

# Power Control in Full Duplex Underlay Cognitive Radio Networks: A Control Theoretic Approach

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**Abstract**—Both cognitive radio (CR) and full duplex transmissions are both effective means to enhance spectrum efficiency and network capacity. In this paper, we investigate the problem of power control in an underlay CR network where the CR nodes are capable of full-duplex (FD) transmissions. The objective is to guarantee the required quality of service (QoS) in the form of a minimum signal-to-interference-plus-noise (SINR) ratio at each CR user and keep the interference to primary users below a prescribed threshold. We design an effective distributed power control scheme that integrates a proportional-integral-derivative (PID) controller and a power constraint mechanism to achieve the above goals. We then develop a hybrid scheme that can switch between FD and half duplex modes. The proposed schemes are validated with extensive simulations.

## I. INTRODUCTION

In recent years, an unprecedented increase in wireless data has been observed, largely due to the proliferation of smartphones, tablets and other wireless devices. The exploding wireless data calls for effective technologies for enhancing spectrum utilization and wireless network capacity. To this end, cognitive radios (CR) have been recognized as one of the key technologies to meet this grand challenge on wireless network capacity. As an effective means of sharing spectrum among licensed (i.e., primary) users (PU) and unlicensed (i.e., secondary) users (SU), CR has been demonstrated to achieve high utilization of the scarce spectrum resource [1], [2].

In CR networks, the most important design factor is to balance the tension between PU protection and SU spectrum access gains [1]. On one hand, the capacity of SUs should be maximized to “squeeze” the most out of the spectrum. On the other hand, the adverse impact to PUs, resulting from sharing spectrum with SUs, should be kept below a tolerable level. Obviously, these are two conflicting goals that should be balanced in the design of CR networks. In the so-called overlay CR networks, PU protection is achieved by spectrum sensing and spectrum access only when the PUs are sensed absent [1]. In the so-called underlay CR networks, both PU and SU transmissions coexist in the same spectrum band, and PU protection is achieved by carefully controlling the power of the SU transmitters [3].

Recently, a breakthrough in wireless communications is full duplex (FD) transmission [4]–[7]. Traditionally, wireless communications are all half duplex (HD) due to the large path loss typical in wireless transmissions. If FD transmission is allowed, the self-interference will be so strong (like the

sun) and the weak received signal from a remote transmitter (like stars) will be completely overwhelmed and cannot be decoded. Recently, encouraging results have been reported on enabling FD wireless transmissions in both single link and a network setting [4]–[7]. The enabler of HD is the recent advances in self-interference suppression (SIS). Various effective SIS techniques have been proposed and tested, such as antenna separation [4], antenna cancellation [5], signal inversion and adaptive cancellation [6], and combined optimal antenna placement and analog cancellation [8]. In [8], the author showed a practical implementation that can suppress self-interference (SI) for up to 80 dB, which should be sufficient for many application environments [9].

In a recent work [9], the authors propose to integrate FD in overlay CR networks. It is demonstrated that an FD-enabled SU can operate in either simultaneous transmit-and-sense mode or simultaneous transmit-and-receive mode. The authors analytically study the performance of the two modes and evaluate the sensing-throughput tradeoff for both modes. In this paper, we investigate the problem of integrating FD in underlay CR networks. We consider a primary network co-located with multiple SU links. The SUs are capable of FD transmissions. As discussed, the key design issue for underlay CR networks is how to design an effective power control scheme to achieve the dual goal of PU protection and SU spectrum access gain maximization.

For PU protection, we consider multiple detection points (DP) in the network for measuring interference from SU transmissions. Based on the measured interference, each SU transmitter adjusts its transmit power to achieve the primary goal of keeping the measured interference to PUs below a prescribed threshold, and the secondary goal of guaranteeing the quality of service (QoS) of SUs in the form of a minimum signal-to-interference-plus-noise ratio (SINR). We develop a distributed power control scheme that consists of a proportional-integral-derivative (PID) controller, for satisfying the SU QoS requirements, and an additional power constraint mechanism, for PU protection. We develop a hybrid HD-FD scheme for harvesting the benefits of both modes under various network settings. The stability and throughput performance of the proposed schemes are validated with simulations.

