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Locating multiple capacitated semi-obnoxious facilities using evolutionary strategies

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a model for the semi-obnoxious, multiple capacitated facility location problem on a Euclidean plane. Even though this problem arises often in public planning, this is the first known paper solving this class of problem for more than one facility in continuous space where capacity is considered. The problem is solved using a bi-objective evolutionary strategy algorithm that seeks to minimize social and non-social costs. The effects of under and over capacitating the facility are included in the cost functions. Two case study problems are solved, one involving the siting of fire stations in a college town, the other locating solid waste transfer stations in a major metropolitan area. The solutions produced by the model are compared to the real-world placement of the facilities. The algorithm yields many non-dominated solutions covering the whole range of values for both objectives in a short amount of time. Also considered are the effects of adding multiple facilities sequentially as happens in a real-world, phased implementation that spans years, as opposed to placing the facilities all at once.

1. Introduction

The semi-obnoxious facility location problem consists of finding the optimal placement of semi-desirable (also called semi-obnoxious) facilities. An undesirable (or obnoxious) facility is defined by Erkut and Neuman as one that generates a disservice to the people nearby while producing an intended product or service (Erkut & Neuman, 1989). What distinguishes the semi-obnoxious facility location problem from other location problems is that the facilities have both desirable and undesirable characteristics. Obnoxious facilities have only undesirable traits, which implies that they should be placed as far away as possible from demand points. Desirable facilities, on the other hand, should be placed as close as possible to the demand sites. Semi-obnoxious facilities are ideally placed wherever the tension between both their desirability and their undesirability is balanced. This implies, on the one hand, that semi-obnoxious facility location problems are inherently multi-objective, and, furthermore, that the objectives are in direct opposition. This is not the case with other location problems, where even if multiple location objectives are considered, these tend to be of the same kind (either both are "pull" or "push" objectives). This places the semi-obnoxious location problem in a class of its own, even if it shares some aspects of the desirable and obnoxious facility classes.

If the only goal is to minimize the noxious effect of the facility, it is best to place it as far away as possible from any population center. However, some obnoxious facilities also provide a service that requires them to be located nearby. In this case, the facility cannot be considered purely obnoxious nor purely desirable. It is best to refer to it as semi-obnoxious (Brimberg & Juel, 1998). Examples of these types of facilities include garbage dump sites, airports, power plants, police stations, hospitals, stadiums, among others. An increasing public concern with the health and environmental effects of these types of facilities has made the problem of locating them one of importance.

The problem addressed in this paper can be described as follows: suppose a city desires to locate one or more capacitated semi-obnoxious facilities. The total number of facilities is not specified beforehand but an upper bound on how many can be constructed is provided. Several types of costs need to be considered. These costs can be separated into those that are related to the undesirable effects of the facility and those that are not. The former will be referred to as social costs, while the latter as non-social costs. Non-social costs include the setup and operating costs of a facility, as well as a distance-related cost function that captures the effects of distance on the service provided by the facilities. The goal is to minimize these costs. Because doing so draws the facilities towards the demand points, this objective is called a "pull" objective. Social costs capture the undesirable effects the facilities have on the neighboring demand sites (noise, traffic, pollution, etc.). These costs

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should also be minimized. Such a goal tends to move the facilities away from the demand sites and is called a "push" objective. Since both objectives work in opposite directions, this problem lends itself naturally to the use of multi-objective models (Erkut & Neuman, 1989). Capacities also need to be considered. Facilities providing a valuable service to the community would likely operate even if the demand exceeds their capacity, but a situation such as that would strain the facility's operations and would increase its negative effects. Think, for example, of a fire station that needs to provide service to more people than it can handle. This would lead to lower response times, more wear and tear on the equipment, and would eventually require hiring additional staff and purchasing new equipment. This situation would represent a significant increase on the operating costs. At the same time, the excessive demand would also lead to more responses to emergency calls, resulting in increased noise and traffic. Hence, the social costs would also rise significantly. This problem is called the multiple, capacitated, semi-obnoxious location problem.

