Alternative Neural Network Approaches to Corporate Bond Rating

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Abstract

This paper explores three of the most well known supervised neural network paradigms — backpropagation, radial basis function and learning vector quantization — for the task of rating U.S. corporate bonds. Using generally available historic data, bonds are assigned to ratings based on a classification scheme. The classification schemes investigated were a binary categorical assignment and an integer classification. Comparisons are made with logistic regression models on both the data set used to create the predictive models and on new data.

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KEY WORDS: bond rating, neural network, backpropagation, radial basis function, learning vector quantization, logistic regression

1.0 INTRODUCTION

One of the important problems in financial investment is the classification of bonds based on the likelihood that the issuing company may default on the promised payment, i.e. the contracted periodic interest payments and the principal repayment. Commercial bond rating organizations such as *Standard & Poor's* or *Moody's* classify bonds according to the degree of default risk. These rating agencies conduct extensive committee analyses of the intrinsic characteristics of the issuing organizations such as the issuer's ability to pay interest and repay capital, willingness to do so, and protective provisions of an issue. The results are published in the form of ratings which are used to reflect the risk of investment in the bonds, define allowable

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purchases by certain investors and have significant effect on the offering yield (interest rate) of a bond issue. An organization with a better rating can usually issue bonds at a more favorable (lower) interest rate than organizations with worse ratings. Ratings are reviewed and reassigned periodically to reflect the dynamics of the financial market and the organization itself.

The nature of the bond rating task is of the classification type since it deals with a set of input variables mapping to a set of discrete and mutually exclusive classes. Here, the different bond issues form the set of input data instances and the various bond ratings form the set of possible classes to which the input bonds can belong. Bonds must be assigned to a class, and to one class only.

1.1 Statistical Approaches to Bond Rating

The process of bond rating as performed by an expert committee of a specific financial agency is rather covert because of confidentiality issues. Accordingly, many researchers have tried to formulate alternative approaches to predict bond ratings, such as statistical approaches and human knowledge processing approaches, with concentration on the statistical approaches. Previous studies focused on linear regression analysis [5, 19], logistic regression analysis [4], and linear discriminant analysis [1, 13]. Many researchers such as [3, 4, 6, 7] have criticized the use of conventional statistical analyses techniques, such as multiple regression models, as having limited success in predicting bond ratings because the application does not adhere to common functional forms.

One of the human knowledge processing approaches is expert systems, which relies on knowledge engineering techniques. It performs the classification task using a rule set formulated by experts. Rules are derived according to the opinions of experts on the effects of each related financial variable. However, it is very difficult to develop a rule based expert system for rating bonds since few experts are available and most knowledge about the process of rating is confidential.

1.2 Neural Networks Approaches to Bond Rating

A neural network does not require the a priori specification of a functional form, rather it attempts to learn from the training input-output examples alone. This motivates its consideration for the bond rating problem. Neural networks have been noted to perform well as classifiers for problems containing complex and imperfect data and relationships, and they are strongly related to the field of statistics [20]. Inputs to the neural network would consist of financial metrics and indicators while the output would be the bond rating. One might apply unsupervised neural networks, and let the clustering learning algorithms (e.g., self organizing map or adaptive resonance theories) find the rating groupings. A supervised learning approach, however, is usually more effective as long as target (teacher) data is available. In the case of bond ratings, plenty of historic data matching the financial variables to the bond ratings assigned by experts exists, so it seems prudent to take advantage of it.

Backpropagation neural networks [14, 18], have been applied to bond ratings in a few previous research studies. Dutta and Shekhar [3] used different numbers of hidden neurons using either six or ten financial variables as inputs:

1. Liabilities / (Cash + Assets)
2. Debt ratio
3. Sales / Net worth
4. Profit / Sales
5. Financial strength
6. Earnings / Fixed costs
7. Past five years' revenue growth rate
8. Projected next five years' revenue growth rate
9. Working capital / Sales
10. Subjective company prospect

They compared the results with a multiple linear regression model. Bond issues of 47 industrial companies were selected at random from the *Value Line Index* [17] and the *Standard & Poor's Bond Guide* [16] and 30 of them were used to train the neural network and obtain the regression coefficients. The remaining 17 were used to test the neural network and regression performance. A linear scale was used to convert the ratings of the bonds. Dutta and Shekhar concluded that regardless of using the first six or all ten financial variables, backpropagation outperformed linear regression in terms of more correct bond ratings. In addition, whenever the neural network model misclassified a bond, it was off by at most one rating class. In contrast, the regression model was often off by several rating classes.

