GPS Analysis of Multiple Sessions with Applications to Admission Control

George Lapiotis, Shiwen Mao, and Shivendra S. Panwar

Abstract— **In this paper we introduce a statistical method to analyze multiplexing of multiple sessions sharing link bandwidth using GPS scheduling. Our method is shown to substantially improve previous upper bounds for GPS scheduling of MMFP sessions, especially as the number of sessions increases. Application of analytical results to admission control indicate that by sharing bandwidth using GPS among traffic classes there are significant gains over systems that statically segregate the link bandwidth. This effect is quantified in several experiments where various combinations of source types are used. The gains are pronounced when bursty sources with stricter QoS requirements are used.**

Keywords—**Generalized Processor Sharing, Admission Control**

I. INTRODUCTION

TO day's Internet is shifting from the single best-effort service to a paradigm with several classes of service with different QoS (Quality of Service) requirements. One of the mechanisms provided by routers and switches for service differentiation is service scheduling, which in effect controls the bandwidth allocation to different service classes or sessions. Generalized Processor Sharing (GPS) is widely used as the scheduling discipline across traffic classes sharing a link. GPS scheduling can be approximately implemented using Weighted Fair Queueing (WFQ), Weighted Round Robin (WRR), and other improved variants of these schedulers. Well-known useful properties inherent to GPS are traffic isolation and bandwidth sharing, while meeting the QoS requirements of individual traffic classes. Most previous work on GPS analysis is mainly focused on general arrival processes, with deterministic or stochastic settings. These results are generally expected to give loose upper bounds because the finer dynamics of the arrival processes are not captured [6], [7], which results in low utilization. It would therefore be more preferable to use approaches that (i) can provide statistical QoS guarantees and (ii) a traffic characterization of finer granularity. In [7] Markov Modulated Fluid Processes (MMFP), were introduced for the traffic characterization of GPS sessions. This source model falls into the category of Markovian processes which has been widely used to model network traffic sources, especially real-time traffic such as voice and video. Moreover, in previous work [1], [2], [11] it is shown that appropriately fitted Markovian processes may capture to adequate extent effects of long-term traffic correlations at time scales of interest. In our previous work [5] the focus was on the combination of GPS with buffer management policies. This paper deals with the analysis of individual queue occupancy distributions of multiple general MMFP sessions fed into a buffer and sharing link bandwidth with GPS scheduling. Particularly, here we turn our attention to the trends of the GPS analysis method for multiple sessions and to its applications to admission control. There are two main contributions: (i) The improvement on previous upper bound approximations of queue occupancy distributions [7] is here numerically shown to increase with the number of sessions. (ii) By applying our analysis in admission control using GPS for bandwidth sharing between service classes with different QoS requirement we observe a considerable increase in the number of admissible sessions (and link utilization) per service class as compared to systems with segregated bandwidth allocation. Findings supporting a similar conclusion also appeared in [3]. We finally note that our method can be extended to account for the case of shared finite buffering of multiple sessions sharing link bandwidth with GPS [8]. Further results on this case are left for future work. The paper is organized as follows: Section II introduces the problem and a theorem which bounds the decomposition approach used in our method. In section III analytical results are validated by comparison with simulations and previous bounds. Section IV describes applications in admission control and quantifies the increase in link utilization and number of admitted sessions when GPS is used among three service classes one of which is voice. We conclude in section V.

II. GPS ANALYSIS FOR MULTIPLE SESSIONS

GPS is a work conserving scheduling discipline in which the n input sessions share a deterministic server with total rate c. A set of parameters $\{\varphi_i\}_{1\leq i\leq n}$, called the GPS assignment, determine the share of service rate that each session receives as follows: the minimum service rate guaranteed to input session i is equal to $c_i = (\varphi_i/\sum_{j=1}^{n}\varphi_j)c$. The residual service of unbacklogged sessions is distributed to the active sessions in proportion to their weights. The input traffic of session i is considered an infinitely divisible fluid that can be described by a continuous process $r_i(t)$. Thus, GPS can be considered a continuous limiting case of the Weighted Round Robin service discipline. In this section we provide an approach to treat

---

George Lapiotis is currently with the Applied Research Department in Telcordia Technologies, 445 South St., Morristown, NJ 07960, USA (E-mail: lapiotis@research.telcordia.com). He was with the Center for Advanced Technologies in Telecommunications (CATT) of the Polytechnic University of Brooklyn when this work was done.

