Distributed Power Control in Full Duplex Wireless Networks

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***Abstract*—In this paper, we consider the problem of distributed power control in a full duplex (FD) wireless network consisting of multiple pairs of nodes, within which each node needs to communicate with its corresponding node. We aim to find the optimal transmit powers for the FD transmitters such that the network-wide capacity is maximized. Based on the high signal-to-interference-plus-noise ratio (SINR) approximation and a more general approximation method for logarithm functions, we develop effective distributed power control algorithms with the dual decomposition approach. The proposed algorithms are validated with simulation studies.**

I. Introduction

In this paper, we investigate the problem of distributed power control for a wireless network where the nodes are capable of full duplex (FD) transmissions. Although full duplex transmission has been used in wireline network for years (e.g., Asymmetric Digital Subscriber Line (ADSL) based on echo cancellation), FD transmission in wireless networks has become feasible only in recent years. Wireless FD systems are made possible by breakthroughs in self-interference cancellation [1], [2], such as propagation-domain suppression (PDIS) [3], [4], analog-domain interference cancellation, and digital domain interference cancellation (ADIC) [5]. Practical FD systems have been demonstrated that can achieve more than 70 dB [5] or 80 dB [6] reduction of self-interference.

FD can be incorporated in a wireless network in two ways: (i) point-to-point mode, where two nodes transmit to each other simultaneously, and (ii) three-node mode, where a node (e.g., a cellular base station (BS)) simultaneously receives from a node and transmits to another node. FD brings about new challenges to the design of power control algorithms. In a traditional half-duplex (HD) network, if a node increases its power, the signal-to-interference-plus-noise ratio (SINR) at its target receiver can be improved, but with larger interference to other receivers. In an FD network, an increased power causes not only larger interference to other receivers, but also larger residual self-interference to the node itself and may even degrade its own SINR. To fully harvest the high potential of FD wireless networks, the power control problem should be carefully addressed with effective algorithms developed.

In this paper, we consider an FD wireless network consisting of multiple node pairs, where the two nodes in each pair transmit to each other (i.e., the point-to-point mode). We first consider the simpler case with a single pair of nodes [12]. Taking advantage of the structure of the formulated optimal power control problem, we show that the optimal solution should be at the boundary of the feasible region. Furthermore, we consider the case of multiple node pairs in the FD network. Based on the high SINR approximation, we can transform the optimal power control problem into a convex problem in the high SINR region. We then develop a Dual Decomposition based distributed algorithm to find the optimal solution to the transformed problem [7]. Finally, we consider the general case and drop the high SINR assumption. Incorporating an iterative approximation method for logarithm functions [8], we are able to obtain a convex transformation of the original power control problem and develop a distributed algorithm. The proposed algorithms are validated with simulations, where fast convergence and large gain over the traditional HD system are demonstrated.

The remainder of this paper is organized as follows. The system model and preliminaries are introduced in Section II. We investigate the case of a single pair of FD nodes in Section III and the general case of an FD network in Section IV. Simulation results are presented in Section V. Section VI reviews related work and Section VII concludes this paper.

II. System Model and Preliminaries

A. Channel and Propagation Models

Consider an FD wireless network that only consists of two transceivers, denoted as nodes a and b, respectively. The received signals at the two transceivers are

$$\begin{cases} y_a = \sqrt{P_b} h_{ba} x_b + \sqrt{P_a} h_{aa} x_a + n_a \\ y_b = \sqrt{P_a} h_{ab} x_a + \sqrt{P_b} h_{bb} x_b + n_b, \end{cases} \tag{1}$$

where x_a and x_b are the normalized transmitted signals from node a and b, respectively; P_a and P_b are the corresponding transmit powers; h_{ab} and h_{ba} are the channel gains from node a to b and from node b to a, respectively; h_{aa} and h_{bb} are the self-interference channel gain at node a and b, respectively; n_a and n_b are the thermal noises.

According to [9], the maximum likelihood channel estimation can be modeled as

$$h = \hat{h} + \sqrt{e}\tilde{h}, \tag{2}$$

where $\hat{h}$ is the estimated channel; $\sqrt{e}\tilde{h}$ is the estimation error that is uncorrelated with the real channel h; and $\tilde{h}$ is i.i.d Gaussian with a zero mean and unit variance.

B. Propagation-Domain Interference Suppression

Self-interference cancellation is the enabler of FD systems. Propagation domain interference suppression (PDIS) can dramatically reduce the self-interference [10]. Applying PDIS, the self-interference terms, e.g., $\sqrt{P_a}h_{aa}x_a$, in received signals (1) will be reduced to a fraction $\sqrt{\kappa P_a}h_{aa}x_a$.

