A Distributed Online Algorithm for Optimal Real-time Energy Distribution in Smart Grid

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***Abstract*—The two-way energy and information flows in a smart gird, together with the smart devices, bring new perspectives to energy management and demand response. This paper investigates a distributed online algorithm for electricity energy distribution in a smart grid environment. We first present a formulation that captures the key design factors such as user utility, grid load smoothing, and energy provisioning cost. The problem is shown to be convex and can be solved with an online algorithm that only requires present information about users and the grid in our prior work. In this paper, we develop a distributed online algorithm which decomposes and solves the online problem in a distributed manner, and prove that the distributed online solution is asymptotically optimal. The proposed distributed online algorithm is also practical and effective for user privacy protection. It is evaluated with trace-driven simulations and shown to outperform a benchmark scheme.**

I. INTRODUCTION

A smart grid is an electrical grid that is enhanced with communications and networking, computing, and signal processing technologies [1]. As the identifying characteristics of the smart grid, the two-way flows of electricity and real-time information, along with the smart devices and smart meters, brings new perspectives to energy management and demand response in the smart grid.

Demand side management is one of the most important problems in smart grid research, which aims to match electricity demand to supply for enhanced energy efficiency and demand profile while considering user utility, cost and price [1]. Researchers have been focusing on peak shift or peak reduction for reducing the grid deployment and operational cost [2], [3], as well as on reducing user or energy provider's cost [4], [5]. In particular, some prior works have jointly considered both the user and energy provider costs, to increase the users' utility as much as possible while keeping the energy provider's cost at a relatively lower level [6]. Another aspect, user's privacy is also emphasized in the demand side management for individual privacy protection in practice. Some researches [7], [8] investigate privacy in smart grid from various aspects and show that an individual's daily life can even be rebuilt with collection of data on power usage.

Given the wide range of smart grid models and the challenge in characterizing the electricity demand and supply processes, the utility, cost, pricing functions, and user's privacy, a general model that can accommodate various application scenarios would be highly desirable. Furthermore, it is important to jointly consider the utilities and costs of the key components of the system to achieve optimized performance for the overall smart grid system.

In this paper, we consider real-time energy distribution in a smart grid system. Energy flows and information communications overlay each other. The distribution control center (DCC) controls the energy distribution of the energy provider (EP), and coordinates the communications between the users and the EP. It also monitors the grid and the entire system for unexpected situations. The smart meter (SM) at the user side will be responsible for the information exchange with the DCC and for scheduling the electrical appliances in the house. The information flows will be carried through a communications network infrastructure, such as a wireless network or a powerline communication system [1].

For optimizing the performance of such a complex network system, the utilities and costs of the three key components, i.e., the users, the grid, and the EP, should be jointly considered. In this paper, we take a holistic approach, to incorporate the key design factors including user's utility and privacy, grid load smoothing, and energy provisioning cost in a problem formulation. To solve the real-time energy distribution problem, we first present an offline solution and then introduce an online algorithm from our prior work for the sake of completeness [9]. The main contribution of this paper is a *distributed online algorithm*, which firstly decomposes the master problem into several subproblems and then solves them locally at each user and the DCC.

The proposed framework is quite general. It does not require any specific models for the electricity demand and supply processes, and only have some mild assumptions on the utility and cost functions (e.g., convex and differentiable). The proposed algorithm can thus be applied to many different scenarios. It combines the advantages of online algorithm which requires no future information for a convergent solution and distributed algorithm, which solves the problem in a distributed way for privacy protection and scalability. The proposed algorithm is amenable to be implemented in a real smart grid system. It is also asymptotically optimal, a highly desirable property. The proposed algorithm is evaluated with trace-driven simulation using energy consumption traces recorded in the field. It achieves comparable performance as the centralized online algorithm and outperforms a benchmark scheme that is also distributed online but with no control for grid load smoothing.

The remainder of this paper is organized as follows. We present the system model in Section II. The problem formu-

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lation with both offline and online solutions are introduced in Section III. The distributed online algorithm is developed and analyzed in Section IV. We present the simulation studies in Section V and review related works in Section VI. Section VII concludes this paper.