Shiwen Mao and Prof. Shivendra S. Panwar can be reached at the Center for Advanced Technologies in Telecommunications, Polytechnic University, 6 Metrotech Center, Brooklyn, NY 11201, USA (E-mail: smao@photon.poly.edu, and panwar@catt.poly.edu, respectively).

0-7803-7097-1/01/$10.00 ©2001 IEEE

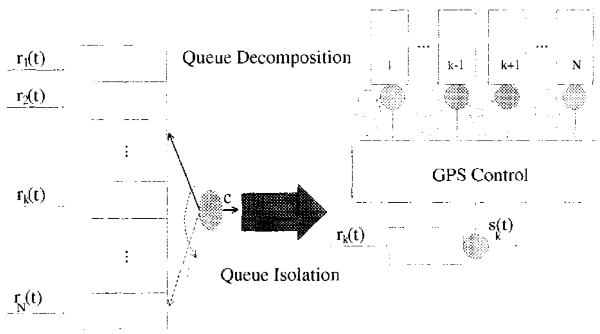

Fig. 1. Queue Decomposition and Isolation in a GPS system with N queues.

the general case of multiple traffic sessions sharing a common buffer and served with GPS scheduling. We initially adopt a decomposition approach [6], [7] to derive the upper bounds for the tail distribution of individual queues for the N-queue case. However, in this work we shall provide a new expression for the service process of each decomposed queue that is a stochastic bound in the case of general sources and is shown by numerical results to substantially improve the upper bound approximation of the tail distribution in [7] when applied in the special case of MFFP sessions.

Consider N general MMFP sources that are inputs to an infinite buffer system with GPS service. The corresponding GPS assignments are $\varphi_1, \varphi_2, ..., \varphi_N$, where without loss of generality we assume $\sum_{i=1}^{N} \varphi_i = 1$. The service process $s_k(t)$ of logical queue k is time-variant and at any time instance t it can be determined by the residual service of the remaining $N-1$ queues and the GPS assignments φ_i, $i = 1$, ..., $k-1$, $k+1$, ...,N. In the general queue decomposition method the GPS server of rate c is divided into a set of N servers with service rates $a_1, a_2, ..., a_N$, so that instead of having a GPS system with N sessions sharing a server, there is a decomposed system consisting of N separate queues, each of which has a dedicated server. The decomposition is such that $\sum_{i=1}^{N} a_i \leq c$ and $\bar{\lambda}_i < a_i$, $\forall i \in 1, 2, ..., N$, where $\bar{\lambda}_i$ is the average incoming traffic rate of session i. In the decomposition step it is assumed that $a_i = c_i$, where c_i is the guaranteed minimum service rate of queue i under the GPS service. Each individual queue's departure process s_k is then approximated using an MMFP. In the second step a single one-queue multiplexing system with modulated service process $\tilde{s}_k$ is considered. Define by $B(t)$ the set of backlogged queues, at any time instant t. Also, define the set of unbacklogged queues $\bar{B}(t)$, which is the complement of $B(t)$, i.e., the union of the two sets at any time is the set of all N logical queues in the system. Then the instantaneous service process $s_k(t)$, $k \in B(t)$ seen by an individual queue k is given by

$$s_k(t) = c_k + \frac{\varphi_k}{\varphi_k + \sum\limits_{l \in B(t), l \neq k} \varphi_l} \sum_{l \in \bar{B}(t)} (c_l - r_l(t)), \quad (1)$$

where $c_k = \varphi_k c$ is the minimum guaranteed rate of session k, and c the total system service capacity. Equation (1) is comprised of c_k increased by the residual service seen by queue k when there are unbacklogged queues in the system. The first term in the product on the right side is the fraction of the total residual service assigned to queue k according to the GPS allocation rule. The second term in the product represents the total available residual service in the system, and is equal to the sum of the residual service of individual unbacklogged queues. Notice that in the fluid model even if a queue l is empty (unbacklogged), part of its available service capacity may be consumed for an instantaneous input rate $r_l(t)$, which is in this case necessarily less than c_l. That is, for $k \in \bar{B}(t)$, $s_k(t) = r_k(t)$. It can be shown that when eq. (1) is applied to the decomposed system, it provides a lower bound on the instantaneous service rate seen by an unbacklogged session in the actual system. By using MMFPs to model the input and output rates of individual queues we can then obtain approximate upper bounds on the buffer occupancy and delay distribution of each individual session at steady-state (see Section III). The critical problem in using (1) is that we do not have an explicit expression that captures $B(t)$ (or $\bar{B}(t)$). As a modeling approach, we first consider decomposed queues, each queue k receiving service equal to its GPS assignment c_k. Define $s_k(t)$ to be the instantaneous service process of any queue k of the real system. In order to obtain an upper bound on the tail distribution of the occupancy of queue k, it is sufficient to show that by using (1) for the decomposed system the following theorem holds:

