On Interference Alignment in Multi-user OFDM Systems

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Abstract—**Multi-user Orthogonal Frequency Division Multiplexing (OFDM) have been widely adopted to combat the detrimental effects of wireless channels and enhance system throughput. Recently, interference alignment is proposed to exploit interference to enable concurrent transmissions of multiple signals. In this paper, we investigate how to incorporate interference alignment in multi-user OFDM systems. We first reveal the unique characteristics and challenges brought about by using interference alignment in diagonal channels. We then derive a performance bound for the multi-user OFDM/interference alignment system under practical constraints (i.e., a finite number of subcarriers), and show how to achieve this bound with a decomposition approach. The superior performance of the proposed scheme is validated with simulations.**

I. Introduction

The past decade has witnessed drastic increase of wireless data traffic, largely due to the so-called "smartphone revolution." As wireless data traffic is explosively increasing, the capacity of existing and future wireless networks will be greatly stressed. Many advanced wireless communication technologies, such as Orthogonal Frequency Division Multiplexing (OFDM), are widely adopted to enhance system throughput [1], while a huge amount of wireless access networks/base stations (BS) are deployed every year to accommodate the compelling need for more capacity. Given the increasing wireless data volume and the increasingly crowded BS deployment, interference is becoming the major factor that limits wireless network performance.

Traditionally, interference is considered harmful and often treated as background noise. As the performance of point-to-point transmission approaching Shannon capacity, there is now considerable interest on exploiting interference for further capacity gains. It is shown that when interference is large, it can be decoded and canceled from the mixed signal (i.e., *interference cancellation*), while when interference is comparable, *interference alignment* can be adopted to enable concurrent transmissions. Although interference is harmful in many cases, it could be beneficial for enhancing system capacity as long as the interference can be aligned.

Interference alignment was proposed in a seminal work [2], and the feasibility condition was investigated in [3]. Since in a large network, there are many users but limited dimensionality, the authors in [4] proposed the "best-effort" interference alignment, and adopted an iterative algorithm to optimize its performance. Although much understandings are gained, the problem of employing interference alignment to enhance the throughput of practical OFDM systems was not fully addressed. An interesting study was presented in [5] on the problem of interference alignment in multi-carrier interference networks. But it is not clear if the approach can be extended to the general case of a large number of subcarriers, such as thousands of subcarriers used in a typical OFDM system [1].

In this paper, we consider the problem of incorporating interference alignment in multi-user OFDM systems. We first examine the fundamental characteristics and practical constraints on adopting interference alignment in a multi-user OFDM system. We show that, for a K user N subcarrier OFDM system, $KN/2$ concurrent transmissions that is achievable for generic structureless channels [2], cannot be achieved for a practical multi-user OFDM network with diagonal channels and a limited number of subcarriers. We then investigate an effective scheme to exploit interference in multi-user OFDM systems. With an integer programming problem formulation, we derive the maximum efficiency of a Multi-user OFDM system with interference alignment under the practical constraint of a limited number of subcarriers. We also show how to achieve the maximum efficiency with a *decomposition approach*, and derive closed-form precoding and decoding matrices for a three-user system. The impact of large channel gain variances on the interference alignment performance is also addressed with an effective solution. The proposed scheme and its enhancement are evaluated with simulations and their superior performance is validated.

The rest of this paper is organized as follows. Section II describes the background and preliminaries. Section III analyzes the multi-user OFDM system with interference alignment. Simulation results are presented in Section IV. Section V concludes the paper.

Notation: in this paper, a capital bold symbol (e.g., $\mathbf{H}$) denotes a matrix, a lower case symbol with an arrow on top (e.g., $\vec{v}$) denotes a vector, and a lower case letter (e.g., v) denotes a scalar. For matrix operations, $[\cdot]^T$ means *transpose* and $[\cdot]^{-1}$ means *inversion*. $\mathbf{H}_{ij}$ and h_{ij} are the *channel gain matrix* and channel gain from the i-th transmitter to the j-th receiver, respectively. $\mathbf{V}_i$ is the *precoding matrix* for transmitter i; $\vec{v}_i^j$ is the j-th column of $\mathbf{V}_i$. $\mathbf{U}_i$ denotes the *interference cancellation matrix* for the i-th receiver, while $\vec{u}_i^j$ is the j-th column of $\mathbf{U}_i$. Let h, v, u denote the entries of $\mathbf{H}$, $\mathbf{V}$, and $\mathbf{U}$, respectively. Note that with our notation, $\mathbf{H}_{ij}$ is the transposed version of the conventional one.