II. System Model

A. Network Structure

We consider a power distribution system in a smart grid environment where one EP supports the power usage of all users. The users could be residential, commercial and industrial energy consumers. Each user deploys an SM to monitor and control the energy consumption of the electrical appliances [1]. All SMs are connected to the DCC of EP through the information infrastructure such as a local area network. In each distribution time cycle, SMs and the DCC exchange some information for users to maximize their utility, EP to minimize the generating cost and the grid to smooth the total power variance. The intended time period for the operation is divided into T time slots, indexed by $t \in \mathbb{T} = \{1, 2, \cdots T\}$ where $\mathbb{T}$ is the set of time slots. Usually, the operation time period is a one-day cycle based on the daily periodical nature of electricity usage, while the division could be 1 hour, 0.5 hour, 15 minutes, etc., according to the users' power demand pattern in consideration of changing demand in different time for a day, and the number of users in an area in consideration of the time needed for communication.

Here, we denote the power consumption of user i at time t as $p_i(t)$ and let $\mathbb{N} = \{1, 2, \cdots, N\}$ be the set of users. We also define a set $\mathbb{P}$ of energy consumption at each time t for each user as

$$\mathbb{P} = [p_{i,min}(t), p_{i,max}(t)], \tag{1}$$

where $p_{i,min}(t)$ is the minimum power demand of user i at time t and $p_{i,max}(t)$ is the maximum power demand as we assume the users are rational. And $\mathbb{P}$ includes all the possible value of power demanded and used, that is, $p_i(t) \in \mathbb{P}$.

B. User Utility Function

We assume that each user behaves independently in the power grid. They have their own preferences and time schedules for using different electrical appliances, for example, different users may set different temperatures for the air conditioner and different users may use the washing machine at different times in a day. Also, the user demand may vary as weather changes, and may have different reactions to different pricing schemes [5]. Therefore, it is challenging to characterize user preferences with a precise mathematical model.

In prior work, user preference is usually represented by a *utility function* [4]. Similarly, we use function $U(p_i(t), \omega_i(t))$ to represent user i's satisfaction level on power consumption. In this paper, $U(\cdot)$ is a general strictly increasing, concave function of the allocated power $p_i(t)$ (e.g., the quadratic utility function used frequently [4]–[6]).

The other parameter $\omega_i(t)$ of the utility function indicates user i's flexibility at time t. A larger $\omega_i(t)$ means higher flexibility. $\omega_i(t)$ could be different for users or vary over time. In a centralized scheduling scheme, the DCC will require the $\omega_i(t)$'s in every updating interval. For users, the utility function is a private information, which can be used possibly to reconstruct many aspects of users' daily life in detail and infringe their privacy [7], [8]. So information about utility function and its parameters should be carefully protected. To this end, a distributed algorithm that does not require exchanging privacy information would be highly appealing.

C. Energy Provision Cost Function

For EPs, when the demand is at normal levels, the generation cost increases only slowly as the demand grows. However, the cost will drastically increase when the load peak approaches the grid capacity. This is because the provider has to transmit more power from the outside or backup batteries to avoid a blackout. Therefore, we could use a general increasing and strictly convex function to approximate the *cost function* for energy provisioning. Similar to [5], [6], we choose a quadratic function to model the provider's cost.

$$C(g(t)) = a \cdot g^2(t) + b \cdot g(t) + c, \tag{2}$$

where $a > 0$ and $b, c \geq 0$ are pre-selected for the power grid and $g(t)$ denotes the total power generated by the EP for time slot t. The EP has to provide sufficient power for users while reducing its cost. We also assume a maximum generating capacity $g_{max}(t)$for EP at time t. Thus we have the following constraint for $g(t)$.

$$\sum_{i \in \mathbb{N}} p_i(t) \leq g(t) \leq g_{max}(t), \forall\, t \in \mathbb{T}. \tag{3}$$

The constraint indicates that $g(t) \in \mathbb{G} = [g_{min}(t), g_{max}(t)]$, where $g_{min}(t) = \sum_{i \in \mathbb{N}} p_i(t)$ and $\mathbb{G}$ is a closed set. Because the *cost function* $C(\cdot)$ is strictly convex and increasing, $C(\cdot)$ is reversible so that the energy provision cost $C(g(t))$ is also bounded in a closed set, i.e., $C(g(t)) \in \mathbb{C}$, for all t. In other words, the EP can control its provision cost by adjusting the amount of power generation.

