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# Analytic study of apparent resist sensitivity in electron-beam lithography

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## ABSTRACT

The nominal definition of resist sensitivity, i.e., the dose required to clear the resist, carries an implicit assumption that the sensitivity is measured for a sufficiently large feature in electron-beam lithography. However, when the feature size is comparable to or less than the backscattering range of electrons, the actual sensitivity to clear the resist (apparent sensitivity) may be different from the nominal sensitivity. The difference becomes much larger as the feature size approaches the forward-scattering range. In this study, a closed-form mathematical expression of the apparent sensitivity is derived as a function of lithographic and pattern parameters. With the expression, one can analyze the dependency of the apparent sensitivity on each parameter and estimate the actual sensitivity for a specific case given the nominal sensitivity without time-consuming simulation or an expensive experiment. Hence, the analytic result in this paper must provide a useful tool to understand and deal with the resist sensitivity.





## I. INTRODUCTION

For clarity of presentation, the two terms, *dose* and *exposure*, are defined first. The dose refers to the energy (or amount of charge) given to a point on the surface of resist and the exposure the energy deposited at a point in the resist.[1] What determines if a point in the resist is developed is the exposure level at the point.

Electron-beam (e-beam) lithography is widely used in writing a pattern of fine features on a substrate.[2–7] It is essential to use a proper dose such that the CD (critical dimension) error in the written pattern is minimized. One of the important metrics in e-beam lithography is *resist sensitivity*,[8–10] which is defined as the dose required for the resist to be fully developed, given a developing process (developer, developing time, etc.). An implicit assumption for this definition is that a certain setup of an e-beam system is employed and a sufficiently large area of resist is exposed by the e-beam such that the entire range of electron scattering is covered. Without the assumption, such a dose may not be uniquely determined since the setup and feature size affect the spatial distribution of exposure ("exposure distribution" hereafter). The main objective of this study is to provide an analytic tool for understanding the behavior of resist sensitivity in general.

When a feature much larger than the electron scattering range is exposed with a uniform dose, the exposure level within the feature except corner and edge regions is the maximum possible exposure that can be received from the dose. Let *nominal sensitivity* (of resist) refer to the dose resulting in the (maximum possible) exposure level required for the full development of resist. The nominal sensitivity can be uniquely determined given a lithographic/developing process. However, as the feature size decreases (or the feature density in a pattern decreases) and/or the e-beam becomes broader, the actual dose, referred to as *apparent sensitivity*, required for the full development may increase beyond the nominal sensitivity due to the proximity effect caused by electron scattering.

The exposure distribution in the resist when a point is exposed by the e-beam is often described by the point spread function (PSF). In this analytic study, the PSF is modeled with two Gaussian functions, which represent the exposure distributions from the forward-scattering and backscattering of electrons. The double-Gaussian model includes three parameters, i.e., the forward-scattering range, the backscattering range, and the ratio of the backscattered energy to the forward-scattered energy. The three parameters (referred to as lithographic parameters) together depict the overall shape of PSF, including its sharpness, and therefore, determine the exposure distribution when a feature or pattern is exposed. In the analytic model, a single feature of rectangle is considered where its length is much larger than the electron scattering range, and hence, only its width affects the exposure distribution.

Closed-form mathematical expressions of the nominal and apparent sensitivities are derived as functions of the lithographic and pattern parameters where the pattern parameters refer to the feature width and its possible reduction for the proximity effect correction.[11,12] The closed-form expressions allow one to analyze the dependency of resist sensitivity, especially the apparent sensitivity, on the parameters and estimate the minimum required dose for the full development of resist given a lithographic setup and a feature, without simulation or experiment. In this paper, the steps of deriving the expressions are described in detail, and the plots of the apparent sensitivity obtained using the expressions for typical cases are also included with extensive discussion.

To the author's knowledge, no closed-form mathematical expression of apparent resist sensitivity was derived in the past. The main contribution of this study is the derivation of such expression explicitly in terms of lithographic and pattern parameters. This enables a new method to study and deal with resist sensitivity, which does not require costly experiment or time-consuming simulation. Also, the detailed analysis of the example cases provides new insights into the relationship between the apparent resist sensitivity and each of the parameters. In addition, the outcomes from this study may be referred to in determining a proper dose for the proximity effect correction.