Kim [7] also compared the backpropagation neural network approach against linear regression, discriminant analysis, logistic analysis, and a rule-based system for bond rating. Training data and test data were prepared from *Standard & Poor's Compustat* financial data tape. The classification comparison was performed on three data sets: (1) training data using 110 companies; (2) test data using 58 companies from the same year as the training data; and (3) prediction test data using 60 companies from the following year. The performance measures used to compare the neural network with regression analysis were the number of accurate classifications and the number of classifications that differ by one rating class. He used eight important financial variables as suggested by Belkaoui [1]:

1. Total assets: indicates the total size of the firm.
2. Total debt: a measure of the total indebtedness of the firm.
3. Long term debt / Total invested capital: a measure of the long-term capital intensity of the firm.
4. Short term debt / Total invested capital: a measure of the short-term capital intensity of the firm.
5. Current assets / Current liabilities: a measure of the total liquidity of the firm.
6. (Net income + Total interest expense) / (Interest expense + Preferred dividend requirement): a measure of debt coverage.
7. Stock price / Common equity per share: a measure of investors' expectations.
8. Subordination (0-1): the most relevant covenant of the indenture.

The neural network was able to classify more bond issues to their actual classes than the other approaches for the given sets of classification and prediction data.

This paper expands beyond these two earlier attempts by considering two additional mainstream supervised neural networks paradigms, viz. radial basis function (RBF) and learning vector quantization (LVQ), along with backpropagation (BP). BP is a multilayer network (with one or more hidden layers) which employs a gradient descent method as the training algorithm. The method is based on the minimization of the total squared error of the output computed by the network. The training algorithm involves three stages: the feed-forward of the input training set, the calculation and backpropagation of the error, and the adjustment of the weights.

The RBF network is similar to the backpropagation network except that the hidden neurons are locally receptive to inputs, rather than globally receptive as in BP [10]. RBF calculates the “closeness” of input **x** to an n-dimensional parameter vector μ_j associated with the j^{th} hidden unit instead of using the weighted sum/sigmoidal activation mechanism in BP. The characteristics of the jth hidden unit response is

$$z_j(\mathbf{x}) = K\left(\frac{||x - \mu_j||}{\sigma_j^2}\right)$$

The normalized Euclidean distance from the center μ_j is used as an argument in the Kernel (K) function. K is rigorously positive and radially symmetric with a unique maximum at its center μ_j and drops off rapidly to zero away from the center. The I-dimensional output layer **y** whose i^{th} component is characterized by

$$y_i(\mathbf{x}) = \sum_{j=1}^{J} w_{ij} z_j(x)$$

The network can be trained by first clustering the data set to the hidden neurons in an unsupervised manner, then using a least squares-based supervised learning technique to train the weights from the hidden neurons to the output neurons. Alternatively, supervised gradient descent training can be used to simultaneously find values of μ_j, σ_j, and w_{ij} that minimize the squared error over the training set.

LVQ is a classification network which was developed by Kohonen as a supervised extension of his self organizing networks, and uses a Euclidean distance metric to group similar input vectors [8]. It is a single layer network with J cluster units. Cluster unit j whose weight vector matches the input pattern **x** most closely according to Euclidean distance $|| x\text{-}w_{ij} ||$ is the winning unit, and its weights are updated during training. If the category represented by jth output unit matches the correct jth class of the training vector, the weight is updated to be more similar to the input vector. Otherwise, the weights are changed to be less similar to the input. Training time for the LVQ network is much faster than either BP or RBF. RBF and LVQ have been used in other research and have been shown to sometimes perform better than BP, so it appears that they might be suitable for the bond rating task.

2.0 RESEARCH METHODOLOGY

2.1 Determination of Financial Inputs and Target Ratings

In selecting the determinants of the bond ratings, financial variables were chosen based on a prominent bond rating organization (*Standard & Poor's*), and the work of Dutta and Shekhar

[3] and Kim [7]. The following are the financial parameters of the firms that were selected as inputs to the models in this paper. Note that the subjective assessment variable used by some other researchers is missing; the neural network predictive models rely only on published factual data. These metrics describe the organization's size, profitability, leverage, liquidity and obligations.

1. Total assets.
2. Total debt.
3. Long-term debt / Total invested capital ratio.
4. Short-term debt / Total invested capital ratio.
5. Current assets / Current liabilities ratio.
6. (Net income + Interest expense) / (Interest expense) or the fixed charge coverage ratio.
7. Total debt / Total assets ratio: a measure of the extent of the total funds that have been supplied by creditors.
8. Profit / Sales ratio: a measure of the firm's profit margin.