The remainder of our paper is organized as follows. We first present the system model and problem statement in Section II. In Section III we develop the power control scheme and

a hybrid HD/FD scheme. Simulation results are presented in Section IV and related work discussed in Section V. Section VI concludes the paper.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

### A. System Model

There is a primary network with active transmissions using a licensed spectrum band. A co-located secondary network consists of  $(s+1)$  SUs, termed  $TR_i$ ,  $i = 1, 2, \dots, s+1$ , where  $s$  is an odd number. The SUs are paired to form  $(s+1)/2$  FD transmission links, i.e.,  $TR_i$  is transmitting to, and simultaneously receiving from  $TR_{i+1}$ , while  $i$  is an odd index. Due to the underlay spectrum sharing policy, the SUs are allowed to use the same spectrum band as the primary network. For protection of the primary network, there are  $p$  detection points (DP) in the primary work that measure the interference from the secondary transmissions. Such interference should be kept below a threshold at the DP locations by effectively controlling the power of the secondary transmitters.

We assume block fading channels. For a given time slot, let  $g_{ij}$  denote the channel gain from  $TR_i$  to  $DP_j$ ;  $h_{ij}$  represent the channel gain from  $TR_i$  to  $TR_j$ ; and  $\sigma_i^2$  be the sum of the total interference from primary transmissions and the noise power at  $TR_i$ . To simplify notation, we assume channel reciprocity, i.e.,  $h_{ij}$  (or  $g_{ij}$ ) is equal to  $h_{ji}$  (or  $g_{ji}$ ) for all  $i, j$ . For each FD link, the self-interference is  $P_i(t)h_{ii}^2$ , where  $P_i(t)$  is the transmit power of  $TR_i$  and  $h_{ii}$  is the channel gain from  $TR_i$ 's transmitting antenna to the receiving antenna. We assume that each  $TR_i$  utilizes SIS, and the residual self-interference is reduced to  $\chi P_i(t)h_{ii}^2$ , where  $\chi$  is a constant in  $[0, 1]$  depending on the specific SIS design. When  $\chi = 0$ , it is the perfect case where the self-interference can be completely canceled; when  $\chi = 1$ , it is the worst case without SIS and FD transmission is not possible. Usually  $\chi$  is a small number, e.g., at least 45 dB across a 40 MHz band and up to 73 dB for a 10 MHz OFDM signal [6].

### B. Problem Statement

For the FD CR network to work properly, two conditions should be satisfied by controlling the transmit power of the  $TR_j$ 's. The first condition is *primary user protection*. That is, the measured interference from secondary transmissions should be kept below a prescribed tolerance level  $D_j$  at each  $DP_j$ . The second condition is *guaranteeing the QoS of SUs*. That is, the SINR at the  $TR_i$ 's should be kept above a prescribed threshold  $\Gamma$ , such that the SUs can be guaranteed with a minimum data rate.

We assume that time is slotted. To achieve these goals, in each time slot  $t$ , a distributed power control algorithm updates the transmit power of each  $TR_i$ , denoted as  $P_i(t)$ , according to the measured radio environment, as

$$P_i(t+1) = P_i(t) + u_i(t), \quad (1)$$

where  $u_i(t)$  is the increment (positive or negative) of power at  $TR_i$  in time slot  $t$ .

We assume that the DPs can detect the interference from the SUs. For example, if the channel gains and the transmit powers from the primary transmitters are known, the DP can estimate the interference from primary transmissions. Alternatively, a quiet period as in IEEE 802.22 WRANs could be enforced for the SUs. Since there is no secondary transmissions in the quiet period, the DPs can measure the interference from primary transmissions. Once the primary interference is known, a DP can estimate secondary interference by subtracting the primary interference from the total interference it receives.

The total interference from the  $TR_i$ 's to a detection point  $DP_j$  is

$$y_j(t) = \sum_{k=1}^{s+1} P_k(t)g_{kj}^2, \quad j = 1, 2, \dots, p. \quad (2)$$

