Smooth Electric Power Scheduling in Power Distribution Networks

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***Abstract*—The emergence of Smart Grid (SG) brings about many fundamental changes in electric power systems. In this paper, we study the problem of smooth electric power scheduling in power distribution networks. We introduce an electricity supply/demand model that takes into account the time-varying demands and their deadlines. We formulate a constrained non-linear programming problem and incorporate the theory of majorization to develop algorithms that can compute smoothness optimal schedules for deferrable load dominant system. An effective heuristic algorithm is also presented by extending the majorization-based algorithm for the general scenario with mixed priority loads and deferrable loads. After obtaining the smooth power schedule, a distributed user benefit maximization load control scheme is used to allocate the scheduled power to individual users, while maximizing their level of satisfaction. The simulation results demonstrate the efficacy of the proposed algorithms on smooth electric power scheduling.**

I. INTRODUCTION

The emergence of Smart Grid (SG) brings about many fundamental changes in electric power systems [1]. Various new power electronics and information techniques are greatly advancing the control and management of energy and resources in the power system. For example, solid sate transformers (SST) can respond to signals from a facility or a household to change the voltage and other electric characteristics. On the user side, smart meters and smart facilities empower the pervasive monitoring and controlling at all levels of power usage in response to power supply and market price fluctuations [1]. The two-way flows of electricity and information in SG are instrumental to the control and optimization of energy and resource allocation in the grid to achieve efficient, green and robust energy systems.

Unlike the traditional grid, in which the electricity supply continuously matches user demands, the next generation power distribution system is based on a network structure [2] and is capable of allowing users to control their load in response to the dynamics in the grid. *Demand response* (DR) is a new technique to balance power generation and demand in the grid [3]. The most important target of DR is to reduce the peak demand by scheduling user requests. With the two-way information flow among provider, users and the market, various DR schemes based on real-time pricing and day-ahead load response concepts have been investigated recently [4]–[7]. Most of existing DR schemes aim to maximize the social warfare or minimize the electricity payment under given demand requirements. Although revealing the intrinsic connection between pricing policies and demand response, the problem of smooth electric power scheduling is not explicitly addressed, which is the key issue in DR. It is shown that the simple off-peak pricing scheme may not be effective in mitigating the demand peak problem, because simply shifting the off-peak period may generate a new *rebound* peak [8].

In this paper, we address the challenging problem of *smooth electric power scheduling* in power distribution networks. The network model is shown in Fig. 1. We assume the end-users are equipped with smart meters and are capable of communicating with the distribution substation and the distribution control center (DCC) through a communication network, and receiving commands from the DCC to adjust the user's electric energy consumption level [9]. The DCC schedules electricity supply on daily basis, which is further divided into multiple time slots. The electricity usage requests at each user are classified into two categories: the *priority load* that must be satisfied in every time slot, and the *deferrable load* that should be satisfied before specific deadlines. Users may set the load for each type according to their preference (e.g., lighting, entertainment, laundry, or charging a plug-in hybrid electric vehicle (PHEV)) [3]. The DCC aggregates the demand profiles from the users through the aggregator [10] and smooths the aggregated electric power supply under the priority load and deferrable load deadline constraints.

We formulate the smooth electric power scheduling problem. The objective is to minimize the power variation during a daily period, based on the concept of day-ahead load response. A deterministic electricity supply/demand model is introduced with cumulative electricity demand/supply curves, which characterizes the demand/supply relationship during the day. We find the formulated problem suits well with the *majorization* theory, which concerns with the comparison and ordering of vectors with respect to the distribution of their elements [11]. Majorization has been used in solving optimization problems in the communications and networking area [12]–[14]. In this paper, we present a majorization-based framework to develop two smooth electric power scheduling algorithms with low computational complexity. After the smooth electric power profile for the entire network is obtained, a user benefit maximization load control algorithm will be executed to allocate the total amount of supply to the individual users, while maximizing their satisfaction of electricity usage. The proposed algorithms can achieve the minimum peak power, thus requiring smaller capacity for the generators, transmission lines and transformers to support the same demand. Since