Considering both PDIS and channel estimation, the received signals in (1) become [12]

$$\begin{cases} y_a = \sqrt{P_b}\hat{h}_{ba}x_b + \sqrt{\kappa P_a}\left(\hat{h}_{aa} + \sqrt{e_{aa}}\tilde{h}_{aa}\right)x_a + n_a \\ y_b = \sqrt{P_a}\hat{h}_{ab}x_a + \sqrt{\kappa P_b}\left(\hat{h}_{bb} + \sqrt{e_{bb}}\tilde{h}_{bb}\right)x_b + n_b. \end{cases} \tag{3}$$

C. Analog and Digital Interference Cancellation

In addition to PDIS, analog and digital interference cancellation (ADIC) can cancel more known self-interference. For example, in the first equation in (3), the first term $\sqrt{P_b}\hat{h}_{ba}x_b$ is the expected signal, and the rest terms are interference and noise. However, $\sqrt{\kappa P_a}\hat{h}_{aa}$ is already known by node a, since the estimated self-interference channel and PDIS of this channel are known at the node. Thus this part of interference can be eliminated by ADIC, and the remaining interference plus noise at node a is $\sqrt{\kappa P_b e_{aa}}\tilde{h}_{aa}x_a + n_a$.

Denoting $\hat{h}^2$ as $\hat{G}$, the received signal power is $P_b\hat{G}_{ba}$, the remaining interference power after interference cancellation is $\kappa P_a e_{aa}$, and the noise power is N_a. Since both κ and e_{aa} are related to self-interference cancellation, we define $\chi = \kappa e_{aa}$ as the *self-interference cancellation coefficient*. The SINRs for transceivers a and b can be written as

$$\text{SINR}_a = \frac{P_b\hat{G}_{ba}}{\chi P_a + N_a} \quad \text{and} \quad \text{SINR}_b = \frac{P_a\hat{G}_{ab}}{\chi P_b + N_b}. \tag{4}$$

We adopt the Shanon formula $C = B\log_2(1 + \text{SINR})$ to approximate the capacity of a channel with bandwidth B. For brevity, we set $B = 1$ for all the transmitters. The total capacity for the pair of nodes, C_{ab}, is

$$C_{ab} = \log\left(1 + \frac{P_b\hat{G}_{ba}}{\chi P_a + N_a}\right) + \log\left(1 + \frac{P_a\hat{G}_{ab}}{\chi P_b + N_b}\right). \tag{5}$$

For an FD network with M pair of FD nodes, $\{a_j, b_j\}_{j=1,2,\cdots,M}$, the sum rate of the entire network, C_{total}, can be written as

$$C_{total} = \sum_{j=1}^{M} C_{a_j b_j} = \tag{6}$$

$$\sum_{j=1}^{M}\left[\log\left(1 + \frac{\hat{G}_{b_j a_j}P_{b_j}}{\chi P_{a_j} + \sum\limits_{l\neq j}\hat{G}_{a_l a_j}P_{a_l} + \sum\limits_{l\neq j}\hat{G}_{b_l a_j}P_{b_l} + N_{a_j}}\right)\right.$$
$$\left. + \log\left(1 + \frac{\hat{G}_{a_j b_j}P_{a_j}}{\chi P_{b_j} + \sum\limits_{l\neq j}\hat{G}_{a_l b_j}P_{a_l} + \sum\limits_{l\neq j}\hat{G}_{b_l b_j}P_{b_l} + N_{b_j}}\right)\right].$$

Algorithm 1: Optimal Power Control Algorithm for a Single FD Pair

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1 Solve $\left.\frac{\partial C_{ab}}{\partial P_a}\right|_{P_b=P_{max}} = 0$ for P_a ;
2 Solve $\left.\frac{\partial C_{ab}}{\partial P_b}\right|_{P_a=P_{max}} = 0$ for P_b ;
3 Substitute (P_a, P_{max}) into (5) to get $C_{ab}(1)$;
4 Substitute (P_{max}, P_b) into (5) to get $C_{ab}(2)$;
5 **if** $C_{ab}(1) \geq C_{ab}(2)$ **then**
6 | Output optimal power allocation $(P_a^*, P_b^*) = (P_a, P_{max})$;
7 **else**
8 | Output optimal power allocation $(P_a^*, P_b^*) = (P_{max}, P_b)$;
9 **end**

III. Optimal Power Control for a Pair of FD Nodes

We first consider the simpler case of a pair of FD nodes, i.e., $M = 1$. The sum rate of the pair, C_{ab}, is given in (5). The power control problem for nodes a and b is

$$\max_{\{P_a, P_b\}} \quad C_{ab} \tag{7}$$

$$\text{s.t.} \quad 0 \leq P_a \leq P_{max} \quad \text{and} \quad 0 \leq P_b \leq P_{max}. \tag{8}$$

Lemma 1. *In the optimal solution to Problem* (7)*, at least one node transmits at its maximum power* P_{max}.