Theorem 1:

$$s'_k(t) \leq s_k(t), \forall t, \quad (2)$$

where $s'_k(t)$ is the instantaneous service process of queue k of the decomposed system. The proof of the theorem may be found in [8]. Given (2) it has already been established in [7] that the tail distribution of a decomposed queue is an upper bound of its tail distribution in the real system. We note however that the modified expression (1) for the service process $s'_k(t)$ used in our derivation of the tail distribution gives a tighter bound than the one used in [7] as will be shown in numerical examples (section IV). In the following section we introduce the application of MMFPs to characterize the input and output processes of the GPS system.

III. THE SYSTEM WITH MMFP SESSIONS

The expression for $s'_k(t)$ can be used to obtain the tail distribution of any individual decomposed queue. We first introduce some notation and develop the necessary steps of the derivation. Consider the system of N independent MMFP sources. Each source k has a state space S_k, irreducible generator M_k, and generates fluid at constant rate λ_{sk} when in state s_k, $s_k \in S_k$. Thus, source k is characterized by the pair $(\mathrm{M}_k, \lambda_k)$. The stationary probability vector of the source is π_k, where $\mathrm{M}_k^T \pi_k = 0$, and the inner product $\langle \pi_k, 1 \rangle = 1$ (1

0-7803-7097-1/01/$10.00 ©2001 IEEE

denotes the unity vector). Its mean arrival rate is $\bar{\lambda}_k$, where $\bar{\lambda}_k = \langle \pi_k, \lambda_k \rangle$. We have already focused on the practical class of GPS assignments c_k such that $\bar{\lambda}_k < c_k$.

In a first step the queues are decomposed and we obtain the departure process $\omega_k(t)$ of each queue k by application of the technique described in [10], where $\omega_k(t)$ is approximated by a MMFP characterized by $(\mathrm{M}_k^{\omega}, \lambda_k^{\omega})$, and defined on a state space L_k. In this step, each decomposed k queue is assumed to be depleted at a constant service rate c_k. A property of the approximate output process is that all its stationary moments are equal to the corresponding moments of the actual output process. Let $R(t)$ be the aggregate MMFP of the independent departure processes of all decomposed queues, excluding the queue of session k. We denote by R the state space of $R(t)$.

In the second step, we set up the governing system of differential equations for the isolated queue of session k assuming its service rate is the modulated process $s_k(t)$ as given in (1). In this unified queue system the state space H_k is the Cartesian product of the session k's source state space S_k and the state space of the aggregate departure process $R(t)$ of the remaining queues in the system, i.e., H_k=S_kx L_1x...xL_{k-1}x L_{k+1}x...xL_N. Let $h = (s_k, \rho)$ denote the state of the system under consideration, $h \in H_k$, $\rho \in R$. We are looking for the stationary state distribution $F^k(x_k) = \{F_h^k(x_k) | h_k \in H_k\}$ where

$$F^k(x_k) = \Pr(H_k = h_k, X_k \leq x_k) \qquad (3)$$
$$(h_k \in H_k, 0 \leq x_k \leq \infty).$$

The ordinary differential equation system governing the occupancy distribution of the isolated queue k, with input MMFP source $(\mathrm{M}_k, \lambda_k)$ and modulated service process given by (1) is

$$\mathrm{D}_k \frac{d}{dx_k} F^k(x_k) = \tilde{\mathrm{M}}_k F^k(x_k), \qquad (4)$$

where $\tilde{\mathrm{M}}_k$ is the transition rate matrix of the states in H_k and D_k is the drift matrix of the queue of session k. M_k can be expressed as the Kronecker sum of the source k generator and the generators of the aggregate departure process $R(t)$ of the remaining $N-1$ queues.

$$\tilde{\mathrm{M}}_k = \mathrm{M}_k \oplus \mathrm{M}_1^{\omega} ... \oplus \mathrm{M}_{k-1}^{\omega} \oplus \mathrm{M}_{k+1}^{\omega} ... \oplus \mathrm{M}_N^{\omega}. \qquad (5)$$

D_k is a diagonal drift matrix whose diagonal element d_h^k at row h, which corresponds to system state h, is given by

$$d_h^k = r_h^k - [c_k + \frac{\varphi_k}{\varphi_k + \sum\limits_{j \in B_h} \varphi_j} \sum_{j' \in \bar{B}_h} (c_{j'} - r_h^{j'})]. \qquad (6)$$

In (6), r_h^k $(r_h^{j'})$ is the input rate of source $k(h)$ at system state h, and $B_h(\bar{B}_h)$ is the set of backlogged (unbacklogged) queues at state h in the decomposed system.