Multi-objective models for semi-obnoxious facilities were being studied as early back as the 70's and 80's (Church & Cohon, 1976; Goicoechea, Hansen, & Duckstein, 1982; Sarin, 1980). The majority of multi-objective models limit themselves to minimizing noxious effects while also minimizing transportation costs, though models with more than two objectives can also be found in the literature (Azadeh, Ahmadzadeh, & Eslami, 2018; Fernández, Redondo, Arrondo, & Ortigosa, 2015; Hammad, Akbarnezhad, & Rey, 2017). Examples of bi-objective models include those developed by (Coutinho-Rodrigues, Tralhão, & Alçada-Almeida, 2012; Heydari & Melachrinoudis, 2012; Melachrinoudis, 1999; Yapicioglu, Smith, & Dozier, 2007).

Research in the location of multiple semi-obnoxious facilities is limited, though there are a few instances available in the literature (Berman & Wang, 2007; Coutinho-Rodrigues et al., 2012; Eiselt & Marianov, 2014; Silva, Alçada-Almeida, & Dias, 2017). Publications on the location of multiple facilities in general, on the other hand, is significant and ranges across a variety of facility types, including fire stations (Yang, Jones, & Yang, 2007), recycling centers (Toso & Alem, 2014), electric vehicle charging stations (Lam, Leung, & Chu, 2014; Zhu, Gao, Zheng, & Du, 2016), distribution centers (Boujelben, Gicquel, & Minoux, 2016; Carlo, David, & Salvat-Dávila, 2017; Klose & Drexl, 2005), among others. Note that some of these facilities might be thought of as semi-obnoxious though they were not considered as such in these papers.

Capacitated models of semi-obnoxious facilities are also scarce. Most of the work done on the subject defines the capacity as a decision variable (Eiselt & Marianov, 2014; Silva et al., 2017; Toso & Alem, 2014), which is reasonable if all facilities are to be placed simultaneously. This is not always the case. When facility placement occurs sequentially, the capacities of the already existing facilities are fixed. Likewise, the capacity of any new facility might be determined beforehand based on criteria other than those included in the model. The literature on capacitated facilities is mostly limited to facilities where undesirable effects are not important, such as hubs (Correia, Nickel, & Saldanha-da-Gama, 2018; Meraklı & Yaman, 2017; Mirabi & Seddighi, 2018), servers (Papadimitriou, Colle, & Demeester, 2018) or competing services facilities (Beresnev & Melnikov, 2018; Nasiri, Mahmoodian, Rahbari, & Farahmand, 2018).

Another characteristic of the multiple semi-obnoxious facility models found in the literature is that they are either discrete—where one chooses from a finite set of locations (Colmenar, Martí, & Duarte, 2018; Coutinho-Rodrigues et al., 2012; Eiselt & Marianov, 2014; Silva et al., 2017; Song, Morrison, & Ko, 2013)—or network based models (Colebrook & Sicilia, 2013; Heydari & Melachrinoudis, 2012). There have been no continuous space models for the multiple facility version of this problem. Several papers address the planar case for a single semi-obnoxious facility (see Fernández et al., 2015; Golpayegani, Fathali, & Moradi, 2017; Melachrinoudis, 1999; Melachrinoudis & Xanthopulos, 2003; Plastria, Gordillo, & Carrizosa, 2013; Yapicioglu et al., 2007), and those using a continuous metric are common for the purely obnoxious facility location problem (Drezner, Drezner, & et al., 2018; Drezner, Kalczynski, & et al., 2018; Kalczynski & Drezner, 2019), as well as for location problems in general (Akyüz, 2017; Callaghan, Salhi, & Nagy, 2017; Drezner, Drezner, et al., 2019; Fernández, Tóth, Redondo, & Ortigosa, 2019; Tian, Zhou, Li, Zhang, & Jia, 2016; Wei & Murray, 2015).

The model presented in this paper seeks to solve the multiple, capacitated, semi-obnoxious facility location problem in Euclidean space. Using a planar model instead of a discrete or network-based model is an obvious simplification of the problem, however, it is still a useful approach. Though the locations proposed by the model might not be accessible in real-life (they might be far removed from a road, for instance), they still represent valid approximations, as will be shown in Section 4. The decision maker can use them to find the nearest valid location and place the facility there. Furthermore, the continuous model provides insight into the location distribution patterns that result from different placement strategies. Discrete or network-based models determine the candidate locations *a priori*, which inherently biases the placement of the facilities, thus obscuring the underlying logic guiding the location decision. Continuous models do not suffer from this bias though they may gain this advantage at the expense of some realism.