Proof: Defining $Z = \left(1 + \frac{P_b\hat{G}_{ba}}{\chi P_a + N_a}\right)\left(1 + \frac{P_a\hat{G}_{ab}}{\chi P_b + N_b}\right)$, then we have $C_{ab} = \log Z$ and $Z > 0$. Consider the weighted sum of the partial derivatives of C_{ab} with respect to P_a and P_b and with weights $1/P_b$ and $1/P_a$, respectively. We have

$$\begin{aligned} &\frac{\partial C_{ab}}{\partial P_a}\cdot\frac{1}{P_b} + \frac{\partial C_{ab}}{\partial P_b}\cdot\frac{1}{P_a} \\ &= \frac{\hat{G}_{ba}\hat{G}_{ab}\chi(2N_aN_b + N_aP_b\chi + N_bP_a\chi)}{Z(\chi P_a + N_a)^2(\chi P_b + N_b)^2} + \\ &\frac{\hat{G}_{ab}N_b}{Z(\chi P_b + N_b)^2 P_b} + \frac{\hat{G}_{ba}N_a}{Z(\chi P_a + N_a)^2 P_a} > 0. \end{aligned} \tag{9}$$

Therefore, the two partial derivatives cannot be both non-positive; we have either $\partial C_{ab}/\partial P_a > 0$ or $\partial C_{ab}/\partial P_b > 0$ (or both) for any power allocation in the feasible region defined in (8). We can always increase C_{ab} by increasing the power for the node with a positive partial derivative, until finally hitting the boundary. Thus the optimal solution will always be on the boundary of the feasible region, i.e., it will be either (P_a, P_{max}) or (P_{max}, P_b). ■

Based on this observation, the algorithm for finding the optimal power allocation for the FD pair is presented in Algorithm 1.

In the case of a single pair of FD nodes, increasing the power of one node, say node a, has both positive and negative effects on the sum rate: the SINR for node b will be larger since the received signal will be stronger; but the SINR for node a itself will be smaller, since the self-interference will also go up. Furthermore, at least one of the nodes should transmit at the maximum power. This situation will occur again when power allocation for $M > 1$ pairs of nodes is considered.

IV. Optimal Power Control for the FD Network

We next consider the optimal power allocation problem for the general case of $M > 1$ pairs of FD nodes. However, the challenge is that problem (6) is not convex in the feasible region, and thus cannot be solved by a convex optimization technique directly. We show how to apply two approximation methods to convert this problem into a convex one.

A. High SINR Approximation and Convexity

In the high SINR region where SINR $\gg 1$, we have $\log(1+\text{SINR}) \approx \log(\text{SINR})$. The sum rate of the FD network for a given power allocation vector $\mathbf{P}$ can be approximated as

$$\begin{aligned} &C^1_{total}(\mathbf{P}) = \sum_{j=1}^{M} C^1_{a_j b_j}(\mathbf{P}) \\ &= \sum_{j=1}^{M} \Big[\log \hat{G}_{a_j b_j} + \log \hat{G}_{b_j a_j} + \log P_{a_j} + \log P_{b_j} - \\ &\quad \log\left(\chi P_{a_j} + \sum_{l \neq j} \hat{G}_{a_l a_j} P_{a_l} + \sum_{l \neq j} \hat{G}_{b_l a_j} P_{b_l} + N_{a_j}\right) - \\ &\quad \log\left(\chi P_{b_j} + \sum_{l \neq j} \hat{G}_{a_l b_j} P_{a_l} + \sum_{l \neq j} \hat{G}_{b_l b_j} P_{b_l} + N_{b_j}\right)\Big]. \end{aligned} \tag{10}$$

The power control problem for the FD network becomes

$$\max_{\mathbf{P}} \; C^1_{total}(\mathbf{P}) \tag{11}$$

$$\text{s.t.} \;\; 0 \le P_{a_j} \le P_{max} \;\text{and}\; 0 \le P_{b_j} \le P_{max}, \forall\, j. \tag{12}$$

We show the problem is convex in the high SINR region.

