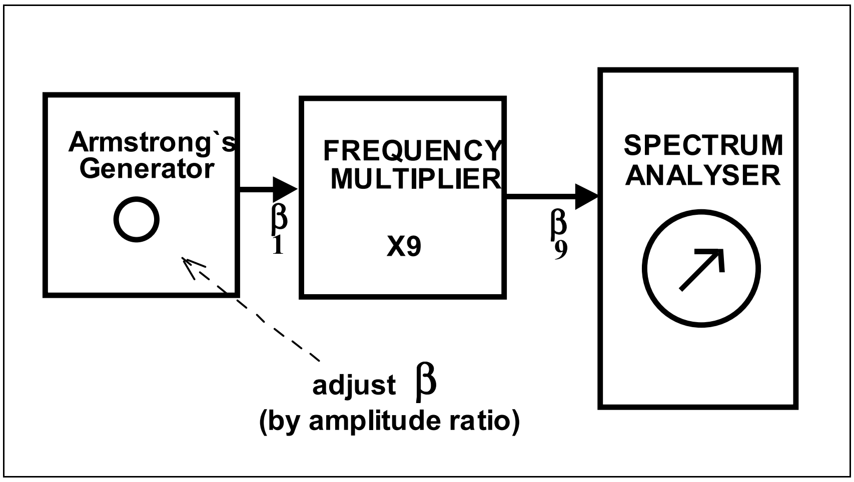


Figure 1: the measurement set-up

EXPERIMENT

The block diagram of Figure 1 is shown modelled in Figure 2 below. This model was examined in the experiment entitled *FM deviation multiplication*, so the setting up details here will be brief.

T1 *set up the model illustrated in Figure 2. Choose a suitable message frequency (this will be below 1 kHz ?). Adjust the phasing in the Armstrong modulator, using the envelope as a guide (see the experiment entitled* ***Armstrong`s phase modulator****).*

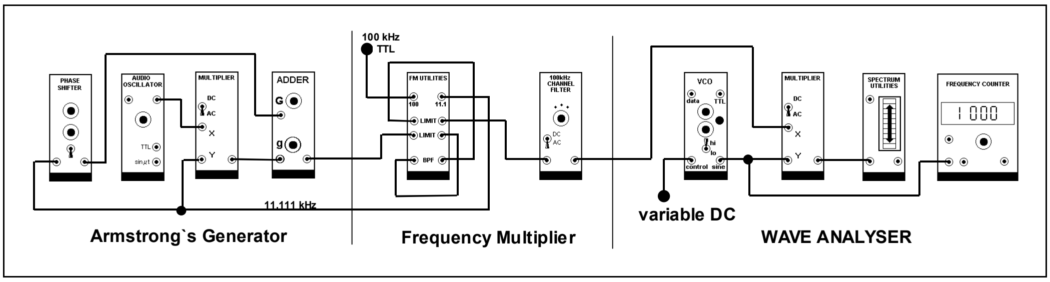


Figure 2: patching details

To ensure the modulator does not introduce spectral distortion, ensure at all times that the phase deviation at the ARMSTRONG MODULATOR is kept well below

1.0 radian. This means that the ratio of DSBSC to carrier, at the ADDER output, must remain less than unity.

Note that the phase deviation at the FREQUENCY MULTIPLIER output will be the Armstrong phase deviation multiplied by a factor of 9.

Let us denote the phase deviation at the ARMSTRONG MODULATOR as β_1 and the phase deviation at the FREQUENCY MULTIPLIER output as β_9, as in Figure 1.

spectral components

You will be using the WAVE ANALYSER to measure the amplitude and frequency of the spectral components of various signals. For this experiment the *absolute* amplitudes of the spectral components are of secondary importance. What will interest you is their *relative amplitudes*. Thus it is not necessary to calibrate the amplitude sensitivity of the WAVE ANALYSER.

locate the 'carrier'

T2 *with β_1 set to zero, locate the unmodulated carrier with the WAVE ANALYSER. It should be at about the TIMS ANALOGUE REFERENCE LEVEL.*

Spectral amplitudes are typically quoted with respect to the amplitude of the unmodulated carrier. Thus it is convenient to set this component to full scale on the measuring equipment. This can be done by tuning to the carrier and manually setting the on-board adjuster RV1, labelled 'SCALING', for this condition.

T3 *adjust the reference signal to full scale deflection on the meter of the SPECTRUM UTILITY module.*

T4 *check that there are no other components of significance within 10 kHz of the carrier.*

T5 *set β_1 to about 0.16 and search for components of significance within 10 kHz of the carrier. Record the frequency and amplitude of all components found.*

The component at the carrier frequency ω, and the components at $(\omega \pm \mu)$, should have been of about equal magnitude.