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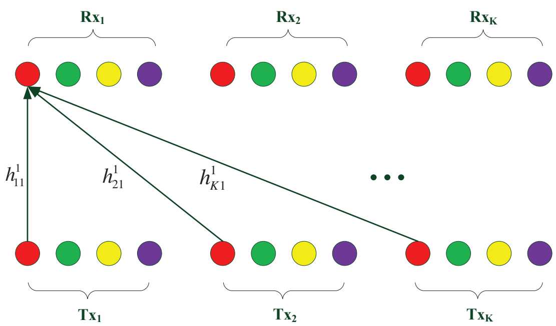

Fig. 1. Multi-user OFDM using interference alignment.

II. Background and Preliminaries

A. Orthogonal Frequency Division Multiplexing

While higher data rates can be achieved by reducing symbol duration, severe inter-symbol-interferences (ISI) will be caused over time dispersive channels. OFDM is an effective approach to combat the destructive effect of channel and to allow transmissions at a high data rate. By dividing the channel into multiple narrow bands, in each of which the signal experiences flat fading, OFDM could effectively mitigate ISI and maintain high data rate transmissions. Interested reader are referred to [1] and the references therein for details.

B. Interference Alignment

It is shown in [2] that in a K-user wireless network, with $(n+1)^q + n^q$ *symbol extensions*, totally $K/2$ normalized *degrees of freedom* (DoF) can be achieved using interference alignment, where $q = (K-1)(K-2)-1$ and $n \in \mathbb{N}$. In single antenna systems, the normalized DoF is 1. With interference alignment, the system throughput is enhanced by a factor of $K/2$ for $K \geq 2$. Note that there is no interference if there is only one user occupying the time or frequency resource.

Observation 1. *The system throughput could be improved if alignable interference is introduced among users.*

This observation is useful for OFDM systems, where the channel gain matrix is *diagonal*. Since the gain of interference alignment is proportional to K, we should have more users transmit in the same time slot or frequency band if the transmitted vectors can be aligned. That is why we call this kind of interference *beneficial interference* in this paper.

III. Multi-user OFDM with Interference Alignment

In this section, we investigate the problem of interference alignment in multi-user OFDM systems. We first examine fundamental characteristics and practical constraints, and then demonstrate how to exploit interference in multi-user OFDM systems. We derive the maximum DoF with interference alignment, as well as closed-form precoding and interference cancellation matrices to achieve the maximum DoF. Finally, we address the impact of large channel gain variances on interference alignment and present an effective solution.

A. Subcarriers versus Antennas

In traditional interference alignment, deploying multiple transmitting antennas allows us to precode data packets and align them at the receiver. Deploying multiple receiving antennas provides a multidimensional signal space, so that interference can be aligned into a sub-space that is orthogonal to the desired signal. Therefore, using multiple antennas can provide the needed dimensions of freedom in the signal space.

In OFDM, we observe that subcarriers can be exploited to function in the same way as antennas in MIMO/interference alignment systems. To some extent, subcarriers can be regarded as a counterpart of antennas. However, there is a distinguishing difference between the two systems: *there is no cross-talk among different subcarriers in OFDM.*

B. Precoding in OFDM

The main idea of interference alignment is to compress the interference space to no more than half of the total received signal space at each receiver, leaving the remaining part of the space for desired signals [2]. This goal is achieved through precoding at every transmitter and zero-forcing interference cancellation at every receiver.

In OFDM, data is transmitted on multiple carriers, as shown in Fig. 1. Suppose there are N subcarriers. Ignoring noise, the received signal $\vec{y}$ is an $N \times 1$ vector given by

$$\vec{y} = \mathbf{H}\vec{x}, \tag{1}$$

where $\vec{x}$ is the desired signal in the form of an $N \times 1$ vector, and $\mathbf{H}$ is the $N \times N$ channel gain matrix between the transmitter and receiver. Since different subcarriers are on different frequency bands, the channel gain matrix is *diagonal* if there is no severe frequency shift. It can be seen from later discussions that this property makes interference alignment in OFDM system quite different from the general channel case.