The stationary tail distribution is obtained by solving the system (4) and has the following usual spectral representation:

$$F^k(x_k) = \sum_i a_i e^{z_i x_k} \varphi_i, \qquad (7)$$

where (z_i, φ_i) are the eigenvalue-eigenvector pairs of $\mathrm{D}_k^{-1} \tilde{\mathrm{M}}_k$ and the coefficients a_i can be determined by applying well-known boundary conditions [9]. We evaluate the accuracy of our analytic approximation in the following section.

IV. APPLICATIONS

We initially apply the analytical results to observe the trend of increasing number of sessions. In a second set of results we investigate the gains of using the analysis in the previous section in admission control.

A. Increasing the Number of Sessions

In all the simulation cases under study the simulation run lengths were such that the 95% confidence interval of an estimated value is within 20% of the value. For simplicity, each input traffic stream k consists of a two-state MMFP source (on-off source) characterized by the state transition rates a_{ki} (i=1,2), input rate λ_k during the on state, and zero input rate during the off state. The average duration of the on state is kept equal to 1 in all cases, i.e., $1/a_{k2} = 1$. To give some intuition we also define: (i) the *source activity factor* ρ_k as the fraction of time source in the ON state, i.e., $\rho_k = a_{k1}/(a_{k1} + a_{k2})$ and (ii) the *normalized source load* u_k as the ratio of average source rate to server capacity, i.e., $u_k = \rho_k \lambda_k / c$. In order to give a more practical interpretation to results we may equate the server capacity of one fluid unit per time unit to the standard OC-3 rate of 155 Mbps. By using a time unit of 0.5 msecs, one unit of fluid then corresponds to 10 KBytes, so $B = 10$ is equivalent to 100 Kbytes (this is a common order of memory at output ports of routers and switches). In figure 2 we demonstrate our results on tail distributions of the queue occupancy in the case of multiple queues (case of nine sources in the figure), as compared to simulations and results of the method in [7] (LZT analysis). We observe the tail distribution of the isolated session's queue. The tail distribution we predict is an upper bound of the simulation result, and a tighter bound than the one in the LZT approach. Moreover, by comparing the tail distributions with the cases of 3 and 6 queues (not included here due to space limitations) we have observed that as the number of sources increases, our prediction of overflow is increasingly closer to that of simulations as compared to the LZT bound.

TABLE I

SOURCE PARAMETERS

Isolated source				Remaining 8 sources			
ρ	u	λ	ϕ	ρ	u	λ	ϕ
0.15	0.03	1.01	1/9	0.03	0.005	0.75	1/9

B. Admission Control

In this set of results we focus on bandwidth sharing using GPS in admission control. Using the analysis of previous sections we quantify the improvement in the number of admissi-

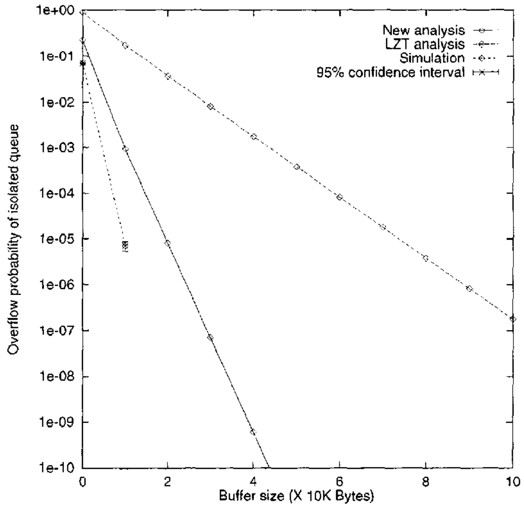

Fig. 2. Comparative results of our approach, simulations, and the LZT bound for the case of nine queues. We provide the tail distribution of an isolated session for asymmetric sources ($c = 4.5$).