Lemma 2. *Problem* (11) *is a convex optimization problem.*

Proof: Let $\tilde{P}_{x_j} = \log P_{x_j}$, which means $P_{x_j} = e^{\tilde{P}_{x_j}}$, for all $x \in \{a, b\}, j \in \{1, 2, \cdots, M\}$. Consider the problem with variables $\tilde{\mathbf{P}} = [\tilde{P}_{a1}, \tilde{P}_{b1}, \cdots, \tilde{P}_{aM}, \tilde{P}_{bM}]$. Taking partial derivative of $C^1_{total}(\tilde{\mathbf{P}})$ with respect to $\tilde{P}_{a_j}$, it follows that

$$\begin{aligned} &\nabla_{a_j} C^1_{total}(\tilde{\mathbf{P}}) = 1 - \\ &\sum_{k \neq j}\left(\frac{s_{a_j a_k}}{\sum_l (s_{a_l a_k} + s_{b_l a_k}) + N_{a_k}} + \frac{s_{a_j b_k}}{\sum_l (s_{a_l b_k} + s_{b_l b_k}) + N_{b_k}}\right), \end{aligned}$$

where $s_{a_j a_j} = \chi e^{\tilde{P}_{a_j}}$, $s_{a_j b_j} = 0$, and $s_{a_j b_l} = \hat{G}_{a_j b_l} e^{\tilde{P}_{a_j}}$.

Taking derivative again for each nonlinear term $-\log(\chi P_{a_j} + \sum_{l \neq j} \hat{G}_{a_l a_j} P_{a_l} + \sum_{l \neq j} \hat{G}_{b_l a_j} P_{b_l} + N_{a_j})$, the Hessian is $\mathbf{H}^{a_j} = \mathbf{s}^T_{a_j} \mathbf{s}_{a_j} - \text{diag}(\mathbf{s}_{a_j})$, where $\mathbf{s}_{a_j} = [s_{a_1 a_j}, \cdots, s_{b_M a_j}]/(\sum_k s_{k a_j} + N_{a_j})$. The overall Hessian is $\sum_l \mathbf{s}^T_l \mathbf{s}_l - \text{diag}(\mathbf{s}_l)$, where k and l represent either a_j and b_j. For a non-zero row vector $\mathbf{t}$, we have

$$\begin{aligned} \mathbf{tHt}^T &= \sum_l \left(\sum_m t_m \frac{s_{ml}}{\sum_k s_{kl} + N_l}\right)^2 - \sum_l \sum_m \frac{t_m^2 s_{ml}}{\sum_k s_{kl} + N_l} \\ &= \sum_l \frac{\sum_m (t_m s_{ml})^2 - \sum_m t_m^2 s_{ml}\left(\sum_k s_{kl} + N_l\right)}{(\sum_k s_{kl} + N_l)^2} \\ &< \sum_l \frac{\sum_m (t_m s_{ml})^2 - \sum_m t_m^2 s_{ml}\left(\sum_m s_{ml}\right)}{(\sum_k s_{kl} + N_l)^2}. \end{aligned}$$

The inequality is due to the omission of some negative terms $-\sum_l \sum_m t_m^2 s_{ml} N_l / (\sum_k s_{kl} + N_l)^2$. For each numerator, letting $a_m = t_m \sqrt{s_{ml}}$ and $b_m = \sqrt{s_{ml}}$, we have $\sum_m (t_m s_{ml})^2 - \sum_m t_m^2 s_{ml} (\sum_m s_{ml}) \le 0$, for all l, according to the Cauchy-Schwarz Inequality (i.e., $(\mathbf{a}^T \mathbf{b})^2 \le (\mathbf{a}^T \mathbf{a})(\mathbf{b}^T \mathbf{b})$). Thus $\mathbf{tHt}^T < 0$, and $C^1_{total}(\tilde{\mathbf{P}})$ is concave.

Consider the solution space of $\mathbf{P}$, it is easy to verify that $\nabla_{a_j} C^1_{total}(\mathbf{P}) = \frac{1}{P_{a_j}} \nabla_{a_j} C^1_{total}(\tilde{\mathbf{P}})$. The transformation from $\mathbf{P}$ to $\tilde{\mathbf{P}}$ only scales the gradient. Thus $C^1_{total}(\mathbf{P})$ is also concave. With linear constraints (12), problem (11) is convex. ∎