Going one step further, we can precode data before transmission. If d packets are to be transmitted in an N subcarrier OFDM system, an $N \times d$ *precoding matrix* $\mathbf{V}$ should be used. The system equation with precoding becomes

$$\vec{y} = \mathbf{H}\mathbf{V}\vec{x}. \tag{2}$$

If we let $d = N$ and $\mathbf{V} = \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix, (2) can be reduced to (1).

In general, we could control what to be transmitted on the subcarriers by adjusting the precoding vector accordingly. For a single user single antenna OFDM system with N subcarriers, the maximum number of packets can be transmitted is N. Note here N is normalized by the QAM (Quadrature Amplitude Modulation) modulation level. However, inspired by the idea of interference alignment, we show that a throughput higher than N can be achieved in the following subsections.

C. Interference Alignment in a Multiuser OFDM System

We assume effective channel estimation and suitable feedback channels for obtaining the channel gains. To incorporate interference alignment in a multiuser OFDM system, we need to answer the following questions.

1) What are the practical constraints for adopting interference alignment in multi-user OFDM systems?
2) What is the maximum throughput that can be achieved?
3) How to achieve the maximum throughput?

1) Dependence of Precoding and Decoding Vectors: We first show the difference on adopting interference alignment between a diagonal and a general channel, as well as the challenges to use interference alignment in the former case.

It was shown in [3] that given M_1 transmitting antennas and M_2 receiving antennas in a K user interference channel, the DoF for each user, denoted by d, must satisfy

$$d \leq (M_1 + M_2)/(K+1). \tag{3}$$

For example, given two transmitting antennas and two receiving antennas in a three-user interference channel, Eqn. (3) indicates that each user could transmit one packet simultaneously. With a generic structureless channel, the throughput $Kd = 3$ can be achieved as follows.

At each receiver, we align the signals from the other two users for decoding the desired signal. It follows that

$$\mathbf{H}_{21}\vec{v}_2 = \mathbf{H}_{31}\vec{v}_3, \;\; \mathbf{H}_{12}\vec{v}_1 = \mathbf{H}_{32}\vec{v}_3, \;\; \mathbf{H}_{13}\vec{v}_1 = \mathbf{H}_{23}\vec{v}_2. \tag{4}$$

Solving the equations in (4), we have

$$\vec{v}_1 = \mathrm{eig}(\mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{31}^{-1}\mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}) \tag{5}$$

$$\vec{v}_2 = \mathbf{H}_{23}^{-1}\mathbf{H}_{13}\vec{v}_1 \;\; \text{and} \;\; \vec{v}_3 = \mathbf{H}_{32}^{-1}\mathbf{H}_{12}\vec{v}_1, \tag{6}$$

where eig($\mathbf{A}$) stands for the eigenvector of matrix $\mathbf{A}$.

This scheme works well for generic structureless channels, but not for the case of diagonal channels. For instance, if two subcarriers (instead of two antennas) are used in OFDM, all the channel gain matrices in (5) and (6) are diagonal. Since the product of diagonal matrices is still diagonal, we have from (5) that either $\vec{v}_1 = [1,0]^T$ or $\vec{v}_1 = [0,1]^T$. If $\vec{v}_1 = [1,0]^T$, we derive $\vec{v}_2 = [a,0]^T$ and $\vec{v}_3 = [b,0]^T$ from (6), where a and b are scalars. To cancel the interference at receiver 1, the cancellation vector $\vec{u}_1$ must assume the form $\vec{u}_1 = [0,c]^T$, where c is a scalar. However, the desired packet is also canceled since $\vec{u}_1$ is orthogonal to $\vec{v}_1$. Therefore, we cannot simultaneously transmit three packets in this system.

The reason behind is that for a diagonal channel, its eigenvectors have only one nonzero entry. If we align interferences at receiver r by letting $\mathbf{H}_{jr}\vec{v}_j = \cdots = \mathbf{H}_{ir}\vec{v}_i$, for $j \neq \cdots \neq i \neq r$, the precoding vectors are dependent to each other. Consequently, when interference is canceled at a receiver, the desired packet will also be canceled.

2) Performance Bound: It is shown in [2] that in a K-user system with $(n+1)^q + n^q$ *symbol extension*, totally $K/2$ normalized DoF can be achieved using interference alignment, where $q = (K-1)(K-2) - 1$ and $n \in \mathbb{N}$. In light of this result, one may think that $KN/2$ concurrent transmissions is achievable in a K-User, N subcarrier OFDM system. However, we will show that this is unachievable for large K in practical systems in the following.