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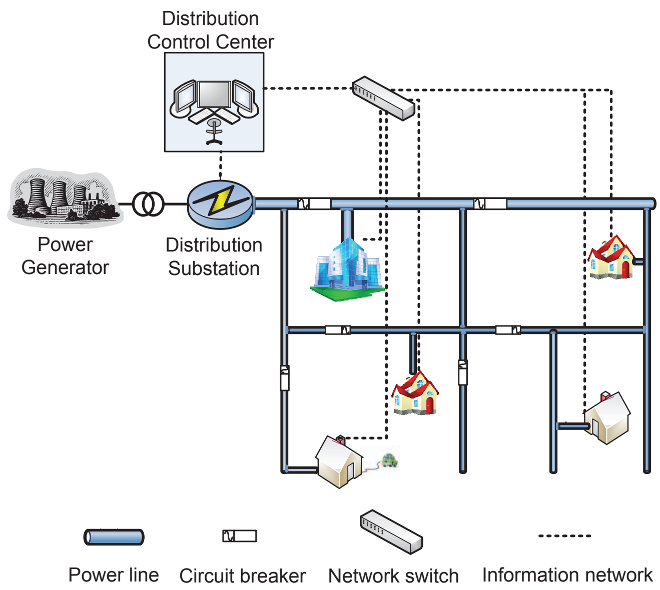


Fig. 1. Illustration of the electricity distribution network.

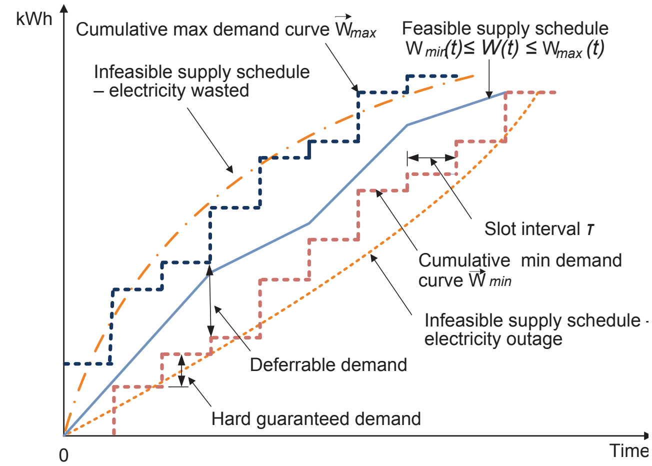


Fig. 2. Cumulative demand and supply curves.

electrical generation and transmission systems are generally designed to accommodate peak electric power [1], the smooth electric power schedule has the potential of optimizing the assets and operation cost of the grid.

The remainder of this paper is organized as follows. We first present the system model and problem statement in Section II. The smooth electric power scheduling algorithms are described in Section III and their performance evaluated in Section IV. Section V concludes this paper.

II. Problem Statement

A. Load Demand Profile

We consider a power distribution network with two-way flows of electricity and information. We assume N users in the power distribution network, which may generate residential, commercial and industrial loads. Let $\mathbb{R} = \{1, 2, \cdots, N\}$ be the set of end users. The electric demand of a user is daily based. Without loss of generality, we assume the one day period is divided into L time slots, each with length τ. Let $p_n(t)$ be the power consumption of user n in time slot t, which is time varying but remains constant within the time slot. Each user n knows its own total daily demand, i.e., $E_n = \sum_{t=1}^{L} p_n(t)\tau$, and wishes to schedule the demand over the one day period [5].

We assume the total demand E_n consists of two parts: the *priority load* and the *deferrable load*. The priority load should be strictly guaranteed in a time slot (e.g., for lighting), while the deferrable load can be served flexibly but with a specific deadline (e.g., charging a household battery or PHEVs). We define $e_{n,p}(t)$ and $e_{n,d}(t)$ as the electric energy for priority load in time slot t, and the deferrable load that must be satisfied by time slot t, respectively. The minimum demand of user n in time slot t, denoted by $e_n^{min}(t)$, is the sum of $e_{n,p}(t)$ and $e_{n,d}(t)$. We finally let $e_n^{max}(t)$ be the maximum possible demand for user n, which is limited by the amount of deferrable loads that have not been satisfied yet, and the capacity of protective relays and switches of the users.