The rest of the paper is organized as follows. The model for the analytic study is depicted in Sec. II. The analytic expressions of the nominal and apparent sensitivities are derived in Secs. III and IV, respectively. Examples of analyzing the dependency of apparent sensitivity on the lithographic and pattern parameters are provided in Sec. V. The results are summarized along with their applicability and significance in Sec. VI.

## II. ANALYTIC MODEL

What dictates the development of resist is the exposure, which depends on, but may not be equal to, the dose due to the proximity effect. Therefore, for the analytic study of resist sensitivity, a closed-form mathematical expression of the exposure distribution needs to be derived first. The exposure distribution can be obtained through a convolution between the dose distribution of a pattern and a PSF. The double-Gaussian model of PSF is employed, and the exposure is assumed to be invariant in the resist-depth dimension ($Z$). Then, the PSF can be expressed as in Eq. (1),

$$ps(x, y) = \frac{1}{\pi(1+\eta)}\left(\frac{1}{\alpha^2}e^{\frac{-(x^2+y^2)}{\alpha^2}} + \frac{\eta}{\beta^2}e^{\frac{-(x^2+y^2)}{\beta^2}}\right), \tag{1}$$

where $\alpha$, $\beta$, and $\eta$ are the forward-scattering and backscattering ranges, and the ratio of the backscattered energy to the forward-scattered energy, respectively.

In this study, a single-feature pattern is considered where the feature is a long rectangle as shown in Fig. 1. The rectangle has its center at the coordinate origin and is vertically long enough that its length is much larger than the electron scattering ranges, i.e., $L \gg \beta$ (which implies that $L \gg \alpha$ since $\beta > \alpha$). When the feature is exposed with a uniform dose, the exposure varies only in the horizontal dimension ($X$) except in the regions close to the top and bottom ends (of the feature). Then, the exposure distribution across the middle ($y = 0$) of the feature can be expressed as a function of $x$ only;[13] i.e., $e(x)$, as in Eq. (2),

$$e(x) = D\int_{\frac{-W}{2}}^{\frac{W}{2}}\int_{-\infty}^{\infty} ps(x-x', y')dy'dx' = D\int_{\frac{-W}{2}}^{\frac{W}{2}} h(x-x')dx', \tag{2}$$

where $D$ is the uniform dose and $h(x)$ is the line spread function (LSF).

The LSF, which describes the exposure distribution when a (infinitely) long line (the $Y$ axis in the case of the coordinate setup in Fig. 1) is exposed with a uniform dose of 1, can be obtained by carrying out the inner integration of $ps(x, y)$ with respect to $y$ in Eq. (2),[14]

$$h(x) = \frac{1}{\sqrt{\pi}(1+\eta)}\left(\frac{1}{\alpha}e^{\frac{-x^2}{\alpha^2}} + \frac{\eta}{\beta}e^{\frac{-x^2}{\beta^2}}\right). \tag{3}$$

From Eqs. (2) and (3), $e(x)$ can be derived as follows:

$$\begin{aligned} e(x) &= D\int_{\frac{-W}{2}}^{\frac{W}{2}} h(x-x')dx' \\ &= \frac{D}{\sqrt{\pi}(1+\eta)}\int_{\frac{-W}{2}}^{\frac{W}{2}}\left(\frac{1}{\alpha}e^{\frac{-(x-x')^2}{\alpha^2}} + \frac{\eta}{\beta}e^{\frac{-(x-x')^2}{\beta^2}}\right)dx' \\ &= \frac{D}{2(1+\eta)}\left[\mathrm{erf}\left(\frac{x+\frac{W}{2}}{\alpha}\right) - \mathrm{erf}\left(\frac{x-\frac{W}{2}}{\alpha}\right)\right. \\ &\quad \left. + \eta\left\{\mathrm{erf}\left(\frac{x+\frac{W}{2}}{\beta}\right) - \mathrm{erf}\left(\frac{x-\frac{W}{2}}{\beta}\right)\right\}\right], \end{aligned} \tag{4}$$

where erf( ) is the error function.

Equation (4) shows the relationship between the dose and exposure in terms of the lithographic and pattern parameters. From the basic properties of the erf( ) function, one can understand that $e(x) \le D$ since what is in the brackets in Eq. (4) is not greater than $2(1+\eta)$ as will be shown later; that is, the energy deposited at a point is at most the e-beam energy given to the point, and the former is usually less than the latter due to the spatial spread of exposure (i.e., the proximity effect) caused by electron scattering. This is the fundamental reason why there can be a significant difference between the nominal and apparent sensitivities.