Then the primary user protection constraint becomes

$$y_j(t) \leq D_j, \quad j = 1, 2, \dots, p. \quad (3)$$

For time slot  $t+1$ , the secondary interference  $y_j(t+1)$  caused by the updated transmit powers should also satisfy (3), i.e.,

$$y_j(t+1) \leq D_j, \quad j = 1, 2, \dots, p. \quad (4)$$

For the second constraint on guaranteeing the QoS of SUs, the SINR at the receiving antenna of  $TR_i$  can be written as

$$\gamma_i(t) = \begin{cases} \frac{P_{i+1}(t)h_{i+1,i}^2}{\sum_{j=1, j \neq i}^{s+1} P_j(t)h_{ji}^2 + \chi_i P_i(t)h_{ii}^2 + \sigma_i^2(t)}, & i \text{ is odd} \\ \frac{P_{i-1}(t)h_{i-1,i}^2}{\sum_{j=1, j \neq i}^{s+1} P_j(t)h_{ji}^2 + \chi_i P_i(t)h_{ii}^2 + \sigma_i^2(t)}, & i \text{ is even,} \end{cases} \quad (5)$$

where  $\chi_j$  is the SIS factor [9]. Recall that  $u_i(t) = P_i(t+1) - P_i(t)$ . From the control point of view, (5) can be regarded as the state equation and  $u_i(t)$  the input. The updated state is

$$\gamma_i(t+1) = \begin{cases} \gamma_i(t) + \frac{h_{i+1,i}^2}{I_i(t)} u_i(t) + \frac{P_i(t+1)h_{i+1,i}^2[I_i(t) - I_i(t+1)]}{I_i(t)I_i(t+1)}, & i \text{ is odd} \\ \gamma_i(t) + \frac{h_{i-1,i}^2}{I_i(t)} u_i(t) + \frac{P_i(t+1)h_{i-1,i}^2[I_i(t) - I_i(t+1)]}{I_i(t)I_i(t+1)}, & i \text{ is even,} \end{cases} \quad (6)$$

where

$$I_i(t) = \sum_{j=1, j \neq i}^{s+1} P_j(t)h_{ji}^2 + \chi_i P_i(t)h_{ii}^2 + \sigma_i^2(t). \quad (7)$$

It is shown that generally  $I_i(t) - I_i(t+1)$  is much smaller than  $I_i(t)I_i(t+1)$  [10]. It follows that (6) can be approximated as

$$\gamma_i(t+1) = \begin{cases} \gamma_i(t) + \frac{h_{i+1,i}^2}{I_i(t)} u_i(t), & i \text{ is odd} \\ \gamma_i(t) + \frac{h_{i-1,i}^2}{I_i(t)} u_i(t), & i \text{ is even.} \end{cases} \quad (8)$$

Let  $\Gamma$  denote the minimum required SINR for SU  $TR_i$ . The SU QoS constraint is

$$\gamma_i(t) \geq \Gamma. \quad (9)$$

The updated  $\gamma_i(t+1)$  should also satisfy condition (9), i.e.,

$$\gamma_i(t+1) \geq \Gamma. \quad (10)$$

Define parameters  $a$  and  $b$  as

$$a = \begin{cases} [u_i(t) + P_i(t)]h_{i+1,i}^2, & i \text{ is odd} \\ [u_i(t) + P_i(t)]h_{i-1,i}^2, & i \text{ is even.} \end{cases} \quad (11)$$

$$b = \sum_{j=1, j \neq i}^{s+1} [u_j(t) + P_j(t)]h_{ji}^2 + \chi_i [u_i(t) + P_i(t)]h_{ii}^2 + \sigma_i^2(t+1), i = 1, 2, \dots, s+1. \quad (12)$$

From (1), (4), and (10), we derive the following system of equations that can be solved for  $u_i(t)$ .

$$\begin{cases} a/b = \Gamma, & i = 1, 2, \dots, s+1 \\ [u_i(t) + P_i(t)]g_{ij}^2 + \sum_{k=1, k \neq i}^{s+1} [u_k(t) + P_k(t)]g_{kj}^2 \leq D_j, & j = 1, 2, \dots, p. \end{cases} \quad (13)$$

If the channel gains vary over time (e.g., in a mobile SU network), we can defined parameters  $a^*$  and  $b^*$  as

$$a^* = \begin{cases} [u_i(t) + P_i(t)]h_{i+1,i}^2(t+1), & i \text{ is odd,} \\ [u_i(t) + P_i(t)]h_{i-1,i}^2(t+1), & i \text{ is even.} \end{cases} \quad (14)$$

$$b^* = \sum_{j=1, j \neq i}^{s+1} [u_j(t) + P_j(t)]h_{ji}(t+1)^2 + \chi_i [u_i(t) + P_i(t)]h_{ii}(t+1)^2 + \sigma_i^2(t+1), i = 1, \dots, s+1. \quad (15)$$

