Additive Cancellation Signal Method for Sidelobe Suppression in NC-OFDM Based Cognitive Radio Systems

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***Abstract*—In this paper, we propose a novel additive cancellation signal (ACS) method for sidelobe suppression in non-contiguous orthogonal frequency division multiplexing (NC-OFDM) based CR systems. The key idea of the proposed method is to dynamically add several additive cancellation symbols on both the primary user (PU) subcarriers and the secondary user (SU) subcarriers, to generate the additive cancellation signals for suppressing the sidelobe power of NC-OFDM signals. Moreover, the ACS method formulates the problem of sidelobe suppression as a quadratically constrained quadratic program (QCQP), and the optimal additive cancellation signal can be obtained by the standard interior-point method. Simulation results show that the proposed ACS method can provide significant sidelobe suppression performance.**

***Index Terms*—Non-contiguous orthogonal frequency division multiplexing (NC-OFDM), cognitive radio, sidelobe suppression.**

I. INTRODUCTION

Recently, cognitive radio (CR) has drawn significant attention from academic and industrial communities to meet the ever-growing needs of spectrum resources and high data rate communication. For CR systems, non-contiguous orthogonal frequency division multiplexing (NC-OFDM) is an attractive physical layer technology due to its considerable high spectrum efficiency, immunity to the frequency selective fading channels, multipath delay spread tolerance and high power efficiency [1]–[3].

Fig. 1 depicts the coexistence of the NC-OFDM based CR system with both secondary users (SUs) and primary users (PUs). For the NC-OFDM based CR system, the SUs are cognitive unlicensed users which detect and utilize the subcarriers located in the unused spectrum band for data transmission. Moreover, the PU subcarriers which are located in the PU spectrum band need to be turned off to create spectrum notches to limit the interference to the PUs in the conventional NC-OFDM based CR system [4].

Although the NC-OFDM based CR system has many advantages, there are still some challenging issues remained unresolved in the design of the NC-OFDM based CR system. One of its main drawbacks is the large spectrum sidelobe. The large spectrum sidelobe introduces interference to the adjacent PUs, resulting in the serious performance degradation of the adjacent PUs [5]. Hence, it is highly desirable to suppress the spectrum sidelobe as much as possible in the NC-OFDM based CR system.

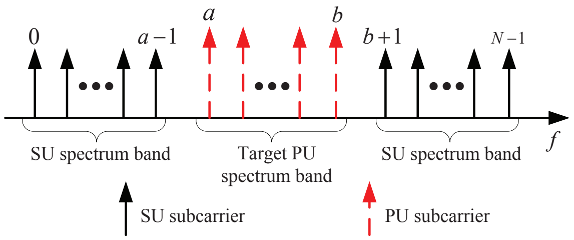

Fig. 1. NC-OFDM based CR system coexisting with the PU and SUs.

Recently, various methods have been proposed to suppress the sidelobe of the NC-OFDM based CR system in the literature, such as active interference cancellation (AIC) [6], extended active interference cancellation (EAIC) [4], constellation adjustment (CA) [7], spectral precoding [8], adaptive symbol transition (AST) [9], additive signal (AS) [10], active point modification (APM) [11], and sidelobe suppression with orthogonal projection (SSOP) [12]. The AIC method utilizes some subcarriers as the guard band to suppress the sidelobe, resulting in the decrease of the spectrum efficiency. The EAIC method destroys the orthogonality of the NC-OFDM based CR system, leading to the degradation of the bit error rate (BER) performance. For the CA method, transmitters must send the selected weights as side information to the receiver for data recovery, resulting in the decrease of data rate. The SP method suffers from the high computational complexity for both the transmitter and receiver. The AST method adds AST blocks between OFDM symbols, which leads to a considerable degradation of the system throughput. For the AS method, it distorts the data symbols to suppress the spectrum sidelobe, results in remarkable degradation of the BER performance. The APM method significantly enlarges the power of NC-OFDM systems, since it extends the constellation points on data subcarriers. The SSOP method adopts several reserved subcarriers for recovering the distorted signal in the receiver, thus, the SSOP method suffers from the decrease of data rate.