It is worth noting that an assumption made in [2] is that symbol extension can be infinitely large. This assumption may not hold true in practical systems. Given a finite bandwidth, the number of subcarriers is the bandwidth divided by the subcarrier spacing. Typically, the value of subcarrier spacing is $10-20$ KHz. Then even for a 100 MHz bandwidth, we can have at most 10^4 subcarriers. For instance, in 802.16m and LTE, the maximum number of IFFT is 2,048, and the maximum number of effective subcarriers is 1,200.

TABLE I
SYSTEM EFFICIENCY

n	*No. of subcarriers*	*No. of packets*	*Normalized DoF d*
When $K=5$ and $q=11$			
1	2,049	2,052	1.002
2	179,195	185,339	1.03
When $K=4$ and $q=5$			
1	33	35	1.06
2	275	339	1.23
3	1,267	1,753	1.38
4	4,149	6,197	1.49
When $K=3$ and $q=1$			
1	3	4	1.333
2	5	7	1.40
3	7	10	1.429
4	9	13	1.444
100	201	301	1.498
1000	2001	3001	1.4998

Therefore, the problem is to maximize system throughput given a finite number of subcarriers, denoted by N_{max}. It is shown in [2] that with $(n+1)^q + n^q$ symbol extensions, the total normalized DoF is $[(n+1)^q+(K-1)n^q]/[(n+1)^q+n^q]$.[1] We aim to maximize the numerator $(n+1)^q + (K-1)n^q$ and have the following formulation.

$$\max_{n,K} \quad (n+1)^q + (K-1)n^q \tag{7}$$

$$s.t. \quad (n+1)^q + n^q \leq N_{max}, \;\; n \in \mathbb{N} \tag{8}$$

$$K \geq 3, K \in \mathbb{N}, \tag{9}$$

where $q = (K-1)(K-2) - 1$. This is an integer non-linear programming problem since all the variables are integers. Constraint (8) indicates that for practical systems, the number of subcarriers $N = (n+1)^q + n^q$ is upper bounded by N_{max}, a system parameter. Although this integer programming problem is NP-hard, by careful inspection, we can find the solution under practical constraints.

In particular, we find the feasible region is very small for practical N_{max} values. Also the objective value is monotone increasing with respect to n and K. In problem (7), we have $q = 11$ when $K = 5$. For each value of n, we can derive the number of subcarriers needed, N_{max}, from (8) for the problem to be feasible, as well as the total DoF of the system (i.e., the objective value (7)). The corresponding normalized DoF d is the ratio of the total DoF and the number of subcarriers required. These numbers are presented in Table I.

Table I shows that if there are $K = 5$ users, $2,049$ and $179,195$ subcarriers are needed when $n = 1$ and $n = 2$, respectively. As discussed, a practical system usually do not

[1] $\sup_n[(n+1)^q + (K-1)n^q]/[(n+1)^q + n^q] = K/2$ as shown in [2].

have more than 10^4 subcarriers. So n can only be 1 in this case, with efficiency $d_{max} = 1.002$. Therefore, interference alignment is not useful in this case, since we can simply allow only one user to transmit over one time-slot or a particular frequency band to get $d = 1$ (i.e., single user OFDM).

If there are $K = 6$ transmitters, we have $q = 19$. Even if $n = 1$, the number of subcarriers needed is $524,289$, which is not feasible for practical systems. Since the number of subcarriers $(n+1)^{(K-1)(K-2)-1} + n^{(K-1)(K-2)-1}$ grows exponentially with $(K^2 - 3K + 1)$, it can be readily concluded that K cannot be more than 4 for interference alignment to be beneficial in multi-user OFDM systems.

Since the objective value of (7) is an monotone increasing function of K, the maximum feasible value $K = 4$ is of particular interest. We have $q = 5$ when $K = 4$. Table I also shows that under this condition, the maximum efficiency for practical system is $d_{max} = 1.38$ for the practical case with at most $2,000$ subcarriers. When $K = 3$, we have $q = 1$. The objective function (7) becomes $3n + 1$, and the constraint (8) becomes $2n + 1 \leq N_{max}$. If the maximum number of subcarriers is $N_{max} = 2,001$, the system achieves its maximum efficiency $d_{max} = 1.4998$. These analysis can be summarized in the following theorem.

