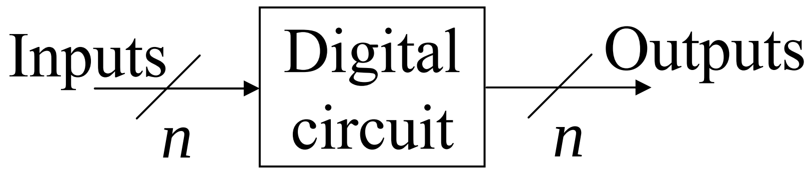
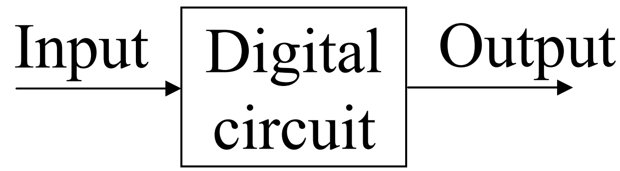
Binary Data Transmission

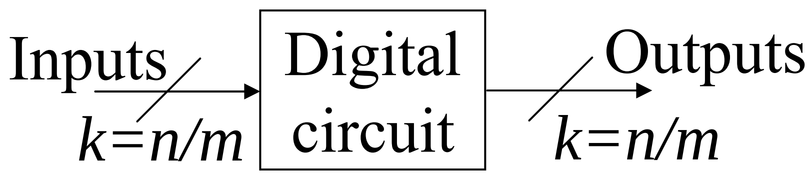
- Data transmission for *n*-bit data words
  - › Parallel
    - all bits at once
    - 1 time step to get all data



  - › Serial
    - one bit at a time
    - *n* time steps to get all data

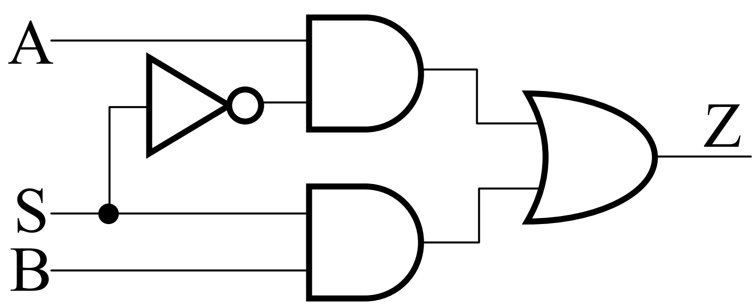


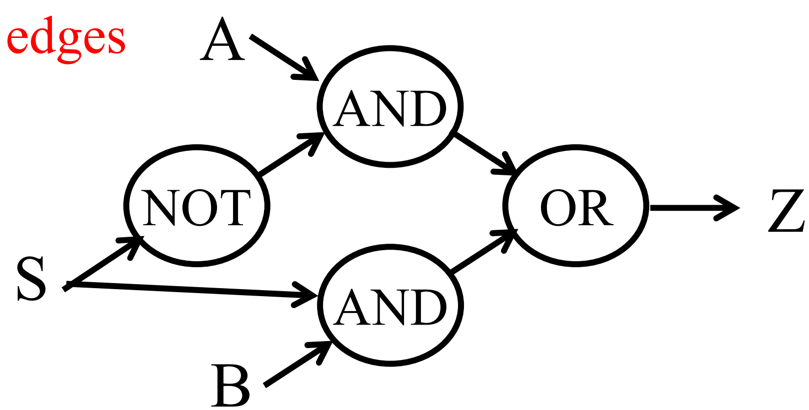
  - › Serial-parallel
    - both serial & parallel components
    - *m* time steps to get all *n* bits, *k* bits at a time
  - › Trade-off:
    - # inputs/outputs (I/O)
    - speed of data transmission
    - Combinational logic has parallel input data and output data



What is Combinational Logic?

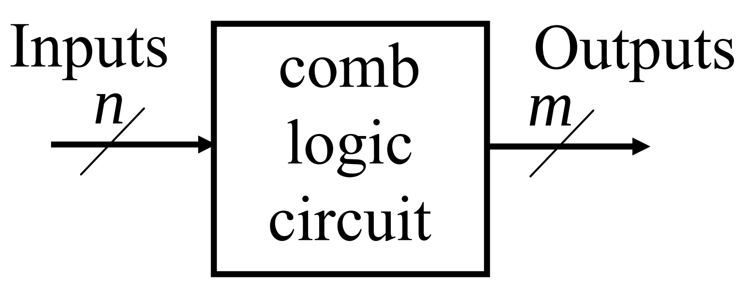
- A collection of logic gates in which there are ***NO*** feedback loops
  - › No feedback loops means there is no path in the circuit on which you will pass through a given gate more than once
  - › Also defined as a circuit that can be represented by a directed acyclic graph (no cycles in the graph)
    - Gates represented by vertices (aka *nodes*)
    - Connections represented by edges