III. Problem Formulation and Centralized Solutions

In this section, we summarize the problem formulation and the centralized offline and online algorithms presented in our prior work [9] for the sake of completeness, and do not claim contribution for this part. The proposed distributed online algorithm will be presented in Section IV and evaluated in Section V.

A. Problem Formulation

We take into account three important parts: users, EP and the grid. Under certain constraints, we aim to maximize user utility, minimize EP cost, and smooth the total power of the grid.

We first consider an offline scenario where the DCC has global information on users' flexibility $\omega_i(t)$ and on the EP's total generated power $g(t)$ for the entire period (i.e., future information is known). Let $P_i(t)$ denote the power usage for user i at time t, for $t \in \mathbb{T}$. We use upper case P in the *offline problem*, where all the necessary constraints are known a priori. In the corresponding *online problem*, which will be examined in Section III-C, we use lower case p for the corresponding variables. A vector with subscript i is used to denote a time sequence, e.g., $\vec{P}_i$ for the power usage by user i for $t \in \mathbb{T}$.

The offline problem (termed Prob-OFF) can be formulated as follows. For $P_i(t) \in \mathbb{P}, g(t) \in \mathbb{G}, \forall i \in \mathbb{N}, t \in \mathbb{T}$, we have [9]

$$\max: \sum_{t=1}^{T}\left[\sum_{i\in\mathbb{N}} U(P_i(t),\omega_i(t)) - C(g(t))\right] - \frac{\alpha T}{2}\text{Var}\left(\sum_{i\in\mathbb{N}}\vec{P}_i\right) \quad (4)$$

$$\text{subject to:} \sum_{i\in\mathbb{N}} P_i(t) \leq g(t), \forall\, t \in \mathbb{T}, \quad (5)$$

where

$$\text{Var}\left(\sum_{i\in\mathbb{N}}\vec{P}_i\right) = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i\in\mathbb{N}}P_i(t) - \frac{1}{T}\sum_{k=1}^{T}\sum_{i\in\mathbb{N}}P_i(k)\right)^2.$$

The objective function (4) consists of three parts. The first part represents users' satisfaction and preference. The second part represents EP's energy provision cost. And the third part represents the load variance of the grid. It is integrated with a parameter $\alpha > 0$, allowing a trade-off between the benefits for the grid operator and user's. All the users' demand and generating power should be included in the set $\mathbb{P}$ and $\mathbb{G}$ as discussed in Sections II-B and II-C.

We next present an algorithm that can solve this offline problem and explain how we can move from the offline approach to an online algorithm. In Section III-C, we then present the corresponding centralized online algorithm that does not require any a priori user/grid information, and show that the online algorithm is asymptotically optimal.

B. Offline Algorithm

In problem Prob-OFF (4), the user power consumption $P_i(t)$'s are independent. Hence the variance term can be rewritten as $\text{Var}(\sum_{i\in\mathbb{N}}\vec{P}_i) = \sum_{i\in\mathbb{N}}\text{Var}(\vec{P}_i)$. It can be verified that Prob-OFF is a convex optimization problem, because the function $U(\cdot)$ is concave and $C(\cdot)$ and $Var(\cdot)$ are both convex. Also due to the convexity of $Var(\cdot)$, we can show that Prob-OFF has a unique solution [9]. Thus, if we carefully define sets $\mathbb{P}$ and $\mathbb{G}$, the Slater's condition can be satisfied [10], which indicates that the KKT conditions are sufficient and necessary for the optimality of Prob-OFF [10]. By solving the KKT conditions, we can obtain the optimal energy distribution for each user at each time slot [9].

In Prob-OFF, all information are assumed to be known a priori. Because of this, its solutions are optimal. However, since it requires future information to obtain the grid variance (the third part in (4)), we cannot solve the KKT conditions at each time slot. For this, we proposed an online algorithm to this problem without using any future information in our prior work [9], which is briefly introduced in the following for the sake of completeness.

C. Online Algorithm

We now present the online algorithm for energy distribution, and summarize the main result that the online solution is asymptotically convergent to the offline optimal solution, i.e., *asymptotically optimal* [9]. The online energy distribution algorithm consists of the following three steps [9].

