# Interference Alignment Improves the Capacity of OFDM Systems

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***Abstract*—Multiuser orthogonal frequency-division multiplexing (OFDM) and multiple-output multiple-output (MIMO) have been widely adopted to enhance the system throughput and combat the detrimental effects of wireless channels. Interference alignment has been proposed to exploit interference to enable concurrent transmissions of multiple signals. In this paper, we investigate how to combine these techniques to further enhance the system throughput. We first reveal the unique characteristics and challenges brought about by using interference alignment in diagonal channels. We then derive a performance bound for the multiuser (MIMO) OFDM interference alignment system under practical constraints and show how to achieve this bound with a decomposition approach. The superior performance of the proposed scheme is validated with simulations.**

***Index Terms*—Interference alignment, multiple-input multiple-output (MIMO), multiuser OFDM, orthogonal frequency-division multiplexing (OFDM).**

## I. Introduction

THE past decade has witnessed a drastic increase in wireless data traffic, largely due to the so-called "smartphone revolution." It is foreseeable that the capacity of existing and future wireless networks will soon be greatly stressed. Many advanced wireless communication technologies, such as orthogonal frequency-division multiplexing (OFDM) and multiple-input multiple-output (MIMO), are widely adopted to enhance the system capacity, whereas a huge amount of wireless access networks/base stations (BSs) are deployed every year to accommodate the compelling need for larger capacity. Given the increasing wireless data volume and the more and more crowded BS deployment, interference is apparently the major factor that limits wireless network performance.

Traditionally, interference is considered harmful and often treated as background noise. As the performance of point-to-point transmission techniques is approaching Shannon capacity, there is now considerable interest in exploiting interference for further capacity gains. It is shown that when interference is large, it can be decoded and canceled from the mixed signal (as in interference cancelation), whereas when interference is comparable, interference alignment can be adopted to enable concurrent transmissions. Although interference is harmful in many cases, it could be beneficial for enhancing system throughput as long as the interference can be aligned. This kind of interference can be termed as beneficial interference.

Interference alignment was first proposed in [2], and the feasibility condition was investigated in [3]. Since interference alignment is expected to greatly enhance the system throughput, many existing works focus on applying interference alignment to practical wireless networks. For instance, interference alignment with limited feedback or incomplete channel state information (CSI) were investigated in [4]–[6]. In a more general scenario, multicell opportunistic interference alignment was examined in [7]. Combined with cognitive radios, in [8], a cognitive interference alignment scheme was presented to suppress both cross- and cotier interferences in two-tier networks. Moreover, in [9] and [10], Xu and Mao investigated the behaviors of primary users and secondary users under a Stackelberg game theory framework, where distributed interference alignment was adopted to enable spectrum leasing in the cognitive radio network. There are also some existing studies that aim to adopt interference alignment in more advanced systems, such as heterogeneous networks [11] and multihop mesh networks [12]. In [13], Gollakota *et al.* also considered combining interference alignment and interference cancelation. In [14], El-Hadidy *et al.* proposed to use multimode MIMO antennas instead of the typical omnidirectional antennas with interference alignment, whereas in [15], the impact of antenna spatial correlation on the performance of interference alignment systems was investigated.

Although many applications of interference alignment have been investigated, few works have studied how to apply interference alignment to OFDM systems due to the difficulty of analyzing throughput with a structured channel (e.g., diagonal channel) [1]. Since in a large network, there are many users but limited dimensionality, in [16], Shen *et al.* proposed the concept of "best effort" interference alignment and adopted an iterative algorithm for optimization. However, how to use interference alignment to enhance the throughput of a practical OFDM system was not fully considered. Another attempt can be found in [17], where the precoder design was studied. However, it only considered the case when the cyclic prefix was insufficient. General precoding design specifically for OFDM

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systems was not discussed. In [18], Shi *et al.* also considered the problem of interference alignment in multicarrier interference networks. However, the number of subcarriers considered was small, typically three subcarriers. Hence, it is not clear if the approach can be extended to the general case of a large number of subcarriers. In a more recent work, in [19], Kafedziski and Javornik proposed two schemes to adopt interference alignment in multicell MIMO OFDM systems. In the first scheme, interference alignment was used to remove the intercell interference, whereas zero-forcing precoding was used to suppress the intracell interference. In the second scheme, interference alignment was also used for intercell interference removal, whereas the OFDMA access scheme was applied for intracell interference cancelation. However, the fundamental performance bound of multiuser MIMO OFDM systems with interference alignment has not been discussed. Some other works tried to understand the performance bounds of OFDM systems with interference alignment, such as [20] and [21]. In [20], Zhang *et al.* derived the necessary and sufficient conditions for the three-user OFDM system with interference alignment in the time domain. However, these conditions cannot be applied to a system with more users or under other conditions. In [21], Ayach *et al.* studied the feasibility of the MIMO-OFDM system with interference alignment over measured channels.

In this paper, we show how interference alignment works in the OFDM system under practical constraints and is mainly concerned about the theoretical bound when interference alignment is incorporated in the OFDM system. Specifically, we consider the problem of incorporating interference alignment in multiuser (MIMO) OFDM systems. We first examine the fundamental characteristics and practical constraints on adopting interference alignment in a multiuser OFDM system. We show that, for a $K$-user $N$-subcarrier OFDM system, $KN/2$ concurrent transmissions that are achievable for generic structureless channels [2] cannot be achieved for a practical multiuser OFDM network with diagonal channels and a limited number of subcarriers. We then investigate effective schemes to exploit interference in multiuser OFDM systems. With an integer programming problem formulation, we derive the maximum efficiency of the multiuser OFDM/interference alignment system. We also show how to achieve the maximum efficiency with a decomposition approach and derive the closed-form precoding and decoding matrices. Finally, we extend the given analysis to the multiple-antenna scenarios. All the proposed schemes are evaluated with simulations, and their superior performance is validated.

*Notation:* In this paper, a capital bold symbol such as $\mathbf{H}$ denotes a matrix, a lowercase symbol with an arrow on top such as $\vec{v}$ denotes a vector, and a lowercase letter such as $v$ denotes a scalar. $[\cdot]^T$ means *transpose*, and $[\cdot]^{-1}$ means *inversion*. $\mathbf{H}_{ij}$ and $h_{ij}$ are the channel gain matrix and channel gain from the $i$th transmitter to the $j$th receiver, respectively. $\mathbf{V}_i$ is the precoding matrix for transmitter $i$; $\vec{v}_i^j$ is the $j$th column of $\mathbf{V}_i$. $\mathbf{U}_i$ denotes the interference cancelation matrix for the $i$th receiver, whereas $\vec{u}_i^j$ is the $j$th column of $\mathbf{U}_i$. Let $h$, $v$, and $u$ denote the entries of $\mathbf{H}$, $\mathbf{V}$, and $\mathbf{U}$, respectively.

Note that with these notations, the entries of $\mathbf{H}_{ij}$ take slightly different ordering from conventional ones. For instance, if transmitter 1 and receiver 2 are both equipped with $M$ antennas, the channel gain is

$$\mathbf{H}_{12} = \begin{pmatrix} h_{11} & h_{21} & \cdots & h_{M1} \\ h_{12} & h_{22} & \cdots & h_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1M} & h_{2M} & \cdots & h_{MM} \end{pmatrix}. \tag{1}$$

The rest of this paper is organized as follows. Section II describes the background and preliminaries. Section III investigates how to adopt interference alignment in a multiuser OFDM system. Section IV extends the analysis to the multiple-antenna scenario. Simulation results are presented in Section V. Section VI concludes this paper.

## II. Background and Preliminaries

### A. OFDM

While higher data rates can be achieved by reducing symbol duration, severe intersymbol interferences (ISIs) will be caused over time-dispersive channels. OFDM is an effective approach to allow transmissions at a high data rate and combat the destructive effect of the channel. By dividing the channel into narrow bands, in which the signal experiences flat fading, OFDM can effectively mitigate ISI and maintain high-data-rate transmissions. See [23] and the references therein for details.

### B. MIMO

With the single-antenna transmission technique being well developed, it is natural to extend to multiple-antenna systems. The MIMO transmission techniques have been rapidly evolving since the last decades. Generally speaking, multiple antennas or an antenna array can be used to attain the *diversity gain*, *multiplexing gain*, or *antenna gain*, and thereby reduce the system error rate, enhance the system throughput, or strengthen the signal-to-interference-and-noise ratio [24], [25]. Given $M_1$ transmitting antennas and $M_2$ receiving antennas, the maximum multiplexing gain is known to be $\min\{M_1, M_2\}$. Throughout this paper, we assume that CSI is perfectly known at each transmitter and receiver as in prior works [2].