This fact can be checked by reference to the curves of Figure 3 below. For:

$$\beta_1 = 0.16$$

then

$$\beta_9 = 1.44$$

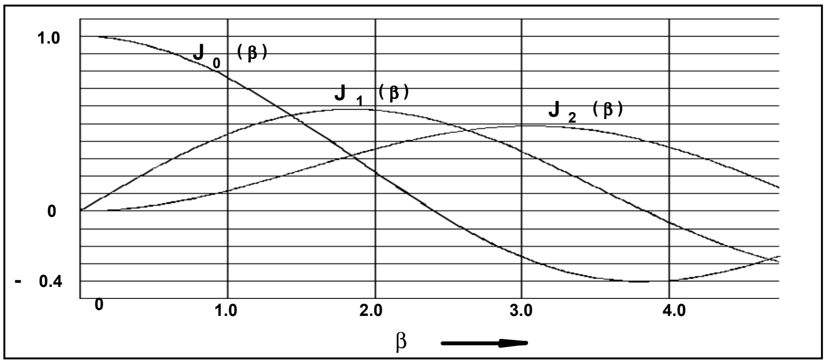


Figure 3: Bessel function plots

You will note that $J_0(\beta)$ is approximately equal to $J_1(\beta)$ when β_9 is about 1.4. In fact they approach equality for $\beta = 1{\cdot}435$.

But the amplitude of the carrier is proportional to $J_0(\beta)$, and that of the first pair of sidebands is proportional to $J_1(\beta)$.

So here is a way of calibrating the ARMSTRONG MODULATOR phase deviation control. Adjust the DSBSC amplitude, at the output of the ADDER in the ARMSTRONG GENERATOR, until these two components are equal.

You have now set

$$\beta_9 = 1.435$$

Note that the method did not involve the measurement of an absolute amplitude, but rather the matching of two amplitudes to equality. So the amplitude sensitivity of the WAVE ANALYSER need not be calibrated.

This amplitude matching method can be applied to determine other values of β_9. From the curves of Figure 3 one could suggest the following pairs:

components	β_9	β_1
carrier and second	1.85	0.21
first and second	2.6	0.29
carrier and second	3.8	0.42

Table 1: equal amplitude sidefrequency pairs

T6 *check some or all of the pairs of sidefrequencies listed in Table 1. These will give other points on the curve of β versus the modulator phase deviation control.*

the method of Bessel zeros

So far the calibration points have been obtained by equating the amplitudes of two spectral components.

There is an even more precise method of obtaining points on the calibration curve. Not only is an absolute amplitude reading not required, but there is only a single measurement to make - and this is a *null* measurement. There is no need for a calibrated instrument.

This is *the method of Bessel zeros*.

Note from Figure 3 that the Bessel functions are oscillatory (but not, incidentally, periodic). In fact they are *damped* oscillatory, which means that successive maxima are monotonically decreasing. But for the moment the important property is that they are oscillatory about zero amplitude, which means that there are values of their argument for which they become zero.

> *There are precise, and multiple values, of β, for which the amplitude of a particular spectral component of an angle modulated signal falls to zero.*

If you can find when the amplitude of a particular spectral component falls to zero, you have a precise measure of β_9, and another point on the calibration curve. It is easier to find a *single* zero, by trimming of the ARMSTRONG MODULATOR phase deviation control [2], than it is to adjust the amplitudes of *two* components to equality.

looking for a Bessel zero

One would normally think of using a WAVE ANALYSER when looking for Bessel zeros. So in the first instance this will be done.

using the WAVE ANALYSER

Table 2 below shows some particular Bessel zeros which you can use experimentally. These can be checked by reference to the curves of Figure 3.

Table 2

Bessel coefficient	side frequency	first zero	second zero
$J_0(\beta)$	central carrier	2.41	5.52
$J_1(\beta)$	first pair	3.83	7.02
$J_2(\beta)$	second pair	5.13	8.4

[2] the ADDER gain control **g** which adjusts the DSBSC amplitude

Each Bessel zero will give a point on the calibration curve.

For a multiplication factor of 9, as you are using, and the Armstrong modulator as the source of the phase deviations, β_9 is restricted to the range 0 to about 3 radians [3]. So only the first carrier zero, and the first sidefrequency pair zero, are available to you. But these are quite sufficient to demonstrate the method.

Note that it is necessary to keep track of *which* zero one is seeking. This is relatively simple when finding the first or second, but care is needed with the higher zeros. The problem will not arise in this experiment.

T7 *whilst monitoring the amplitude of the component at carrier frequency, increase the phase deviation control on the ARMSTRONG MODULATOR from zero until the amplitude is reduced to zero. Measure the amplitude ratio of the DSBSC and carrier at the ADDER output. This should be about (2.4/9.0) = 0·27. Check this expected value against your tables. Explain any disagreement between measured and expected values.*

T8 *with β_9 as for the previous Task, locate either of the first pair of sidefrequencies (100 kHz ± message frequency). Increase the phase deviation control on the ARMSTRONG MODULATOR until the amplitude of the chosen component is reduced to zero. Measure the amplitude ratio of the DSBSC and carrier at the ADDER output. This should be about (3.8/9.0) = 0·42. Explain any disagreement between measured and expected values.*

without a WAVE ANALYSER

In practical engineering one is often (always ?) looking for ways and means of simplifying procedures, and avoiding the use of expensive equipment, especially when in the field.

The Bessel zero method of frequency deviation calibration is extremely precise, but appears to need an expensive SPECTRUM ANALYSER for its execution. But this need not be so, especially if one is content to use only the zeros of the carrier component.