## III. NOMINAL SENSITIVITY

Consider a long feature of width $W$ as shown in Fig. 1, which is exposed with a uniform dose $D$. When the feature is large, i.e., $W \gg \alpha, \beta$, all of the four erf terms in Eq. (4) are nearly constant in the regions where $|x|$ is not close to $\frac{W}{2}$; that is, the exposure distribution within (and outside) the feature is flat (uniform) except in edge regions as illustrated in Fig. 2. That uniform exposure level, denoted by $E_m$, is the maximum achievable exposure given $D$. In the edge regions, the exposure level decreases rapidly toward 0 as $|x|$ increases (moving from the inside to the outside of the feature). Outside the feature (where $|x|$ is substantially greater than $\frac{W}{2}$), the

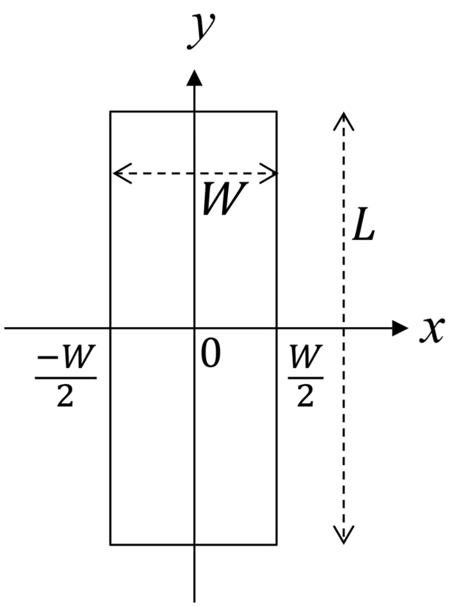


**FIG. 1.** Rectangular feature with width $W$ and length $L$, which is long along the $Y$ dimension. The $Y$ axis is aligned such that the horizontal center of feature is at $x = 0$ and the left and right edges are at $x = \frac{-W}{2}$ and $x = \frac{W}{2}$, respectively.

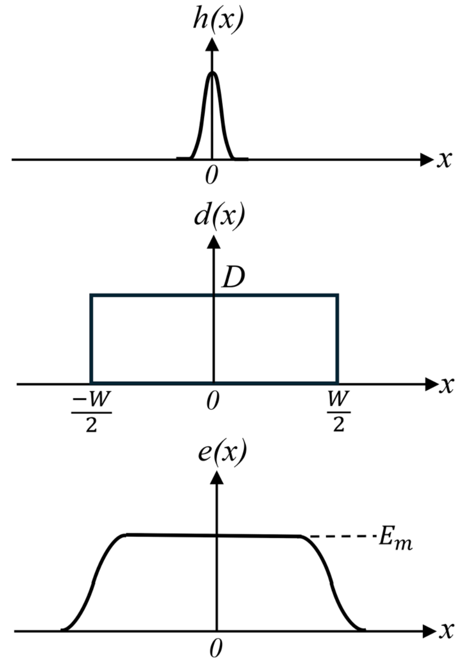


**FIG. 2.** Line spread function $h(x)$, dose distribution $d(x)$, and exposure distribution $e(x)$ for a rectangular feature with width $W$ where $W \gg \alpha, \beta$.

first and third erf terms are almost equal to the second and fourth erf terms, respectively, and hence, the exposure level is practically 0. Let $E_0$ denote the minimum exposure level required to develop the resist fully. Then, the nominal sensitivity, denoted by $D_n$, refers to the (uniform) dose to achieve the exposure level of $E_0$ within a large feature ($W \gg \alpha, \beta$). For convenience, the center point ($x = 0$) within the feature is considered in analyzing the resist sensitivity.