B. Cumulative Demand and Supply Curves

At the beginning of a day, the DCC will aggregate the individual demand profiles received by communicating with the smart meters and smart facilities via the two-way information network [9]. Let the total minimum electricity demand in time slot t be $E_{min}(t) = \sum_{n\in\mathbb{R}} e_n^{min}(t)$, where $E_{min}(L) = \sum_{n\in\mathbb{R}} E_n = \Phi$ (since the daily aggregated demand of all users should finally be satisfied by end of the day). We define the *cumulative minimum demand curve* $\vec{W}_{min}$ as $W_{min}(t) = \sum_{l=1}^{t} E_{min}(l)$, $1 \leq t \leq L$. We define the *cumulative maximum demand curve* $\vec{W}_{max}$ to represent the maximum amount of electricity demand that can be consumed up to t as

$$\begin{aligned} & W_{max}(t) \qquad (1) \\ = \; & \min\{W_{min}(t-1) + \sum_{n\in\mathbb{R}} [e_n^{max}(t) + \Delta e_n(t-1)], \Phi\}, \\ & 1 \leq t \leq L \end{aligned}$$

where $\Delta e_n(t) = e_n^{max}(t) - e_n^{min}(t)$ is the deferrable load that can be served in slot t but with deadlines later than t.

For given demand curves $\vec{W}_{min}$ and $\vec{W}_{max}$, we aim to find a feasible electricity schedule $\vec{W}$, which is the *cumulative supply* of electricity to the users that satisfies constraints $W_{min}(t) \leq W(t) \leq W_{max}(t)$, for all $1 \leq t \leq L$, and $W(L) = \Phi$. The three cumulative curves are illustrated in Fig. 2. Obviously, the three cumulative curves are all nondecreasing over time.

The proposed demand and supply model is quite general. It does not assume any mathematical model for either the supply or the demand. It is more practical than the complex statistical models for supply and demand used in the literature [1]. The cumulative curves represent the demand/supply status in the power distribution network. In each time slot t, $W_{min}(t)$ tracks the priority load and the deferrable load with deadline t, while $W_{max}(t)$ represents an upper bound of the possible consumption by time t. The slope of $W(t)$, denoted by $P(t)$, corresponds to the scheduled electric power. The DCC aims to find an optimal schedule $W(t)$ for every time slot t to achieve a specific control target. A feasible power supply schedule $\vec{P} = [P(1), P(2), \cdots, P(L)]$ ensures that $\vec{W}$ lies in between $\vec{W}_{min}$ and $\vec{W}_{max}$ for all the L time slots, thus preventing both outages events and energy waste.

It can be seen from Fig. 2 that the feasible electric power schedule may not be unique. Among various feasible schedules, we are interested in the one that distributes electricity most smoothly among the L time slots, i.e., the *smoothness optimal schedule*. Once the DCC obtains the smoothness optimal schedule, it can announce the schedule to the smart meters and smart utilities at the users' premises via the communication network, and the users can shape their demand to match the schedule (assuming cooperative users). Therefore, we can achieve smooth electricity generation, transmission and consumption, which is highly preferable for the grid design and operation [1].

C. Smooth Power Scheduling Problem

Based on the demand and supply model, we formulate the smooth power scheduling problem in this section. Let $\bar{P} = \Phi/(L\tau)$ be the average power consumption in the power distribution network through the daily period. The scheduled power for each time slot is $P(t) = W(t)/\tau$. The smoothness optimal schedule minimizes the variations of the supplied power over the entire period, i.e.,

$$\begin{aligned} \text{maximize:} \quad & \mathcal{S}(\vec{P}) \\ \text{subject to:} \quad & W_{min}(t) \le W(t) \le W_{max}(t), \text{ for all } t \\ & W(L) = W_{min}(L) = W_{max}(L) = \Phi \\ & P(t) \ge E_p(t)/\tau, \end{aligned} \tag{2}$$

where $\mathcal{S}(\vec{P})$ is the smoothness of a schedule $\vec{P}$, $E_p(t) = \sum_{n \in \mathbb{R}} e_{n,p}(t)$ is the electric energy required to satisfy the total priority load in time slot t.

Generally, smoothness can be measured by different metrics, such as variance, cumulative absolute difference, etc. Each smoothness measure leads to a different objective function in Problem (2), while the solution to the problem will then depend on the specific form of the objective function. In addition, the smoothness measures are generally nonlinear, making the problem nontrivial to solve. In this paper, we resort to a mathematic theory of majorization [11], which explicitly addresses the unique mathematical notion for the smoothness. Applying majorization theory, we will see that for any arbitrary smoothness objective function in Problem (2) that satisfies the Schur-convex properties [11], the problem can be solved by an universal algorithm in polynomial time. For brevity in the deduction, we use variance in the rest of the paper, while the solution algorithms developed in Section III apply to any objective function that is Schur-convex.