In this project, only the top six ratings of bonds used by *Standard & Poor's Bond Guide* [16] were selected as classes due to the fact that bonds with lower ratings than these are rarely attractive to investors. The following gives the description of each bond class:

1. **AAA** has an extremely strong capacity to pay interest and repay principal.
2. **AA** has a very strong capacity to pay interest and repay principal, which differs in merely a small degree from the higher rated issues.
3. **A** has a strong capacity to pay interest and repay principal. It is more susceptible to adverse effects of changes in economic condition than debt in higher rated categories.
4. **BBB** has adequate capacity to pay interest and repay principal.
5. **BB** has speculative debt facing major uncertainties which could lead to inadequate capacity to pay interest and repay principal.
6. **B** has greater vulnerability to default but currently has the capacity to meet interest payment and principal repayment.

For BP and RBF, two output encodings were investigated; integer values over the range (1, 6) using a single output neuron and binary outputs using six output neurons (one for each class). For LVQ, only the binary encoding was used, because the integer encoding is not appropriate for this network type. For regression, only the the integer encoding with a single dependent variable was used. This is summarized in Table 1. Note that the encoding is for the target values only, and that the network may output any continuous value between 1 and 6 (for the integer coding) or between 0 and 1 (for the binary coding).

2.2 Selection of Data Sets

Bond ratings and values of the financial variables for a set of non-financial industrial bonds were taken from the S*tandard & Poor's Bond Guide* [16]. Data from the companies with financial indicators in the outlier range (very high or very low) were discarded.

Three data sets were prepared as described below:

A) **Training Data Set**: Bond issues of 60 companies were randomly selected from the 1996 issues listed in *Standard & Poor's*. This data set contained ten of each bond class (AAA through B) and is shown in Appendix 1.

B) **Testing Data Set I**: Bond issues of 30 companies were again randomly selected from the 1997 issues listed in *Standard & Poor's*. This data set contained five of each bond class and is shown in Appendix 2.

C) **Testing Data Set II**: 30 bond issues randomly chosen from the training data set were combined with the 30 bond issues from Testing Data Set I. This data set contained ten of each bond class and was used to compare among the three neural network paradigms and the statistical approach. This data set was used to balance performance on training and testing and is shown in Appendix 3.

3.0 EXPERIMENTS AND RESULTS

Neural networks can be sensitive to the number of layers and neurons in their architecture. Therefore, before comparisons with logistic regression can be made, studies on the appropriate choice of architecture for each paradigm were carried out. Full factorial experiments using five replications[3] were performed, then the results were analyzed using analysis of variance (ANOVA). The results reported in sections 3.1 and 3.2 were for the binary encoding scheme, i.e. networks with six output neurons.

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[3] Replications involve using the same architecture and learning parameters but with different sets of random initial weights.

3.1 The Backpropagation Neural Network

The experiments tested multiple levels of hidden neurons in a single hidden layer. Training used a learning rate of 0.15 for the output layer and 0.3 for the hidden layer. The evaluation criteria was the number of correct classifications and the number of classifications that differed by one rating class, which were obtained from training the network on the training set and testing using Testing Data Set I. The results were analyzed by ANOVA and Scheffe's simultaneous inference. Table 2 shows the results for Training Data Set and Table 3 shows the results for Testing Data Set I.

As expected, the training set improved with more hidden neurons but the test set classification rate declined. Using the results above, a more detailed study was made using between one and eight hidden neurons with the ANOVA table shown in Table 4. The results conclude that there is no statistical difference between these numbers of neurons and classification accuracy for the test set, but the best classification results were for three hidden neurons, so that is chosen for the final network.

3.2 The Radial Basis Function Network

An experimental method similar to that employed in section 3.1 was used here. The same number of hidden neurons as for the BP were initially tried (1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25) and results indicated that from four to eight hidden neurons needed to be studied in more detail. The results were also similar to BP in that more hidden neurons resulted in better training set accuracy at the expense of testing set accuracy. Table 5 shows the results of the ANOVA on these experiments. Though no statistical difference exists for the test set, seven hidden neurons were chosen, as that yielded the best classification accuracy.