Boolean (Logic) Equations

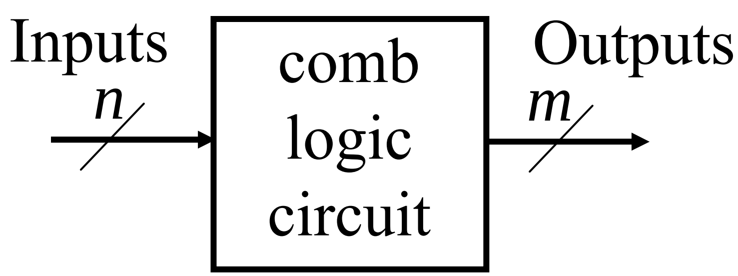
- Any *n*-input, *m*-output combinational logic circuit can be completely described by a set of *m* logic equations
  - One logic equation for each output
  - Gives the output responses to all 2^n possible combinations of input values



$$O_1=f_1(I_1,I_2,\ldots,I_n)$$
$$O_2=f_2(I_1,I_2,\ldots,I_n)$$
$$\ldots$$
$$O_m=f_m(I_1,I_2,\ldots,I_n)$$

Truth Tables

- Any *n*-input, *m*-output combinational logic circuit can be completely described by a truth table
  - › Gives the output responses to all 2^n possible combinations of input values
  - › Therefore, truth tables and logic equations contain the same information
- Two logic equations (or two combinational logic circuits) are *equivalent* if they produce the same truth tables



Inputs	Outputs
00…00	$v_1 \ldots v_m$
00…01	$v_1 \ldots v_m$
00…10	$v_1 \ldots v_m$
…	…
11…11	$v_1 \ldots v_m$

Representations of Logic Functions

- Truth Table
- Boolean (or logic) equations
- Sum-of-Products (SOP)
  - Z=AB+A'C
  - AND is product
  - OR is sum
- SOP canonical form

  Z=A'B'C+A'BC+ABC'+ABC
  - All literals are present in all product terms
- Minterm (a 1 in a TT row)
  - $Z=\Sigma_{A,B,C}(1,3,6,7)$

Literal *– a single variable or the complement of a variable*

Product term *– a single literal or a logic product of multiple literals*

A	B	C	Z	Row value	Minterm
0	0	0	0	0	A'B'C'
0	0	1	1	1	A'B'C
0	1	0	0	2	A'BC'
0	1	1	1	3	A'BC
1	0	0	0	4	AB'C'
1	0	1	0	5	AB'C
1	1	0	1	6	ABC'
1	1	1	1	7	ABC