The model discussed introduces important novelties to the semi-obnoxious facility literature. The cost functions (both for social and non-social costs) attempt to capture the effects of under and over capacitating a facility, which offers more flexibility than imposing the capacity as a constraint.

The remainder of this paper is organized as follows. Section 2 describes the proposed model, while Section 3 describes the solution methodology used. Computational experiments are discussed next, and the last section presents our conclusions.

2. Optimization model

In this section, a new bi-criterion model for the multiple capacitated semi-desirable facility location problem on a Euclidean plane is presented. The first criterion seeks to minimize the total non-social costs (which include the cost of setting up and operating a facility as well as the distance-based cost which varies from problem to problem), while the second one reflects the undesirable effects of the facility's location, also known as social cost.

The basic notation is described first.

Sets:

I: set of facilities, indexed over $i = 1, 2, \cdots, n$ and represented by vector $\mathbf{x}$

J: set of demand sites, indexed over $j = 1, 2, \cdots, S$ and represented by vector $\mathbf{a}$

Parameters:

N: maximum number of permitted facilities
S: number of demand sites
$a_j = (x_{j,1}, x_{j,2})$ – coordinates of demand site j
c_i: capacity limit for facility i
SC_i: annual equivalent setup cost for facility i
FC_i: fixed annual operating cost for facility i
$d_{i,1}$, $d_{i,2}$: threshold distances for obnoxious effects for facility i
M_i: full capacity social cost for locations near facility i
m_i: rate at which social cost decreases with distance from facility i

Decision variables:

$x_i = (x_{1,i}, x_{2,i})$: coordinates of facility i
n: number of facilities to be placed
l_i: current operating load of facility i
$y_{ij} \in \{0, 1\}$: whether demand site j is assigned to facility i.