Algorithm 1: Centralized Online Algorithm

Step 1: For each $i \in \mathbb{N}$, initialize $\hat{p}_i(0) \in \mathbb{P}$.
Step 2: In each time slot t, the DCC solves the following convex optimization problem (termed Prob-ON). For $p_i(t) \in \mathbb{P}, g(t) \in \mathbb{G}, \forall i \in \mathbb{N}$,

$$\max: \sum_{i\in\mathbb{N}} U(p_i(t),\omega_i(t)) - C(g(t)) - \frac{\alpha}{2}\sum_{i\in\mathbb{N}}(p_i(t) - \hat{p}_i(t-1))^2 \quad (6)$$

$$\text{subject to:} \sum_{i\in\mathbb{N}} p_i(t) \leq g(t), \forall t \in \mathbb{T}. \quad (7)$$

Let $\vec{p}^*(t)$ denote the solution to Prob-ON, where each element $p_i^*(t)$ represents the optimal power allocation to user i.
Step 3: Update $\hat{p}_i(t)$ for all $i \in \mathbb{N}$ as follows.

$$\hat{p}_i(t) = \hat{p}_i(t-1) + \frac{\alpha}{t+\alpha}\cdot(p_i^*(t) - \hat{p}_i(t-1)). \quad (8)$$

Comparing to (4), the variance term is approximated by $\sum_{i\in\mathbb{N}}(p_i - \hat{p}_i(t-1))^2$ in (6). Similar to Prob-OFF, problem Prob-ON is also a convex optimization problem satisfying Slater's condition. Then we can write its KKT conditions as follows,

$$\begin{cases} U'(p_i^*(t),\omega_i(t)) - \alpha\,(p_i^*(t) - \hat{p}_i(t-1)) - \lambda^*(t) = 0 \\ -C'(g(t)) + \lambda^*(t) = 0 \\ \lambda^*(t)\left(\sum_{i\in\mathbb{N}} p_i^*(t)/g(t) - 1\right) = 0 \\ \lambda^*(t) \geq 0, \ \ \forall\, t, \end{cases} \quad (9)$$

where $\lambda^*(t)$ is the Lagrange multiplier. In (9), only information for time slot t is needed to solve the equations. This allows us to solve the problem in each time slot without needing any future information. The solution will be optimal if it converges to the optimal Prob-OFF solution, where future information is needed. The following theorem confirms this conjecture (see [9] for a complete proof).

Theorem 1. *The online optimal solution converges asymptotically and almost surely to the offline optimal solution [9].*

Although the formulation in [9] is slightly different with our problem in this paper, the conditions of the theorem are still satisfied in our model. Therefore, the theorem still holds true. It presents a strong result, based on which we can solve Prob-ON instead of Prob-OFF but with an equally good result. However, Prob-ON is still solved in a centralized manner, which means at each time slot, the DCC still requires accurate utility functions of all users (with their preference parameters $\omega_i(t)$). As discussed, utility functions are important user privacy information. It will be highly appealing to develop a distributed algorithm that can preserve user privacy, but still achieve the optimal performance. We present such a distributed algorithm in the next section.

IV. Distributed Algorithm

In this section, we firstly decompose problem Prob-ON in a distributed manner so that the DCC and the users can solve each subproblem independently. We then present a distributed algorithm to solve the online problem. Thus, a user's utility function will not be shared with the DCC or other users.

A. Problem Decomposition

Firstly, the objective function (6) can be rewritten as

$$\sum_{i\in\mathbb{N}}\left[U(p_i(t),\omega_i(t))-\frac{\alpha}{2}(p_i(t)-\hat{p}_i(t-1))^2\right]-C(g(t)), \tag{10}$$

where the first part is a function of $p_i(t)$ and $\omega_i(t)$ (information available at user i), and the second part is a function of $g(t)$ (information available at the EP). But we cannot decompose the problem simply in this way, because constraint (7) contains both $p_i(t)$ and $g(t)$.

The Lagrangian of problem Prob-ON can be written as [10]

$$\begin{aligned}&L(\vec{p}(t),g(t),\lambda(t))\\&=\sum_{i\in\mathbb{N}}\left[U(p_i(t),\omega_i(t))-\frac{\alpha}{2}(p_i(t)-\hat{p}_i(t-1))^2\right]-\\&\quad C(g(t))-\lambda(t)\left(\sum_{i\in\mathbb{N}}p_i(t)-g(t)\right)\\&=\sum_{i\in\mathbb{N}}\Big[U(p_i(t),\omega_i(t))-\frac{\alpha}{2}(p_i(t)-\hat{p}_i(t-1))^2-\\&\quad\lambda(t)p_i(t)\Big]-\left(C(g(t))-\lambda(t)g(t)\right),\end{aligned} \tag{11}$$

where $\lambda(t)$ is the Lagrange multiplier. In (11), functions of $p_i(t)$ and $g(t)$ are separated. For each $p_i(t)\in\mathbb{P}$, define

$$\begin{aligned}&T_i(\lambda(t))=\\&\max\left\{U(p_i(t),\omega_i(t))-\frac{\alpha}{2}(p_i(t)-\hat{p}_i(t-1))^2-\lambda(t)p_i(t)\right\}.\end{aligned} \tag{12}$$