All that is needed is an oscillator close to the carrier frequency, a multiplier, and a pair of headphones.

The principle is that the unmodulated FM signal and the local oscillator are multiplied together, and the difference component of the product monitored with the headphones. The local oscillator is adjusted to give a convenient difference frequency - say about 1 kHz.

Now, while concentrating on this tone, the transmitter frequency deviation is increased from zero. The message should be a single tone. The amplitude of the 1 kHz tone will decrease, until it falls to zero when β is at the first zero of $J_o(\beta)$ - approximately $\beta = 2{\cdot}4$.

It is true that under this condition there will be other tones present in the headphones. But by ‘suitable’ choice of message frequency these will lie above the 1 kHz tone

[3] distortion is discussed in the experiment entitled ***Armstrong`s frequency modulator***.

that is being monitored, and one can, with experience, ignore them. The key to the method lies in choosing a 'suitable' message frequency.

Suffice to say, the method is used with success in practice.

There is no need for a true MULTIPLIER; almost any non-linear device will do, typically an overloaded transistor amplifier which will generate intermodulation products, including the wanted 1 kHz difference component.

T9 *demonstrate the method of setting a Bessel zero, using only a listening device and a non-linear element (there is a rectifier in the UTILITIES module), as described above.*

TUTORIAL QUESTIONS

There are suitable tutorial questions in the Chapter entitled *Analysis of the FM spectrum*.

FM DEMODULATION WITH THE PLL

FM DEMODULATION WITH THE PLL

ACHIEVEMENTS: *introduction to the PLL as an FM demodulator*

PREREQUISITES: *an understanding of the contents of the Chapter entitled* ***Analysis of the FM spectrum*** *(this Volume) is desirable, but not essential. A familiarity with the analysis of a PLL will allow some quantitative measurements to be made and interpreted.*

PREPARATION

the phase locked loop - PLL.

The phase locked loop is a non-linear feedback loop. To analyse its performance to any degree of accuracy is a non-trivial exercise. To illustrate it in simplified block diagram form is a simple matter. See Figure 1.

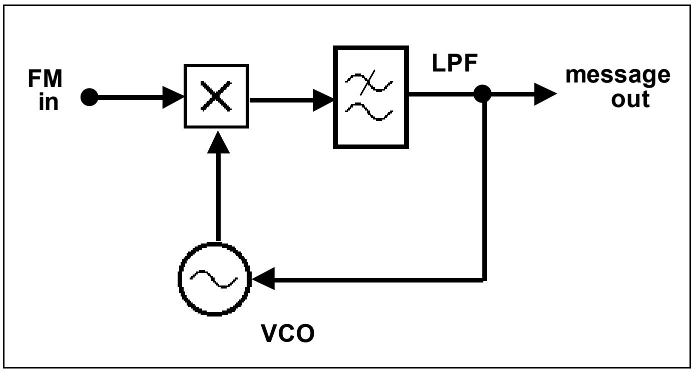


Figure 1: the basic PLL

This arrangement has been used in an earlier experiment (this Volume), namely that entitled *Carrier acquisition and the PLL*, where an output was taken from the VCO. As an FM demodulator, the output is taken from the LPF, as shown.

It is a simple matter to describe the principle of operation of the PLL as a demodulator, but another matter to carry out a detailed analysis of its performance.

It is complicated by the fact that its performance is described by non-linear equations, the solution to which is generally a matter of approximation and compromise.

The principle of operation is simple - or so it would appear. Consider the arrangement of Figure 1 in open loop form. That is, the connection between the filter output and VCO control voltage input is broken.

Suppose there is an unmodulated carrier at the input.

The arrangement is reminiscent of a product, or multiplier-type, demodulator. If the VCO was tuned precisely to the frequency of the incoming carrier, ω_0 say, then the output would be a DC voltage, of magnitude depending on the phase difference between itself and the incoming carrier.

For two angles within the 360^0 range the output would be precisely zero volts DC.

Now suppose the VCO started to drift slowly off in frequency. Depending upon which way it drifted, the output voltage would be a slowly varying AC, which if slow enough looks like a varying amplitude DC. The sign of this DC voltage would depend upon the direction of drift.

Suppose now that the loop of Figure 1 is closed. If the sign of the slowly varying DC voltage, now a VCO *control voltage*, is so arranged that it is in the direction to urge the VCO back to the incoming carrier frequency ω_0, then the VCO would be encouraged to 'lock on' to the incoming carrier.

This is the principle of carrier acquisition. This was examined in the experiment entitled *Carrier acquisition and the PLL*, where this same description was used.

Next suppose that the incoming carrier is frequency modulated. For a low frequency message, and small deviation, you can imagine that the VCO will endeavour to follow the incoming carrier frequency. What about wideband FM ? With 'appropriate design' of the lowpass filter and VCO circuitry the VCO will follow the incoming carrier for this too.

The control voltage to the VCO will endeavour to keep the VCO frequency locked to the incoming carrier, and thus will be an exact copy of the original message.

Rather than attempt to analyse the operation of the arrangement of Figure 1 as a demodulator, you will make a model of it, and demonstrate that it is able to recover the message from an FM signal.