B. Decomposition and Distributed Algorithm

We next apply the *Dual Decomposition* technique to develop a distributed power control algorithm for the FD network [7]. Rewrite the sum rate approximation (10) as

$$\begin{aligned} &C^1_{total}(\mathbf{P}) = \sum_{j=1}^{M} C^1_{a_j b_j}(\mathbf{P}) \\ &= \sum_{j=1}^{M} \left[\log\left(\frac{\hat{G}_{a_j b_j} P_{a_j}}{\chi P_{b_j} + m_{b_j}}\right) + \log\left(\frac{\hat{G}_{b_j a_j} P_{b_j}}{\chi P_{a_j} + m_{a_j}}\right)\right], \end{aligned} \tag{13}$$

where

$$\begin{cases} m_{a_j} = \sum_{l \neq j} \hat{G}_{a_l a_j} P_{a_l} + \sum_{l \neq j} \hat{G}_{b_l a_j} P_{b_l} + N_{a_j} \\ m_{b_j} = \sum_{l \neq j} \hat{G}_{a_l b_j} P_{a_l} + \sum_{l \neq j} \hat{G}_{b_l b_j} P_{b_l} + N_{b_j} \end{cases} \tag{14}$$

are the received signals for nodes a_j and b_j when both of them are idle, i.e., the interference plus noise at nodes a_j and b_j from other nodes, respectively.

Problem (11) can be rewritten as maximizing (13) subject to constraints (12) and (14). Define Lagrange multipliers $\boldsymbol{\mu}$ and consider the *Lagrangian* only including the coupled constrains.

Algorithm 2: Distributed Optimal Algorithm for M Pair of FD Nodes in the High SINR Region

Initialize μ_{a_j} and μ_{b_j} to some non-negative values, for all j ;
repeat
  Each FD pair receives power updates from other nodes ;
  Each FD pair computes $m_{a_j}(t)$ and $m_{b_j}(t)$, and solves (17) to update μ_{a_j}, μ_{b_j}, ν_{a_j}, and ν_{b_j} ;
  Each FD pair solves (15) for $\{P^*_{a_j}, P^*_{b_j}\}$;
  Each FD pair distributes $\{P^*_{a_j}, P^*_{b_j}\}$ to the entire network ;
until *convergence*;

We have

$$L^1(\mathbf{P},\boldsymbol{\mu}) = C^1_{total}(\mathbf{P}) + \sum_{j=1}^{M}\left[\mu_{a_j}\left(m_{a_j} - \sum_{l\neq j}\hat{G}_{a_l a_j}P_{a_l} - \sum_{l\neq j}\hat{G}_{b_l a_j}P_{b_l} - N_{a_j}\right) + \mu_{b_j}\left(m_{b_j} - \sum_{l\neq j}\hat{G}_{a_l b_j}P_{a_l} - \sum_{l\neq j}\hat{G}_{b_l b_j}P_{b_l} - N_{b_j}\right)\right]$$
$$= \sum_{j=1}^{M}\left(C^1_{a_j b_j} - \nu_{a_j}P_{a_j} - \nu_{b_j}P_{b_j}\right) + \sum_{j=1}^{M}(\mu_{a_j}m_{a_j} + \mu_{b_j}m_{b_j}),$$

where $C^1_{a_j b_j}$ is the capacity for node pair $\{a_j, b_j\}$; $\nu_{a_j} = \sum_{l=1}^{M}\mu_{a_l}\hat{G}_{a_j a_l} + \sum_{l=1}^{M}\mu_{b_l}\hat{G}_{a_j b_l}$; and $\nu_{b_j} = \sum_{l=1}^{M}\mu_{a_l}\hat{G}_{b_j a_l} + \sum_{l=1}^{M}\mu_{b_l}\hat{G}_{b_j b_l}$.

The subproblem for each pair of nodes $\{a_j, b_j\}$ is

$$\max_{\{P_{a_j},P_{b_j}\}} C^1_j(P_{a_j}, P_{b_j}) = C^1_{a_j b_j}(P_{a_j}, P_{b_j}) - \nu_{a_j}P_{a_j} - \nu_{b_j}P_{b_j} \tag{15}$$

$$\text{s.t. } 0 \leq P_{a_j} \leq P_{max},\ 0 \leq P_{b_j} \leq P_{max},\ \forall\, j \tag{16}$$

The optimal solution $\{P^*_{a_j}, P^*_{b_j}\}$ can be solved by local *Karush-Kuhn-Tucher* (KKT) Conditions or using the subgradient method.

The master problem is

$$\min_{\boldsymbol{\mu}} C^1_{total}(\boldsymbol{\mu}) = \sum_{j=1}^{M}C^1_j(\boldsymbol{\mu}) + \sum_{j=1}^{M}(\mu_{a_j}m_{a_j} + \mu_{b_j}m_{b_j}) \tag{17}$$

$$\text{s.t. } \boldsymbol{\mu} \geq \mathbf{0}. \tag{18}$$

The gradient method can be used to solve the master problem by using a centralized controller or by flooding the power information to all other nodes as in [11].

The distributed power control algorithm for the FD network is presented in Algorithm 2.