ble sessions when bandwidth is shared among multiple classes using GPS. Within a single class FIFO service is assumed among individual flows. The eight types of sources used in the experiments, are listed in Table II. The voice and compressed voice average active and silent states are drawn from ITU standard models [13] modified to include the UDP, IP, and RTP headers appended for carrying voice over IP (VoIP) networks. For the FTP source we matched the moments of FTP sources from [4]. Although we admittedly do not try to scale our MMFP sources to capture exact long term correlations of data traffic, we are able to derive some general trends with a simple parameter matching. A source with a high activity factor ρ is loosely characterized as *smooth*, while one with a low activity factor is loosely characterized as *bursty*. Between two sources with the same average rate the burstier one has a greater expected number of transitions to the active state during any arbitrary time interval. The Loss column in the table refers to the loss requirement of that source. We conducted five experiments by multiplexing three types of sources in each experiment. In the first four experiments voice and bursty source 1 were combined with another type of source. In the fifth experiment we combined voice, compressed voice, and FTP. The combination of the sources for the experiments are:

- Experiment 1: voice, bursty source 1, and bursty source 2, with service rate 10.1;
- Experiment 2: voice, bursty source 1, and bursty source 3, with service rate 10.1;
- Experiment 3: voice, bursty source 1, and smooth source 1, with service rate 10.1;
- Experiment 4: voice, bursty source 1, and smooth source 2, with service rate 10.1;
- Experiment 5: voice, compressed voice, and ftp, with service rate 40.1.

For each experiment, we calculated the admissible region of the system using (i) GPS bandwidth sharing using the analysis in the previous sections, (method I) (ii) GPS bandwidth sharing using the analysis in [7] (method II), and (iii) segregated bandwidth allocation to each traffic class (method III). For reasons of comparison in all cases the number of admissible sources is optimal at any given total system bandwidth. For each of the three systems the average link utilization was obtained as the ratio of the total input source rate over the link rate for each point of the three-dimensional admissible region. The results are summarized in Tables III and IV. The Mean column represents the relative improvement in link utilization averaged over all points obtained by using our method as compared to the other two methods. The Maximum column represents the relative improvement at the point where it reaches a maximum over the entire admissible region. From the results of Table IV it can be concluded that by using GPS in admission control there is substantial gain in link utilization as compared to a system where bandwidth is segregated among the traffic classes. The maximum gain of 38.35% was observed in Experiment 2 over method III. The results in Table III indicate small improvements over method II in the average and substantial improvements at some operating points of the system, with a maximum improvement of 26.86% in Experiment 2 also. In general larger gains are observed for burstier sources with stricter loss requirements. To better demonstrate the results we include the improvement in number of admissible sources over method III in Fig. 3. The graph corresponds to Experiment 5 where voice, compressed voice, and FTP traffic are multiplexed. The improvement corresponds to as many as 10 voice, 10 compressed voice, and 25 ftp sources. In the case of method II the improvement was 10 voice, 9 compressed voice, and 9 ftp sources. As also observed in [3] the improvement in admissible number of sources is expected to increase as the total link capacity of the system. In our case this is also supported by the fact that our method derives better queue occupancy results as the number of sources (whether traffic classes or sessions) increases. Finally, the tradeoff for the gains in our method comes at the cost of higher complexity over the other two methods, especially when traffic classes include a high degree of source heterogeneity. However, in the most usual applications traffic classes have a small number of groups of homogeneous sources, which may be reduced to solutions of manageable complexity. Such state reduction methods were introduced in [12].

V. CONCLUSIONS

In this paper a method for the statistical GPS analysis of multiple sessions was presented. This method improves previous upper bounds of the tail distributions of the sessions in the GPS system. It was further shown that the improvement

TABLE II

SOURCE PARAMETERS FOR THE SOURCES USED IN THE EXPERIMENTS

Source	ρ	u	λ	Loss
Voice	0.387	0.053	5.54	1e-2
Compressed voice	0.387	0.010	1.0	1e-3
FTP	0.026	0.082	127.2	1e-5
Smooth source 1	0.909	0.072	0.8	1e-3
Smooth source 2	0.909	0.072	0.8	1e-5
Bursty source 1	0.167	0.026	1.6	1e-3
Bursty source 2	0.091	0.022	2.4	1e-3
Bursty source 3	0.091	0.022	2.4	1e-5

TABLE III

IMPROVEMENT IN LINK UTILIZATION OVER METHOD II.