EXPERIMENT

FM demodulation

There is an FM signal at TRUNKS. It is based on a nominal 100 kHz carrier. You will model the PLL, and recover the message from the FM signal.

T1 *make a model of the PLL of Figure 1. Use the RC-LPF in the UTILITIES module. Remember to set up the VCO module in 100 kHz VCO mode. In the first instance set the front panel* GAIN *control to its mid-range position.*

T2 *examine, with your oscilloscope, the FM signal at TRUNKS. Identify those features which suggest it could indeed be an FM signal.*

T3 *connect the FM signal at TRUNKS to the PLL.*

The PLL may or may not at once lock on to the incoming FM signal. This will depend upon several factors, including:

- the frequency to which the PLL is tuned
- the capture range of the PLL
- the PLL loop gain - the setting of the front panel GAIN control of the VCO

You will also need to know what method you will use to verify that lock has taken place.

When you have satisfied yourself that you understand the significance of these considerations then you should proceed.

T4 *make any necessary adjustments to the PLL to obtain lock, and record how this was done. Measure the amplitude and frequency of the recovered message (if periodic), or otherwise describe it (speech or music ?). Are any of these measurements dependent upon the setting of the VCO* GAIN *control ?*

If you are familiar with the analysis of the PLL you should complete the next task.

T5 *measure the properties of each element of the PLL, and then predict some of its properties as a demodulator. If the message was a single tone, from its amplitude can you estimate the frequency deviation of the FM signal ?*

more measurements

If you have two VCO modules you can make your own FM signal. You will then have access to both the original message and the demodulator output. This will allow further measurements.

T6 *set up an FM signal, using a VCO, as described in the experiment entitled* ***Introduction to FM using a VCO****. Use any suitable message frequency, and a frequency deviation of say 5 kHz.*

T7 *compare the waveform and frequency of the message at the transmitter, and the message from the demodulator.*

T8 *check the relationship between the message amplitude at the transmitter, and the message amplitude from the demodulator.*

T9 *as a further confidence check, use the more demanding two-tone signal as a test message. The two tones can come from an AUDIO OSCILLATOR and the 2.033 kHz message from the MASTER SIGNALS module, combined in an ADDER.*

TUTORIAL QUESTIONS

Even if you are unable to complete any of the following questions in detail, you should read them, and a text book on the subject, so as to obtain some appreciation of the behaviour of the PLL, especially as an FM demodulator.

Q1 *define capture range, lock range, demodulation sensitivity of the PLL as an FM demodulator. What other parameters are important ?*

Q2 *how does the sensitivity of the VCO, to the external modulating signal, determine the performance of the demodulator ?*

Q3 *what is the significance of the bandwidth of the LPF in the phase locked loop ?*

THE COSTAS LOOP

THE COSTAS LOOP

ACHIEVEMENTS: *using the Costas loop for carrier acquisition from, and demodulation of, a DSBSC signal.*

PREREQUISITES: *familiarity with the quadrature modulator (as, for example, in the experiment entitled **Phase division multiplex,** in this Volume, would be an advantage.*

ADVANCED MODULES: *BIT CLOCK REGEN and SPECTRUM UTILITIES are both optional.*

EXTRA MODULES: *a total of three MULTIPLIER, two PHASE SHIFTER, and two TUNEABLE LPF modules.*

PREPARATION

the basic loop

Read about the Costas loop in your text book.

This loop, and its variations, is much-used as a method of carrier acquisition (and simultaneous message demodulation) in communication systems, both analog and digital.

It has the property of being able to derive a carrier from the received signal, even when there is no component at carrier frequency present in that signal (eg, DSBSC). The requirement is that the amplitude spectrum of the received signal be symmetrical about this frequency.

The basic Costas loop is illustrated in Figure 1.

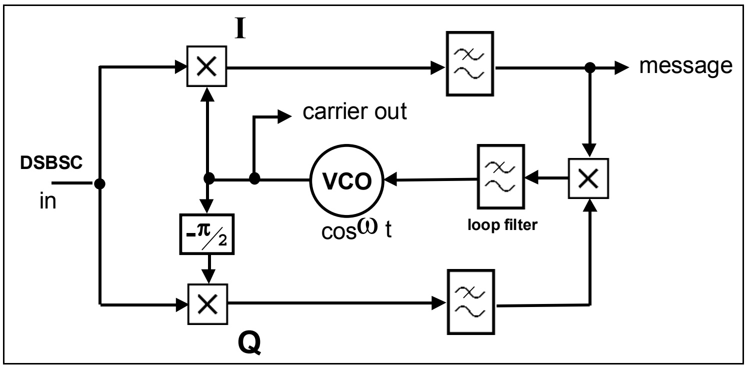


Figure 1: the Costas loop

The Costas loop[1] is based on a pair of quadrature modulators - two multipliers fed with carriers in phase-quadrature. These multipliers are in the in-phase (I) and quadrature phase (Q) arms of the arrangement.

Each of these multipliers is part of separate synchronous demodulators. The outputs of the modulators, after filtering, are multiplied together in a third multiplier, and the lowpass components in this product are used to adjust the phase of the local carrier source - a VCO - with respect to the received signal.