C. Approximation in a General Network

Note that Algorithm 2 is based on the high SINR assumption, which may not be true in a general FD network. We can exclude low SINR transceivers to enforce this assumption though. In this section, we present an alternative approach to relax the high SINR assumption, with is a more general approximation for the power allocation problem.

Following the approach in [8], the tightest lower bound for $\log(1+x)$ is $\alpha \log x + \beta$, which intersects $\log(1+x)$ at x_0 when the coefficient α and β are chosen as follows.

$$\alpha = \frac{x_0}{1+x_0},\ \ \beta = \log(1+x_0) - \frac{x_0}{1+x_0}\log x_0. \tag{19}$$

Therefore we can approximate the network-wide sum rate as

$$C^2_{total}(\mathbf{P},\boldsymbol{\alpha},\boldsymbol{\beta}) = \sum_{j=1}^{M} C^2_{a_j b_j}(\mathbf{P},\boldsymbol{\alpha},\boldsymbol{\beta})$$
$$= \sum_{j=1}^{M}\left[\alpha_{a_j}\log\left(\frac{\hat{G}_{b_j a_j}P_{b_j}}{\chi P_{a_j} + m_{a_j}}\right) + \beta_{a_j} + \alpha_{b_j}\log\left(\frac{\hat{G}_{a_j b_j}P_{a_j}}{\chi P_{b_j} + m_{b_j}}\right) + \beta_{b_j}\right]. \tag{20}$$

The power control problem for M node pairs becomes

$$\max_{\mathbf{P}}\ C^2_{total}(\mathbf{P},\boldsymbol{\alpha},\boldsymbol{\beta}) \tag{21}$$
$$\text{s.t. constraints (12) and (14).}$$

It can be shown that $C^2_{total}(\boldsymbol{\lambda})$ is also concave, with a similar approach as in section IV-A. This is because the non-negative and constant scalars $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ do not change the concavity perperty. Hence $C^2_{total}(\mathbf{P})$ is concave for non-negative $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$; the method of Dual Decomposition can be applied to find the optimal solution for given $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

To further improve the optimality of the solution, we iteratively update $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in each step τ as follows, until the optimal solution for the original problem is achieved.

$$\begin{cases} x_{a_j}(\tau) = \frac{\hat{G}_{b_j a_j}P^*_{b_j}(\tau)}{\chi P^*_{a_j}(\tau) + m_{a_j}(\tau)} \\ \alpha_{a_j}(\tau) = \frac{x_{a_j}(\tau)}{1+x_{a_j}(\tau)} \\ \beta_{a_j}(\tau) = \log(1+x_{a_j}(\tau)) - \frac{x_{a_j}(\tau)}{1+x_{a_j}(\tau)}\log(x_{a_j}(\tau)). \end{cases} \tag{22}$$

Specifically, in each iteration, we use the updated power assignment $\mathbf{P}(\tau)$ to find $m_{a_j}(\tau)$ and $m_{b_j}(\tau)$. We then use them to compute $x_{a_j}(\tau)$ as in (22), and use $x_{a_j}(\tau)$ to update $\alpha_{a_j}(\tau)$ and $\beta_{a_j}(\tau)$. Finally the new power assignment $\mathbf{P}(\tau+1)$ can be derived based on $\boldsymbol{\alpha}(\tau)$ and $\boldsymbol{\beta}(\tau)$, and so forth.

The distributed algorithm for the general network/SINR case is presented in Algorithm 3.

V. Simulation Results

In this section, we validate the proposed algorithms with simulations. We first examine the case of one pair of FD nodes with unbiased noise. We use an HD network with the same setting as a benchmark; a simplified version of the proposed algorithm is used to find the optimal powers for the HD only network. Note that in the FD network, it is possible that the optimal power for some nodes are zero, indicating that such nodes operate in the HD mode. In the simulations, we assume log-normal block fading channels, with zero mean and 10 dB standard deviation. The path loss exponent is 4. The noise powers are randomly generated around -110 dBW, unless otherwise specified. The results are presented for a time slot

Algorithm 3: Distributed Optimal Algorithm for M Pair of FD Nodes in the General Case

Set $\tau = 0$, and $\boldsymbol{\alpha}(0) = \mathbf{1}$, $\boldsymbol{\beta}(0) = \mathbf{0}$;
repeat
  Initialize λ_{a_j} and λ_{b_j} to some non-negative values, and set $m_{a_j} = 0$ and $m_{b_j} = 0$, for all j ;
  repeat
    Each FD pair receives updated powers from other nodes ;
    Each FD pair computes m_{a_j} and m_{b_j}, and solves the master problem to update μ_{a_j}, μ_{b_j} and ν_{a_j}, ν_{b_j} ;
    Each FD pair solves the subproblem for $\{P^*_{a_j}, P^*_{b_j}\}$;
    Each FD pair distributes $\{P^*_{a_j}, P^*_{b_j}\}$ to the entire network ;
  until *convergence*;
  Each FD pair updates $\alpha_{a_j}(\tau)$, $\alpha_{b_j}(\tau)$, $\beta_{a_j}(\tau)$, and $\beta_{b_j}(\tau)$ as in (22) ;
  $\tau \leftarrow \tau + 1$;
until *convegence*;