### C. Interference Alignment

It is shown in [2] that in a $K$-user wireless network, with $(n+1)^q + n^q$ symbol extensions, in total, $K/2$ normalized *degrees of freedom* (DoF) can be achieved using interference alignment, where $q = (K-1)(K-2) - 1$, and $n \in \mathbb{N}$. In single-antenna systems, the normalized DoF is 1. With interference alignment, the system throughput is enhanced by a factor of $K/2$ for $K \geq 2$. Note that there is no interference if there is only one user occupying the time or frequency resource.

*Observation 1:* The system throughput could be improved if alignable interference is introduced among users.

This observation is useful for OFDM systems, where the channel gain matrix is diagonal. Since the gain of interference alignment is proportional to $K$, we should have more users

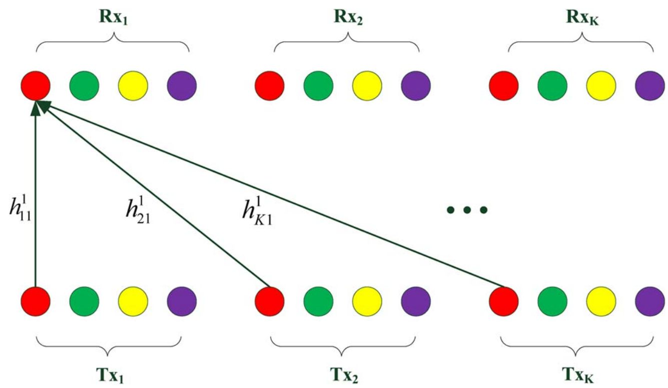


Fig. 1. Multiuser OFDM using interference alignment.

transmit at the same time slot or frequency band if the transmitted vectors can be aligned. That is why we call this kind of interference *beneficial interference* in this paper.

## III. Multiuser Orthogonal Frequency Division Multiplexing With Interference Alignment

Here, we investigate the problem of interference alignment in multiuser OFDM systems. The system model is shown in Fig. 1. We first examine fundamental characteristics and practical constraints and then demonstrate how to exploit interference in multiuser OFDM systems. We derive the maximum throughput when interference alignment is adopted, as well as closed-form precoding and decoding matrices to achieve the maximum throughput.

### A. Subcarriers Versus Antennas

In traditional interference alignment, deploying multiple transmitting antennas allows us to precode data packets and align them at the receiver. Deploying multiple receiving antennas provides multidimensional signal space; therefore, interference can be aligned into subsignal space that is orthogonal to the desired signal. Therefore, deploying multiple antennas can provide the needed freedom in the signal space.

In OFDM systems, we observe that subcarriers can function in similar ways as antennas in MIMO/interference alignment systems, since subcarriers could also provide multidimensional signal space. To some extent, subcarriers can be regarded as a counterpart of antennas. Hence, we could compress the interference at each receiver in no more than half of the subcarriers and leave the other half of subcarriers free from interference.

However, note that there is a distinguishing difference between the two systems: There is no crosstalk among different subcarriers in OFDM.

### B. Precoding in OFDM

The main idea of interference alignment is to compress the interference space to no more than half of the total received signal space at each receiver, leaving the remaining part of the space clean for desired signals [2]. This goal is achieved through precoding at every transmitter and zero-forcing interference cancelation at every receiver.

In OFDM systems, data are transmitted over multiple carriers between transmitters and receivers, as shown in Fig. 1. Since, in OFDM systems, subcarriers can also provide multidimensional signal space for the transmitter and the receiver as multiple antennas, we could precode over multiple subcarriers to achieve interference alignment for the OFDM system as multiple antennas for the MIMO system. Suppose there are $N$ subcarriers. Ignoring noise, if there is no precoding, the received signal for each receiver $\vec{y}$ is an $N \times 1$ vector given by

$$\vec{y} = \mathbf{H}\vec{x} \tag{2}$$

where $\vec{x}$ is the desired signal in the form of an $N \times 1$ vector, and $\mathbf{H}$ is the $N \times N$ channel gain matrix between the transmitter and the receiver. Since different subcarriers have different frequencies, the channel gain matrix is *diagonal* if there is no severe frequency shift. It can be seen from later discussions that this property makes interference alignment in the OFDM system quite different from the general channel case.

Going one step further, we can precode the data before transmission. If $d$ packets are to be transmitted in an $N$-subcarrier OFDM system, an $N \times d$ *precoding matrix* $\mathbf{V}$ could be used. The system equation is rewritten as follows:

$$\vec{y} = \mathbf{HV}\vec{x}. \tag{3}$$

If we let $d = N$ and $\mathbf{V} = \mathbf{I}_N$, where $\mathbf{I}_N$ is an $N \times N$ identity matrix, (3) is reduced to (2).

In general, we could control what to be transmitted on the subcarriers by adjusting the precoding matrix accordingly. For a single-user single-antenna OFDM system with $N$ subcarriers, the maximum number of packets that can be transmitted is $N$. Note that, here, $N$ is normalized by the quadrature amplitude modulation level. However, inspired by the idea of interference alignment, we show that a throughput higher than $N$ can be achieved in the following sections.

### C. Interference Alignment in a $K$-User OFDM System

As discussed, we consider the problem of interference alignment in multiuser OFDM systems. Basically, we aim to answer the following questions.

1) What are the practical constraints for adopting interference alignment in such systems?
2) What is the maximum throughput that can be achieved?
3) How is the maximum throughput (i.e., deriving closed-form precoding and decoding matrices) achieved?

*1) Dependence of Precoding and Decoding Vectors in Diagonal Channels:* Here, we show the difference in applying interference alignment between a diagonal channel and a general channel, as well as the challenges to adopt interference alignment in the former case.

It was shown in [3] that given $M_1$ transmitting antennas and $M_2$ receiving antennas in a $K$-user interference channel, the DoF for each user, which is denoted by $d$, must satisfy

$$d \le (M_1 + M_2)/(K + 1). \tag{4}$$

For example, given two transmitting and receiving antennas in a three-user interference channel, (4) indicates that each user could transmit one packet simultaneously. With a generic structureless channel, the throughput $Kd = 3$ can be achieved as follows.

At each receiver, we align the signals from the other two users. Recall the channel gain matrices as defined in (1) and let the user $i$ signal be $\vec{v}_i$, $i = 1, 2, 3$. It follows that

$$\mathbf{H}_{21}\vec{v}_2 = \mathbf{H}_{31}\vec{v}_3, \mathbf{H}_{12}\vec{v}_1 = \mathbf{H}_{32}\vec{v}_3, \mathbf{H}_{13}\vec{v}_1 = \mathbf{H}_{23}\vec{v}_2. \tag{5}$$

Solving (5), we have

$$\vec{v}_1 = \mathrm{eig}(\mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{31}^{-1}\mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}) \tag{6}$$

$$\vec{v}_2 = \mathbf{H}_{23}^{-1}\mathbf{H}_{13}\vec{v}_1 \tag{7}$$

$$\vec{v}_3 = \mathbf{H}_{32}^{-1}\mathbf{H}_{12}\vec{v}_1 \tag{8}$$

where $\mathrm{eig}(\mathbf{A})$ stands for the eigenvector of matrix $\mathbf{A}$.

This scheme works well for generic structureless channels, but not for the case of diagonal channels. For instance, if two subcarriers (instead of two antennas) are used in OFDM, all the channel gain matrices in (6)–(8) are diagonal. Since the product of diagonal matrices is still diagonal, we have from (6) that

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If $\vec{v}_1 = [1, 0]^T$, we derive $\vec{v}_2 = [c_1, 0]^T$ from (7) and $\vec{v}_3 = [c_2, 0]^T$ from (8), where $c_1$ and $c_2$ are scalars. To cancel the interference at receiver 1, the *cancelation vector* $\vec{u}_1$ must be $\vec{u}_1 = [0, c]^T$, where $c$ is also a scalar. However, the desired packet is also canceled since $\vec{u}_1$ is orthogonal to $\vec{v}_1$. Therefore, we cannot simultaneously transmit three packets in this system.

The reason behind is that for a diagonal channel, its eigenvectors have only one nonzero entry. If we align interferences at receiver $r$ by letting $\mathbf{H}_{jr}\vec{v}_j = \cdots = \mathbf{H}_{ir}\vec{v}_i$, for $j \neq \cdots \neq i \neq r$, the precoding vectors are dependent on each other. Consequently, when interference is canceled at a receiver, the desired packet will also be canceled.

*2) Interference Alignment With Multiuser OFDM—Performance Bound:* It is shown in [2] that in a $K$-user system with $(n+1)^q + n^q$ symbol extensions, in total, $K/2$ normalized DoF can be achieved using interference alignment, where $q = (K-1)(K-2) - 1$, and $n \in \mathbb{N}$. In light of this result, one may think that $KN/2$ concurrent transmissions is achievable in a $K$-user $N$-subcarrier OFDM system. However, we will show that this is unachievable for large $K$ in practical systems in the following.