The operation is such as to maximise the output of the I arm, and minimize that from the Q arm. The output of the I arm happens to be the message, and so the Costas loop not only acquires the carrier, but is a (synchronous) demodulator as well.

A complete analysis of this loop is non-trivial. It would include the determination of conditions for stability, and parameters such as lock range, capture range, and so on. A simplified analysis is given in Appendix 1 to this experiment.

phase ambiguity

Although the Costas loop can provide a signal at carrier frequency, there remains a 180^0 phase uncertainty.

A phase ambiguity of 180^0 in many (typically analog) situations is of no consequence - for example, where the message is speech. In digital communications it will give rise to data inversion, and this may not be acceptable - but there are methods to overcome the problem.

See Tutorial Question Q9.

experiment philosophy

In most of the experiments involved with demodulation a stolen carrier is used. This allows full attention to be paid to the performance of the demodulator. Considerations of how to acquire a carrier from the received signal are ignored.

In this experiment, following a similar principle, attention will be paid to the means of acquiring a carrier from a DSBSC signal, without paying attention to the subsequent performance of the device for which the carrier is required (eg, a demodulator).

However, you could combine the two if you like.

measurements

The experiment to follow is described in outline only. It will take you only to the point at which the carrier is acquired.

Thus, before the experiment, you should prepare a list of those performance attributes with which you may be interested, with some suggestions as to how these might be measured.

[1] Costas, J.P. 'Synchronous Communications'. Proc.IRE, **44**, pp1713-1718, Dec.1956

EXPERIMENT

setting up the Costas loop

T1 *obtain a DSBSC signal. There should be one or more at TRUNKS, each based on a carrier at or near 100 kHz. Alternatively, if you have a fourth MULTIPLIER module, you could generate your own.*

For the Costas loop:

1. use TUNEABLE LPF modules for the filters in the I and Q arms. Set them both to their WIDE range, and TUNE them to their widest bandwidth.
2. use the RC LPF in the UTILITIES module to filter the control signal to the VCO (although you might find the LOOP FILTER in a BIT CLOCK REGEN module to be preferable).
3. before patching in the PHASE SHIFTER set the on-board toggle switch to the HI range. Then set it to approximately 90^0 using a 100 kHz sine wave.
4. before inserting the VCO set the on-board FSK/VCO switch to VCO. Select the HI frequency range with the front panel toggle switch.
5. if making your own DSBSC use an AUDIO OSCILLATOR for the message. You will find the loop will lock using any frequency within the tuning range, but for measurement purposes something well above the cut-off of the RC filter may be found more convenient.

T2 *model the Costas loop of Figure 1. A suitable model is shown in Figure 2.*

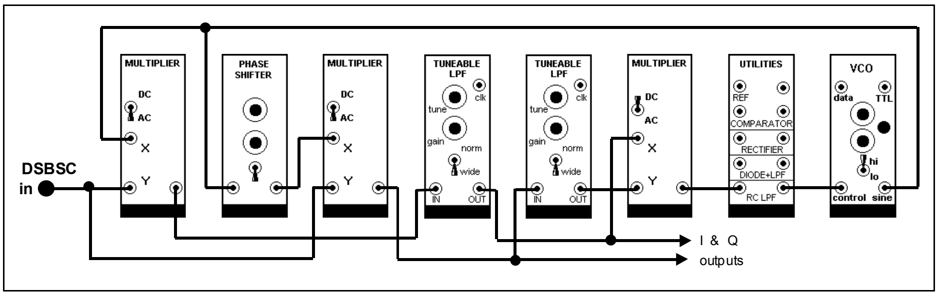


Figure 2: model of the Costas loop of Figure 1

T3 *look for a DSBSC signal at TRUNKS. If there is more than one, select one based on a 100 kHz carrier (**hint**: examine it with one arm of the Costas loop, with a stolen 100 kHz carrier from the TIMS MASTER SIGNALS module).*

T4 *check the amplitudes at all module interfaces. Check the gain of the TUNEABLE LPF modules in the I and Q arms so that the third MULTIPLIER is not overloaded (will the input amplitudes to this module change between the lock and not-locked condition ?).*

It is now time to lock the loop to the carrier of the incoming signal. There are various techniques to be adopted in the laboratory (where a reference carrier is available) while performing the alignment technique described in the next Task. Two of these are:

1. watch the reference carrier and the VCO on two channels of the scope.
2. watch the outputs of the filters in the I and Q arms.

Make your choice. Then:

T5 *synchronise the oscilloscope to either the reference carrier, or the output of the I channel, according to whichever of the above options you have chosen.*

T6 *disable the feedback loop by turning the GAIN of the VCO fully anti-clockwise.*

T7 *tune the VCO to within a few hundred Hertz (preferably less !) of 100 kHz, using the FREQUENCY COUNTER.*

T8 *slowly increase the VCO GAIN until the VCO locks to the DSBSC carrier, as indicated by the oscilloscope traces becoming stationary with respect to each other* ***or*** *by observing that the FREQUENCY COUNTER now reads 100.000 kHz.*

T9 *observe the demodulated output from the filter of the I arm. If lock has been achieved, but the demodulated waveform (the message) is other than sinusoidal, fine tune the VCO while still locked. The frequency won`t change (it is locked to the carrier) but this will result in a 'cleaner' and smaller control signal to the VCO, and a maximum amplitude minimum-distortion demodulated output.*

You will notice that lock is achieved when the VCO GAIN setting is above a certain minimum value. If the gain is increased 'too far', the lock will eventually be lost. From the behaviour of the VCO output signal (or otherwise) during this procedure, can you explain the meaning of 'too far' ?