3.3 The Learning Vector Quantization Network

The appropriate architecture for LVQ was determined using a two factor experimental design. The first factor is the type of LVQ training, L: LVQ1 only and LVQ1 followed by LVQ2. LVQ1 is a simple modification by DeSieno [2] to the original LVQ by Kohonen [8] that penalizes neurons for "winning" too often, therefore resulting in smoother training. LVQ2 is a further refinement that changes the weights for both the "winning" neuron and the first "runner-up," and this again, tends to fine tune the network during training [9]. The second factor is the number of Kohonen neurons, N (6, 12, 18). The number of Kohonen neurons is chosen to be a multiple of the number of the output neurons (6) as recommended by NeuralWare software [12]. The rest of the experimental design and the classification criteria were the same as in sections 3.1 and 3.2. Table 6 shows the ANOVA results for the training and the testing data sets for both factors and the interaction term (N x L). For the training set, as expected, accuracy increased with number of neurons and the combined LVQ1 and LVQ2 training strategy was superior to the LVQ1 alone. Accuracy declined with an increase in Kohonen neurons for the test set. The interaction term was also significant. The final network selected had six Kohonen neurons and the combined training.

3.4 Regression Analysis

Three types of logistic regression were used to classify bonds (using the training data set to fit the model and Testing Data Set II to test the model): (1) Logistic regression with a first order model, (2) Stepwise logistic regression with a first order model, (2) Stepwise logistic regression with first and second orders and interactions. Logistic regression was used instead of the more common multiple regression because the outputs were categorical rather than continuous. Table 7 shows the results of these regressions. The first order model was the simplest and the most accurate, so it was chosen for comparison to the neural networks.

3.5 Comparison of Neural Network Approach with Logistic Regression Approach

The best neural networks from the preceding sections were compared with the first order logistic regression and BP and RBF networks with the integer output encoding using Testing Data Set II. Also, a regular multiple regression with the first order terms and all second order interactions was performed, as less experienced analysts might not use the more appropriate logistic regression. Table 8 shows the results of these comparisons. It can be seen that LVQ and RBF are inferior to BP and logistic regression. The comparison among the three network paradigms with six output neurons (the binary encoding scheme) are compared statistically in Table 9, showing that BP is statistically superior to the others. It is also seen that multiple regression is worse than logistic regression (and is not really a proper choice).

Table 10 and 11 present the complete classification tables of the bond ratings (actual versus predicted ratings) by BP with one output and the first-order logistic regression respectively. There is not much difference between the best neural network paradigm and the best statistical approach in these experiments. The BP network performs slightly better than the logistic regression in terms of correct classification. When the methods misclassify a bond, the logistic regression misses by more classes slightly more often than BP.

4.0 CONCLUSIONS

For assignment of bond ratings, neither a neural network or a regression model have great accuracy. It seems that this may be one area that is better performed by experienced and specialized experts. However, if a mathematical approach is desired, either the backpropagation paradigm or a logistic regression model is a good choice. BP is comparable to logistic regression in classification accuracy, but the model building issues are quite different. Smith and Mason give a comprehensive discussion of the relative merits of backpropagation neural networks and regression for prediction of costs [15]. To summarize, regressions are much more straightforward to construct and validate, and are computationally quicker and easier to replicate. Neural networks are more flexible in the form of relationship they can model and may be more resistant to imperfect data. Both types of modeling can benefit from appropriate choice of input financial measures and a large quantity of both training and validation data. There may be promise in combining the mathematics of neural networks or regression with a rule base replicating the bond rating assignment capability of an expert.

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Table 1. Output assignment for bond ratings.

Rating	Continuous Output*	Binary Output Pattern#
AAA	6	1 0 0 0 0 0
AA	5	0 1 0 0 0 0
A	4	0 0 1 0 0 0
BBB	3	0 0 0 1 0 0
BB	2	0 0 0 0 1 0
B	1	0 0 0 0 0 1

* Used for BP and RBF networks

\# Used for BP, RBF and LVQ networks and regression.

Table 2. Correct BP classifications on Training Data Set.

	Number of Hidden Neurons												
Rep	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**
1	26	28	39	34	41	51	54	48	41	48	51	44	46
2	24	29	39	41	40	49	44	46	44	53	53	40	52
3	19	32	39	43	47	44	46	49	40	48	50	46	49
4	27	33	41	47	47	52	43	45	47	46	43	52	50
5	28	31	34	44	49	41	43	50	45	48	44	46	47
Total	124	153	192	209	224	237	230	238	217	243	241	228	244
Ave	24.8	30.6	38.4	41.8	44.8	47.4	46.0	47.6	43.4	48.6	48.2	45.6	48.8
%	41.3	51.0	64.0	69.7	74.7	79.0	76.7	79.3	72.3	81.0	80.3	76.0	81.3

Table 3. Correct BP classifications on Testing Data Set I.