A similar system of equations can be solved to determine  $u_i(t)$  as

$$\begin{cases} a^*/b^* = \Gamma, & i = 1, 2, \dots, s+1 \\ [u_i(t) + P_i(t)]g_{ij}^2(t+1) + \sum_{k=1, k \neq i}^{s+1} [u_k(t) + P_k(t)]g_{kj}^2(t+1) \leq D_j, & j = 1, 2, \dots, p. \end{cases} \quad (16)$$

### III. POWER CONTROL SCHEMES

In this section, we develop a power control scheme for adapting the transmit power of the secondary users [11]. The goal is to achieve the SU QoS requirement while satisfying PU protection constraint as given in (13). The proposed scheme is a distributed algorithm in the sense that each  $TR_i$  adjusts its power  $P_i$  independently. However, each  $TR_i$  needs to know the minimum tolerable interference level and the maximum measured SU interference at all the DPs.

#### A. A PID Controller

First, we consider the SU QoS constraint, while ignoring the PU protection constraint. The goal is to drive  $\gamma_i(t)$  to converge to the SU QoS requirement  $\Gamma$ , for all  $i$ . The difference between these two parameters should be considered and should be reduced as small as possible. Another consideration is that the error signal  $e_i(t)$  should be related to the power  $P_i(t)$ , which is the parameter that we need to determine for each  $TR_i$ . Therefore  $P_i(t)$  is used as the reference input. As we can see, the ratio of  $\Gamma$  and  $\gamma_i(t)$  can be an indicator for the control error, and  $\gamma_i(t) \propto P_i(t)$  if all other parameters remain the same. Thus, we use  $\frac{\Gamma}{\gamma_i(t)}P_i(t)$  as the feedback. The error  $e_i(t)$  should be the difference of feedback and  $P_i(t)$  and we have the diagram of the PID controller as in Fig. 1.

The PID controller collects the SINR of each TR at every time slot and uses it as feedback for the controller. For each

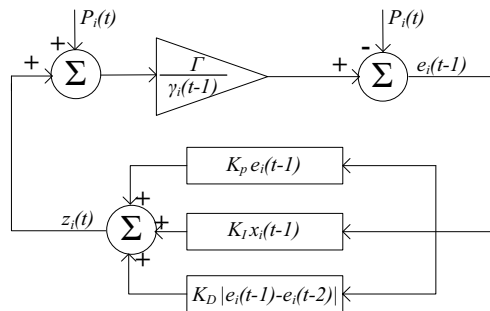


Fig. 1. The PID controller design.

time slot, let  $z_i(t)$  denote the power increment from  $P_i(t)$  to  $P_i(t+1)$ . With feedback  $\frac{\Gamma}{\gamma_i(t)}P_i(t)$ , the PID controller controls the system as

$$e_i(t-1) = \left\{ \frac{\Gamma}{\gamma_i(t-1)} - 1 \right\} P_i(t) \quad (17)$$

$$x_i(t-1) = x_i(t-2) + e_i(t-1) \quad (18)$$

$$z_i(t) = K_P e_i(t-1) + K_I x_i(t-1) + K_D |e_i(t-1) - e_i(t-2)|, \quad (19)$$

where  $e_i(t-1)$ ,  $x_i(t-1)$  and  $|e_i(t-1) - e_i(t-2)|$  represent the proportional, integral and derivative parts, respectively;  $K_p$ ,  $K_I$ , and  $K_D$  are the corresponding coefficients. Proper coefficients should be designed to achieve a stable and convergent control process for adjusting the  $P_i(t)$ 's to achieve the required minimum SINR  $\Gamma$  for each SU [12].

#### B. Power Control Constraint

Next we take into account the PU protection constraint. The objective of this constraint is to prevent the SU transmission powers from violating the interference tolerance at the DPs. This constraint actually represents a relationship between  $P_i(t)$  and  $D_j$ , for all  $i$  and  $j$ .

We first introduce the following two parameters.

$$D_{min} = \min_{j=1,2,\dots,p} D_j \quad (20)$$

$$y_{max}(t-1) = \max_{j=1,2,\dots,p} y_j(t-1). \quad (21)$$

$D_{min}$  is the minimum tolerance value among all the DPs, and  $y_{max}(t)$  is the maximum measured interference among all DPs. Since  $D_{min}$  is a constant and  $y_{max}(t-1) \propto P_i(t-1)$ , the additional power constraint should also be proportional to  $P_i(t-1)$ . We follow a similar approach as in prior work [3] to introduce the following additional constraint on the power adjustment  $z_i(t)$ .

$$c_i(t) = \theta(t)P_i(t-1) - P_i(t), \quad (22)$$

where  $\theta(t) = D_{min}/y_{max}(t-1)$ .