From Eq. (4), noting that $\mathrm{erf}(-s) = -\mathrm{erf}(s)$, the exposure at the center of feature is obtained as follows:

$$e(0) = \frac{D}{2(1+\eta)}\left[\mathrm{erf}\left(\frac{W}{2\alpha}\right) - \mathrm{erf}\left(\frac{-W}{2\alpha}\right) + \eta\left\{\mathrm{erf}\left(\frac{W}{2\beta}\right) - \mathrm{erf}\left(\frac{-W}{2\beta}\right)\right\}\right]$$
$$= \frac{D}{(1+\eta)}\left[\mathrm{erf}\left(\frac{W}{2\alpha}\right) + \eta\,\mathrm{erf}\left(\frac{W}{2\beta}\right)\right]. \quad (5)$$

When $W \gg \alpha, \beta$, $\mathrm{erf}\left(\frac{W}{2\alpha}\right) \approx 1$ and $\mathrm{erf}\left(\frac{W}{2\beta}\right) \approx 1$. Hence,

$$e(0) = \frac{D}{1+\eta}(1+\eta) = D = E_m. \quad (6)$$

For the full development of resist, the exposure level within the feature, $e(0)$, i.e., $E_m$, needs to be (at least) $E_0$, which must be achieved when the dose is $D_n$. Therefore, from Eq. (6),

$$D_n = E_0. \quad (7)$$

$D_n$ is the resist sensitivity often referred to with the implicit assumption that the feature is sufficiently large. However, in reality, a feature may not be large enough that the dose required for the feature to be fully developed deviates from $D_n$. The deviation, which stems from electron scattering, depends on the lithographic and pattern parameters and is analyzed quantitatively in Sec. IV.

## IV. APPARENT SENSITIVITY

In practice, the condition that $W \gg \alpha, \beta$ may not be met, e.g., $W$ is comparable to $\alpha$ or $\beta$. Also, for the proximity effect correction, the width of feature to be exposed may be reduced. Let $\Delta W$ denote the width reduction on each side of the feature; i.e., the exposed width is $W - 2\Delta W$, while the target width is still $W$ (refer to Fig. 3). Then, the exposure distribution $e(x)$ can be derived from Eq. (2) and (3) as follows:

$$e(x) = D\int_{-\left(\frac{W}{2}-\Delta W\right)}^{\frac{W}{2}-\Delta W} h(x - x')dx'$$
$$= \frac{D}{2(1+\eta)}\left[\mathrm{erf}\left(\frac{x + \left(\frac{W}{2} - \Delta W\right)}{\alpha}\right) - \mathrm{erf}\left(\frac{x - \left(\frac{W}{2} - \Delta W\right)}{\alpha}\right)\right.$$
$$\left. + \eta\left\{\mathrm{erf}\left(\frac{x + \left(\frac{W}{2} - \Delta W\right)}{\beta}\right) - \mathrm{erf}\left(\frac{x - \left(\frac{W}{2} - \Delta W\right)}{\beta}\right)\right\}\right]. \quad (8)$$

When $W$ is not much greater than $\alpha$ and $\beta$, the four erf terms in Eq. (8) vary with $x$ within the feature in general. Therefore, the exposure distribution within the feature may not be flat even in the center region depending on $\alpha, \beta, W$, and $\Delta W$ as illustrated in Fig. 3. In such a case, the exposure level is highest at the center; i.e., $e(0)$ is the maximum. Also, $e(0)$ can be lower than $E_m$. Then, the dose required for the full development of resist would be higher than the nominal sensitivity, which is the apparent sensitivity denoted by $D_a$.

Referring to Eq. (8), it is not difficult to see from the requirement of $e(0) = E_0$ that the apparent sensitivity $D_a$ can be expressed as follows:

$$D_a = \frac{(1+\eta)E_0}{\mathrm{erf}\left(\frac{\frac{W}{2}-\Delta W}{\alpha}\right) + \eta\,\mathrm{erf}\left(\frac{\frac{W}{2}-\Delta W}{\beta}\right)}. \quad (9)$$

Since $0 < \mathrm{erf}(s) < 1$ for $s > 0$, the denominator of Eq. (9) is less than $1 + \eta$ (unless $W \gg \alpha, \beta$). Therefore, $D_a > E_0 = D_n$; that is, the resist sensitivity measured for a feature, i.e., the apparent sensitivity, is higher than the nominal sensitivity. Only when the feature size ($W$) becomes much larger than the scattering ranges ($\alpha, \beta$) or, equivalently, the PSF becomes much sharper (narrower) than the feature, the apparent sensitivity approaches the nominal sensitivity from above. Hence, in general, the apparent sensitivity can be significantly different (higher) than the nominal sensitivity for a feature.