For $g(t)\in\mathbb{G}$, define

$$K(\lambda(t))=\max\left\{\lambda(t)g(t)-C(g(t))\right\}. \tag{13}$$

We can reformulate problem Prob-ON to the *Lagrange dual problem* as follows [10].

$$\begin{aligned}\text{minimize:}\quad & F(\lambda(t))\\ \text{subject to:}\quad & \lambda(t)\geq 0,\end{aligned} \tag{14}$$

where

$$\begin{aligned}F(\lambda(t))&=\max\left\{L(\vec{p}(t),g(t),\lambda(t))\right\}\\&=\sum_{i\in\mathbb{N}}T_i(\lambda(t))+K(\lambda(t)).\end{aligned} \tag{15}$$

Therefore problem Prob-ON is decomposed into two parts: (i) the first one is an optimization problem $T_i(\lambda(t))$ in (12) for each user to solve, and (ii) the other one is also an optimization problem $K(\lambda(t))$ for the EP to solve in (13). Then we have the following theorem.

Theorem 2. *The online optimal energy distribution solution could be obtained by solving its associated dual problem.*

Proof: For the online problem Prob-ON that has a concave objective function and convex constraints, we have *strong duality* for proper sets $\mathbb{P}$ and $\mathbb{G}$. This means the optimal duality gap between the Prob-ON and the dual problem is zero (see Chapter 5 in [10]). Thus to find the optimal solutions, the dual problem can be solved instead. ■

The above theorem presents an effective means to solve the online distribution problem. It decomposes the problem into many subproblems solved by the SM at each user and by the DCC, respectively. In this way, we can protect the users' privacy while still achieving the optimal distribution results as well as scalability.

B. Distributed Online Algorithm

According to Theorem 2, we can solve the dual problem (14) to acquire the optimal solutions. Because of constraint (7), $T_i(\lambda(t))$ and $K(\lambda(t))$ are coupled by the Lagrange multiplier $\lambda(t)$; $\lambda(t)$ is associated with both the user utility maximization problem (12) and the EP cost minimization problem (13). As the dual variable, it is also a key parameter for solving the dual problem. In our case, the dual function $F(\lambda(t))$ is differentiable. So we apply the following gradient method [11] to acquire the dual variable $\lambda(t)$.

$$\lambda(t+1)=\left[\lambda(t)-\delta(g(t)-\sum_{i\in\mathbb{N}}p_i(t))\right]^+, \tag{16}$$

where δ is the step-size and $[\cdot]^+$ is the projection onto the nonnegative orthant.

At each time slot t, the DCC employs (16) to update $\lambda(t)$ and broadcasts the new value $\lambda(t+1)$ to all the users. We present the distributed online algorithm, Algorithm 2, to solve the dual problem (14) as well as problem Prob-ON. The algorithm is composed of two parts: (i) a three-step Algorithm 2.a for all the users, and (ii) a two-step Algorithm 2.b executed by the DCC.

Algorithm 2.a: Distributed Online Algorithm for the Users

Step 1: For each $i\in\mathbb{N}$, initialize $\hat{p}_i(0)\in\mathbb{P}$.
Step 2: In time slot t, the SM of each user does the following:
1) Receives the updated $\lambda(t)$ from the DCC;
2) Solves problem (12) for user utility maximization;
3) Sends solution $p_i^*(t)$ to the DCC as energy demand.
Step 3: Update $\hat{p}_i(t)$ for all $i\in\mathbb{N}$ as in (8).

Algorithm 2.b: Distributed Online Algorithm for the DCC

Step 1: For each $i\in\mathbb{N}$, initialize $\hat{p}_i(0)\in\mathbb{P}$. Choose any $\lambda(0)\geq 0$.