It is worth noting that an assumption made in [2] is that the symbol extensions can be infinitely large. This assumption may not hold true in practical systems. Given a finite bandwidth, the number of subcarriers is the bandwidth divided by the subcarrier spacing. Typically, the value of subcarrier spacing is 10–20 kHz. Then, even for a 100-MHz bandwidth, we can have, at most, $10^4$ subcarriers. For instance, in IEEE 802.16 m and 3GPP Long-Term Evolution (LTE), the maximum number of inverse fast Fourier transform is 2048, and the maximum number of effective subcarriers is 1200.

TABLE I
SYSTEM EFFICIENCY

| $n$ | *No. of subcarriers* | *No. of packets* | *Normalized DoF d* |
|---|---|---|---|
| When $K = 5$ and $q = 11$ | | | |
| 1 | 2,049 | 2,052 | 1.002 |
| 2 | 179,195 | 185,339 | 1.03 |
| When $K = 4$ and $q = 5$ | | | |
| 1 | 33 | 35 | 1.06 |
| 2 | 275 | 339 | 1.23 |
| 3 | 1,267 | 1,753 | 1.38 |
| 4 | 4,149 | 6,197 | 1.49 |
| When $K = 3$ and $q = 1$ | | | |
| 1 | 3 | 4 | 1.333 |
| 2 | 5 | 7 | 1.40 |
| 3 | 7 | 10 | 1.429 |
| 4 | 9 | 13 | 1.444 |
| 100 | 201 | 301 | 1.498 |
| 1000 | 2001 | 3001 | 1.4998 |

Therefore, the problem is to maximize system throughput given a finite number of subcarriers, which is denoted by $N_{\max}$. It is shown in [2] that with $(n+1)^q + n^q$ symbol extensions, the total normalized DoF is $[(n+1)^q + (K-1)n^q]/[(n+1)^q + n^q]$. Hence, we aim to maximize $(n+1)^q + (K-1)n^q$ and have the following formulation:

$$\max_{\{n,K\}} \quad (n+1)^q + (K-1)n^q \tag{9}$$

$$\text{s.t.} \quad q = (K-1)(K-2) - 1 \tag{10}$$

$$(n+1)^q + n^q \leq N_{\max}, n \in \mathbb{N} \tag{11}$$

$$K \geq 3, K \in \mathbb{N}. \tag{12}$$

The physical meaning of problem (9) is that we try to maximize the unnormalized DoF given a finite number of subcarriers. Note that all the variables are integers. Constraint (11) indicates that for practical OFDM systems, the number of subcarriers $N = (n+1)^q + n^q$ is upper bounded by $N_{\max}$. Although this integer programming problem is NP-hard in general, by careful inspection, we can find the solution under practical constraints.

In particular, we find that the feasible region is very small for practical $N_{\max}$ values. Moreover, the objective value is monotone with respect to the two variables $n$ and $K$. In problem (9), assuming $K = 5$, we have $q = 11$ from (10). For each value of $n$, we can derive the number of subcarriers needed, i.e., $N_{\max}$, from (11) for the problem to be feasible, as well as the throughput of the system [i.e., the objective value of (9)]. The corresponding degree of freedom, i.e., $d$, is the ratio of the throughput and the number of subcarriers required. These numbers are presented in Table I.

Table I shows that if there are $K = 5$ users, 2049 and 179 195 subcarriers are needed when $n = 1$ and $n = 2$, respectively. As discussed, a practical system usually does not have more than $10^4$ subcarriers. Hence, $n$ can only be 1 in this case, with efficiency $d_{\max} = 1.002$. Therefore, interference alignment is not useful in this case, since we can simply allow only one user to transmit over one time slot or a particular frequency band to get $d = 1$ (i.e., single-user OFDM).

If there are $K = 6$ transmitters, we have $q = 19$. Even if $n = 1$, the number of subcarriers needed is 524 289, which is not feasible for practical systems. Since the number of subcarriers $(n+1)^{(K-1)(K-2)-1} + n^{(K-1)(K-2)-1}$ exponentially grows with $(K^2 - 3K + 1)$, it can be readily concluded that $K$ cannot be more than 4 for interference alignment to be beneficial in multiuser OFDM systems.

Since the objective value of (9) is a monotonically increasing function of $K$, the maximum feasible value $K = 4$ is of particular interest. We have $q = 5$ when $K = 4$. Table I also shows that under this condition, the maximum efficiency for a practical system is $d_{\max} = 1.38$ for the practical case with, at most, 2000 subcarriers. When $K = 3$, we have $q = 1$. Objective function (9) becomes $3n + 1$, and constraint (11) becomes $2n + 1 \leq N_{\max}$. If the maximum number of subcarriers is $N_{\max} = 2001$, the system achieves its maximum efficiency $d_{\max} = 1.4998$.

The above analysis can be summarized as follows.

*Lemma 1:* For a practical multiuser OFDM system with less than 2002 subcarriers, the maximum efficiency is $d_{\max} = 1.4998$, which is achieved when there are $K = 3$ users using $N = 2001$ subcarriers.

However, in later discussions, we will show that this conjecture does not hold true.

*3) Interference Alignment With Multiuser OFDM—Realization:* It is shown in [2] how to design the precoding matrices to transmit $3n + 1$ packets over $2n + 1$ symbol extensions in a three-user interference channel (i.e., for a three-user system, we have $q = 1$ and $N = (n+1)^q + n^q = 2n + 1$). We will derive the precoding/decoding procedure for interference alignment with multiuser OFDM and prove its efficacy in this section.

The precoding matrices proposed in [2] for the case of three users are as follows:

$$\mathbf{V}_1 = \mathbf{A}, \mathbf{V}_2 = \mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}.\mathbf{V}_3 = \mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B} \qquad (13)$$

where

$$\begin{cases} \mathbf{A} = [\vec{w}\mathbf{T}\vec{w}\mathbf{T}^2\vec{w}, \ldots, \mathbf{T}^n\vec{w}] \\ \mathbf{B} = [\mathbf{T}\vec{w}\mathbf{T}^2\vec{w}, \ldots, \mathbf{T}^n\vec{w}] \\ \mathbf{C} = [\vec{w}\mathbf{T}\vec{w}\mathbf{T}^2\vec{w}, \ldots, \mathbf{T}^{n-1}\vec{w}] \\ \mathbf{T} = \mathbf{H}_{21}\mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{H}_{31}^{-1} \\ \vec{w} = [11, \ldots, 1]^T. \end{cases} \qquad (14)$$

Thus, the received signal at receiver 1 is

$$\vec{y}_1 = \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{31}\mathbf{V}_3\vec{x}_3. \qquad (15)$$

In the general case, since the data streams are independent of each other, the received mixed signal spans $3n + 1$ dimensions of the space. In interference alignment with a multiuser OFDM, the received signal spans only $2n + 1$ dimensions of space. Solving these $2n + 1$ equations will yield the desired packets. However, the challenge is, if $2n + 1$ is too large, we may not be able to solve these equations efficiently (as can be seen from later discussions). This problem can be addressed with a decomposition approach as given in the following theorem.

*Theorem 1:* For an $N$-subcarrier OFDM system, we can divide the subcarriers into $\lfloor N/(2n+1) \rfloor$ groups, where $n \in \mathbb{N}$, and precode and decode the groups separately to achieve the interference alignment gain.