We first consider the case where deferrable load is the dominant component [15], i.e. $E_p(t) \approx 0$. Problem (2) is then reduced to Problem (3).

$$\begin{aligned} \text{minimize:} \quad & \textstyle\sum_{t=1}^{L} \left[P(t) - \bar{P}\right]^2 / L \\ \text{subject to:} \quad & W_{min}(t) \le W(t) \le W_{max}(t), \text{ for all } t \\ & W(L) = \Phi \\ & P(t) \ge 0. \end{aligned} \tag{3}$$

This problem fits well with the *majorization* theory, since the objective function is Schur-convex [16]. Applying majorization theory, we will design a smooth electric power scheduling algorithm for solving Problem (3) in Section III-A. We will then extend the solution for solving Problem (2) in Section III-B.

Proposition 1. *The objective function of Problems (2) and (3) is Schur-convex.*

Proof: The proof is omitted due to lack of space. Interested readers are referred to [16] for details. ■

III. Smooth Electric Power Scheduling

A. SEPS-DL Algorithm

We first develop a smooth electric power scheduling for deferrable load algorithm (SEPS-DL) based on majorization. With Proposition 1, we convert the optimization Problem (3) into an ordering problem of vectors, each representing a feasible schedule. Thus, we solve Problem (3) by finding the most evenly distributed electric power schedule that is feasible for the entire period. Obviously, the most evenly distributed schedule is $\vec{P}^{opt} = [\Phi/(L\tau), \cdots, \Phi/(L\tau)]$, corresponding to having the average power consumption $\bar{P}$ in each time slot. However, due to time varying user demands, $\vec{P}^{opt}$ may not be feasible. In general, each feasible schedule is piece-wise linear with a set of *power changing points*, where the scheduled power increases or decreases to prevent outage events or electric energy waste.

The proposed SEPS-DL algorithm can generate a feasible piece-wise linear schedule, which keeps each piece as long as possible into the future and keeps the power variation as small as possible. The algorithm is illustrated in Fig. 3. Starting from t_{start}, SEPS-DL first computes two probe lines:

- One probe line from t_{start} to the next corner point of $W_{max}(t)$, which can go the furthest into the future without causing outage events or energy waste (e.g., lines P_1P_2 in Case 1 and P_5P_6 in Case 2 of Fig. 3). The power of this probe line is $P_{max}(t) = \frac{W_{max}(t) - W(t_{start})}{t - t_{start}}$.
- The other probe line from t_{start} to the next corner point of $W_{min}(t)$, which can go the furthest into the future without causing outage events or energy waste (e.g., lines P_1P_3 in Case 1 and P_5P_7 in Case 2). The power of this probe line is $P_{min}(t) = \frac{W_{min}(t) - W(t_{start})}{t - t_{start}}$.

All feasible schedules should reside in between the two probe lines in order to go farther. Moreover, when the two probe lines are ended, they must hit *both* on either $W_{max}(t)$ or $W_{min}(t)$. Otherwise, we can always adjust one of the probe lines to make it go even further into the future. For example, see lines P_1P_3

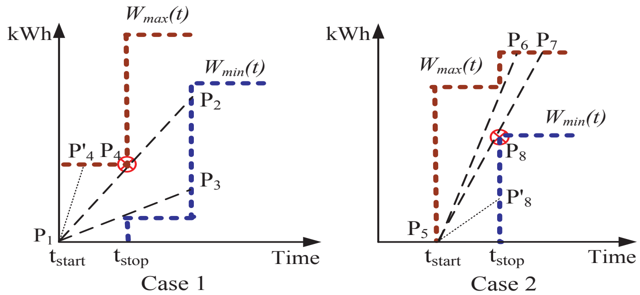


Fig. 3. Illustrate the operation of the SEPS-DL algorithm.

and P_1P_4' in Case 1 of Fig. 3. We can use line P_1P_2 (which goes farther into the future) to replace line P_1P_4', and both probe lines hit $W_{min}(t)$ eventually. In Case 2 in the figure, both probe lines P_5P_6 and P_5P_7 hit $W_{max}(t)$.

If both probe lines hit $W_{min}(t)$ (i.e., Case 1 in Fig. 3), any feasible schedule for this interval will also hit $W_{min}(t)$, since it must lie in between the two probe lines. We then trace back the upper probe line (i.e., line P_1P_2) to find the latest time when the schedule just satisfies the maximum demand (i.e., point P_4 at time t_{stop}). Then segment P_1P_4 will be chosen as the schedule for the interval $[t_{start}, t_{stop})$, with power $\frac{W_{max}(t_{stop})-W(t_{start})}{t_{stop}-t_{start}}$.