According to (22), once the maximum interference  $y_{max}(t-1)$  exceeds the minimum tolerance  $D_{min}$ , the constraint will reduce the transmit power with a proportion of  $\theta(t)$ , which will drive the maximum interference back to  $D_{min}$ .

Eqn. (22) enforces an additional constraint to the power increment  $z_i(t)$  for the SUs, so as to satisfy the PU protection

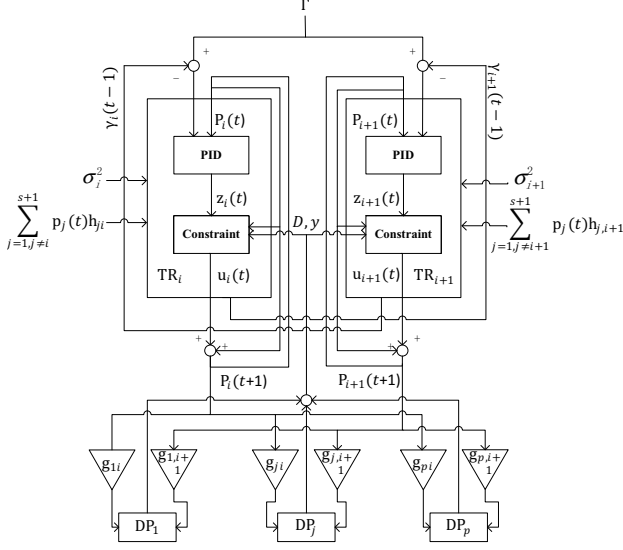


Fig. 2. System control block diagram.

constraint as given in (3). Because the PU protection is a fundamental condition for spectrum sharing, the constraint  $c_i(t)$  cannot be violated. Therefore, we have the final allowed power increment  $u_i(t)$  in time slot  $t$  for  $TR_i$  as

$$u_i(t) = \min\{z_i(t), c_i(t)\}, \quad i = 1, 2, \dots, s+1. \quad (23)$$

With such adjustment, the transmit power can be limited in a safe range that does not lead to severe interference to the primary network, while trying to achieve the minimum required SINR for the SUs. The overall diagram of the proposed power controller is illustrated in Fig. 2.

### C. Hybrid HD-FD Operation

Recall that the SIS factor  $\chi$  depends on the particular SIS design and is a small value in  $[0, 1]$ . Clearly,  $\chi$ , along with other network dynamics such as the channel gains, the number and locations of SUs and DPs, and the prescribed control goals (i.e.,  $\Gamma$  and  $D_j$ 's), all have big impact on the system performance. So in a practical underlay CR network, it is not true that FD transmissions will always achieve a better performance; when  $\chi$  is large, the residual self-interference will be so large that HD transmissions will be a better choice. Therefore, a hybrid scheme that can switch between FD and HD modes depending on the system parameters and states would be highly desirable. In the following, we derive the condition for switching between HD and FD modes.

We use Shannon's capacity to approximate the throughput of an SU, i.e.,  $C = B \log_2(1 + SINR)$ . Since bandwidth  $B$  is a constant for all SUs, we use the spectrum efficiency  $\log_2(1 + SINR)$  for comparing the efficiency of the two operation modes in the following. Let  $\gamma_i^{FD}$  and  $\gamma_i^{HD}$  denote the SINRs of  $TR_i$  in the FD mode and HD mode, respectively. We can derive the average throughput for the SU pair in the

HD mode, denoted as  $R_i^{HD}$ , as follows.

$$R_i^{HD} = \begin{cases} \frac{1}{2} \left[ \log_2(1 + \gamma_i^{HD}) + \log_2(1 + \gamma_{i+1}^{HD}) \right], & i \text{ is odd} \\ \frac{1}{2} \left[ \log_2(1 + \gamma_i^{HD}) + \log_2(1 + \gamma_{i-1}^{HD}) \right], & i \text{ is even,} \end{cases} \quad (24)$$

where,

$$\gamma_i^{HD} = \begin{cases} \frac{P_{i+1}(t)h_{i+1,i}^2}{\sum_{j=1, j \neq i}^{s+1} P_j(t)h_{ji}^2 + \sigma_i^2(t)}, & i \text{ is odd} \\ \frac{P_{i-1}(t)h_{i-1,i}^2}{\sum_{j=1, j \neq i}^{s+1} P_j(t)h_{ji}^2 + \sigma_i^2(t)}, & i \text{ is even.} \end{cases} \quad (25)$$