In this paper, we propose a novel scheme, named as additive cancellation signal (ACS), for sidelobe suppression in the NC-

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OFDM based CR system. For the proposed ACS method, several additive cancellation symbols are dynamically added on both the PU subcarriers and the SU subcarriers, to generate the appropriate cancellation signal to minimize the sidelobe suppression. Then, the ACS method formulates the problem of sidelobe suppression as a quadratically constrained quadratic program (QCQP) [13], and the optimal additive cancellation signal can be obtained by the standard interior-point method. Simulation results show that the proposed ACS method offers significant sidelobe suppression performance.

The rest of this paper is organized as follows. We briefly describe the NC-OFDM based CR system in Section II. In Section III, we propose the additive cancellation signal method to suppress the sidelobe in detail. Simulation results are shown in Section IV, followed by conclusions in Section V.

II. SYSTEM MODEL

For the NC-OFDM based CR system, the SUs can detect and utilize the unoccupied spectrum band of the PUs (i.e., SU spectrum band) for data transmission. Thus, the SUs utilize the SU subcarriers located in the SU spectrum band for data transmission. As shown in Fig. 1, for a conventional NC-OFDM based CR system with N subcarriers, among which $L=(b-a+1)$ subcarriers are occupied by the PUs (from the a-th to b-th subcarriers) and the remaining $N-L$ subcarriers (from the 0-th to $(a-1)$-th subcarriers, and from the $(b+1)$-th to $(N-1)$-th subcarriers) are utilized by the SUs for data transmission. Denote $\mathcal{R}=\{0,1,\cdots,a-1,b+1,b+2,\cdots,N-1\}$ as the subset that consists of indexes of the SU subcarriers, and $\mathcal{R}^c=\{a,\cdots,b\}$ is the subset that consists of indexes of the PU subcarriers. Moreover, we define a diagonal carrier selection matrix $\mathbf{Q}\in\mathbb{R}^{N\times N}$ with the (k,k)-th element as

$$Q_{kk}=\begin{cases}1, & k\in\mathcal{R},\\ 0, & k\notin\mathcal{R}.\end{cases} \tag{1}$$

For a NC-OFDM symbol $\mathbf{X}=[X(0),X(1),\cdots,X(N-1)]^T$, it is obvious that $X(k)=0$ for $k\notin\mathcal{R}$. Then, the time-domain OFDM signal $\mathbf{x}=[x(0),x(1),\cdots,x(N-1)]^T$ can be generated by employing N-point inverse discrete Fourier transform (IDFT) to $\mathbf{X}$, i.e.,

$$\begin{aligned}x(n)=&\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j2\pi kn\triangle f/f_s}\\ =&\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j2\pi kn/N},n=0,1,\cdots,N-1,\end{aligned} \tag{2}$$

where $\triangle f$ is the frequency interval between adjacent subcarriers, and $f_s=N\triangle f$ is the total bandwidth.

To measure the spectrum sidelobe power in the PU spectrum band, we calculate the spectrum at the sample points $\{f_0,f_1,\cdots,f_{M-1}\}$, and the sampled sidelobe at frequency f_m is given as

$$S_d(m)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}x(n)e^{-j2\pi nf_m/f_s},m=0,1,\cdots,M-1. \tag{3}$$

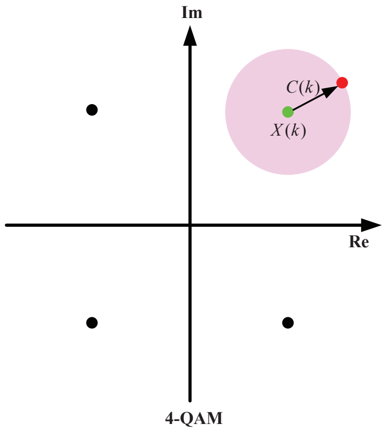

Fig. 2. Constellation region on SU subcarriers for the ACS method with 4-QAM.

Substituting (2) into (3), we have

$$\begin{aligned}S_d(m)=\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}X(k)e^{j2\pi n(k\triangle f-f_m)},\\ m=0,1,\cdots,M-1.\end{aligned} \tag{4}$$

Thus, (4) can be rewritten in matrix form as

$$\mathbf{S}_d=\mathbf{P}_d\mathbf{X}, \tag{5}$$

where the sampled sidelobe matrix $\mathbf{S}_d$ is denoted as $\mathbf{S}_d=[S_d(0),S_d(1),\cdots,S_d(M-1)]^T$. $\mathbf{P}_d$ is a M-by-N matrix with the (m,k)-th element as $\frac{1}{N}\sum\limits_{n=0}^{N-1}e^{j2\pi kn/N}e^{-j2\pi nf_m/f_s}$.