If both probe lines hit $W_{max}(t)$ (i.e., Case 2 in Fig. 3), any feasible schedule for this interval will also hit $W_{max}(t)$. We then trace back the lower probe line (i.e., line P_5P_7) to find the latest time when the schedule just satisfies the minimum demand (i.e., point P_8 at time t_{stop}). Then segment P_5P_8 will be chosen as the schedule for the interval $[t_{start}, t_{stop})$, with power $\frac{W_{min}(t_{stop})-W(t_{start})}{t_{stop}-t_{start}}$.

After the schedule for $[t_{start}, t_{stop})$ is determined, we set $t_{start} = t_{stop}$ and repeat the above procedure to find the schedule for the next time interval.

The proposed SEPS-DL algorithm is very easy to implement, with a complexity of $\mathcal{O}(L^2)$. It can be shown that SEPS-DL is smoothness optimal, with a unique optimal power schedule. It also achieves the smallest peak power. The proofs are omitted due to lack of space. Interested readers are referred to [16] for details.

B. Extension to the General Case

We next extend SEPS-DL to solve the general case Problem (2). With the priority load, a feasible power schedule should satisfy $P(t) \geq E_p(t)/\tau$ in every time slot. The presence of priority load enforces new constraints on the feasibility of the schedules. For example, consider the aggregated cumulative priority load curves and deferrable load curves shown in Fig. 4(a) and Fig. 4(b). According to the definition of feasible power supply schedule in SEPS-DL, both segments 1 and 2 in Fig. 4(c) are feasible. However, it can be seen that segment 1 actually cannot provide enough power to satisfy the priority load in time slot t_2, since its slope is smaller than the required slope (i.e., that of segment 0 in Fig. 4(a)).

To solve this problem, we develop the general smooth electric power scheduling algorithm (GSEPS), which is based on SEPS-DL. Specifically, SEPS-DL assumes no priority load. During the execution, the generated power segment is compared with the priority load. If the SEPS-DL generated power segment $P^*(t)$ is less than the priority load in time slot t, GSEPS will increase $P^*(t)$ to the priority load (e.g., see segment 3 in Fig. 4). Then SEPS-DL will continue to compute further segments of the schedule by setting the new starting point to t, until the entire schedule is computed.

C. Power Allocation Among Individual Users

After the smooth electric power schedule is obtained, the DCC announces the schedule to all the users and requests them to control their loads to match the supplied electric energy $W^*(t) = P^*(t) \times \tau$ in each time slot t. To divide the total supply $W^*(t)$ among the N users, we assume a *benefit function* $U_n(p_n(t))$ for each user n, which is a nondecreasing concave function [6] and represents the level of satisfaction of the user when receiving $p_n(t)$ in time slot t. We then develop an algorithm that maximizes the sum of the benefit functions of all users in the power distribution network. The maximization of the total benefit of the users under the smooth schedule constraint can be formulated for each time slot t as follows:

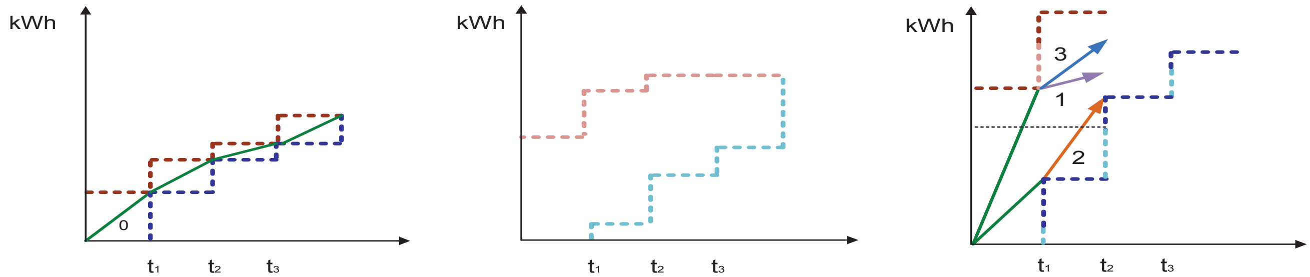
$$\begin{aligned} \text{maximize:} \quad & \textstyle\sum_{n\in\mathbb{R}} U_n(p_n(t)) \qquad\qquad (4) \\ \text{subject to:} \quad & p_n^{min}(t) \leq p_n(t) \leq p_n^{max}(t), \text{ for all } n \\ & \textstyle\sum_{n\in\mathbb{R}} p_n(t) = W^*(t)/\tau. \end{aligned}$$