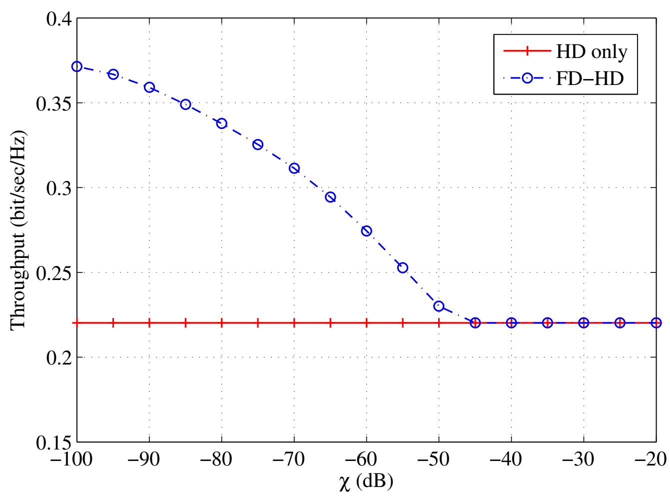

Fig. 1. Optimal throughput of a single pair of FD nodes versus χ.

within which the channel gains do not vary. The peak power is set as $P_{max} = 5$ mW.

We first demonstrate the impact of the self-interference coefficient χ and noise on FD transmissions in Figs. 1 and 2. In Fig. 1, we plot the normalized throughput as a function of χ. It can be seen that, as χ is increased, the normalized throughput of the FD network converges to that of the HD network. Particularly, when χ is less than -45 dB, FD achieves significant throughput gain over HD; the more effective the self-interference cancellation, the higher the throughput gain.

In Fig. 2, we fix $\chi = -80$ dB and plot the normalized system throughput for increasing noise levels. In this simulation, the FD thoughput is always higher than that of HD for the entire range of noise levels simulated. As the noise level is decreased, especially when the average noise power is lower than -120 dB, the FD network throughput approaches that of the HD network. This is because when the noise is extremely low, if one of the nodes, say node b, has $P_b \to 0$,

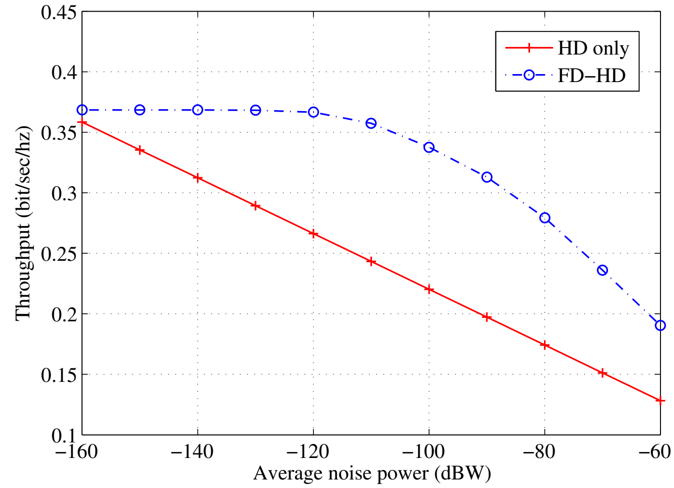

Fig. 2. Optimal throughput of a single pair of FD nodes versus noise level.

the denominator of SINR$_b$ (see (4)) will be reduced close to zero. The throughput of node b will be dramatically increased, which dominates the reduction in the throughput of node a. Thus the pair will work in the HD mode.

Next, we demonstrate the performance of the proposed algorithms for an FD network of $M = 4$ node pairs. The general approximation algorithm shown in Algorithm 3 is used to obtain the results shown in Figs. 3 and 4. The convergence of the eight transmit powers are plotted in Fig. 3. With the proposed algorithm, all the transmit powers converge to the optimal power allocation after several iterations. After convergence, node 2 transmits at the maximum power P_{max}, nodes 3, 4, 7, and 8 are not allowed to transmit (with transmit power zero), and nodes 1, 5, and 6 assumes some transmit power lower than P_{max}. This is consistent with the single pair of node case in Lemma 1 (i.e., at least one node transmits at the peak power). Among the four node pairs, two of them operates in the FD mode (1–2 and 5–6), while two of them are turned off (3–4 and 7–8), in order to achieve a larger sum rate. As the channels vary over time slots, the node pairs with power zero in this time slot will get their turns to transmit in some future time slots.