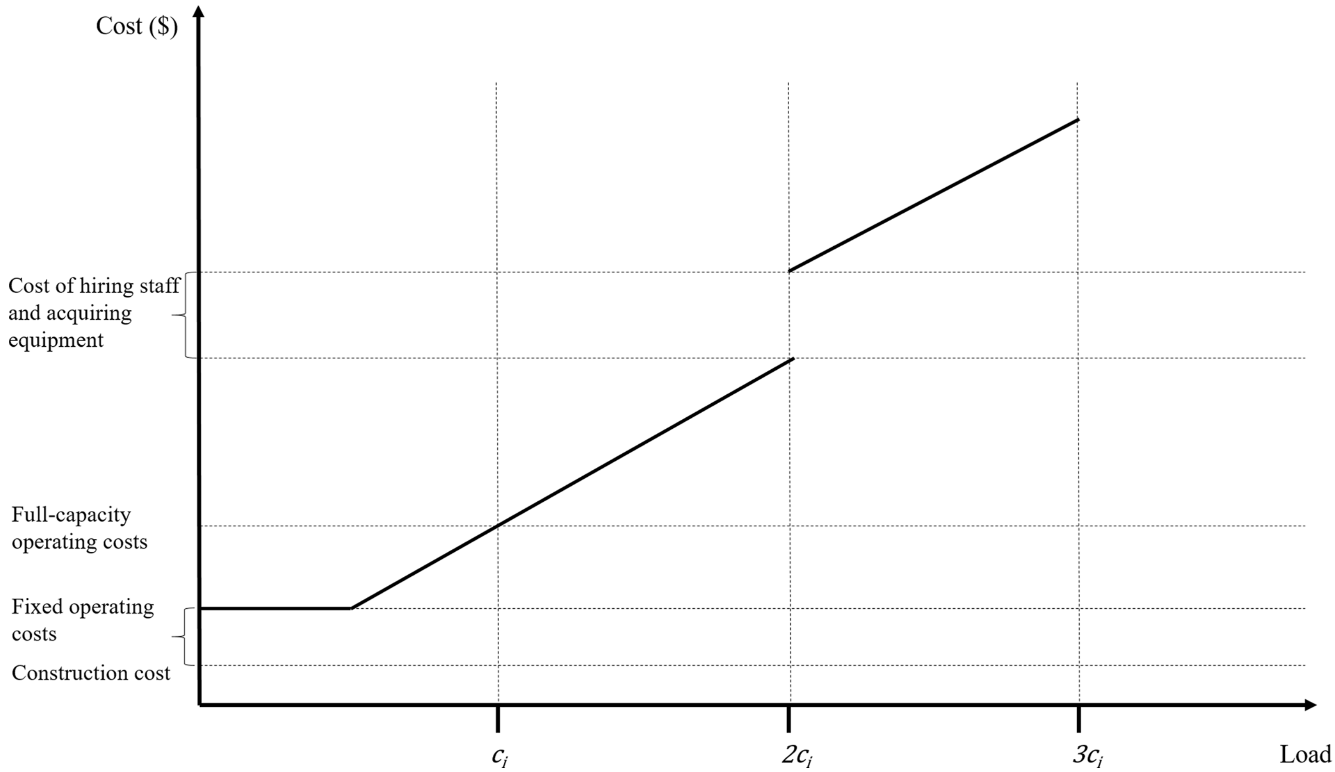


Fig. 1. Capacitated operating cost function (c_i is the capacity of facility i).

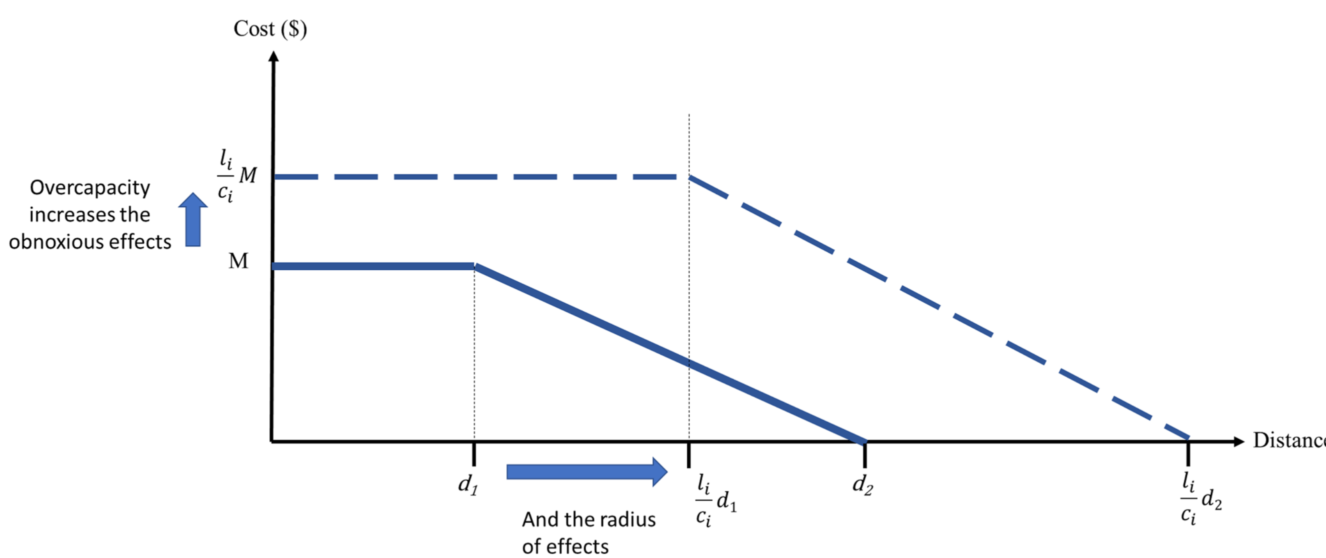


Fig. 2. Social Cost Function.

2.1. Minimization of non-social costs

In general, semi-obnoxious facility location problems seek to minimize a function of the distance between the facilities and their assigned demand sites. Typically, this function represents the cost of transportation. However, in some placement problems transportation cost is not a major concern. For example, in fire station placement, there is greater concern with response times, which are a function of distance.

In capacitated problems, total setup and operating costs are a function of the current load. If the facility is operating below full capacity, total costs might be reduced (less wear and tear on equipment, fewer staff, etc.), while costs increase as the load increases. If capacity is exceeded costs continue to increase regularly up until the point where additional investments are required to expand the facility, acquire more equipment, or hire additional staff. See Fig. 1 for clarification.