The closed-form expression in Eq. (9) enables an efficient way of evaluating the apparent sensitivity given the lithographic and pattern parameters and understanding the dependency of the apparent sensitivity on each parameter without simulation or experiment. Hence, it provides a useful tool to the users of e-beam lithography.

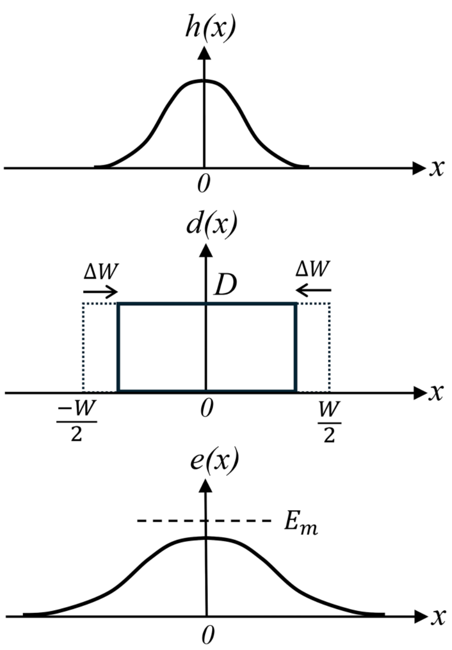


**FIG. 3.** Line spread function $h(x)$, dose distribution $d(x)$, and exposure distribution $e(x)$ for a rectangular feature with a width reduction of $\Delta W$ where $W$ is comparable to $\alpha$.

## V. ANALYSIS

With the analytic results in this study, one can understand the behavior of (apparent) resist sensitivity and estimate the apparent sensitivity given the nominal sensitivity for various cases. In this section, the dependency of the apparent sensitivity on each of the lithographic and pattern parameters ($\alpha, \beta, \eta, W$, and $\Delta W$) is examined in relation to the nominal sensitivity where the ratio of $D_a$ to $D_n$, referred to as the *relative* apparent sensitivity, is employed. Note that $\frac{D_a}{D_n} = \frac{D_a}{E_0}$.

In Fig. 4, the dependency of the relative apparent sensitivity on the forward-scattering range ($\alpha$) is considered for two different sizes of features with $\beta = 500$ nm, $\eta = 0.8$, and $\Delta W = 0$ nm. When the feature size (width) is much larger ($W = 2000$ nm) than both scattering ranges ($\alpha$ and $\beta$), the relative apparent sensitivity remains very close to 1; i.e., the apparent sensitivity is almost the same as the nominal sensitivity as expected. Note that for the range of $\alpha$ considered, the exposure level within the feature stays at $E_m$. In the case of a small feature ($W = 20$ nm), the relative apparent sensitivity is greater than 1 since the exposure level within the feature is lower than $E_m$ (the exposure contributions to each point from both types of scattering are only partial). When $\alpha$ is much smaller than $W$ ($\alpha < 5$ nm), most points of the feature receive a full contribution of exposure from the forward scattering, and therefore, the relative apparent sensitivity stays unchanged. As $\alpha$ increases beyond this range, it becomes more comparable to $W$ resulting in a smaller contribution of exposure from the forward scattering. Hence, the dose needs to be increased to achieve $E_0$; i.e., the apparent sensitivity increases. In other words, the actual resist



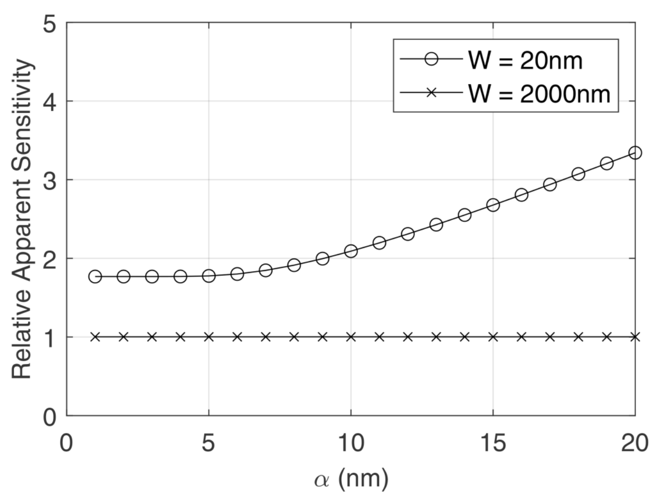


**FIG. 4.** Relative apparent sensitivity ($\frac{D_a}{D_n}$) as a function of the forward-scattering range (1 nm $\le \alpha \le$ 20 nm) : $\beta = 500$ nm, $\eta = 0.8$, and $\Delta W = 0$ nm.