The convergence of the sum rate is presented in Fig. 4. We find that the sum rate also converges after several iterations. With Algorithm 3, the sum rate is always non-decreasing across the iterations. The optimal sum rate of the FD network is about 1.6 times of that of the HD network, demonstrating the benefits of FD transmissions and distributed power control.

VI. Related Work

We briefly review related work on FD in this section. There have been several seminal works on both the theoretic aspect (such as deriving the capacity region [1]) and practical design of FD wireless networks [2]. A practical design and implementation has been reported in [2], including the antenna structure, several kinds of self-interference cancellation techniques, and MAC layer design. The authors showed that up to 73dB

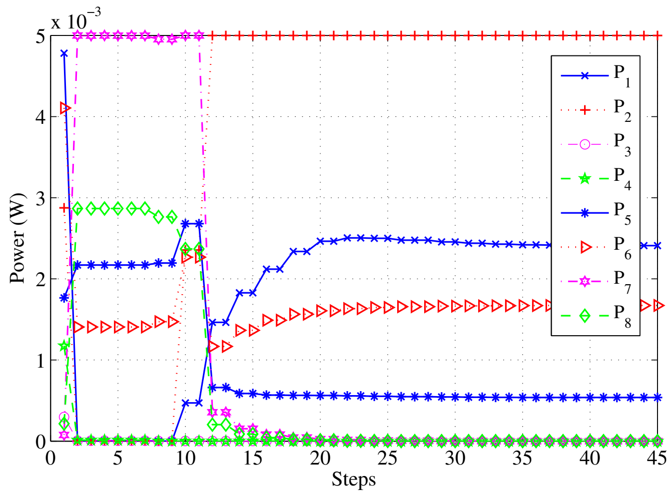

Fig. 3. Convergence performance of the proposed algorithm for four pairs of FD nodes.

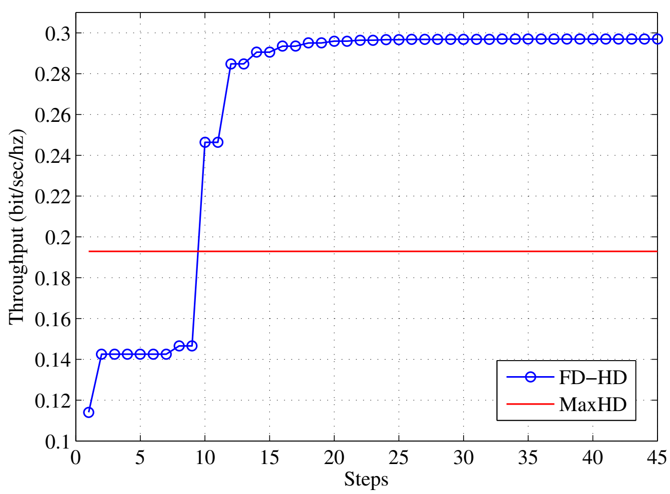

Fig. 4. Convergence of the sum rate of four pairs of FD nodes.

suppression of self-interference for a 10MHz OFDM signal and at least 45dB reduction at a 40MHz channel are achievable. Various self-interference cancellation schemes have been investigated in prior works [3]–[5], such as PDIS, Analog Self-Interference Cancellation, which reduces the self-signal by the design of distance between transmitting antenna and receiving antenna, and Digital Self-Interference Cancellation, which cancels the self-signal by subtracting the transmitted signal from the receiving signal.

In [12], the authors studied power allocation for a single pair of FD nodes equipped with MIMO. It was assumed that the power for each single antenna at the same node were equal. The two transmit powers for the FD pair were then optimized to achieve the maximum throughput. In [6], the details of self-interference cancellation was reviewed and the paper was then focused on antenna design, including the arrangement of the position of transmitting and receiving antennas. In a recent work [13], we studied power control for an underlay cognitive ratio network with FD transmissions, and developed a centralized scheme with a control-theoretic approach. In another recent work [14], we investigated incorporation of FD transmissions in cognitive femtocell networks.

VII. Conclusion

In this paper, we developed distributed algorithms for optimal power control in FD wireless networks. For the case of a single pair of nodes, we presented a simple algorithm that can compute the optimal powers. For the case of multiple pair of FD nodes, we first developed a distributed algorithm by applying the high SINR approximation, and then proposed a distributed algorithm based on an iterative approximation method for the logarithm function. The proposed algorithms are validated with simulation studies.

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