In the FD mode, the throughput for the SU pair is

$$R_i^{FD} = \begin{cases} \log_2(1 + \gamma_i^{FD}) + \log_2(1 + \gamma_{i+1}^{FD}), & i \text{ is odd} \\ \log_2(1 + \gamma_i^{FD}) + \log_2(1 + \gamma_{i-1}^{FD}), & i \text{ is even,} \end{cases} \quad (26)$$

where  $\gamma_i^{FD}$  is given in (5).

In each time slot  $t$ , we estimate the expected throughput for each SU pair in both the FD and HD modes and decide which mode to adopt for the time slot. The cross-over point for the two modes is derived by solving the following equation.

$$R_i^{HD} = R_i^{FD}. \quad (27)$$

From (27), we can derive a ratio  $T_i$  as follows.

$$T_i = \begin{cases} \frac{\sqrt{(1+\gamma_i^{HD})(1+\gamma_{i+1}^{HD})}}{(1+\gamma_i^{FD})(1+\gamma_{i+1}^{FD})}, & i \text{ is odd} \\ \frac{\sqrt{(1+\gamma_i^{HD})(1+\gamma_{i-1}^{HD})}}{(1+\gamma_i^{FD})(1+\gamma_{i-1}^{FD})}, & i \text{ is even.} \end{cases} \quad (28)$$

Thus, we have the following proposition for determining the operation mode for  $TR_i$  in the hybrid scheme.

**Proposition 1.**  $TR_i$  should operate in the HD mode if  $T_i \geq 1$ , and it should operate in the FD model if  $T_i < 1$ .

## IV. SIMULATION STUDY

### A. Simulation Configuration

To evaluate the performance of the proposed power control scheme for FD underlay CR networks, we conduct extensive simulations using a MATLAB implementation. We use one network in a  $100 \times 100$  m<sup>2</sup> area and another network in a  $1000 \times 1000$  m<sup>2</sup> area. The outdoor channel model  $h = 40 \log_{10}(d) + 10$  dB is used in all the simulations, where  $d$  is the distance between the transmitter and receiver. In each simulation, the noises powers  $\sigma^2$  are i.i.d. random variables evenly distributed in a fixed range, while the range may change in different simulations. As discussed, the performance of FD systems are greatly affected by  $\chi$ . We choose  $\chi = 10^{-7}$  in most of the simulations, unless otherwise specified.

There are eight TRs and four DPs in each of the networks. We assume the DPs can communicate with each other to obtain information about detected interference levels at the DPs (i.e.,  $y_j(t-1)$ ) and broadcast the maximum detected interference (i.e.,  $y_{max}(t-1)$ ) to all TRs through a control channel. The control goals  $\Gamma$  and  $D_j$ 's are known to all TRs in advance. Such information is used as input to the control scheme executed at each TR to adjust its transmit power.

### B. Controller Performance

In this section, we evaluate the performance of the proposed controller in the FD and HD modes. First, we simulate the  $100 \times 100 \text{ m}^2$  network under fixed  $\Gamma$  and fixed  $\chi$ . We set  $\Gamma = 0.8$  and  $\chi = 10^{-7}$  in the simulation. Noise  $\sigma^2$  is uniform distributed in  $[1.2 \times 10^{-7}, 2.4 \times 10^{-7}] \text{ W}$ . DP's tolerance limit is set as  $D_j = 1 \times 10^{-7} \text{ W}$  for all  $j$ .

Take  $TR_5$  as an example. The evolutions of its SINR  $\gamma_5(t)$  and transmit power  $P_5(t)$  are plotted in Fig. 3 and Fig. 4, respectively. It can be seen that both the SINR and power curves quickly converge to the neighborhood of the stable values, and then fluctuate around the stable values. In Fig. 4, it can be seen that a higher transmit power is used in the FD mode to overcome the residual self-interference in order to achieve the target SINR value  $\Gamma$ . In both cases, the control of the power adjustment is jointly done by both the PU protection constraint (22) and the SU QoS constraint (19).