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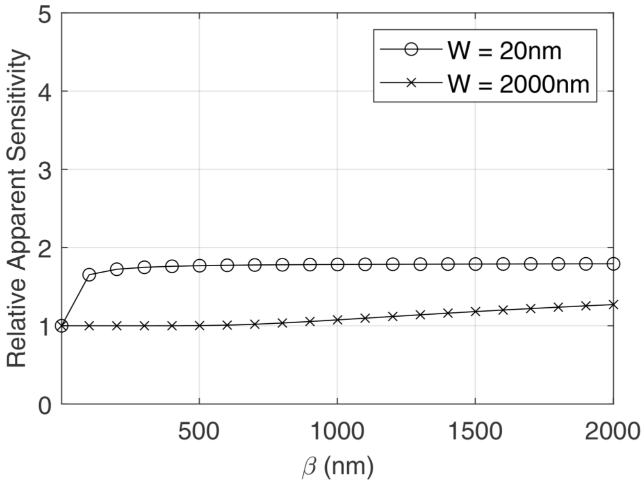


**FIG. 5.** Relative apparent sensitivity ($\frac{D_a}{D_n}$) as a function of the backscattering range ($1\,\text{nm} \le \beta \le 2000\,\text{nm}$): $\alpha = 1\,\text{nm}$, $\eta = 0.8$, and $\Delta W = 0\,\text{nm}$.

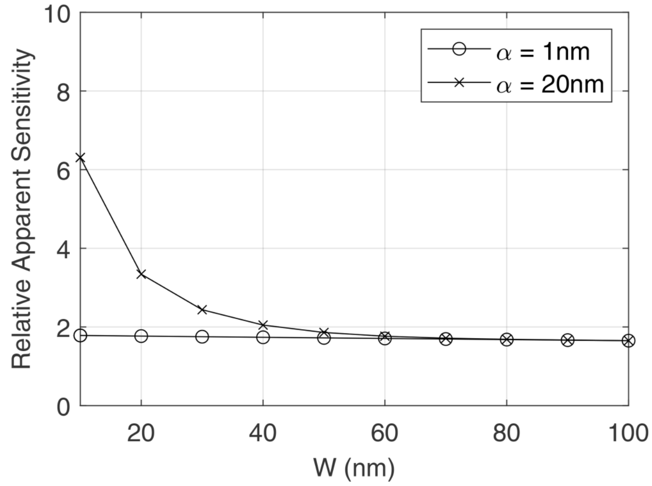


**FIG. 7.** Relative apparent sensitivity ($\frac{D_a}{D_n}$) as a function of the feature width ($10\,\text{nm} \le W \le 100\,\text{nm}$): $\beta = 500\,\text{nm}$, $\eta = 0.8$, and $\Delta W = 0\,\text{nm}$.

sensitivity can be significantly higher than the nominal sensitivity for a broad beam and/or a small feature.

In Fig. 5, the dependency of the relative apparent sensitivity on the backscattering range $\beta$ is plotted for two different feature sizes with $\alpha = 1\,\text{nm}$, $\eta = 0.8$, and $\Delta W = 0\,\text{nm}$. In the case of $W = 2000\,\text{nm}$, the condition is that $W \gg \alpha, \beta$ is satisfied when $\beta$ is well below 1000 nm, and therefore, there is no visible difference between the apparent and nominal sensitivities in the plot, i.e., the relative apparent sensitivity of 1. However, as $\beta$ increases becoming comparable to $W$, the exposure contribution to the points within the