Theorem 1. *For a practical multi-user OFDM system with number of subcarriers less than* $2,002$*, the maximum efficient is* $d_{max} = 1.4998$*, which is achieved when there are* $K = 3$ *users using* $N = 2,001$ *subcarriers.*

3) Achieving the Maximum Throughput: It is shown in [2] how to design the precoding matrices to transmit $3n+1$ packets over $2n + 1$ symbol extensions in a three-user interference channel (i.e., for a three-user system, we have $q = 1$ and $N = (n + 1)^q + n^q = 2n + 1$). We will derive the precoding and decoding procedure for interference alignment with multi-user OFDM and prove its efficacy in this section.

The precoding matrices proposed in [2] for the case of three users are as follows.

$$\mathbf{V}_1 = \mathbf{A},\ \mathbf{V}_2 = \mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C},\ \mathbf{V}_3 = \mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B}, \tag{10}$$

where $\mathbf{A} = [\vec{w}\ \mathbf{T}\vec{w}\ \mathbf{T}^2\vec{w}\ \cdots\ \mathbf{T}^n\vec{w}]$, $\mathbf{B} = [\mathbf{T}\vec{w}\ \mathbf{T}^2\vec{w}\ \cdots\ \mathbf{T}^n\vec{w}]$, $\mathbf{C} = [\vec{w}\ \mathbf{T}\vec{w}\ \mathbf{T}^2\vec{w}\ \cdots\ \mathbf{T}^{n-1}\vec{w}]$, $\mathbf{T} = \mathbf{H}_{21}\mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{H}_{31}^{-1}$, and $\vec{w} = [1\ 1\ \cdots\ 1]^T$. The received signal at receiver 1 is:

$$\vec{y}_1 = \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{31}\mathbf{V}_3\vec{x}_3. \tag{11}$$

In the general channel case, since the data streams are independent of each other, the received mixed signal spans $3n + 1$ dimensions of the space. In interference alignment with multi-user OFDM, the received signal spans only $2n+1$ dimensions of the space. Solving these $2n+1$ equations would yield the desired packets. However, the challenge is, if $2n+1$ is too large, we may not be able to effectively solve these equations (as can be seen in Section III-C4). This problem can be addressed with the following theorem.

Theorem 2. *For an* N *subcarrier OFDM system, we can divide the subcarriers into* $\lfloor N/(2n+1) \rfloor$ *groups, where* $n \in \mathbb{N}$*, and precode and decode the groups separately to achieve the interference alignment gain.*

Proof: Recall that the channel gain matrix in OFDM is diagonal. Generally, if every user tries to transmit d packets over the N subcarriers, we have

$$\mathbf{HV} = \begin{pmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_N \end{pmatrix} \begin{pmatrix} v_{11} & \cdots & v_{1d} \\ v_{21} & \cdots & v_{2d} \\ \vdots & \ddots & \vdots \\ v_{N1} & \cdots & v_{Nd} \end{pmatrix}.$$

The precoding vectors must satisfy the conditions given in (10). Let the precoding matrix assume the following form

$$\mathbf{V} = \mathrm{diag}\{\widetilde{\mathbf{V}}_1, \widetilde{\mathbf{V}}_2, \cdots, \widetilde{\mathbf{V}}_g\}, \tag{12}$$

where $g = N/(2n+1)$ is the number of groups and $\widetilde{\mathbf{V}}_i$ is the precoding matrix for group i with dimensions $(2n+1) \times (n+1)$ or $(2n+1) \times n$ (i.e., user 1 sends $(n+1)$ packets, and each of the other users sends n packets over $(2n+1)$ subcarriers.) Without loss of generality, we assume N is dividable by $2n+1$. Rewriting $\mathbf{H}$ in the form of multiple diagonal sub-matrices with the same dimensions, we have

$$\mathbf{HV} = \mathrm{diag}\{\widetilde{\mathbf{H}}_1\widetilde{\mathbf{V}}_1, \widetilde{\mathbf{H}}_2\widetilde{\mathbf{V}}_2, \cdots, \widetilde{\mathbf{H}}_g\widetilde{\mathbf{V}}_g\}. \tag{13}$$

For instance, when $N = 6$ and $n = 1$, we have for transmitter 1

$$\mathbf{HV} = \begin{pmatrix} h_1v_{11} & h_1v_{12} & 0 & 0 \\ h_2v_{21} & h_2v_{22} & 0 & 0 \\ h_3v_{31} & h_3v_{32} & 0 & 0 \\ 0 & 0 & h_4v_{41} & h_4v_{42} \\ 0 & 0 & h_5v_{51} & h_5v_{52} \\ 0 & 0 & h_6v_{61} & h_6v_{62} \end{pmatrix}. \tag{14}$$