_T10_ _open and close the connection from the DSBSC signal to the input of the Costas loop, and show that carrier acquisition is lost and regained. Although lock may appear to happen 'instantaneously' it will in fact take a finite number of carrier cycles after the connection is made. Note that the phase difference between the reference and recovered carrier takes one of two values, 180^0 apart. This phase ambiguity of the acquired carrier is associated with many carrier acquisition schemes._

_T11_ _examine the other DSBSC at TRUNKS (if any). If they are not based on a 100 kHz carrier you will have to plan a different approach than the one suggested above. How will you know when lock has been achieved ?_

measurements

There are many measurements and observations that could now be made. This will depend upon the level of your course work.

Of practical interest would be a knowledge of the loop acquisition time under different conditions, lock range, holding range, conditions for stability, and so on. These dynamic measurements require more sophisticated instrumentation than you probably have.

Thus it is suggested that you confine your observations to checking that the loop actually works (already done), and some less sophisticated measurements.

VCO simulation

A technique of interest is to replace the VCO signal with a 'stolen' carrier connected, via a PHASE SHIFTER, into the loop. This simulates the locked VCO [2], and allows static observations of all points of the loop for various values of the phase angle α.

Appendix A to this experiment gives an **exact analysis** of this condition.

In particular, the control signal to the VCO can be monitored.

You are looking for the condition where the magnitude of the control signal is a minimum. This must be the condition when final lock is achieved, since any other value would tend to move the VCO until it was met.

It is best to use a message frequency as high as possible so as not to confuse the measurement of the DC control signal with the unavoidable unwanted terms.

The analysis shows that every time a signal is processed by a multiplier followed by a filter there is an amplitude reduction of the signal under observation of one half due to the analytic process, and a further half due to the 'k factor' of each TIMS

[2] you may not agree with this !

MULTIPLIER module. Squaring the message introduces another reduction of one half.

Remember, then, that you will be looking for quite small signals, especially the DC control to the VCO.

This can be measured very conveniently with the SPECTRUM UTILITIES module. This is a meter which responds to DC or slowly-varying AC. Refer to the *Advanced Modules User Manual* for more details.

Your measurements under these conditions will confirm the predictions of the analysis. You could show that:

1. the message appears at the output of both the I and Q lowpass filters.
2. the AC term, at the output of the third multiplier, before removal by filtering, is at twice the message frequency
3. by adjusting the phase α until the DC from the filter at the output of the 'third' multiplier is reduced to zero,
    a) the I-filter output is maximized
    b) the Q-filter output is minimized (see Tutorial Question Q8)

TUTORIAL QUESTIONS

Q1 *a Costas loop can acquire a carrier from a received signal which itself contains no term at carrier frequency. Describe another scheme which can do this.*

Q2 *what are the required properties of the lowpass filter from which the VCO control signal is output ?*

Q3 *what properties of the Costas loop differentiate it from the phase locked loop ?*

Q4 *do any of the multipliers in a Costas loop need to be DC coupled ?*

Q5 *if you have achieved lock, it will be regained if:*

a) the inputs to the I and Q filters are swapped

b) the outputs from the I and Q arms are swapped

What will happen to the I and Q outputs in each case ?

Q6 *what would happen if the polarity of the control signal to the VCO is reversed ?*

Q7 *would you anticipate any differences in performance if the sinusoidal message was replaced with speech ?*

Q8 *if you used a filter from the 100 kHz CHANNEL FILTERS module to simulate a channel (for added realism) you may have had difficulty in achieving a deep null from the output of the Q-filter. How could this be ?*

Q9 *in a digital communications system the phase ambiguity introduced by a Costas loop for carrier acquisition need not necessarily be unacceptable. For example,*

a) some line codes would not be affected.

b) a training sequence may be used.

Explain.

Q10 *in the block diagram of Figure 1 there is a phase shifter of 90^0 ($\pi/2$). How would the performance of the loop be affected if this was set to 80^0 ?*

APPENDIX A

a simplified analysis

A simplified analysis of the Costas loop (Figure 1) starts by assuming that a stable lock has already been achieved.

This in turn assumes that the VCO is operating at the correct frequency, but that its relative phase is unknown. Call the angle α the phase difference between the received carrier and the VCO.

Let the received DSBSC be derived from the message m(t), and based on a carrier frequency of ω rad/s. This then is also the frequency of the VCO when locked.

The 'k (= ½) factor' of the TIMS MULTIPLIER modules has been included.