Since the controlled power of  $TR_5$  has achieved both PU protection and SU QoS goals, the power adjustment  $u_5(t)$  will stay within a narrow range around 0. In Fig. 5, we present the PU protection performance by plotting the PU interference tolerant  $D$  and the maximum measure interference  $y_{max}(t)$  at the DPs for the FD and HD modes. It can be seen that with the proposed power control scheme, the maximum DP detected interference  $y_{max}$  is kept below  $D$  for all the time slots. Therefore the PU protection goal is well achieved by the proposed power control scheme. In the meantime, the controlled power remains around 0.09 W for the FD mode and 0.085 W for the HD mode (see Fig. 4), which are sufficient to satisfy the required SINR  $\Gamma = 0.8$  for  $TR_5$ , as shown in Fig. 3. Since the controlled power of  $TR_5$  has achieved both PU protection and SU QoS goals, the power adjustment  $u_5(t)$  will stay within a narrow range around 0.

### C. Throughput Performance

In this section, we evaluate the achievable throughput by the proposed power control scheme. We focus on the proposed hybrid scheme in the simulations, with which the operating mode for each TR is determined as given in Section III-C. The large  $1000 \times 1000 \text{ m}^2$  network with eight TRs and four DPs is used in the following simulations.

In the simulation, we increase  $\chi$  from 0.000005 to 0.005 with step size of 0.000005. With each  $\chi$  value, we simulate the system for 200 time slots each with a random noise level, which has been shown to be sufficient long for convergence in our previous simulations. We plot the average throughput of the 200 time slots for each  $\chi$  value in Fig. 6 for the FD, HD and hybrid schemes.

It can be seen in Fig. 6 that the hybrid scheme achieves the highest throughput for the entire range of  $\chi$ . In particular, when  $\chi \leq 3.43 \times 10^{-4}$ , SIS is very effective and most of the TRs operate in the FD mode. The hybrid scheme achieves the same throughput as FD, which is higher than that of HD. As  $\chi$  is increased, the advantage of FD transmissions diminishes and HD begins to achieve higher throughput than FD. When  $3.43 \times 10^{-4} \leq \chi \leq 1.3 \times 10^{-3}$ , some TRs operate

in the FD mode and some others in the HD mode. When  $\chi \geq 1.3 \times 10^{-3}$ , all the TRs operate in the HD mode since the residual self-interference is so strong, there is no benefit for using FD transmissions. The proposed hybrid scheme compares the gains of FD and HD, and always chooses the better operating mode to achieve the highest throughput for the entire range of  $\chi$ .

Finally, we investigate the impact of noise level. In the simulation, we set  $\chi$  to 0.0005 and increase the noise power  $\sigma^2$  from  $10^{-6} \text{ W}$  to  $10^{-3} \text{ W}$ . The throughput results for the three schemes are presented in Fig. 7. As expected, the hybrid scheme achieves the highest throughput among the three, and the throughput decreases when noise is increased for all the three schemes. However, the influence of noise on throughput is different for the three schemes. As shown in (5), the FD mode has one extra interference source, i.e., the residual self-interference, making it less sensitive to the varying noise power. This is why the throughput of HD decreases faster than FD. The hybrid scheme can use FD instead of HD even though the  $\chi$  value is larger in this simulation. The hybrid scheme always achieves the highest throughput in all the scenarios simulated.

## V. RELATED WORK

FD transmission is a new technology to push the limit of single channel communications. In [5], the authors proposed basic concepts such as RF and digital cancellations and discusses potential MAC and network gains with full-duplexing. In [8], the authors presented the design and implementation of a real-time 64-subcarrier 10 MHz full-duplex OFDM physical layer, and demonstrated up to 80 dB self-interference suppression with experiments. In [6], the authors presented a full duplex radio design using signal inversion and adaptive cancellation, as well as a full duplex MAC design and evaluation results with a testbed of 5 prototype FD nodes. In [7], a MIMO FD design was presented.

Feedback control has found wide application in communication and networking systems. A modern overview of functionalities and tuning methods for PID controllers was presented in [12]. In [13], a proportional (P) controller was developed for streaming videos to stabilize the received video quality as well as the bottleneck link queue, for both homogeneous and heterogeneous video systems. In [11], the author presented a PID based power adjustment algorithm that was later extended in [3], which developed a PID control for power control in underlay CR networks.