*Proof:* Recall that the channel gain matrix in OFDM is diagonal. Generally, if every user tries to transmit $d$ packets over the $N$ subcarriers, we have

$$\mathbf{HV} = \begin{pmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_N \end{pmatrix} \begin{pmatrix} v_{11} & \cdots & v_{1d} \\ v_{21} & \cdots & v_{2d} \\ \vdots & \ddots & \vdots \\ v_{N1} & \cdots & v_{Nd} \end{pmatrix}.$$

The precoding vectors must satisfy the conditions given in (13) and (14). Let the precoding matrix assume the following form:

$$\mathbf{V} = \begin{pmatrix} \widetilde{\mathbf{V}}_1 & 0 & \cdots & 0 \\ 0 & \widetilde{\mathbf{V}}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widetilde{\mathbf{V}}_g \end{pmatrix} \qquad (16)$$

where $g = N/(2n+1)$ is the number of groups, and $\widetilde{\mathbf{V}}_i$ is the precoding matrix for group $i$ with dimensions $(2n+1) \times (n+1)$ or $(2n+1) \times n$ (i.e., user 1 sends $(n+1)$ packets, and each of the other users sends $n$ packets over $(2n+1)$ subcarriers). Without loss of generality, we assume that $N$ is dividable by $2n + 1$. Rewriting $\mathbf{H}$ in the form of multiple diagonal submatrices with the same dimensions, we have

$$\mathbf{HV} = \begin{pmatrix} \widetilde{\mathbf{H}}_1\widetilde{\mathbf{V}}_1 & 0 & \cdots & 0 \\ 0 & \widetilde{\mathbf{H}}_2\widetilde{\mathbf{V}}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widetilde{\mathbf{H}}_g\widetilde{\mathbf{V}}_g \end{pmatrix}. \qquad (17)$$

For instance, when $N = 6$ and $n = 1$, we have, for transmitter 1

$$\mathbf{HV} = \begin{pmatrix} h_1v_{11} & h_1v_{12} & 0 & 0 \\ h_2v_{21} & h_2v_{22} & 0 & 0 \\ h_3v_{31} & h_3v_{32} & 0 & 0 \\ 0 & 0 & h_4v_{41} & h_4v_{42} \\ 0 & 0 & h_5v_{51} & h_5v_{52} \\ 0 & 0 & h_6v_{61} & h_6v_{62} \end{pmatrix}. \qquad (18)$$

If there are three users, we can let $\mathbf{H}_{21}\mathbf{V}_2 = \mathbf{H}_{31}\mathbf{V}_3$ at receiver 1 to get

$$\begin{pmatrix} h_{21}^{(1)}v_2^{(1)} & 0 & \cdots & 0 \\ h_{21}^{(2)}v_2^{(2)} & 0 & \cdots & 0 \\ h_{21}v_2^{(3)} & 0 & \cdots & 0 \\ 0 & h_{21}^{(4)}v_2^{(4)} & \cdots & 0 \\ 0 & h_{21}^{(5)}v_2^{(5)} & \cdots & 0 \\ 0 & h_{21}^{(6)}v_2^{(6)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{21}^{(N-2)}v_2^{(N-2)} \\ 0 & 0 & \cdots & h_{21}^{(N-1)}v_2^{(N-1)} \\ 0 & 0 & \cdots & h_{21}^{(N)}v_2^{(N)} \end{pmatrix} = \begin{pmatrix} h_{31}^{(1)}v_3^{(1)} & 0 & \cdots & 0 \\ h_{31}^{(2)}v_3^{(2)} & 0 & \cdots & 0 \\ h_{31}^{(3)}v_3^{(3)} & 0 & \cdots & 0 \\ 0 & h_{31}^{(4)}v_3^{(4)} & \cdots & 0 \\ 0 & h_{31}^{(5)}v_3^{(5)} & \cdots & 0 \\ 0 & h_{31}^{(6)}v_3^{(6)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{31}^{(N-2)}v_3^{(N-2)} \\ 0 & 0 & \cdots & h_{31}^{(N-1)}v_3^{(N-1)} \\ 0 & 0 & \cdots & h_{31}^{(N)}v_3^{(N)} \end{pmatrix}$$

which indicates

$$\begin{pmatrix} h_{21}^{(i)}v_2^{(i)} \\ h_{21}^{(i+1)}v_2^{(i+1)} \\ h_{21}^{(i+2)}v_2^{(i+2)} \end{pmatrix} = \begin{pmatrix} h_{31}^{(i)}v_3^{(i)} \\ h_{31}^{(i+1)}v_3^{(i+1)} \\ h_{31}^{(i+2)}v_3^{(i+2)} \end{pmatrix}, \ i = 1, 4, \ldots, N-2. \quad (19)$$

Since the given conditions can also be obtained by separately encoding the $N/(2n+1)$ groups of subcarriers, we could decompose the problem into a number of subproblems, i.e., one for each group, and precode and decode the groups separately.

It remains to show how to decode the packets for this scheme. Without loss of generality, we also assume $K = 3$. If this scheme is adopted, each time, we sequentially take out $2n+1$ subcarriers. The received signal at receiver 1 is

$$\begin{aligned} \vec{y}_1 &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{31}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{T}\mathbf{C}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\left(\vec{x}_2 + \vec{x}_3\right) \\ &= \left(\mathbf{H}_{11}\mathbf{V}_1\mathbf{H}_{21}\mathbf{V}_2\right)\cdot\left(\vec{x}_1\vec{x}_2 + \vec{x}_3^T\right). \end{aligned} \quad (20)$$

Taking the inverse of matrix $(\mathbf{H}_{11}\mathbf{V}_1\mathbf{H}_{21}\mathbf{V}_2)$ and discarding the packets from transmitters 2 and 3, we can recover the desired packets $\vec{x}_1$. Note that we exploit the *commutative* property of diagonal matrices in (20).

At receiver 2, the received signal is

$$\begin{aligned} \vec{y}_2 &= \mathbf{H}_{12}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{32}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{12}(\vec{w}\mathbf{B})\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= \mathbf{H}_{12}\vec{w}x_1^{(1)} + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{12}\mathbf{B}\begin{pmatrix} x_1^{(2)} + x_3^{(1)} \\ \vdots \\ x_1^{(n+1)} + x_3^{(n)} \end{pmatrix} \\ &= \left(\mathbf{H}_{22}\mathbf{V}_2\mathbf{H}_{12}\vec{w}\mathbf{H}_{12}\mathbf{B}\right) \\ &\quad \cdot \left(\vec{x}_2, x_1^{(1)}, x_1^{(2)} + x_3^{(1)}, \ldots, x_1^{(n+1)} + x_3^{(n)}\right)^T. \end{aligned} \quad (21)$$

Taking the inverse of matrix $(\mathbf{H}_{22}\mathbf{V}_2\mathbf{H}_{12}\vec{w}\mathbf{H}_{12}\mathbf{B})$, we get $\vec{x}_2$.

At receiver 3, the received signal is

$$\begin{aligned} \vec{y}_3 &= \mathbf{H}_{13}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{23}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{13}(\mathbf{C}\mathbf{T}^n\vec{w})\vec{x}_1 + \mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{13}\mathbf{C}\begin{pmatrix} x_1^{(1)} + x_2^{(1)} \\ \vdots \\ x_1^{(n)} + x_2^{(n)} \end{pmatrix} + \mathbf{H}_{13}\mathbf{T}^n\vec{w}x_1^{(n+1)} + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\ &= \left(\mathbf{H}_{33}\mathbf{V}_3\mathbf{H}_{13}\mathbf{C}\mathbf{H}_{13}\mathbf{T}^n\vec{w}\right) \\ &\quad \cdot \left(\vec{x}_3, x_1^{(1)} + x_2^{(1)}, \ldots, x_1^{(n)} + x_2^{(n)}, x_1^{(n+1)}\right)^T. \end{aligned} \quad (22)$$

Taking the inverse of matrix $(\mathbf{H}_{33}\mathbf{V}_3\mathbf{H}_{13}\mathbf{C}\mathbf{H}_{13}\mathbf{T}^n\vec{w})$, we can decode $\vec{x}_3$. After decoding each group separately, we then combine the decoded data. The theorem is thus proved. ■

Note that the proof of Theorem 1 also leads to an algorithm to achieve interference alignment gains for any large $N \in \mathbb{N}$.

*4) Practical Issue of Large Channel Variance:* Here, we examine another practical problem of adopting interference alignment for a multiuser OFDM.

A necessary condition to achieve interference alignment in OFDM is that the channel gain is drawn from a continuous distribution. As a result, if the variance of the channel is large, some of the channel gains can be very small in certain conditions, whereas some other channel gains can be very large. When precoding over all the subcarriers, after taking the inverse of the channel gain matrix, some entries of the precoding matrix could be $10^4$ times (or even more) larger than some others. The result is that the power of one subcarrier could be $10^8$ times (or even more) larger than that of another subcarrier. Given certain power constraints, the error performance of the system will suffer from great degradation, which makes interference alignment less useful.

In our proposed scheme, if the channel variance is large, there is also a certain chance that some entries of $\mathbf{T}$ can be much larger than the others, since $\mathbf{T} = \mathbf{H}_{21}\mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{H}_{31}^{-1} = \mathbf{H}_{21}\mathbf{H}_{32}\mathbf{H}_{13}\mathbf{H}_{12}^{-1}\mathbf{H}_{23}^{-1}\mathbf{H}_{31}^{-1}$. If we precode and decode over large $n$, since the last columns of $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ are all obtained by multiplying $\mathbf{T}^n$, the situation could be further exacerbated. The consequences are as follows.

1) Since some of the entries can be extremely small, the decoding matrices can be close to singular. Thus, the desired signal cannot be decoded.
2) Even if the decoding matrices are invertible, due to the transmitter power constraint, the system error performance could be rather poor.