If there are three users, we can let $\mathbf{H}_{21}\mathbf{V}_2 = \mathbf{H}_{31}\mathbf{V}_3$ at receiver 1 to have

$$\begin{pmatrix} h_{21}^1v_2^1 & 0 & \cdots & 0 \\ h_{21}^2v_2^2 & 0 & \cdots & 0 \\ h_{21}^3v_2^3 & 0 & \cdots & 0 \\ 0 & h_{21}^4v_2^4 & \cdots & 0 \\ 0 & h_{21}^5v_2^5 & \cdots & 0 \\ 0 & h_{21}^6v_2^6 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{21}^{N-2}v_2^{N-2} \\ 0 & 0 & \cdots & h_{21}^{N-1}v_2^{N-1} \\ 0 & 0 & \cdots & h_{21}^Nv_2^N \end{pmatrix}$$

$$= \begin{pmatrix} h_{31}^1v_3^1 & 0 & \cdots & 0 \\ h_{31}^2v_3^2 & 0 & \cdots & 0 \\ h_{31}^3v_3^3 & 0 & \cdots & 0 \\ 0 & h_{31}^4v_3^4 & \cdots & 0 \\ 0 & h_{31}^5v_3^5 & \cdots & 0 \\ 0 & h_{31}^6v_3^6 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{31}^{N-2}v_3^{N-2} \\ 0 & 0 & \cdots & h_{31}^{N-1}v_3^{N-1} \\ 0 & 0 & \cdots & h_{31}^Nv_3^N \end{pmatrix},$$

which leads to

$$\begin{pmatrix} h_{21}^{i} v_2^{i} \\ h_{21}^{i+1} v_2^{i+1} \\ h_{21}^{i+2} v_2^{i+2} \end{pmatrix} = \begin{pmatrix} h_{31}^{i} v_3^{i} \\ h_{31}^{i+1} v_3^{i+1} \\ h_{31}^{i+2} v_3^{i+2} \end{pmatrix}, \ i = 1, 4, \cdots, N-2. \quad (15)$$

Since the above conditions can also be obtained by separately encoding the $N/(2n+1)$ groups of subcarriers, we could decompose the problem into a number of subproblems, one for each group, and precode and decode the groups separately.

It remains to show how to decode the packets for this scheme. Without loss of generality, we also assume $K = 3$ as in [2]. If the above decomposition approach is adopted, each time we sequentially take out $2n+1$ subcarriers. The received signal at receiver 1 is

$$\begin{aligned} \vec{y}_1 &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{31}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{T}\mathbf{C}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}(\vec{x}_2 + \vec{x}_3) \\ &= (\mathbf{H}_{11}\mathbf{V}_1 \ \mathbf{H}_{21}\mathbf{V}_2) \cdot \\ &\quad \left(x_1^1, \cdots x_1^{n+1}, x_2^1 + x_3^1, \cdots, x_2^n + x_3^n\right)^T. \qquad (16) \end{aligned}$$

Taking the inverse of matrix $(\mathbf{H}_{11}\mathbf{V}_1 \ \mathbf{H}_{21}\mathbf{V}_2)$ and discard the packets from transmitters 2 and 3, we can recover the desired packets $\vec{x}_1$. Note that we exploit the *commutative* property of diagonal matrices in (16).

At receiver 2, the received signal is:

$$\begin{aligned} \vec{y}_2 &= \mathbf{H}_{12}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{32}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{12}(\vec{w} \ \mathbf{B})\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= \mathbf{H}_{12}\vec{w}x_1^1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{12}\mathbf{B} \times \\ &\quad \left(x_1^2 + x_3^1, \cdots, x_1^{n+1} + x_3^n\right)^T \\ &= (\mathbf{H}_{22}\mathbf{V}_2 \ \mathbf{H}_{12}\vec{w} \ \mathbf{H}_{12}\mathbf{B}) \cdot \\ &\quad \left(x_2^1, \cdots, x_2^n, x_1^1, x_1^2 + x_3^1, \cdots, x_1^{n+1} + x_3^n\right)^T. \ (17) \end{aligned}$$

Taking the inverse of matrix $(\mathbf{H}_{22}\mathbf{V}_2 \ \mathbf{H}_{12}\vec{w} \ \mathbf{H}_{12}\mathbf{B})$, we get $\vec{x}_2$.