## VI. CONCLUSION

In this paper, we investigated the design of distributed power controllers for underlay CR networks, where FD transmissions were exploited to improve network capacity. Taking the SIS factor into consideration, we investigated the design of a power control scheme that integrates a PID controller and a power constraint mechanism, and develop a hybrid FD-HD scheme to achieve the dual goals of PU protection and SU QoS

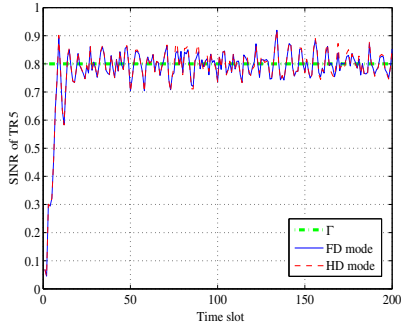


Fig. 3. Evolution of the SINR at  $TR_5$  when  $\Gamma = 0.8$  and  $\chi = 10^{-7}$ .

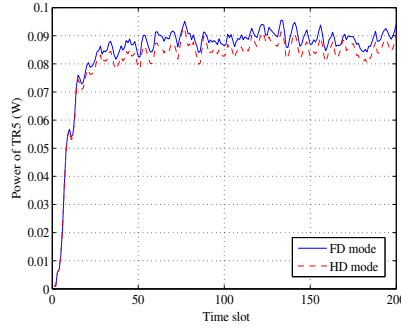


Fig. 4. Evolution of the transmit power at  $TR_5$  when  $\Gamma = 0.8$  and  $\chi = 10^{-7}$ .

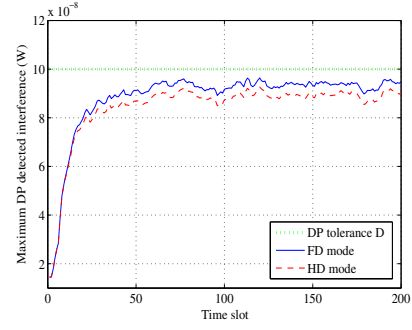


Fig. 5. Maximum measured interference among all the DPs  $\Gamma = 0.8$  and  $\chi = 10^{-7}$ .

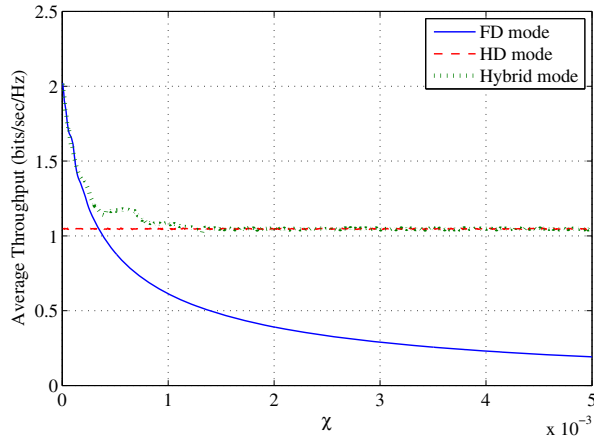


Fig. 6. Average throughput of the FD, HD, and hybrid modes for different values of  $\chi$ .

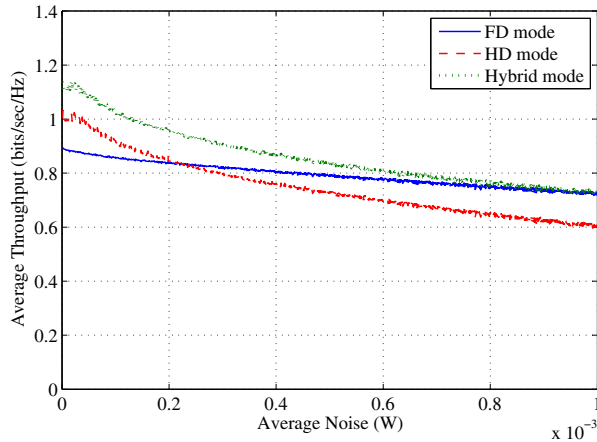


Fig. 7. Average throughput of the FD, HD, and hybrid modes for different values of  $\sigma^2$ .

provisioning. The stability and throughput performance of the proposed schemes were validated with simulations.

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