In fact, even if $n = 1$, there is still chance that some matrices are not invertible. These are the reasons why we cannot precode and decode for large $N$. This issue also demonstrates the importance of the proposed decomposition theorem (see Theorem 1).

Take $\mathbf{V}_1$ for instance. The constraint is that the power on one subcarrier cannot be $10^a$ (e.g., $a = 3$) times larger than the power on another subcarrier. If the constraint is violated, the system is considered to be in the outage state. Let

$$\mathbf{T} = \mathrm{diag}\{t_1, t_2, \ldots, t_{2n+1}\} \tag{23}$$

where $t_i = h_{21}^{(i)} h_{32}^{(i)} h_{13}^{(i)} / (h_{12}^{(i)} h_{23}^{(i)} h_{31}^{(i)}), i = 1, 2, \ldots, (2n+1)$. $t_1, t_2, \ldots, t_{2n+1}$ can be regarded as independent and identically distributed random variables. Let $t$ denote the common distribution of $t_1, t_2, \ldots, t_{2n+1}$. Define $t_{(1)}, t_{(2)}, \ldots, t_{(2n+1)}$ as the order statistics of $t_1, t_2, \ldots, t_{2n+1}$ with $t_{(1)} = \min_i t_i$, $t_{(2n+1)} = \max_i t_i$.

Let $\gamma = t_{(2n+1)}/t_{(1)}$. From (13) and (14), we have $\gamma^{2n} \le 10^a$; thus

$$\gamma \le 10^{a/(2n)} \tag{24}$$

which means $t_{(2n+1)}$ cannot be $10^{a/(2n)}$ times larger than $t_{(1)}$.

On the other hand, since $\gamma_{\max} = 10^{a/(2n)}$, we have

$$1 - \left(\Pr\left\{t \ge \frac{t_{(2n+1)}}{10^{\frac{a}{2n}}}\right\}\right)^{2n+1} \le \Pr\left\{t_{(1)} \le \frac{t_{(2n+1)}}{\gamma}\right\} \le 1. \tag{25}$$

It can be seen that $\Pr\{t \ge (t_{(2n+1)}/10^{a/2n})\}$ is a decreasing function of $n$. With the power of $2n+1$, $\Pr\{t_{(1)} \le t_{(2n+1)}/\gamma\}$ will quickly converge to 1. That means, with large $n$, $P(t_{(2n+1)} \ge \gamma t_{(1)}) = 1$. Therefore, with large $n$, constraint (24) will not be satisfied.

Next, we show how large $n$ could be for given constraint (24). The joint probability density function (pdf) of $t_{(1)}$ and $t_{(2n+1)}$ is found as follows:

$$f_{t_{(1)}t_{(2n+1)}}(x, y) = \frac{\partial^2 F_{t_{(1)}t_{(2n+1)}}(x, y)}{\partial x \partial y} \tag{26}$$

where $F_{t_{(1)}t_{(2n+1)}}(x, y)$ is the joint cumulative distribution function of $t_{(1)}$ and $t_{(2n+1)}$. By the definition of partial derivative, we have

$$\begin{aligned} f_{t_{(1)}t_{(2n+1)}}&(x, y) \\ =& \frac{\partial}{\partial y}\left\{\lim_{\Delta x \to 0}\left[F_{t_{(1)}t_{(2n+1)}}(x+\Delta x, y) - F_{t_{(1)}t_{(2n+1)}}(x, y)\right]/\Delta x\right\} \\ =& \lim_{\Delta x 0, \Delta y \to 0}\left[F_{t_{(1)}t_{(2n+1)}}(x+\Delta x, y+\Delta y)\right. \\ & - F_{t_{(1)}t_{(2n+1)}}(x, y+\Delta y) - F_{t_{(1)}t_{(2n+1)}} \\ & \left.\times (x+\Delta x, y) + F_{t_{(1)}t_{(2n+1)}}(x, y)\right]/(\Delta x \Delta y) \\ =& \lim_{\Delta x \to 0, \Delta y \to 0}\left[\Pr\left\{x \le t_{(1)} \le x+\Delta x, t_{(2n+1)} \le y+\Delta y\right\}\right. \\ & \left. - \Pr\left\{x \le t_{(1)} \le x+\Delta x, t_{(2n+1)} \le y\right\}\right]/(\Delta x \Delta y) \\ =& \lim_{\Delta x \to 0, \Delta y \to 0} \Pr\left\{x \le t_{(1)} \le x+\Delta x, \right. \\ & \left. y \le t_{(2n+1)} \le y+\Delta y)/(\Delta x \Delta y\right\}. \end{aligned} \tag{27}$$

To calculate the probability of the last equality, for any $x < y$, we can divide the $x$-axis into five disjoint intervals as: $I_1 = (-\infty, x)$, $I_2 = (x, x+\Delta x)$, $I_3 = (x+\Delta x, y)$, $I_4 = (y, y+\Delta y)$, and $I_5 = (y+\Delta y, \infty)$. For each $t_i$, the probability that it falls into each interval can be calculated as follows:

$$\begin{cases} p_1 = \Pr\{t_i \in I_1\} = F_t(x) \\ p_2 = \Pr\{t_i \in I_2\} = F_t(x+\Delta x) - F_t(x) \\ p_3 = \Pr\{t_i \in I_3\} = F_t(y) - F_t(x+\Delta x) \\ p_4 = \Pr\{t_i \in I_4\} = F_t(y+\Delta y) - F_t(y) \\ p_5 = \Pr\{t_i \in I_5\} = 1 - F_t(y+\Delta y). \end{cases} \tag{28}$$

To make $(x \le t_{(1)} \le x+\Delta x, y \le t_{(2n+1)} \le y+\Delta y)$ happen, the statistics $\{t_1, t_2, \ldots, t_{2n+1}\}$ must have exactly 1 sample falling into interval $I_2$, 1 falling into interval $I_4$, $(2n-1)$ falling into interval $I_3$, and 0 elsewhere, which is a multinomial problem. Hence, we have

$$\begin{aligned} \Pr&\left\{x \le t_{(1)} \le x+\Delta x, y \le t_{(2n+1)} \le y+\Delta y\right\} \\ &= \binom{2n+1}{0, 1, (2n-1), 1, 0} p_1^0 p_2^1 p_3^{(2n-1)} p_4^1 p_5^0. \end{aligned} \tag{29}$$

It follows that

$$\begin{aligned} f_{t_{(1)}t_{(2n+1)}}&(x, y) \\ =& \lim_{\Delta x \to 0, \Delta y \to 0}\left\{\frac{(2n+1)!}{(2n-1)!}\frac{F_t(x+\Delta x) - F_t(x)}{\Delta x}\right. \\ & \times \frac{F_t(y+\Delta y) - F_t(y)}{\Delta y} \\ & \left.\times \left[F_t(y) - F_t(x+\Delta x)\right]^{2n-1}\right\} \\ =& (2n+1)(2n) f_t(x) f_t(y) \left[F_t(y) - F_t(x)\right]^{2n-1}. \end{aligned} \tag{30}$$

Since $t_i = h_{21}^{(i)} h_{32}^{(i)} h_{13}^{(i)} / (h_{12}^{(i)} h_{23}^{(i)} h_{31}^{(i)}), i = 1, 2, \ldots, 2n+1$, and each $h^{(i)}$ is a random variable, the distribution of $t_i$ is difficult to be explicitly found. Here, we continue our analysis by approximating $t_i$ as a uniform distributed or Rayleigh distributed random variable.

If $t_i$ is approximated as a uniform distributed random variable and $t_i \in (0, 1)$, we have

$$\Pr\left\{t_{(1)} \le \frac{t_{(2n+1)}}{\gamma}\right\} \tag{31}$$

$$= \int_0^1 \int_0^{\frac{y}{\gamma}} (2n+1)(2n)(y-x)^{2n-1} dx dy. \tag{32}$$

$$= \left[1 - \left(1 - \frac{1}{\gamma}\right)^{2n}\right] \ge 1 - (1 - 10^{-\frac{a}{2n}})^{2n} \tag{33}$$

where the last inequality is a direct result of (24). Taking the derivative of (33), it can be found that $P_{outage} = \Pr\{t_{(1)} \le t_{(2n+1)}/\gamma\}$ is an increasing function of $n$. For $a = 3$, if $n = 1$, $P_{outage} = 0.0622$; if $n = 2$, $P_{outage} = 0.5431$; and if $n = 3$,

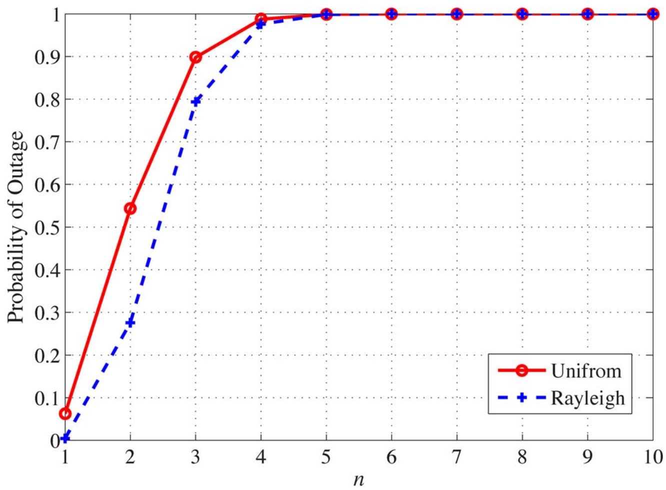


Fig. 2. Probability of system outage.