	Number of Hidden Neurons												
Rep	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**
1	13	13	13	11	13	10	9	10	10	12	11	7	10
2	11	9	15	12	9	7	9	9	12	11	11	10	10
3	10	16	11	11	11	11	9	11	12	9	8	9	7
4	11	14	9	11	9	13	11	9	11	11	11	11	10
5	14	11	13	13	11	9	10	9	10	12	11	11	9
Total	59	63	61	58	53	50	48	48	55	55	52	48	46
Ave	11.8	12.6	12.2	11.6	10.6	10.0	9.6	9.6	11.0	11.0	10.4	9.6	9.2
%	39.3	42.0	40.7	38.7	35.3	33.3	32.0	32.0	36.7	36.7	34.7	32.0	30.7

Table 4. Analysis of variance results for BP networks from one to eight hidden neurons.

ANOVA Table : Training Set					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	99.65	24.9125	2.45	0.0693
No. of Neurons	7	1966.375	280.9107	27.62	0.0001
Error	28	284.75	10.1696		
Total	39	2350.775			
ANOVA Table : Testing Data Set I					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	2.56	0.6625	0.15	0.9617
No. of Neurons	7	11.975	1.7107	0.39	0.9028
Error	28	124.15	4.4339		
Total	39	138.775			

Table 5. Analysis of variance results for RBF networks from four to eight hidden neurons.

ANOVA Table : Training Set					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	13.36	3.34	0.38	0.8189
No. of Neurons	4	18.96	4.74	0.54	0.7081
Error	16	140.24	8.76		
Total	24	172.56			
ANOVA Table : Testing Data Set I					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	8.24	2.06	0.91	0.4809
No. of Neurons	4	11.44	2.86	1.27	0.3242
Error	16	36.16	2.26		
Total	24	55.84			

Table 6. Analysis of variance results for LVQ networks with different learning algorithms and different number of Kohonen neurons.

ANOVA Table : Training Set					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	3.13	0.78	0.14	0.9673
No. of Neurons	2	950.60	475.30	82.18	0.0001
Learning	1	563.33	563.33	97.41	0.0001
N x L	2	76.06	38.03	6.58	0.0064
Error	20	115.67			
Total	29	1708.80			
ANOVA Table : Testing Data Set I					
SOV	**DF**	**SS**	**MS**	**F Value**	**Pr >F**
Block	4	4.2	1.05	0.45	0.7734
No. of Neurons	2	128.60	64.30	27.36	0.0001
Learning	1	2.13	2.13	0.91	0.3521
N x L	2	17.27	8.63	3.67	0.0500
Error	20	47.00	2.35		
Total	29	199.20			

Table 7. Classification results of different logistic regressions on Testing Set II.

Logistic Regression	Numbers of Correct Classifications	Classification Accuracy (%)
Logistic-1st order	32	53.33
Stepwise logistic- 1st order	26	43.33
Stepwise logistic-1st, 2nd order and interactions	28	46.67

Table 8. Comparison among various neural networks and regression approach on Testing Set II.

Accuracy (%)	BP 6 outputs	BP 1 outputs	RBF 6 outputs	RBF 1 outputs	LVQ	Multiple regression	Logistic regression
Correct	51.9	56.7	38.3	23.3	36.7	48.3	53.3
Miss 1 Class	30.3	30.3	27.7	44.0	38.7	28.3	28.3

Table 9. ANOVA of neural networks with six output neurons on Testing Set II.

SOV	DF	SS	MS	F Value	Pr >F
Block	4	78.9444	19.7361	0.51	0.73
Paradigm	2	502.2163	250.1082	6.48	0.0212
Error	8	308.8578	38.6072		
Total	14				

Table 10. Classification of bonds by backpropagation with one output neuron on Testing Set II*.

Actual Rating	Predicted Rating					
	AAA	**AA**	**A**	**BBB**	**BB**	**B**
AAA	6	3	0.8	0.2	0	0
AA	0.8	4.8	2.6	1.4	0.4	0
A	0	0.8	5.4	1.8	0.8	0.8
BBB	0	0	1.2	4.8	3.2	0.8
BB	0	0	0	3	5.8	1.2
B	1	0	1	0	1.4	6.6

* Values are averaged over five replications.

Table 11. Classification of bonds by 1st order logistic regression on Testing Set II.

Actual Rating	Predicted Rating					
	AAA	**AA**	**A**	**BBB**	**BB**	**B**
AAA	8	0	1	1	0	0
AA	0	6	3	1	0	0
A	1	2	4	2	0	1
BBB	0	1	3	4	1	1
BB	0	1	1	2	4	2
B	1	0	0	1	2	6

Biographical Sketches

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