Moreover, the total sidelobe power SL in the target spectrum band can be measured by the sum of the sampled energy at frequency points $\{f_0,f_1,\cdots,f_{M-1}\}$, i.e.,

$$SL=\sum_{m=0}^{M-1}|S_d(m)|^2=||\mathbf{S}_d||_2^2. \tag{6}$$

For the conventional NC-OFDM based CR system, the PU subcarriers are turned off at the occupied PU band to create spectrum notches to limit interference to the PUs. However, the total sidelobe power of the NC-OFDM signals can still cause nonnegligible interference when the PU subcarriers are turned off [4]. Therefore, we propose a novel additive cancellation signal method to suppress the sidelobe power in the NC-OFDM based CR system, which can significantly reduce the interference from the SUs to the PUs.

III. ADDITIVE CANCELLATION SIGNAL METHOD

In this section, we propose a novel additive cancellation signal (ACS) method for the sidelobe suppression in the NC-OFDM based CR system. Different to the conventional NC-OFDM based CR scheme which turns off the PU subcarriers to suppress sidelobe power to limit the interference to PUs, the ACS method utilizes both the SU subcarriers and PU subcarriers to generate the additive cancellation signals for sidelobe suppression. For the ACS method, several additive cancellation symbols are dynamically added on both the PU subcarriers and the SU subcarriers, to generate the appropriate additive cancellation signal to minimize the sidelobe power. Moreover, the proposed ACS method does not need to utilize some spectrum bands as the guard band, thus, the spectral efficiency of the ACS method can be maintained.

For the proposed ACS method, the constellation points on SU subcarriers can be extended, and Fig. 2 illustrates the constellation extension when the 4-quadrature amplitude modulation (QAM) is employed. Denote $\mathbf{C} = [C(0), C(1), \cdots, C(N-1)]$ as the additive cancellation symbols added to the original NC-OFDM symbol $\mathbf{X}$, thus, the transmitted NC-OFDM symbol $\widehat{\mathbf{X}}$ of the ACS method can be expressed as

$$\widehat{\mathbf{X}} = \mathbf{X} + \mathbf{C}. \tag{7}$$

Since the constellation extension on SU subcarriers can lead to BER performance degradation, we denote the error vector magnitude (EVM) of data constellations as

$$EVM = \sqrt{\frac{\sum_{k\in\mathcal{R}} |C(k)|^2}{\sum_{k\in\mathcal{R}} |X(k)|^2}} = \frac{||\mathbf{QC}||_2}{||\mathbf{QX}||_2}. \tag{8}$$

It is obvious that the BER of the NC-OFDM based CR system increases when the EVM increases, thus, the EVM must be restricted to be less than or equal to the error parameter ϵ, i.e.,

$$||\mathbf{Q}(\widehat{\mathbf{X}} - \mathbf{X})||_2 = ||\mathbf{QC}||_2 \leq \epsilon, \tag{9}$$

where the error parameter ϵ is defined as

$$\epsilon = \text{EVM}_0 \cdot ||\mathbf{SX}||_2. \tag{10}$$

Moreover, EVM_0 denotes the maximum allowed EVM, which can be determined according to the specific NC-OFDM based CR system. When the proposed ACS method satisfies the EVM constraint, then acceptable BER performance at the receiver can be achieved.

Therefore, the sidelobe suppression problem can be formulated as an optimization problem, i.e.,

$$\begin{aligned} \min_{\mathbf{C}} \quad & ||\mathbf{P}_d\mathbf{X} + \mathbf{P}_d\mathbf{C}||_2^2 && \text{(11a)} \\ \text{subject to :} \quad & ||\mathbf{QC}||_2 \leq \epsilon && \text{(11b)} \\ & ||\mathbf{Q}(\mathbf{X} + \mathbf{C})||_2 \geq ||\mathbf{QX}||_2 && \text{(11c)} \\ & ||(\mathbf{I} - \mathbf{Q})\mathbf{C}||_2 \leq \beta, && \text{(11d)} \end{aligned}$$

where (11a) is the objective function which minimizes the sidelobe power of the NC-OFDM based CR system with the proposed ACS method. (11b) represents EVM constraint of the NC-OFDM based CR system. In (11c), the data power of the proposed ACS method should be equal to or larger than the data power of the conventional NC-OFDM based CR system, because the average power on data subcarriers must be kept nondecreasing in the practical NC-OFDM based CR system. Moreover, the power on PU subcarriers must be restrained, and (11d) represents the power constraint of PU subcarriers with the ACS method, where β denotes the maximum power. Obviously, (11a) is a convex function, while (11b) and (11d) are convex sets. However, (11c) defines a nonconvex set, thus, the sidelobe suppression problem (11) is a nonconvex optimization problem and it requires nondeterministic polynomial time to obtain the optimal solution.