$P_{outage} = 0.8978$. For a system with many subcarriers, it indicates that we can only precode over $n = 1$.

If $t_i$ is approximated as a Rayleigh distributed random variable with pdf $f(x \mid \sigma) = (x/\sigma^2)\exp(-(x^2/2\sigma^2)), x \geq 0$, then

$$f_{t_{(1)}t_{(2n+1)}}(x,y) = (2n+1)(2n)\frac{xy}{\sigma^4}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \times \left(\exp\left(-\frac{x^2}{2\sigma^2}\right) - \exp\left(-\frac{y^2}{2\sigma^2}\right)\right)^{2n-1}.$$

There is no closed-form solution of $\Pr\{t_{(1)} \leq t_{(2n+1)}/\gamma\}$ in this case. The numerical results are shown in Fig. 2. It can be seen that the conclusion still holds, i.e., we can only precode over $n = 1$.

Recall that Lemma 1 shows $d_{\max} = 1.4998$ when $K = 3$ and $n = 1000$. Here, we can see that this maximum DoF cannot be achieved under practical settings. Hence, we have the following theorem.

*Theorem 2:* For a practical multiuser OFDM system with less than 4149 subcarriers, the maximum DoF is $d_{\max} = 1.33$, which is achieved when there is three-transmitter/receiver-pair precoding over three subcarriers each time.

## IV. Multiuser Multiple-Input–Multiple-Output Orthogonal Frequency Division Multiplexing With Interference Alignment

In previous sections, we have considered applying interference alignment to OFDM systems. Since a MIMO transmission technique can also be adopted to enhance the system throughput, we consider incorporating interference alignment to MIMO-OFDM systems in this section.

Suppose we have $M$ antennas at both the transmitter and receiver sides and $N$ subcarriers in total. The signals received at the $i$th receiver on subcarrier $n$ can be represented as

$$\vec{y}_i(n) = \mathbf{H}_{ii}(n)\mathbf{V}_i(n)\vec{x}_i(n) + \sum_{j\neq i}\mathbf{H}_{ji}(n)\mathbf{V}_j(n)\vec{x}_j(n) \quad (34)$$

where $\mathbf{H}_{ij}(n)$, $\mathbf{V}_i(n)$, and $\vec{x}_i(n)$ are the channel matrix from transmitter $i$ to receiver $j$, precoding matrix at transmitter $i$, and data at transmitter $i$, respectively; all of them are at subcarrier $n$. From (34), we can see that the signals received can be represented as a matrix, with each column being the signals received from each subcarrier, i.e., $\mathbf{Y}_i = [\vec{y}_i(1)\vec{y}_i(2), \ldots, \vec{y}_i(n)]$, or we could vectorize this matrix so that we get the following simpler form:

$$\vec{y}_i = \mathbf{H}_{ii}\mathbf{V}_i\vec{x}_i + \sum_{j\neq i}\mathbf{H}_{ji}\mathbf{V}_j\vec{x}_j. \quad (35)$$

Since each antenna pair could operate on any subcarrier and there is no crosstalk between subcarriers, the wireless channel $\mathbf{H}_{ij}$ between transmitter $i$ and receiver $j$ is of the form in (36), shown at the bottom of the page.

*Theorem 3:* For a MIMO-OFDM system with $N$ subcarriers and $M$ antennas at each transmitter and receiver side, we can divide the subcarriers into $\lfloor N/(2n+1)\rfloor$ groups, where $n \in \mathbb{N}$, and precode and decode the groups separately to achieve the interference alignment gain.

*Proof:* In Theorem 1, we have actually established that for a system of diagonal channels, we could separately precode and decode each group of subcarriers. Now, consider the case when all the devices are equipped with multiple antennas. We can still divide the subcarriers into different groups and then precode and decode them separately, since we are able to distinguish the signals from different antennas and different subcarriers. In other words, upon receiving a signal, the receiver has knowledge of from which antenna and which subcarrier it gets the signal. Hence, by properly adjusting the order of the data transmitted, the channel is essentially of the form in (37), shown at the bottom of the next page. We can readily identify that (37) is actually in the block diagonal form with the $i$th block corresponding to the channels associated with the $i$th subcarrier. Within each block, we have standard MIMO channels. Letting $V$, with dimension $MN \times d$, assume the form of (16), by similar arguments as in Theorem 1, we could precode and decode the groups separately to achieve the interference alignment gain. ■

*Lemma 2:* All the channel matrices and matrix $\mathbf{T}$ are invertible.

*Proof:* As in (38), shown below, the inverse of a block matrix can be found by calculating the inverse of each block.

$$\mathbf{H}_{ij} = \begin{pmatrix} h_{ij}^{1,1} & 0 & 0 & \cdots & h_{ij}^{N+1,1} & 0 & \cdots & h_{ij}^{(M-1)N+1,1} & 0 & \cdots \\ 0 & h_{ij}^{2,2} & 0 & \cdots & 0 & h_{ij}^{N+2,2} & \cdots & 0 & h_{ij}^{(M-1)N+2,2} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \end{pmatrix} \quad (36)$$

Since, for each block, we have a standard MIMO channel matrix and each of its entries is drawn from a continuous random distribution, each block is invertible with probability 1. Hence, each channel matrix is invertible. Since the product of invertible matrices is still invertible, according to (14), matrix $\mathbf{T}$ is invertible. Thus

$$\begin{pmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{pmatrix}^{-1} = \begin{pmatrix} B_1^{-1} & 0 & 0 \\ 0 & B_2^{-1} & 0 \\ 0 & 0 & B_3^{-1} \end{pmatrix}. \tag{38}$$

■

*Theorem 4:* For a MIMO-OFDM system with $N$ subcarriers and $M$ antennas at each transmitter and receiver side, the maximum gain is $(4/3)M$.

*Proof:* According to Theorem 3, we could precode and decode over groups of subcarrier. Moreover, according to our previous results, we can only precode and decode over three subcarriers. Hence, subcarrier-wise, the normalized DoF is 4/3.

We next show that $(4/3)M$ is the maximum achievable DoF. First, we notice that by dividing the subcarriers into groups of three, taking $\mathbf{H}_{11}$ for instance, it is transformed from (39) to (40), as shown below. With the establishment of Lemma 2, following the proof of Theorem 1, and replacing the scalars with blocks, we readily have the maximum gain of $(4/3)M$. Thus

$$\mathbf{H}_{11} = \begin{pmatrix} h_{11}^{11} & 0 & 0 & h_{11}^{41} & 0 & 0 \\ 0 & h_{11}^{22} & 0 & 0 & h_{11}^{52} & 0 \\ 0 & 0 & h_{11}^{33} & 0 & 0 & h_{11}^{63} \\ h_{11}^{14} & 0 & 0 & h_{11}^{44} & 0 & 0 \\ 0 & h_{11}^{25} & 0 & 0 & h_{11}^{55} & 0 \\ 0 & 0 & h_{11}^{36} & 0 & 0 & h_{11}^{66} \end{pmatrix} \tag{39}$$

$$\mathbf{H}_{11} = \begin{pmatrix} h_{11}^{11} & h_{11}^{21} & 0 & 0 & 0 & 0 \\ h_{11}^{12} & h_{11}^{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{11}^{33} & h_{11}^{44} & 0 & 0 \\ 0 & 0 & h_{11}^{34} & h_{11}^{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{11}^{55} & h_{11}^{65} \\ 0 & 0 & 0 & 0 & h_{11}^{56} & h_{11}^{66} \end{pmatrix}. \tag{40}$$