Problem (4) is a convex optimization problem, which can be solved effectively with a convex optimization solver. To solve Problem (4), we develop a distributed user benefit maximization load control algorithm (DUBMLC) based on *dual decomposition* [16], which is omitted due to lack of space.

IV. Simulation Study

In this section, we evaluate the proposed algorithms by simulating an electric power distribution network with 250 independent users. We assume a daily period slotted into $L = 144$ time slots (i.e., $\tau = 10$ min). The demand for each user during the period is randomly distributed from 35 kWh to 50 kWh. The DCC aggregates the load profiles and generates the cumulative supply/demand curves at the beginning of the period. We adopt a benefit function $U_n(t) = k_1 q_n(t) - \frac{1}{2}k_2 q_n(t)^2$ [17], where $q_n(t) \in [0, 1]$ is the normalized value of power supply $p_n(t)$. With this $U_n(\cdot)$, Problem (4) becomes a quadratic programming problem, which can be effectively solved with the proposed distributed algorithm. Without loss of generality, we set $k_1 = k_2 = 1$ in the simulations.

We first examine the performance of SEPS-DL and GSEPS. For SEPS-DL, all the electric energy demand is deferrable. For GSEPS, we assume 50% of the demand is deferrable and the deadlines are randomly distributed during the daily period. The cummulative demand curves and the computed schedules are plotted in Fig. 5. We find that both electric power schedules lies in between $\vec{W}_{min}$ and $\vec{W}_{max}$, meaning they are feasible and satisfy the demand of the users in the entire period. In some time slots, e.g. slots 70 to 80 and 110 to 120, GSEPS



(a) Cumulative priority load curves (b) Cumulative deferrable load curves (c) Aggregated cumulative load curves

Fig. 4. Smooth power scheduling with priority load. Note that although not deferrable, the cumulative priority load can also be represented with two curves as shown in (a), where the maximum and minimum curves meet at the corners.

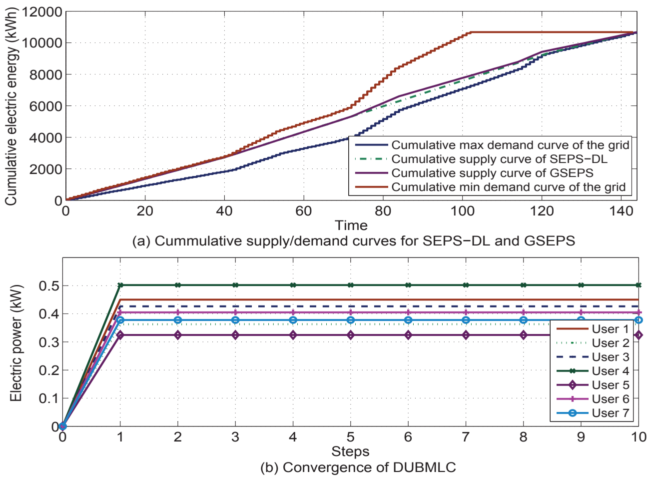


Fig. 5. Comparison of the SEPS-DL, GSEPS and DUMLC schedules.

requests a larger electric power than SEPS-DL. This is due to the hard requirement for the priority load, which temporally forces GSEPS to increase the electric power supply.

After the smooth power schedule is obtained, DUBMLC is executed to divide the power to individual users in each time slot. For better illustration, we only plot the power convergence curves for seven users in Fig. 5. The other users have similar behavior. We observed that all the powers converge to the optimal allocations very quickly, and after one step there is no significant variation in the individual user powers.

V. Conclusions

In this paper, we addressed the problem of smooth electric power scheduling in a power distribution network. We introduced a deterministic model to characterize the complex relationship between demand and supply. A constrained nonlinear optimization problem is formulated aiming to minimize the electric power variation and satisfy user power usage quality. We developed majorization-based algorithms for deriving smooth schedules, and a distributed algorithm for allocating the power supply among the users. The performance of the proposed schemes is validated with simulations.

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