To reformulate the sidelobe suppression problem as a convex optimization problem, we need to rewrite (11c) as follows

$$(\mathbf{X} + \mathbf{C})^H \mathbf{Q} (\mathbf{X} + \mathbf{C}) \geq \mathbf{X}^H \mathbf{QX}. \tag{12}$$

We define $\Re(\cdot)$ as the real part, and $<,>$ denotes the inner product of two complex vectors. Then, according to (12), we can obtain

$$\begin{aligned} \Re\left(\langle \mathbf{X}, \mathbf{QC} \rangle\right) &= \Re\left(\mathbf{X}^H \mathbf{QC}\right) \\ &= \mathbf{X}^H \mathbf{QC} + (\mathbf{X}^H \mathbf{QX})^H \geq -\frac{1}{2}||\mathbf{QC}||_2^2. \end{aligned} \tag{13}$$

Since $\mathbf{C}$ satisfies the EVM constraint (11b), according to (13), we can obtain the convex linear inequality as

$$\Re\langle \mathbf{X}, \mathbf{QC} \rangle \geq -\epsilon^2/2. \tag{14}$$

Therefore, the sidelobe suppression problem can be formulated as an optimization problem, i.e.,

$$\begin{aligned} \min_{\mathbf{C}} \quad & ||\mathbf{P}_d\mathbf{X} + \mathbf{P}_d\mathbf{C}||_2^2 && \text{(15a)} \\ \text{subject to :} \quad & ||\mathbf{QC}||_2 \leq \epsilon && \text{(15b)} \\ & \Re\langle \mathbf{X}, \mathbf{QC} \rangle \geq -\epsilon^2/2 && \text{(15c)} \\ & ||(\mathbf{I} - \mathbf{Q})\mathbf{C}||_2 \leq \beta, && \text{(15d)} \end{aligned}$$

Obviously, the sidelobe suppression problem (15) is a convex quadratically constrained quadratic program (QCQP), and the ACS method obtains the optimal solution by the standard interior-point method [13]. Since the standard interior-point method is a well-known and widely used optimization method, we do not discuss the standard interior-point method in the paper.

IV. SIMULATION RESULTS

In this section, some simulations have been conducted to evaluate the capability of the proposed ACS method, including the sidelobe suppression and BER performances, where the NC-OFDM based CR system is considered with $N = 64$ subcarriers. Moreover, the target spectrum band occupied by PUs is from $29\Delta f$ to $38\Delta f$, in which 37 frequency sampling points for the evaluation of the total sidelobe power are placed with the equivalent space of $\frac{\Delta f}{4}$. Thus, $a = 29$, $b = 38$, $M = 37$, and the remaining spectrum band is utilized by SUs. In addition, 10^4 independent NC-OFDM symbols modulated by 4-QAM are randomly generated, and the Welch method with Blackman window is used to calculate the power spectrum density of the transmitted signals. Besides, for the proposed ACS methods, the power threshold of PU subcarriers is set to be $\beta = \mu||\mathbf{SX}||_2$ in the simulations, where μ is the parameter to adjust the power threshold β and it is set to be $\mu = 0.05$ in the simulations. For comparison, we also conduct simulations using the conventional method which simply turns off the subcarriers in the target band, the AS method [10], and the APM method [11], respectively. In the simulations, the ACS method solves the convex optimization problem (15) using the public software CVX [14].