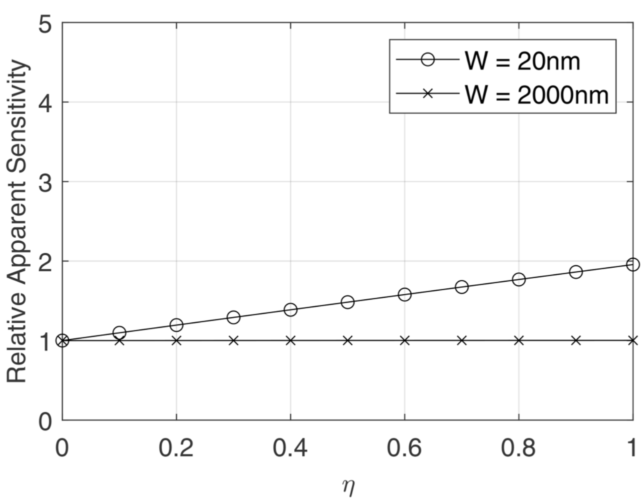


**FIG. 6.** Relative apparent sensitivity ($\frac{D_a}{D_n}$) as a function of the ratio of the backscattered energy to the forward-scattered energy ($0 \le \eta \le 1$): $\alpha = 1\,\text{nm}$, $\beta = 500\,\text{nm}$, and $\Delta W = 0\,\text{nm}$.

feature from backscattering decreases gradually such that the relative apparent sensitivity slowly increases. When $W = 20\,\text{nm}$, $\beta$ becomes comparable to $W$ quickly as it increases. Therefore, the relative apparent sensitivity increases fast early on and then levels out to be about 1.8 beyond $\beta = 100\,\text{nm}$. When $\beta$ is much larger than $W$, the second term in the denominator of Eq. (9) is nearly 0, but the first term is 1 since $W \gg \alpha$. Hence, the relative apparent sensitivity is $1 + \eta = 1.8$. The effect of $\beta$ on the (relative) apparent sensitivity would be less for a smaller $\eta$.

In Fig. 6, the dependency of the relative apparent sensitivity on $\eta$, the ratio of the backscattered energy to the forward-scattered energy, is examined for two different feature sizes with $\alpha = 1\,\text{nm}$, $\beta = 500\,\text{nm}$, and $\Delta W = 0\,\text{nm}$. For $W = 2000\,\text{nm}$, $W \gg \alpha, \beta$ independent of $\eta$, and therefore, the relative apparent sensitivity stays at 1 for the whole range of $\eta$. Another way to understand this is to see that the denominator in Eq. (9) becomes $1 + \eta$ since both erf terms are 1 in this case. Then, the relative apparent sensitivity remains to be 1 while $\eta$ is varied. For $W = 20\,\text{nm}$, the forward scattering of electrons still fully contributes to the exposure at the center of feature [the first term in the denominator of Eq. (9) is nearly 1] since $\alpha \ll W$, but the backscattering contribution is very low since $\beta \gg W$ (the second term is practically 0). Therefore, the relative apparent sensitivity can be expressed as $1 + \eta$, i.e., increases linearly with $\eta$ as seen in the figure.

In Fig. 7, the dependency of the relative apparent sensitivity is plotted as a function of the feature size ($W$) for two different cases of $\alpha$ with $\beta = 500\,\text{nm}$, $\eta = 0.8$, and $\Delta W = 0\,\text{nm}$. In general, for a smaller feature (in isolation), the exposure level within the feature is lower since more electron energy is deposited outside the feature due to electron scattering. This becomes more visible when the feature size is close to $\alpha$. Hence, a smaller feature requires a higher dose to be fully developed, i.e., a higher apparent sensitivity. As $W$ increases, it is expected that the apparent sensitivity becomes closer to the nominal sensitivity. In the figure, this can be observed in the

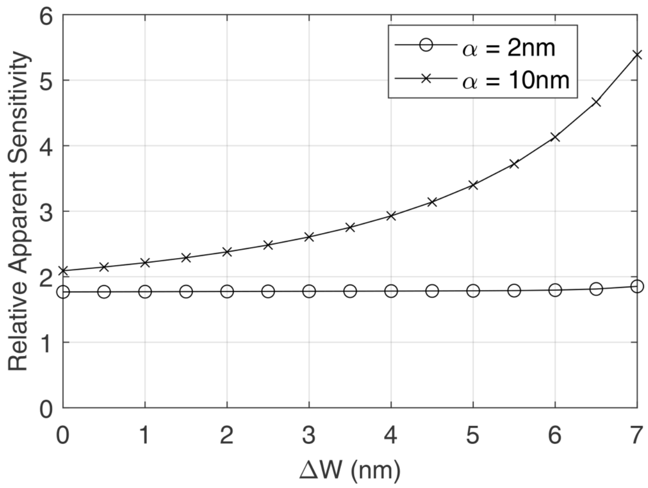


**FIG. 8.** Relative apparent sensitivity ($\frac{D_a}{D_n}$) as a function of the feature-width reduction ($0 \leq \Delta W \leq 7$ nm): $\beta = 500$ nm, $\eta = 0.8$, and $W = 20$ nm.