Other Representations

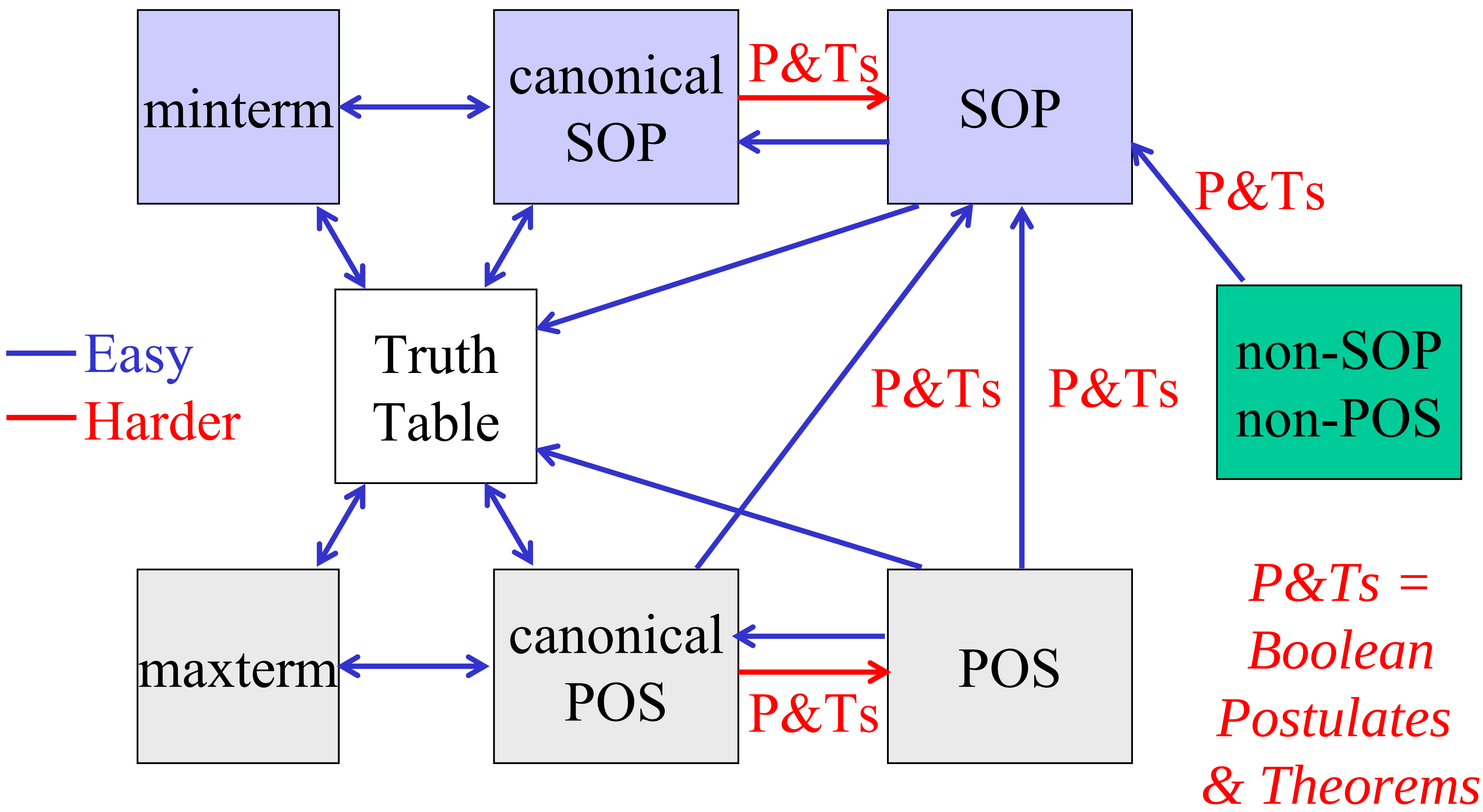
- Product-of-Sums (POS)
  - › Z=(A+C)•(A'+B)
- POS canonical form

  Z=(A+B+C)•(A+B'+C)
  •(A'+B+C)•(A'+B+C')
  - › All literals are present in all sum terms
- Maxterm (a 0 in a TT row)
  - › $Z=\Pi_{A,B,C}(0,2,4,5)$
    - Note this is all TT rows not in minterm expression for this example
- POS representations are less often used than SOP

***Sum term** – a single literal or a logic sum of multiple literals*

A	B	C	Z	Row value	Maxterm
0	0	0	0	0	A+B+C
0	0	1	1	1	A+B+C'
0	1	0	0	2	A+B'+C
0	1	1	1	3	A+B'+C'
1	0	0	0	4	A'+B+C
1	0	1	0	5	A'+B+C'
1	1	0	1	6	A'+B'+C
1	1	1	1	7	A'+B'+C'

Conversion Between Representations



Conversion Between Representations

- Minterm to truth table
  - › convert decimal minterm to binary

    $Z = \Sigma_{A,B,C}(1,3,6,7)$

    $= \Sigma_{A,B,C}(001,011,110,111)$
  - › place 1s in truth table entry for each minterm
    - Pay attention to input ordering
  - › place 0s in all other entries
- Maxterm to truth table
  - › convert decimal maxterm to binary

    $Z = \Pi_{A,B,C}(0,2,4,5)$

    $= \Pi_{A,B,C}(000,010,100,101)$
  - › place 0s in truth table entry for each maxterm
    - Pay attention to input ordering
  - › place 1s in all other entries

Conversion Between Representations

- Minterm to canonical SOP
  - › convert decimal minterm to binary

    $Z = \Sigma_{A,B,C}(1,3,6,7)$

    $= \Sigma_{A,B,C}(001,011,110,111)$
  - › replace 1s and 0s with variable and complement of variable, respectively

    $= \Sigma_{A,B,C}(A'B'C,A'BC,ABC',ABC)$
    - Be sure to maintain input ordering
  - › then sum