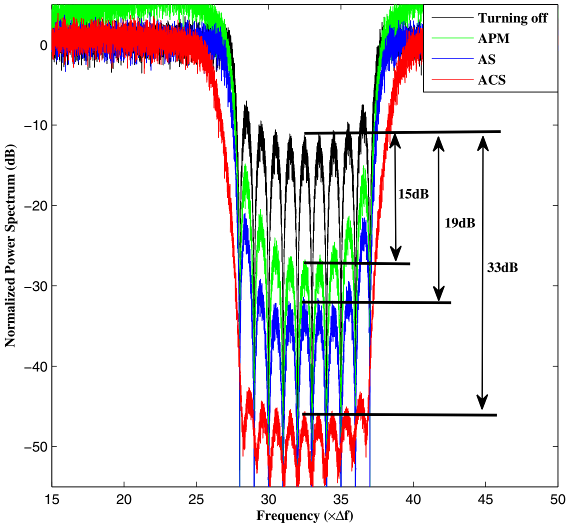

Fig. 3. Power spectrum density of the NC-OFDM signals with the ACS method when $\mathrm{EVM}_0 = -5\mathrm{dB}$.

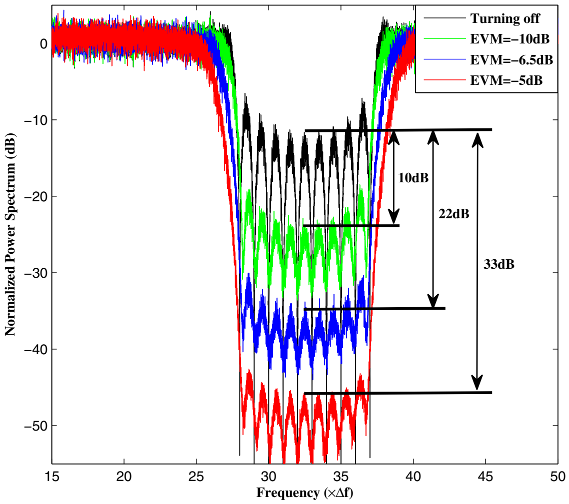

Fig. 4. Power spectrum density of the NC-OFDM signals with the ACS method when EVM_0 is different.

Fig. 3 illustrates the normalized power spectrum of the NC-OFDM signals with the proposed ACS method when $\mathrm{EVM}_0 = -5\mathrm{dB}$. Seen from Fig. 3, the turning off method can achieve a spectrum notch of only 13dB in the target spectrum band from $29\Delta f$ to $38\Delta f$, which means a considerable interference to the PUs. Compared with the turning off method, the APM method and AS method offer much better sidelobe suppression performances, which can improve the sidelobe suppression performance by 15dB and 19dB, respectively. Moreover, it is obvious that the proposed ACS method can improve the sidelobe suppression performance by 33dB, compared with the turning off method. Thus, the proposed ACS method offers much better sidelobe suppression performance than the turning off method, APM method, and AS method.

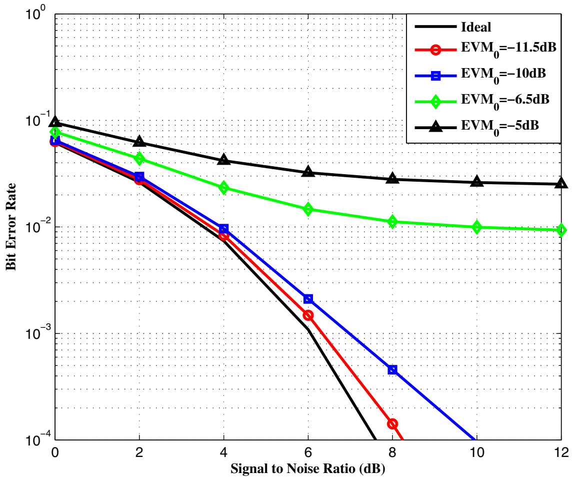

Fig. 5. BER performance of the ACS method when EVM_0 is different.

Fig. 4 depicts the the normalized power spectrum of the NC-OFDM signals with the proposed ACS method when EVM_0 is different. It is obvious that the sidelobe suppression performance of the ACS method improves when the value of EVM_0 increases. For example, compared with the turning off method, when $\mathrm{EVM}_0 = -10\mathrm{dB}$, $-6.5\mathrm{dB}$ and $-5\mathrm{dB}$, the ACS method can improve the sidelobe suppression performance by 10dB, 22dB and 33dB, respectively.