■

We next show how to achieve this gain. We design $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ as follows:

$$\mathbf{V}_1 = \mathbf{A}, \mathbf{V}_2 = \mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}, \mathbf{V}_3 = \mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B} \tag{41}$$

where

$$\begin{cases} \mathbf{A} = [\vec{w}\mathbf{T}\vec{w}\mathbf{T}^2\vec{w}, \ldots, \mathbf{T}^{(n+1)M-1}\vec{w}] \\ \mathbf{B} = [\mathbf{T}^M\vec{w}\mathbf{T}^{M+1}\vec{w}, \ldots, \mathbf{T}^{(n+1)M-1}\vec{w}] \\ \mathbf{C} = [\mathbf{T}^{M-1}\vec{w}\mathbf{T}^M\vec{w}, \ldots, \mathbf{T}^{(n+1)M-2}\vec{w}] \\ \mathbf{T} = \mathbf{H}_{12}^{-1}\mathbf{H}_{32}\mathbf{H}_{31}^{-1}\mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13} \\ \vec{w} = [11, \ldots, 1]^T. \end{cases} \tag{42}$$

It can be observed that

$$\mathbf{A} = \left[\vec{w}\mathbf{T}\vec{w}, \ldots, \mathbf{T}^{M-1}\vec{w}\mathbf{B}\right] \tag{43}$$

$$= [\vec{w}\mathbf{T}\vec{w}, \ldots, \mathbf{T}^{M-2}\vec{w}\mathbf{C}\mathbf{T}^{(n+1)M-1}\vec{w}]. \tag{44}$$

At receiver 1, the received signals can be written as

$$\begin{aligned} \vec{y}_1 &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{31}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{31}\mathbf{H}_{32}^{-1}\mathbf{H}_{12}\mathbf{T}\mathbf{C}\vec{x}_3 \\ &= \mathbf{H}_{11}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{21}\mathbf{H}_{23}^{-1}\mathbf{H}_{13}\mathbf{C}(\vec{x}_2 + \vec{x}_3) \\ &= (\mathbf{H}_{11}\mathbf{V}_1\mathbf{H}_{21}\mathbf{V}_2) \cdot \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 + \vec{x}_3 \end{pmatrix}. \end{aligned} \tag{45}$$

For signals at receiver 2, we have

$$\begin{aligned} \vec{y}_2 &= \mathbf{H}_{12}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{32}\mathbf{V}_3\vec{x}_3 \\ &= \mathbf{H}_{12}(\vec{w}\mathbf{T}\vec{w}, \ldots, \mathbf{T}^{M-1}\vec{w}\mathbf{B})\vec{x}_1 + \mathbf{H}_{22}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{12}\mathbf{B}\vec{x}_3 \\ &= (\mathbf{H}_{22}\mathbf{V}_2\mathbf{H}_{12}(\vec{w}\mathbf{T}\vec{w}, \ldots, \mathbf{T}^{M-1}\vec{w})\mathbf{H}_{12}\mathbf{B}) \\ &\quad \times \begin{pmatrix} \vec{x}_2 \\ x_1^{(1)} \\ \vdots \\ x_1^{(M)} \\ x_1^{(M+1)} + x_3^{(1)} \\ \vdots \\ x_1^{((n+1)M)} + x_3^{(nM)} \end{pmatrix}. \end{aligned} \tag{46}$$

$$\mathbf{H}_{ij} = \begin{pmatrix} h_{ij}^{1,1} & \cdots & h_{ij}^{M,1} & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & 0 & \cdots & 0 & \cdots & 0 \\ h_{ij}^{1,M} & \cdots & h_{ij}^{M,M} & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & \cdots & h_{ij}^{M(N-1)+1,M(N-1)+1} & \cdots & h_{ij}^{MN,M(N-1)+1} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \cdots & h_{ij}^{M(N-1)+1,MN} & \cdots & h_{ij}^{MN,MN} \end{pmatrix} \tag{37}$$

Similarly, for signals at receiver 3, we have

$$
\begin{aligned}
\vec{y}_3 &= \mathbf{H}_{13}\mathbf{V}_1\vec{x}_1 + \mathbf{H}_{23}\mathbf{V}_2\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\
&= \mathbf{H}_{13}(\vec{w}\mathbf{T}\vec{w},\ldots,\mathbf{T}^{M-2}\vec{w}\mathbf{C}\mathbf{T}^{(n+1)M-1}\vec{w})\vec{x}_1 \\
&\quad + \mathbf{H}_{13}\mathbf{C}\vec{x}_2 + \mathbf{H}_{33}\mathbf{V}_3\vec{x}_3 \\
&= \begin{pmatrix} \mathbf{H}_{33}\mathbf{V}_3 \\ \mathbf{H}_{13}\mathbf{C} \\ \mathbf{H}_{13}(\vec{w}\mathbf{T}\vec{w},\ldots,\mathbf{T}^{M-2}\vec{w}) \\ \mathbf{H}_{13}\mathbf{T}^{((n+1)M-1)}\vec{w} \end{pmatrix}^T \\
&\quad \times \begin{pmatrix} \vec{x}_3 \\ x_1^{(M)} + x_2^{(1)} \\ \vdots \\ x_1^{((n+1)M-1)} + x_2^{(nM)} \\ x_1^{(1)} \\ \vdots \\ x_1^{(M-1)} \\ x_1^{((n+1)M)} \end{pmatrix}. \qquad (47)
\end{aligned}
$$

From (45)–(47), we can see that the desired signals are all free from interferences.

We can also calculate the probability of system outage when multiple antennas are deployed. Hence, we need to find the probability of $\Pr\{t_{(1)} \leq (t_{((2n+1)M)}/\gamma)\}$. With similar arguments, the joint pdf of $t_{(1)}$ and $t_{((2n+1)M)}$ can be found as

$$
\begin{aligned}
f_{t_{(1)}t_{((2n+1)M)}}&(x,y) \\
&= \lim_{\Delta x \to 0, \Delta y \to 0} P(x \leq t_{(1)} \leq x + \Delta x \\
&\qquad y \leq t_{((2n+1)M)} \leq y + \Delta y)/(\Delta x \Delta y) \\
&= \binom{(2n+1)M}{0, 1, (2n+1)M-2, 1, 0} p_1^0 p_2^1 p_3^{(2n+1)M-2} p_4^1 p_5^0. \quad (48)
\end{aligned}
$$

If $t_i$ is approximated as a uniform distributed variable in the range of (0,1), the probability $\Pr\{t_{(1)} \leq (t_{((2n+1)M)}/\gamma)\}$ can be found as follows:

$$
\begin{aligned}
\Pr&\left\{t_{(1)} \leq \frac{t_{((2n+1)M)}}{\gamma}\right\} \\
&= (2nM+M-1)\times(12nM+M)\int_0^1\int_0^{\frac{y}{\gamma}}(y-x)^{2nM+M-2}dxdy \\
&= 1 - \left(1 - \frac{1}{\gamma}\right)^{(2n+1)M-1} \\
&\geq 1 - \left(1 - 10^{-\frac{a}{2(n+1)M-2}}\right)^{(2n+1)M-1}.
\end{aligned}
$$

If $t_i$ is approximated as a Rayleigh distributed variable, there is no closed-form solution for probability $\Pr\{t_{(1)} \leq (t_{((2n+1)M)}/\gamma)\}$. The joint pdf of $t_{(1)}$ and $t_{((2n+1)M)}$ can be derived as

$$
\begin{aligned}
f_{t_{(1)}t_{((2n+1)M)}}&(x,y) \\
&= ((2n+1)M)\,((2n+1)M-1)\,\frac{xy}{\sigma^4} \times \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \\
&\quad \times \left[\exp\left(-\frac{x^2}{2\sigma^2}\right) - \exp\left(-\frac{y^2}{2\sigma^2}\right)\right]^{(2n+1)M-2}.
\end{aligned}
$$

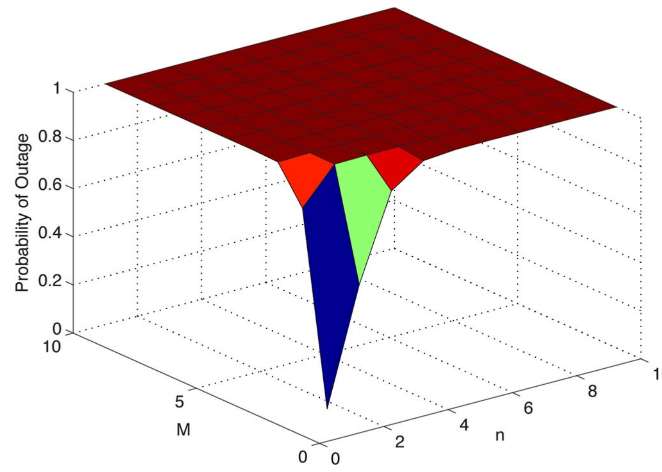


Fig. 3. Probability of system outage with multiple antennas for uniform distribution.