At receiver 3, the received signal is:

$$\begin{aligned} \vec{y}_3 &= \mathbf{H}_{13}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{23}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{13}(\mathbf{C} \ \mathbf{T}\vec{w})\vec{x}_1 + \mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{13}\mathbf{C}\left(x_1^1 + x_2^1, \cdots, x_1^n + x_2^n\right)^T + \\ &\quad \mathbf{H}_{13}\mathbf{T}^n\vec{w}x_1^{n+1} + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= (\mathbf{H}_{33}\mathbf{V}_3 \ \mathbf{H}_{13}\mathbf{C} \ \mathbf{H}_{13}\mathbf{T}^n\vec{w}) \cdot \\ &\quad \left(x_3^1, \cdots, x_3^n, x_1^1 + x_2^1, \cdots, x_1^n + x_2^n, x_1^{n+1}\right)^T. \ (18) \end{aligned}$$

Taking the inverse of matrix $(\mathbf{H}_{33}\mathbf{V}_3 \ \mathbf{H}_{13}\mathbf{C} \ \mathbf{H}_{13}\mathbf{T}^n\vec{w})$, we can decode $\vec{x}_3$. After decoding each group separately, we then combine the decoded data. The theorem is thus proved. ■

The proof of Theorem 2 also leads to an algorithm to achieve interference alignment gains for any large $N \in \mathbb{N}$.

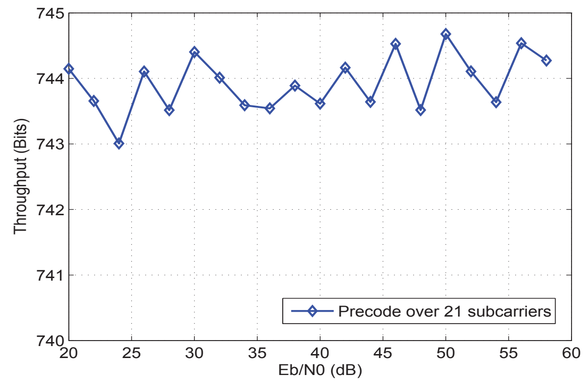

Fig. 2. System throughput with groups of 21 subcarriers (i.e., when $n = 10$).

4) Practical Issue of Large Channel Variance: We next examine another practical issue of adopting interference alignment in multi-user OFDM, and propose an effective solution.

A necessary condition to achieve interference alignment in OFDM is that the channel gain is drawn from a continuous distribution. If the variance of the channel is large, some of the channel gains can be very small, while some others can be very large in certain conditions. When precoding over all the subcarriers, the consequence is two-fold: (i) Since some of the entries can be extremely small, the decoding matrices can be close to singular. Thus the desired signal cannot be decoded. (ii) Even if the matrices are invertible, some entry of the precoding matrix could be much larger than some others. Consequently, the power of one subcarrier could be much larger than that of another. Due to the transmitter power constraint, the system error performance could be rather poor.

To this end, the proposed decomposition approach is helpful since the matrix sizes can be greatly reduced. However, there is still a certain chance that some entries of $\mathbf{T}$ can be much larger than others (see (10)), when the channel variance is large. If we precode and decode over large n, since the last column of $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ are all obtained by multiplying $\mathbf{T}^n$, the situation could be further exacerbated. Even if $n = 1$ (i.e., groups of three subcarriers), there is still a non-negligible chance that some matrices are not invertible. These are the reasons why we cannot precode and decode for large N.

To address the first issue, we can adopt the proposed decomposition theorem and choose the group size to be three by letting $n = 1$. To overcome the second obstacle, we can replace the extremely large numbers in the precoding matrices with small numbers. The reason is that if one entry of the precoding matrix is extremely large, one of the user's corresponding subcarrier must be experiencing deep fading. As a result, on one hand it would be very hard to detect the signal at its desired receiver; on the other hand, the subcarrier in deep fading causes less interference to other receivers. Rather than detecting the signal painstakingly, a better strategy is to simply abandon it for the sake of overall network performance. Although perfect alignment is not achieved in this case, the system performance could still be greatly enhanced under such practical scenarios. Also note that, in the proposed solution,

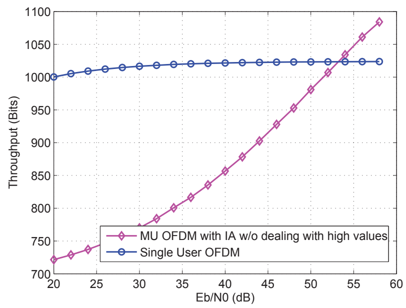

Fig. 3. System throughput without dealing the high value in precoding matrices.