Fig. 5 illustrates the the BER performance of the proposed ACS method when EVM_0 is different. Seen from Fig. 5, compared with the turning off method, there is a BER performance degradation for the proposed ACS method, because the ACS method extends the constellation points to suppress the spectrum sidelobe. Moreover, it is obvious that the BER performance of the ACS method improves when the value of EVM_0 decreases. For example, when $SNR = 6\mathrm{dB}$, the BERs of the ACS method with $\mathrm{EVM}_0 = -11.5\mathrm{dB}$, $-10\mathrm{dB}$, $-6.5\mathrm{dB}$ and $-5\mathrm{dB}$ are 1.088×10^{-3}, 1.483×10^{-3}, 1.469×10^{-2}, and 3.23×10^{-2}, respectively. Therefore, there is a trade-off between the BER performance and the sidelobe suppression. This means that the sidelobe suppression performance can be improved by increasing the value of EVM_0, however, it results in the degradation of the BER performance. Thus, the ACS method can provide sufficient sidelobe suppression performance and BER performance by adjusting the value of EVM_0.

V. CONCLUSIONS

In this paper, we propose a novel additive cancellation signal method for sidelobe suppression in NC-OFDM based

CR systems. The proposed method adds several additive cancellation symbols on both the PU subcarriers and SU subcarriers, to generate the additive cancellation signals to minimize the sidelobe power in NC-OFDM based CR systems. Simulation results show that the proposed ACS method can provide significant sidelobe suppression performances.

VI. ACKNOWLEDGEMENT

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REFERENCES

[1] Z. Li and X.-G. Xia, “Time-domain interference cancellation for ALAMOUTI-coded cooperative OFDM systems with insufficient CP,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 6, pp. 3131-3136, Jul. 2010.

[2] T. Jiang, C. Ni, and L. Guan, “A novel phase offset SLM scheme for PAPR reduction in Alamouti MIMO-OFDM systems without side information,” *IEEE Signal Processing Letters*, vol. 20, no. 4, pp. 383-386, Apr. 2013.

[3] L. Guan, T. Jiang, D. Qu, and Y. Zhou, “Joint channel estimation and PTS to reduce peak-to-average-power radio in OFDM systems without side information,” *IEEE Signal Processing Letters*, vol. 17, no. 10, pp. 883-886, Oct. 2010.

[4] D. Qu, Z. Wang, and T. Jiang, “Extended active interference cancellation for sidelobe suppression in cognitive radio OFDM systems with cyclic prefix,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1689-1695, May. 2010.

[5] D. Qu, J. Ding, T. Jiang, and X. Sun, “Detection of non-contiguous OFDM symbols for cognitive radio systems without out-of-band spectrum synchronization,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 693-701, Feb. 2011.

[6] H. Yamaguchi, “Active interference cancellation technique for MB-OFDM cognitive radio,” 34*th IEEE European Microwave Conference*, vol. 2, pp. 1105-1108, Oct. 2004.

[7] D. Li, X. Dai, and H. Zhang, “Sidelobe suppression in NC-OFDM systems using constellation adjustment,” *IEEE Communications Letters*, vol. 13, no. 5, pp. 327-329, May. 2009.

[8] W.-C. Chen, and C.-D. Chung, “Spectral precoding for cyclic-prefixed OFDMA with interleaved subcarrier allocation,” *IEEE Transactions on Communications*, vol. 61, no. 11, pp. 4616-4629, Nov. 2013.

[9] H. Mahmoud and H. Arslan, “Sidelobe suppression in OFDM-based spectrum sharing systems using adaptive symbol transition,” *IEEE Communications Letters*, vol. 12, no. 2, pp. 313-315, Feb. 2008.

[10] I. Cosovic and T. Mazzoni, “Sidelobe suppression in OFDM spectrum sharing systems via additive signal method,” *IEEE Vehicular Technology Conference*, pp. 2692-2696, Apr. 2007.

[11] Y. Zhou and T. Jiang, “Active point modification for sidelobe suppression with PAPR constraint in OFDM systems,” *ACM/Springer Journal of Wireless Networks*, vol. 19, no. 7, pp. 1653-1663, Oct. 2013.

[12] J. Zhang, X. Huang, A. Cantoni, and Y. Guo, “Sidelobe suppression with orthogonal projection for multicarrier systems,” *IEEE Transactions on Communications*, vol. 60, no. 2, pp. 589-599, Feb. 2012.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[14] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming (web page and software).” [Online]. Available: http://cvxr.com/cvx, Oct. 2013.