With these considerations in mind, the minimization of non-social costs takes the following form.

$$\text{Minimize } w_1(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{S} f(d(\mathbf{x}_i, \mathbf{a}_j))y_{ij} + \sum_{i=1}^{n}[SC_i + FC_i + g(l_i, c_i)] \tag{1}$$

where

$d(\cdot)$ is the Euclidean distance between each fixed point and its corresponding facility.
$f(d(\cdot))$ is a function representing a distance dependent cost.

$g(l_i, c_i)$ is a function representing a load and capacity dependent cost (variable cost) in one year.

2.2. Minimization of social cost

The obnoxious effects are modeled using an extension of the piecewise function developed in Yapicioglu et al. (2007) with a modification to account for capacity. If the facility's capacity is exceeded, the obnoxious effects, as well as the radius of these effects, increases. Likewise, if the facility is operating at less-than-full capacity, the social cost decreases accordingly. Fig. 2 shows the social cost function used in this paper. Note, however, that any social cost function could be easily substituted.

The social cost function takes the following form:

$$\text{Minimize } \; w_2(x)$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{S}\left(\frac{l_i}{c_i}\right)\begin{cases} M_i & \text{if } d(x_i, a_j) \le \left(\frac{l_i}{c_i}\right)d_{i,1} \\ M_i - m_i(d(x_i, a_j)) & \text{if } \left(\frac{l_i}{c_i}\right)d_{i,1} \le d(x_i, a_j) \le \left(\frac{l_i}{c_i}\right)d_{i,2} \\ 0 & \text{if } \left(\frac{l_i}{c_i}\right)d_{i,2} \le d(x_i, a_j) \end{cases} \tag{2}$$

M, m, d_1, d_2 are provided based on the specifics of the problem.

Note that the obnoxious effects of a facility can extend to fixed points that are not assigned to it.

2.3. Constraints

The two objectives mentioned above are subject to the following constraints:

$$\sum_{i=1}^{n} y_{ij} = 1, \; \forall\, j = 1, \cdots, S \tag{3}$$

$$n \le N \tag{4}$$

where N is the upper bound on the number of facilities allowed.

Constraint (3) ensures that each site is assigned to only one facility. Constraint (4) bounds the number of facilities that can be placed. Note that the number of facilities (n) to be placed is a decision variable that is bound from above by the parameter N. The number of facilities in a solution are not determined beforehand and the optimization algorithm is free to add or remove facilities as needed.

2.4. Allocation strategy

Demand points are assigned to their nearest facility regardless of the facility's capacity. Excess capacity is penalized by the cost functions which increase or decrease based on the total work load.

3. Evolution strategies (ES)

As Yapicioglu et al. (2007) showed, meta-heuristics, such as the particle swarm optimization (PSO) used there, are effective in solving the location problem in continuous space. In this paper, an Evolution Strategies (ES) algorithm is used instead of a PSO. Evolution Strategies are population-based evolutionary algorithms developed by Rechenberg and Schwefel commonly used for continuous spaces (Beyer & Schwefel, 2002). ES algorithms are good at exploring continuous search spaces because they rely on a crossover mechanism together with a perturbation mutation which together provide a good balance between global and local search capabilities (Beyer & Schwefel, 2002). The ES is also more controllable than PSO because the configuration parameters of the algorithm evolve together with the solutions. PSO algorithms have also been shown to converge on the region of optimality very quickly, but then struggle to continue the search at a more fine-grained level, at which point ES algorithms tend to outperform them (Angeline, 1998; Shi & Eberhart, 1999).

Individual solutions consist of the solution variables $x_i \in \mathbb{R}, (1 \le i \le n)$ together with the strategy parameters σ_i, $(1 \le i \le n)$. By including the parameters in the solution encoding, they are self-adapted during the search. For a detailed discussion of evolution strategies, the reader is referred to (Back & Schwefel, 1996).