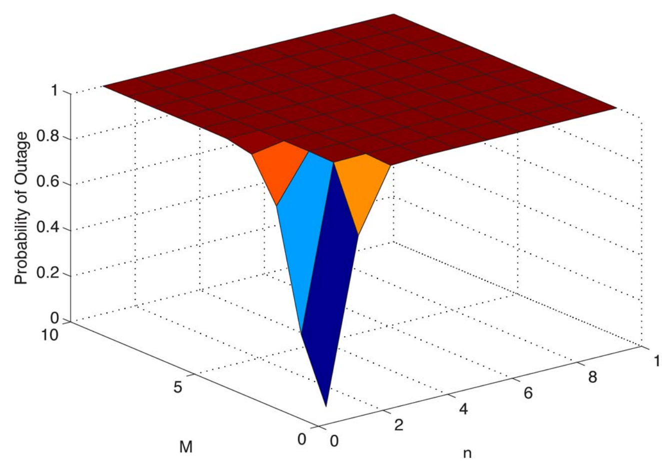


Fig. 4. Probability of system outage with multiple antennas for Rayleigh distribution.

Figs. 3 and 4 show the probabilities of system outage for uniform and Rayleigh distributions, respectively. Here, we also set $a = 3$. Hence, the power on one subcarrier cannot be $10^3$ times larger than that on any other subcarrier. We can see that when $n = 1$ and $M = 2$, the system outage probabilities are 0.8505 and 0.2758 for uniform and Rayleigh distributions, respectively. For $n = 1$ and $M = 3$, the outage probabilities are even higher, i.e., 0.9962 and 0.7937, respectively. For $n = 2$ and $M = 2$, the outage probabilities are unacceptably as high as 0.9981 and 0.9971, respectively. From the simulation results to be presented in the following section, we will see that a high outage probability is undesirable.

## V. Simulation Study

Simulations are conducted to evaluate the performance of the proposed schemes and verify the benefits brought about by incorporating interference alignment in multiuser (MIMO) OFDM systems. We consider the case of three users. The number of subcarriers is 255. Each transmitter precodes over $(2n+1)M$ subcarriers. Block-fading channels are used in the simulations, where channel gains are piecewise constants for

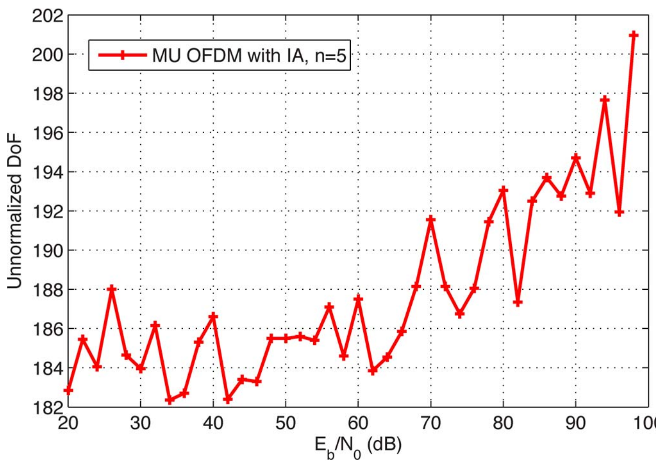


Fig. 5. System throughput when $n = 5$.

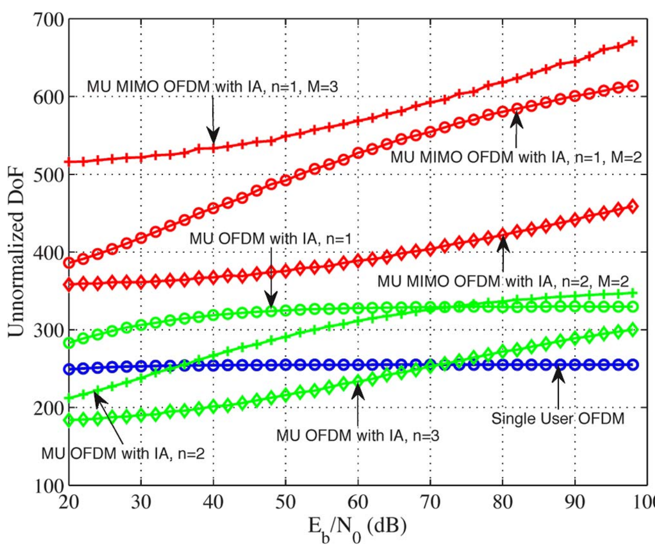


Fig. 6. System throughput comparison when the channel variance is large.

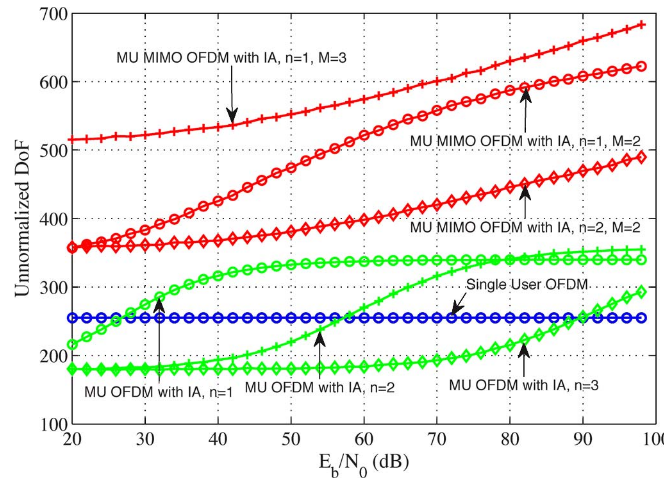


Fig. 7. System throughput comparison when the channel variance is small.

the duration of each time slot drawn from a certain distribution. Binary phase-shift keying is used as the modulation scheme. Hence, we transmit 1 bit on each subcarrier, and we measure how many bits are successfully decoded at the receivers. This way, we are essentially calculating the number of interference-free channels in the system (we call it *unnormalized DoF* hereafter).

Fig. 5 shows the system throughput when $n = 5$. We can see that the system performance is generally unstable. Comparing Fig. 5 with Figs. 6 and 7, we find that since $n$ is too large, the system performance is degraded. This confirms our result that we could not precode over a large amount of subcarriers.

Figs. 6 and 7 show the performances of different schemes when the channel is drawn from a uniform distribution on [0,1] and [0.9, 1], respectively. Comparing these two figures, we can see that when the channel variance is small, higher system throughput can be achieved. This conforms to our discussions about the precoding matrix in Section III-C4. It can also be observed that the trends and comparative relationships are similar in Figs. 5 and 6.

We can see in Fig. 7 that when $n = 1$, multiuser OFDM with interference alignment can achieve an unnormalized DoF of 339.98. Compared with the highest throughput of single-user OFDM of 255, the DoF has been improved by a factor of approximately 1.33 by incorporating interference alignment. When $n = 2$, we can see from both figures that the throughput of multiuser OFDM with interference alignment has degraded when the signal-to-noise ratio (SNR) is in the range of [0, 78] dB. That verifies our theorem that under certain power constraint, we can only precode over three subcarriers. Same conclusions also hold for $n = 3$ of multiuser OFDM with interference alignment, which exhibits poorer performance in the SNR range of [20, 100] dB.

For the case of multiuser MIMO OFDM with interference alignment, when $n = 1$ with small channel variance, the highest unnormalized DoF is 622.7, which is 2.44 times the unnormalized DoF of the single-user OFDM system. The reason why it is slightly less than 2.66 is also due to the big differences among the elements of the precoding matrices. For $n = 2$ and $M = 2$, we can see that the performance is worse than that of $n = 1$ and $M = 2$. When the devices are equipped with three antennas, we let $n = 1$ and precode over three subcarriers. The highest unnormalized DoFs are 671.2 and 683.208 for large and small channel variance cases, respectively, which are 2.63 and 2.68 times that of the single-user OFDM system. However, the maximum gain is supposed to be four times that of the single-user OFDM system. The performance degradation is also due to the big difference among the elements of the precoding matrices.

## VI. Conclusion

In this paper, we have investigated the problem of how to exploit interference in OFDM systems. We provided an analysis and developed effective schemes in incorporating interference alignment with multiuser (MIMO) OFDM to enhance system throughput. With an integer programming formulation, we derived the maximum efficiency for multiuser (MIMO) OFDM/interference alignment systems and showed how to achieve the maximum efficiency under practical constraints. The performance of the proposed schemes was validated with simulations. The proposed decomposition algorithm and the main results in this paper may serve as guidance for practical OFDM system design.

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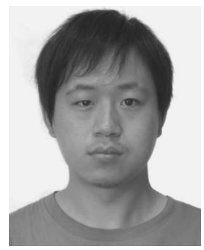

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