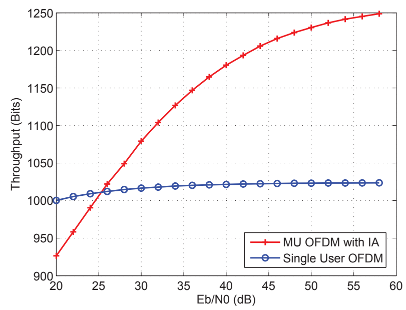

Fig. 4. System throughput comparison when the channel variance is large.

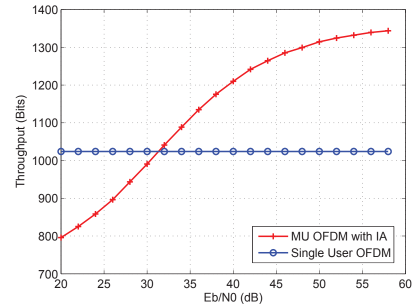

Fig. 5. System throughput comparison when the channel variance is small.

although $d = 4/3 = 1.33$ is slightly less than 1.38,[2] the computational complexity is greatly reduced.

IV. Simulation

Simulations are conducted to evaluate the performance of the proposed scheme and verify the benefits brought about by incorporating interference alignment in multi-user OFDM systems. We consider the case of three users. The number of subcarriers is 1,024. Block fading channels are used in the simulations, where channel gains are piece-wise constants for one or more time slots drawn from a certain distribution. BPSK is used as the modulation scheme.

In Fig. 2, we present the system throughput when $n = 10$ and the subcarrier group size is 21 under large channel gain variances. The enhancement described in Section III-C4 is not used in this simulation. It can be seen that some matrices cannot be inverted. The system performance is unstable and poor. As discussed, even if we choose $n = 1$ with group size three, there is still a non-negligible chance that some matrices are not invertible. In the following simulations, we set $n = 1$.

In Fig. 3, we compare the proposed scheme with single user OFDM when the channel variance is big. Again, the enhancement described in Section III-C4 is not used in this simulation. We find that if we do not deal with the entries whose values are extremely high, the system performance is degraded. For instance, when $E_b/N_0 = 30$ dB, the throughput of the proposed scheme is less than 800 bits, while the throughput of single user OFDM is around 1,010 bits with the same E_b/N_0.

Therefore, in the following two simulations, we adopt the enhancement described in Section III-C4. A threshold λ is chosen. Each time when we decode the three subcarriers, if an entry of the precoding matrix is λ times greater than the minimum of all the entries, we replace the entry with a smaller number (i.e., $+1$ if the entry is positive and -1 if it is negative). It can be seen in Fig. 4 that our proposed scheme works quite well and enhances the system throughput by a factor of up to 1.25 when the channel variance is big.

[2]For a three-user system, we have $q = 1$. If $n = 1$, the number of subcarriers needed is $(n+1)^q + n^q = 3$. The number of packets transmitted is $(n+1)^q + (K-1)n^q = 4$. Then we have $d = (4 \times g)/(3 \times g) = 1.33$, where g is the number of groups.

When the channel variance is small, it can be seen in Fig. 5 that the system throughput is further enhanced. The normalized DoF is about $d = 1.31$ in this scenario. This number is slightly less than 1.33 as derived in Section III-C4, since when picking λ, we destroyed the orthogonality between the desired signal and the interference. Another cause of the discrepancy between the analysis and simulation results is noise, since noise is not considered in the interference alignment analysis.

V. Conclusions

In this paper, we investigated how to enhance multi-user OFDM system throughput by exploiting interference. We first examine the practical constraints and challenges of incorporating interference alignment in multi-user OFDM systems. With an integer programming formulation, we then derive the maximum efficiency for multi-user OFDM/interference alignment systems, and show how to achieve the maximum efficiency under practical constraints. The practical issue of large variance in the channel gain is addressed. The performance of the proposed scheme is validated with simulations.

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