The evolutionary strategy used in this paper is a (μ, λ)-strategy with discrete dual recombination of the solution variables, and intermediate dual recombination of the parameters, followed by a normally distributed perturbation mutation. The (μ, λ) notation denotes that the population at time t consists of μ individuals and that λ offspring solutions are generated at each time step. Of those, the μ best are preserved for the next generation. Discrete dual recombination means that each child variable is chosen with equal probability from each parent. Intermediate dual recombination means that the child's parameters (the σ_i corresponding to each x_i) are the average of the two parents' parameters. The mutation operator is defined as follows:

$$x_i(t+1) = x_i(t) + N(0, \sigma_i), \; i = 1, \cdots, n$$

Solutions are encoded as arrays of size $4n$, where the first $2n$ elements represent the x and y coordinates of the solution's facilities, and the following $2n$ elements represent σ_{x_i} and σ_{y_i}. The value of n varies across solutions based on the number of facilities that have been added. Consider a solution where there are two facilities ($n = 2$), the first located at (5,4) and the second at (3,2) and the strategy parameters are $\sigma_{x1} = 0.5$, $\sigma_{y1} = 0.25$, $\sigma_{x2} = 0.3$, $\sigma_{y2} = 0.35$. The encoding for this solution would be an array with $4n$ elements: $[5, 4, 3, 2, 0.5, 0.25, 0.3, 0.35]$. If a new facility is added to the solution the new array would contain 12 elements. The initial positions of the facilities, as well as the initial values of the strategy parameters, are randomly generated when a new facility is placed.

This configuration of the evolutionary strategy was chosen after preliminary trials showed that it performed better than the alternatives. The pseudocode for the proposed ES is shown in Fig. 3.

The Pareto Front was constructed by adding the child solution and comparing it, one by one, with those in the Pareto set. Dominated solutions are removed from the Pareto set. If a solution in the set dominates the child solution, then the child solution is removed from the Pareto set. Trial runs of the algorithm showed that this approach produces good Pareto fronts spanning the entire range of values for both objective functions. Though runtime increases due to the larger size of the Pareto front, it does not occur on such a scale as to require implementing more advanced techniques to reduce its size.

4. Computational experience

The bi-objective ES was used to solve two single facility placement problems from the literature to test its performance, and two capacitated multi-facility location problems from the real world: the placement of fire stations in Auburn, Alabama, a college town in the United States, and the placement of solid waste transfer stations in Mexico City. The results from our model are compared to the real-life placement of the facilities. For the multi-facility problems, two different placement strategies were used. The first was a sequential placement where a facility is added, its best location is determined and then its position is fixed before the next facility is added. This strategy attempts to roughly emulate the placement of facilities over time as demand grows. The second strategy placed all facilities simultaneously.

The first two problems are labeled problem 1.1 and 4 in Yapicioglu et al. (2007), while the two multi-facility instances were developed by the authors. The main characteristics of each problem are shown in Table 1.

For the single facility problems, the facility parameters were set as in Yapicioglu et al. (2007). For the multi-facility problems, the parameters are shown in Table 2. All facilities were assumed to be equal

```
Procedure ES{
   t=0
   Generate random population P(t)
   Evaluate solutions in P(t)
   Initialize Pareto Front
   While not done{
         For i = 1 to lambda{
               Select two parents at random from P(t)
               Recombine parents to generate child i
               Mutate child i
               Evaluate child i
               Add child i to Pareto Front
               For each solution sol in Pareto Front
                     If child i dominates sol then
                           Remove sol from Pareto Front
                     Else
                           Remove child i from Pareto Front
         }
         P(t+1) = Update Population
         t++
   }
}
```

Fig. 3. Pseudocode for the ES algorithm.

Table 1
Problem Characteristics.

Problem	Demand points	Maximum facilities	Source of problem data
Problem 1.1	7	1	(Yapicioglu et al., 2007)
Problem 4	45	1	(Skriver & Andersen, 2003)
Auburn	33	9	
Mexico City	7876	16	

Table 2
Facility Parameters.

	Auburn	Mexico City
c_i	1500 requests	1000 tons
M	1000	100
m	1	1
d1	0.5 miles	0.5 miles
d2	1 mile	1 mile
Setup cost	$110,000	$2,500
Operating cost	$1,200,000	$10,000

though the model can also accept heterogenous facilities.