Define the signals into the multipliers of the I and Q arms as I and Q. Then:

$$I = m(t).k.\cos\omega t.\cos(\omega t + \alpha) \qquad \text{........ A-1}$$

$$Q = m(t).k.\cos\omega t.\sin(\omega t + \alpha) \qquad \text{........ A-2}$$

Equations (A-1) and (A-2) may be expanded, and only the low frequency terms retained, to obtain the signals from the lowpass filters. These go into the 'third' multiplier. Let these be named I_{LF}: and Q_{LF}. Then:

$$I_{LF} = ½.m(t).k.\cos\alpha \qquad \text{........ A-3}$$

$$Q_{LF} = ½.m(t).k.\sin\alpha \qquad \text{........ A-4}$$

After these are multiplied together, the output of the 'third' multiplier is:

$$\textit{'third' mult out} = ½.¼.m^2(t).k^2.\sin 2\alpha \qquad \text{........ A-5}$$

No matter what the message m(t), the square of it will be positive, and contain a DC component, which can be filtered off.

If the message is a sine wave, and the DSBSC amplitude is unity, then:

$$\textit{filter output} = \frac{1}{16} k^2 \sin 2\alpha . \qquad \text{........ A-6}$$

The DC from the filter has a magnitude which is a function of the phase error α. This DC is the control signal to the VCO. It can change sign, according to the magnitude of α. Providing the loop is stable the tendency will be to shift the phase of the VCO until α is reduced to zero, since only then will the VCO come to rest.

message output

The message appears at the output of each of the I and Q filters. But under lock condition the phase error α will be zero, and eqns. A-3 and A-4 tell us that the message amplitude at the output of the I filter will be maximized, and minimized at the output of the Q filter.

Appendix A
to Volume A2

Tables of Bessel coefficients

Tables of Bessel coefficients

arg => order	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	0.9975	0.9900	0.9776	0.9604	0.9385	0.9120	0.8812	0.8463	0.8075	0.7652
1	0.0499	0.0995	0.1483	0.1960	0.2423	0.2867	0.3290	0.3688	0.4059	0.4401
2	0.0012	0.0050	0.0112	0.0197	0.0306	0.0437	0.0588	0.0758	0.0946	0.1149
3	0.0000	0.0002	0.0006	0.0013	0.0026	0.0044	0.0069	0.0102	0.0144	0.0196
4	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0006	0.0010	0.0016	0.0025
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0	0.7196	0.6711	0.6201	0.5669	0.5118	0.4554	0.3980	0.3400	0.2818	0.2239
1	0.4709	0.4983	0.5220	0.5419	0.5579	0.5699	0.5778	0.5815	0.5812	0.5767
2	0.1366	0.1593	0.1830	0.2074	0.2321	0.2570	0.2817	0.3061	0.3299	0.3528
3	0.0257	0.0329	0.0411	0.0505	0.0610	0.0725	0.0851	0.0988	0.1134	0.1289
4	0.0036	0.0050	0.0068	0.0091	0.0118	0.0150	0.0188	0.0232	0.0283	0.0340
5	0.0004	0.0006	0.0009	0.0013	0.0018	0.0025	0.0033	0.0043	0.0055	0.0070
6	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0005	0.0007	0.0009	0.0012
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00
00	0.1666	0.1104	0.0555	0.0025	-0.0484	-0.0968	-0.1424	-0.1850	-0.2243	-0.2601
1	0.5683	0.5560	0.5399	0.5202	0.4971	0.4708	0.4416	0.4097	0.3754	0.3391
20	0.3746	0.3951	0.4139	0.4310	0.4461	0.4590	0.4696	0.4777	0.4832	0.4861
3	0.1453	0.1623	0.1800	0.1981	0.2166	0.2353	0.2540	0.2727	0.2911	0.3091
4	0.0405	0.0476	0.0556	0.0643	0.0738	0.0840	0.0950	0.1067	0.1190	0.1320
5	0.0088	0.0109	0.0134	0.0162	0.0195	0.0232	0.0274	0.0321	0.0373	0.0430
6	0.0016	0.0021	0.0027	0.0034	0.0042	0.0052	0.0065	0.0079	0.0095	0.0114
7	0.0002	0.0003	0.0004	0.0006	0.0008	0.0010	0.0013	0.0016	0.0020	0.0025
8	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90	4.00
0	-0.2921	-0.3202	-0.3443	-0.3643	-0.3801	-0.3918	-0.3992	-0.4026	-0.4018	-0.3971
1	0.3009	0.2613	0.2207	0.1792	0.1374	0.0955	0.0538	0.0128	-0.0272	-0.0660
2	0.4862	0.4835	0.4780	0.4697	0.4586	0.4448	0.4283	0.4093	0.3879	0.3641
3	0.3264	0.3431	0.3588	0.3734	0.3868	0.3988	0.4092	0.4180	0.4250	0.4302
4	0.1456	0.1597	0.1743	0.1892	0.2044	0.2198	0.2353	0.2507	0.2661	0.2811
5	0.0493	0.0562	0.0637	0.0718	0.0804	0.0897	0.0995	0.1098	0.1207	0.1321
6	0.0136	0.0160	0.0188	0.0219	0.0254	0.0293	0.0336	0.0383	0.0435	0.0491
7	0.0031	0.0038	0.0047	0.0056	0.0067	0.0080	0.0095	0.0112	0.0130	0.0152
8	0.0006	0.0008	0.0010	0.0012	0.0015	0.0019	0.0023	0.0028	0.0034	0.0040
9	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008	0.0009
10	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90	5.00
0	-0.3887	-0.3766	-0.3610	-0.3423	-0.3205	-0.2961	-0.2693	-0.2404	-0.2097	-0.1776
1	-0.1033	-0.1386	-0.1719	-0.2028	-0.2311	-0.2566	-0.2791	-0.2985	-0.3147	-0.3276
2	0.3383	0.3105	0.2811	0.2501	0.2178	0.1846	0.1506	0.1161	0.0813	0.0466
3	0.4333	0.4344	0.4333	0.4301	0.4247	0.4171	0.4072	0.3952	0.3811	0.3648
4	0.2958	0.3100	0.3236	0.3365	0.3484	0.3594	0.3693	0.3780	0.3853	0.3912
5	0.1439	0.1561	0.1687	0.1816	0.1947	0.2080	0.2214	0.2347	0.2480	0.2611
6	0.0552	0.0617	0.0688	0.0763	0.0843	0.0927	0.1017	0.1111	0.1209	0.1310
7	0.0176	0.0202	0.0232	0.0264	0.0300	0.0340	0.0382	0.0429	0.0479	0.0534
8	0.0048	0.0057	0.0067	0.0078	0.0091	0.0106	0.0122	0.0141	0.0161	0.0184
9	0.0011	0.0014	0.0017	0.0020	0.0024	0.0029	0.0034	0.0040	0.0047	0.0055
10	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0010	0.0012	0.0015
11	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0004
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	5.10	5.20	5.30	5.40	5.50	5.60	5.70	5.80	5.90	6.00
0	-0.1443	-0.1103	-0.0758	-0.0412	-0.0068	0.0270	0.0599	0.0917	0.1220	0.1506
1	-0.3371	-0.3432	-0.3460	-0.3453	-0.3414	-0.3343	-0.3241	-0.3110	-0.2951	-0.2767
2	0.0121	-0.0217	-0.0547	-0.0867	-0.1173	-0.1464	-0.1737	-0.1990	-0.2221	-0.2429
3	0.3466	0.3265	0.3046	0.2811	0.2561	0.2298	0.2023	0.1738	0.1446	0.1148
4	0.3956	0.3985	0.3996	0.3991	0.3967	0.3926	0.3866	0.3788	0.3691	0.3576
5	0.2740	0.2865	0.2986	0.3101	0.3209	0.3310	0.3403	0.3486	0.3559	0.3621
6	0.1416	0.1525	0.1637	0.1751	0.1868	0.1986	0.2104	0.2223	0.2341	0.2458
7	0.0592	0.0654	0.0721	0.0791	0.0866	0.0945	0.1027	0.1113	0.1203	0.1296
8	0.0209	0.0237	0.0267	0.0300	0.0337	0.0376	0.0418	0.0464	0.0513	0.0565
9	0.0064	0.0074	0.0086	0.0099	0.0113	0.0129	0.0147	0.0166	0.0188	0.0212
10	0.0017	0.0021	0.0024	0.0029	0.0034	0.0039	0.0045	0.0053	0.0061	0.0070
11	0.0004	0.0005	0.0006	0.0007	0.0009	0.0011	0.0013	0.0015	0.0017	0.0020
12	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0005
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