Step 2: In each time slot t, the DCC does the following:

1) Updates the value of $\lambda(t)$ using (16) and broadcasts to the users;

2) Solves problem (13) and generates energy;

3) Receives $p_i^*(t)$ from all users to distribute the energy accordingly.

In Algorithm 2, we see an interaction between the users and the EP, enabled by the dual variable $\lambda(t)$. It is not only the necessary parameter to solve both (12) and (13), but $\lambda(t)$ couples user and EP decisions. The DCC has no information about user utilities, while $\lambda(t)$ instead conveys information from users to the DCC. By updating $\lambda(t)$ as in (16), the new value contains new information from both users and the EP. Thus, by using Algorithm 2, the online problem can be solved in a distributed fashion with comparable results to the centralized online algorithm. Furthermore, from Theorems 1 and 2, the distributed solution from Algorithm 2 will also converge asymptotically to the offline optimal solution.

It is also worth noting that the information exchanged between users and the DCC does not require any information on user utility. Consider practical data communication network for the smart grid, less transmitted data brings about higher security, reliability and efficiency. This also helps simplify the communication protocol design and implementation for the smart grid. Furthermore, the computation load is offloaded from the DCC to each user's SM, and the computation at the DCC is simplified, leading to much resource and time savings so that a larger number of users could be supported. In conclusion, the distributed online Algorithm 2 could be very useful in practice.

V. Performance Evaluation

In this section, we evaluate the proposed distributed online algorithm (DOA) with trace-driven simulations. The simulation data and parameters are acquired from the traces of power consumption in the Southern California Edison (SCE) area recorded in 2011 [12]. We first study the performance of DOA on convergence comparing to the centralized online algorithm (COA) proposed in our prior work [9] (see Section III-B). We then compare the distribution solutions between DOA and COA. We have shown the superiority of COA over a state-of-the-art scheme in [9], and will use COA for comparison purpose.

Consider a power distribution system in a small area with $N = 20$ users. The updating period is 15 minutes. Note that 15 minutes is a practical choice for COA, because it is applied in a centralized manner with more transmitted information and more complicated calculations. And 15 minutes is also short enough to capture the users' change of demand. With DOA, a shorter update period is also practical. We will show results within a 24-hour period for an evaluation of the daily operations. We choose users' utility function from a function set $\mathbb{U}$, in which the functions are generated as the widely used quadratic expression (see [4], [5]) with $\omega_i(t) \in (0,1)$ randomly selected.

$$U(p_i(t),\omega_i(t)) = \begin{cases} \omega_i(t)p_i(t) - \frac{1}{8}p_i(t)^2, & \text{if } 0 \le p_i(t) \le 4\omega_i(t) \\ 4\omega_i(t), & \text{if } p_i(t) \ge 4\omega_i(t). \end{cases} \quad (17)$$

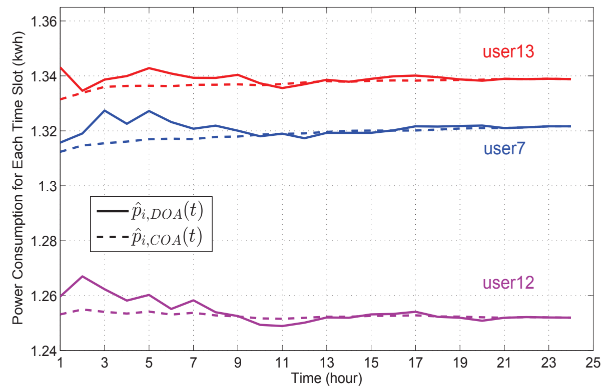

Fig. 1. Convergence of $\hat{p}_i(t)$ for DOA and COA under different numbers of users.

We also assume user's energy demand $p_i(t)$ is selected from a set $\mathbb{P} = [1.0, 3.0]$, for all i. The maximum generating power $g_{max}(t)$ is set to the maximum total power demand of all the users, i.e., $g_{max}(t) = \sum_{i\in\mathbb{N}} p_{i,max}(t)$, implying the generated power is equal to the total power demand. The initial value of $\lambda(t)$ in Algorithm 2 is randomly picked from set $(0, 1)$. The parameters in the energy provisioning cost function (2) are chosen as $a = 0.05$, $b = c = 0$. These parameters are determined after a careful study of the SCE trace characteristics. Parameter α in the updating function (8) is set as $\alpha = 1$ in the simulations. In [9], we have shown that $\alpha = 1$ is a proper value for fast convergence.