Operating costs are annual and setup costs are annual equivalent costs. The setup and operating costs for the Auburn problem were estimated using the city of Auburn's 2017 budget (available at https://www.auburnalabama.org/budget/). The construction cost of a fire station is assumed to be $1,500,000 and is annualized using a 4% annual interest rate over 20 years. Operating costs are obtained by dividing the annual cost of the fire department (a little over $6,000,000) divided by the five fire stations currently operating in Auburn. For the Mexico City problem, costs are per ton, and were obtained from the Environmental Protection Agency (EPA, 2002).

The ES was developed in VB.NET. The single facility problems were run on an Intel Core i7-8550U CPU at 1.8 GHz with 16 GB of RAM and Windows 10, while the multiple facility problems were run on an Intel Xeon 3.5 GHz CPU, with and 64 GB of RAM and Windows 7.

4.1. Cost functions

The non-social cost functions used for each problem were the following:

For the fire station problem, the distance-based function used to calculate the non-social cost was based on the findings of Challands (2010) where the cost of fire damage as a function of response time is studied. We modeled the cost of fire damage as a linear function with slope equal to 4000, representing a $4000 increase in damage costs for every minute that passes between the beginning of the fire and the arrival of the firefighting squad. To calculate the arrival time, we assumed that a fire truck travels at an average speed of 35 mph.

For the transfer station problem, the distance-based function was calculated using the per ton costs found in Environmental Protection Agency (EPA, 2002). The total cost included transportation from fixed sites to their assigned transfer station, and transportation costs from each transfer station to its nearest available final disposal site.

4.2. Results

4.2.1. Single facility problems

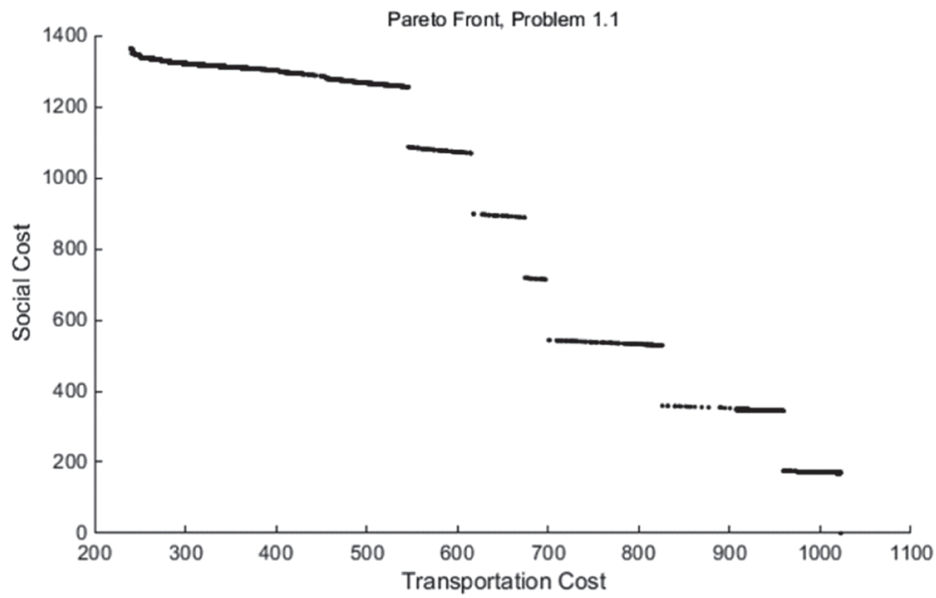
For the single facility problems, the ES approach found more non-dominated solutions than the bi-PSO approach presented in Yapicioglu et al. (2007) and the approach of Skriver and Andersen (2003). Figs. 4 and 5 show the results obtained by the ES algorithm compared to the PSO algorithms.

Table 3 shows a comparison of the results obtained using the PSO and the ES in terms of number of solutions in the Pareto set and algorithm runtime (in seconds).