case of $\alpha = 20$ nm. Also, it is seen that the apparent sensitivity can be much larger than the nominal sensitivity for a small feature. When $\alpha = 1$ nm, most points receive a full contribution of exposure from the forward scattering of electrons since $W \gg \alpha$ for the entire range of $W$ considered. However, the exposure contribution from the backscattering is negligible since $W \ll \beta$. Therefore, the denominator in Eq. (9) is 1, and, accordingly, the relative apparent sensitivity remains constant at 1.8, which is $1 + \eta$.

In Fig. 8, the dependency of the relative apparent sensitivity is observed as a function of the feature-width reduction ($\Delta W$) for two different cases of $\alpha$ with $\beta = 500$ nm, $\eta = 0.8$, and $W = 20$ nm. The behaviors of the relative apparent sensitivity in this graph can be easily understood by noting that an increase in $\Delta W$ is equivalent to a decrease in $W$. As $\Delta W$ increases, the exposed width ($W - 2\Delta W$) of the feature decreases. In the case of $\alpha = 2$ nm, the effective (exposed) width remains to be much larger than $\alpha$ for the whole range of $\Delta W$ considered. Hence, the relative apparent sensitivity remains constant close to $1 + \eta$. On the other hand, when $\alpha = 10$ nm, the effective width gets closer to $\alpha$, and therefore, the relative apparent sensitivity increases with the increasing rate becoming larger as $\Delta W$ increases.

## VI. SUMMARY

In e-beam lithography, resist sensitivity is an important metric often referred to in practice, which is typically defined as the dose required for the full development of resist. An implicit assumption behind this definition is that the sensitivity is measured for a sufficiently large feature. However, the actual sensitivity (apparent sensitivity) for a feature can be significantly different from the nominal sensitivity. This difference fundamentally stems from the proximity effect; i.e., the maximum possible exposure level may not be always achieved. In order to enable a quantitative analysis of resist sensitivity, a closed-form mathematical expression of the apparent sensitivity has been derived with the double-Gaussian model of LSF in this study. The expression allows one to analyze the dependency of the apparent sensitivity on the lithographic and pattern parameters without any simulation or experiment. Also, given the nominal sensitivity of a resist, one can estimate the actual sensitivity of the resist for a feature and a lithographic setup. This can be helpful in determining a proper dose for the proximity effect correction. While five specific cases are considered in illustrating the usefulness of the closed-form expression, one can easily carry out a similar analysis for any other case. Therefore, this paper must be a useful reference for the community of e-beam lithography.

It is worthwhile to point out that the derivation of the closed-form expression is based on the assumed analytic model, i.e., the double-Gaussian PSF and a single feature of long line. Therefore, its applicability may depend on the deviation from the model.

If the actual PSF deviates significantly from the double-Gaussian model, the closed-form expression can introduce a non-negligible error and, therefore, would not be applicable. However, as long as the PSF can be analytically modeled and is integrable, a closed-form formula may be obtained through the same derivation procedure.

The closed-form expression is derived for a single feature of long line. However, the result can be extended without much difficulty for a uniform pattern of multiple features, such as an L/S (line/space) pattern. This only requires a simple integration of exposure contributions from other features in the pattern. On the other hand, the expression is not applicable to nonuniform patterns directly or through a simple adjustment. Nevertheless, it would be interesting to see if the concept of feature density can be exploited in extending the applicability of the expression for nonuniform patterns.

The apparent sensitivity is defined at the center of feature, i.e., the dose required for the resist to be fully developed at least at the center. Since the exposure is highest at the center (assuming a uniform dose), it is a conservative definition. If it is necessary or desirable to require more than just the center point to be fully developed, the adjusted expression of the apparent sensitivity can be readily obtained by selecting a reference point away from the center of feature.



## AUTHOR DECLARATIONS

### Conflict of Interest

The author has no conflicts to disclose.

### Author Contributions

**Soo-Young Lee:** Conceptualization (lead); Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (lead); Visualization (lead); Writing – original draft (lead).

## DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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