Firstly, we show the convergence of $\hat{p}_i(t)$ from both COA and DOA. $\hat{p}_i(t)$ is a key variable in the online algorithm. Its convergence indicates that the gap between the solutions of the online and offline problems approaches zero (see the updating function (8)). In Fig. 1, $\hat{p}_{i,COA}(t)$ and $\hat{p}_{i,DOA(t)}$ are both convergent. For COA, we see a fairly quick convergence with a short transient period. For DOA, it shows relatively slower convergence with more variance before it achieves the stable value. This is because comparing to COA, DOA has another iteration function introduced by (16), which is used to update $\lambda(t)$. Since the initial value $\lambda(0)$ is randomly set, it requires extra time for the convergence of $\hat{p}_{i,DOA}(t)$. Also in Fig. 1, we notice the coincidence of two curves for the last several time slots. This can be explained by Theorem 2, which indicates identical solutions from DOA and COA.

One important benefit of the DOA is variance control, which is inherited from COA. In Fig. 2, we plot the actual grid load (AGL) and total power consumption achieved by DOA, COA and the dynamic pricing algorithm (DPA) proposed in [6], which is also based on utility maximization. DPA considers both users and EP as we do in our paper, but it has no consideration on the load variance. In this simulation, the actual grid load is the summation of 20 independent users'

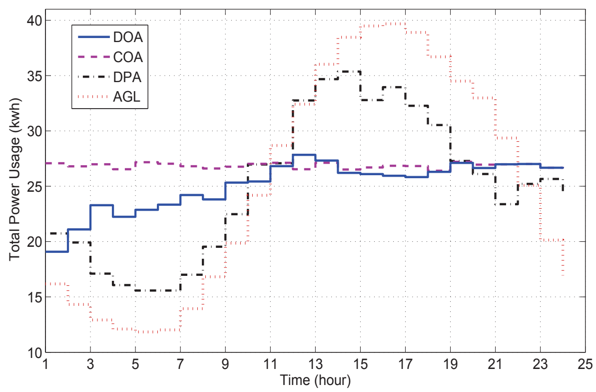

Fig. 2. Actual grid load and total power consumption by DOA, COA and DPA.

consumption generated by the average real load in the SCE trace on a hot day (i.e., Sept. 1, 2011) [12]. From Fig. 2, we find DOA convergent to COA after several time slots of fluctuation. For peak reduction over AGL, DOA achieves almost the same result as COA, which is 29.8% towards 31.7%. However, DPA only achieves a 10.9% gain in this scenario with 20 users.

VI. Related Work

Smart grid, characterized by the two-way flows of electricity and information, is envisioned to replace the existing power grid in the future [13], [14]. A comprehensive review on smart grid technologies and research can be found in [1], where major topics on smart grid are discussed in three areas: infrastructure, management and protection.

Within the three areas, demand side management or demand response has been attracting considerable research efforts [2], [4], [5], [15], [16]. Researchers mainly work on demand profile shaping, user utility maximization and cost reduction. For example, machine learning is used in [4] to develop a learning algorithm for energy costs reduction and energy usage smoothing, while [15] aims to balance the user cost and waiting time. Online algorithms [17], which are widely used in wireless communications and networking, are also deployed in some works [6], [9]. In [6], the authors propose an dynamic pricing algorithm based on utility maximization in a distributed way. And [9] presents an online algorithm which achieves the optimal energy distribution and variance control without any future information.

On the other hand, for a practical consideration, user's privacy is emphasized more and more by many authors [7], [8]. In [7], the author studies how high resolution user electricity information can be used to reconstruct a user's daily life and preference. In [8], the privacy in smart grid is examined from definition to different concerns in detail.

VII. Conclusion

In this paper, we present a study of optimal distributed online energy distribution in smart grid. With a formulation that captures the key design factors of the system, we extend our prior work of a centralized online algorithm, by decomposing the problem into many subproblems that can be solved in a distributed manner, thus protecting users' privacy and achieving scalability. We also show that the distributed online solution converges to the centralized online optimal solution, and to the optimal offline solution asymptotically. The proposed distributed online algorithm is evaluated with trace-driven simulations and outperforms an existing benchmark.

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