	Mean	Maximum
Experiment 1	3.89%	15.35%
Experiment 2	4.38%	26.86%
Experiment 3	2.28%	11.51%
Experiment 4	3.87%	15.35%
Experiment 5	6.90%	22.23%

increases as the number of sessions increases. When comparing admission control using a system where traffic classes share bandwidth using GPS and a system where bandwidth is segregated among traffic classes statically, we conclude that there are significant gains in the first case. This indicates that in a differentiated services environment where GPS implementations such as WFQ are used, admission control algorithms would benefit by taking the GPS multiplexing effect into account. The GPS gains are expected to further increase at higher link rates. We expect that the complexity of our method is manageable for realistic admission control applications such as Voice over IP. Finally, we note that the extension of our buffer occupancy results to obtain approximate upper bounds on delay distributions are straightforward [8]. This topic, and the more difficult topic of GPS analysis combined with buffer management for admission control of multiple sessions, are left for future work.

TABLE IV

IMPROVEMENT IN LINK UTILIZATION OVER A SEGREGATED BANDWIDTH SYSTEM (METHOD III).

	Mean	Maximum
Experiment 1	16.07%	26.85%
Experiment 2	19.94%	38.35%
Experiment 3	9.10%	19.18%
Experiment 4	9.12%	19.18%
Experiment 5	12.88%	23.13%

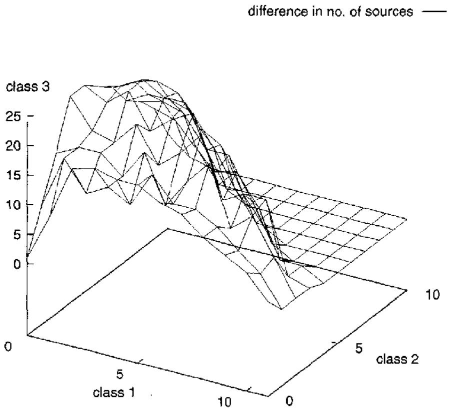

Fig. 3. Gains in number of admitted sources over a segregated bandwidth system (method III) in Experiment 5.

REFERENCES

[1] Bo Ryu and Anwar Elwalid, " The Relevance of Short-Range Dependence of VBR Video Traffic to Real-Time Traffic Engineering," *ACM Sigcomm '96*, Stanford, CA, August 1996.

[2] N. Likhanov and R. R. Mazumdar, "Cell loss asymptotics in buffers fed with a large number of independent stationary sources," *IEEE Infocom '98*, pp. 339-345, 1998.

[3] J. Qiu and E.W. Knightly, "Inter-Class Resource Sharing using Statistical Envelope Curves," *IEEE Infocom '99*, pp. 1404-1411, 1999.

[4] Kihong Park, Gitae Kim, Mark Crovella, "On the relationship between file sizes, transport protocols, and self similar network traffic," Proceedings *Proceedings of ICNP '96*, 1996.

[5] G. Lapiotis and S. Panwar, "Quality of Service Analysis of Shared Buffer Management Policies Combined with Generalized Processor Sharing,", in the *Proceedings IEEE Globecom '99*, Rio, Brasil, 1999.

[6] Z.-L. Zhang, D. Towsley, and J. Kurose, "Statistical analysis of Generalized Processor Sharing Scheduling Discipline," *IEEE J. on Sel. Areas in Comm.*., Vol. 13, No. 6, pp. 1-10, Aug. 1995.

[7] F. Lo Presti, Z.-L. Zhang, and D. Towsley, "Bounds, Approximations and applications for a two-queue GPS system," *Proc. IEEE Infocom '96*, Vol. 3, pp. 1310-1317, 1996.

[8] G. Lapiotis, "Stochastic Analysis of Joint Buffer Management and Service Scheduling in High-Speed Network Nodes," Ph.D. Thesis, Polytechnic University, January 1999.

[9] D. Anick, D. Mitra, and M. M. Sondhi, "Stochastic theory of a data-handling system with multiple sources," *The Bell Syst. Tech. J.*, Vol. 61, No. 8, Oct. 1982.

[10] A. Elwalid, and D. Mitra, "Analysis and design of rate-based congestion control of high-speed networks, I: stochastic flow models, access regulation," *Queueing Systems*, Vol. 9, pp. 29-64, 1993.

[11] A. T. Andersen and B. F. Nielsen, "An application of superpositions of two state Markovian sources to the modeling of self-similar behavior," *Proc. IEEE Infocom '97*, 1997.

[12] Thomas E. Stern and Anwar I. Elwalid, "Analysis of Separable Markov-Modulated Rate Models for Information Handling Systems," *Adv. Appl. Prob.*, pp. 105-139, Vol. 23, 1991.

[13] ITU-T Recommendation P.59, *Artificial Conversational Speech*, International Telecommunications Union, 3/93.