    $= A'B'C+A'BC+ABC'+ABC$
- Canonical SOP to minterm
  - › just reverse the procedure above

Conversion Between Representations

- Maxterm to canonical POS
  - › convert decimal maxterm to binary

    $Z = \Pi_{A,B,C}(0,2,4,5)$

    $= \Pi_{A,B,C}(000,010,100,101)$
  - › replace 0s and 1s with variable and complement of variable, respectively, and sum

    $= \Pi_{A,B,C}(A+B+C,\ A+B'+C,\ A'+B+C,\ A'+B+C')$
    - Be sure to maintain input ordering
  - › then take the product of the individual sum terms

    $= (A+B+C)\bullet(A+B'+C)\bullet(A'+B+C)\bullet(A'+B+C')$

    Note that these last 2 steps are the dual of those for minterm to canonical SOP
- Canonical POS to maxterm
  - › just reverse the procedure above

Conversion Between Representations

- POS to SOP
  - multiply like in regular algebra then apply P&Ts

$Z = (A+C)\bullet(A'+B)$

$= A\bullet(A'+B)+C\bullet(A'+B)$ using P5b

$= AA'+AB+CA'+CB$ using P5b (SOP but not minimal)

$= 0+AB+A'C+CB$ using P6b

$= AB+A'C+CB$ using P2a

$= AB+A'C$ using T9a

- Canonical POS to SOP
  - use same procedure

Conversion Between Representations

- SOP to canonical SOP
  - › Replace all missing variables in each product term with X+X', where X is the missing variable
    - Recall:
      - X+X' = 1, *and*
      - Y•1=Y, so we don't change the product term

Z = AB+A'C

= A•B•1+A'•1•C using P2b

= A•B•(C+C')+A'•(B+B')•C using P6a

  - › then multiply

= ABC+ABC'+A'BC+A'B'C using P5b

= A'B'C+A'BC+ABC'+ABC using P3a

Conversion Between Representations

- Canonical SOP to minimal SOP
  - › apply P&Ts

    Z = A'B'C+A'BC+ABC'+ABC

    = A'•(B'C+BC)+A•(BC'+BC) using P5b

    = A'•(C)+A•(BC'+BC) using T6a

    = A'•(C)+A•(B) using T6a

    = A'C+AB no change, just removed ( ) and •

    = AB+A'C using P3a

- Same for SOP to minimal SOP
- ***Problem*:** How to know when it's minimal?

Conversion Between Representations

- Non-SOP/non-POS to SOP
  - › apply P&Ts

$$Z = (((A'B)'C)'+D')' = \overline{\overline{\overline{\overline{A}\cdot B}\cdot C}+\overline{D}}$$

$$= \overline{\overline{(\overline{\overline{A}\cdot B})\cdot C}}\cdot\overline{\overline{D}}$$ using DeMorgan T8a

$$= ((\overline{\overline{A}\cdot B})\cdot C)\cdot D$$ using T3

$$= ((\overline{\overline{A}}+\overline{B})\cdot C)\cdot D$$ using DeMorgan T8b

$$= ((A+\overline{B})\cdot C)\cdot D$$ using T3

Note: this is a POS

$$= (A+B')\cdot C\cdot D$$ no change, just removed ( )

$$= ACD+B'CD$$ (SOP) using P5b

Conversion Between Representations

- SOP to truth table:
  - › Place a logic 1 in each truth table output entry whose input value satisfies a given product term = 1
    - A *k*-variable product term will produce 2^{n-k} 1s in the truth table where *n* is the total number of input variables
  - › Repeat for all product terms
- POS to truth table:
  - › Place a logic 0 in each truth table output entry whose input value satisfies a given sum term = 0
    - A *k*-variable sum term will produce 2^{n-k} 0s in the truth table where *n* is the total number of input variables
  - › Repeat for all sum terms

Using Truth Tables to Prove Theorems

- Consensus Theorem
  - T9a: X•Y+X'•Z+Y•Z = X•Y+X'•Z
  - T9b: (X+Y)•(X'+Z)•(Y+Z) = (X+Y)•(X'+Z)

T9a:

X	Y	Z	Output
0	0	0	0
0	0	1	**1**
0	1	0	0
0	1	1	**1**
1	0	0	0
1	0	1	0
1	1	0	**1**
1	1	1	**1**

T9b:

X	Y	Z	Output
0	0	0	**0**
0	0	1	**0**
0	1	0	1
0	1	1	1
1	0	0	**0**
1	0	1	1
1	1	0	**0**
1	1	1	1