arg => order	6.10	6.20	6.30	6.40	6.50	6.60	6.70	6.80	6.90	7.00
0	0.1773	0.2017	0.2238	0.2433	0.2601	0.2740	0.2851	0.2931	0.2981	0.3001
1	-0.2559	-0.2329	-0.2081	-0.1816	-0.1538	-0.1250	-0.0953	-0.0652	-0.0349	-0.0047
2	-0.2612	-0.2769	-0.2899	-0.3001	-0.3074	-0.3119	-0.3135	-0.3123	-0.3082	-0.3014
3	0.0846	0.0543	0.0240	-0.0059	-0.0353	-0.0641	-0.0918	-0.1185	-0.1438	-0.1676
4	0.3444	0.3294	0.3128	0.2945	0.2748	0.2537	0.2313	0.2077	0.1832	0.1578
5	0.3671	0.3708	0.3731	0.3741	0.3736	0.3716	0.3680	0.3629	0.3562	0.3479
6	0.2574	0.2686	0.2795	0.2900	0.2999	0.3093	0.3180	0.3259	0.3330	0.3392
7	0.1392	0.1491	0.1592	0.1696	0.1801	0.1908	0.2015	0.2122	0.2230	0.2336
8	0.0621	0.0681	0.0744	0.0810	0.0880	0.0954	0.1031	0.1111	0.1194	0.1280
9	0.0238	0.0266	0.0297	0.0330	0.0366	0.0405	0.0446	0.0491	0.0539	0.0589
10	0.0080	0.0091	0.0104	0.0118	0.0133	0.0150	0.0168	0.0189	0.0211	0.0235
11	0.0024	0.0028	0.0032	0.0037	0.0043	0.0049	0.0057	0.0065	0.0073	0.0083
12	0.0006	0.0008	0.0009	0.0011	0.0013	0.0015	0.0017	0.0020	0.0023	0.0027
13	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008
14	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001