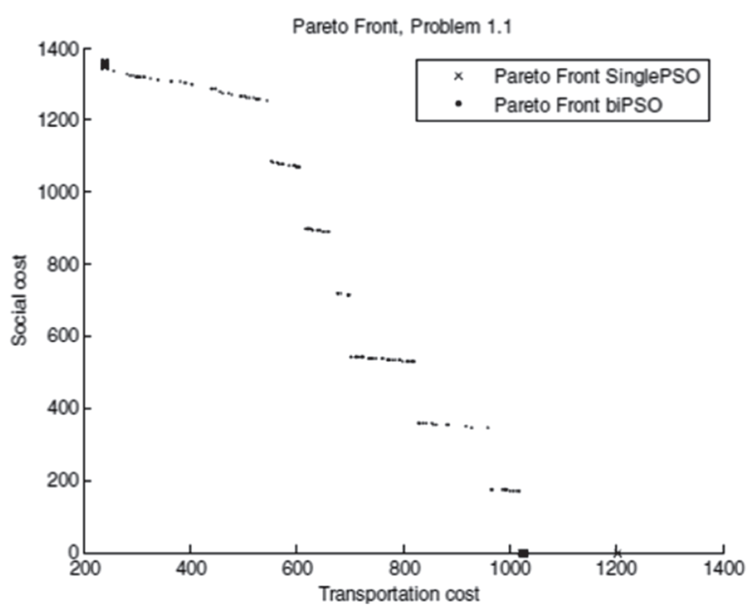
In terms of computational time, the ES algorithm runs much slower than the PSO as reported in Yapicioglu et al. (2007) for both problems. This is due in part to the slower CPU used for the ES. Another reason for this difference is that the ES manages a significantly larger Pareto set than the PSO, as shown by the larger number of non-dominated solutions found by the ES. It seems that the runtime is being driven mainly by the Pareto set because though Problem 4 is much larger than Problem 1.1, the runtime for the ES for both problems is very similar. Though the runtimes for the ES are larger than those for the PSO, they are still reasonable for the type of problem being addressed, since an urban planning problem such as this does not require finding solutions very quickly. These results show that the ES is a very effective optimizer for this problem.

4.2.2. Multiple facility problems

The multiple facility problems were solved using two different facility placement strategies. The first strategy is called "Simultaneous placement" and the search proceeded by adding and removing facilities (up to the maximum allowed) as part of the solution mutation. The



a) ES



b) PSO (Yapicioglu et al., 2007)

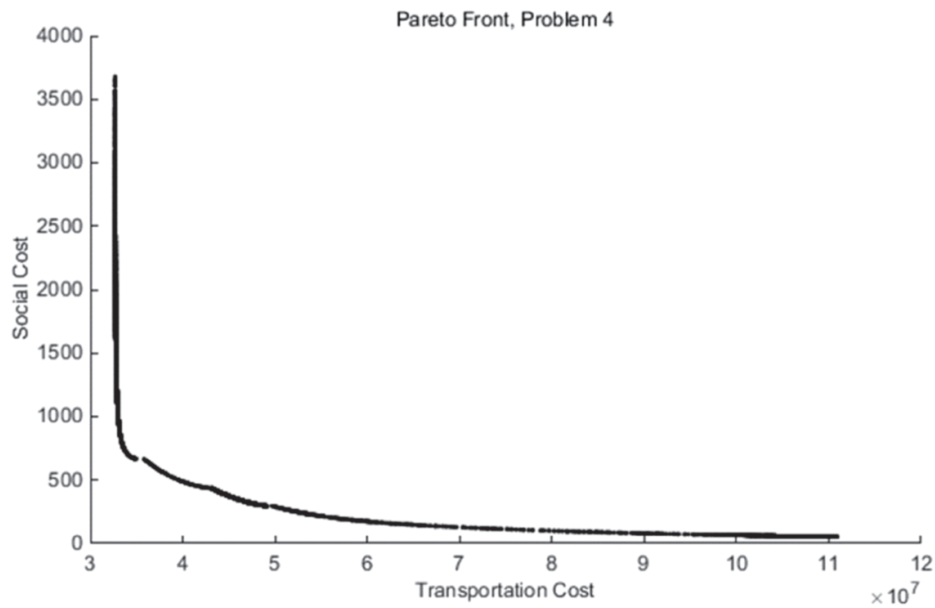
Fig. 4. Pareto Front Problem 1 Comparison.

second strategy is called "Sequential placement". In it, a new facility was placed after a certain number of generations and all previous facilities were fixed. The latter strategy most resembles the placement procedure used in real-life, where facilities are added one at a time as needed. The results produced using both strategies were then compared among themselves and with the actual locations of the facilities from these real-world examples.

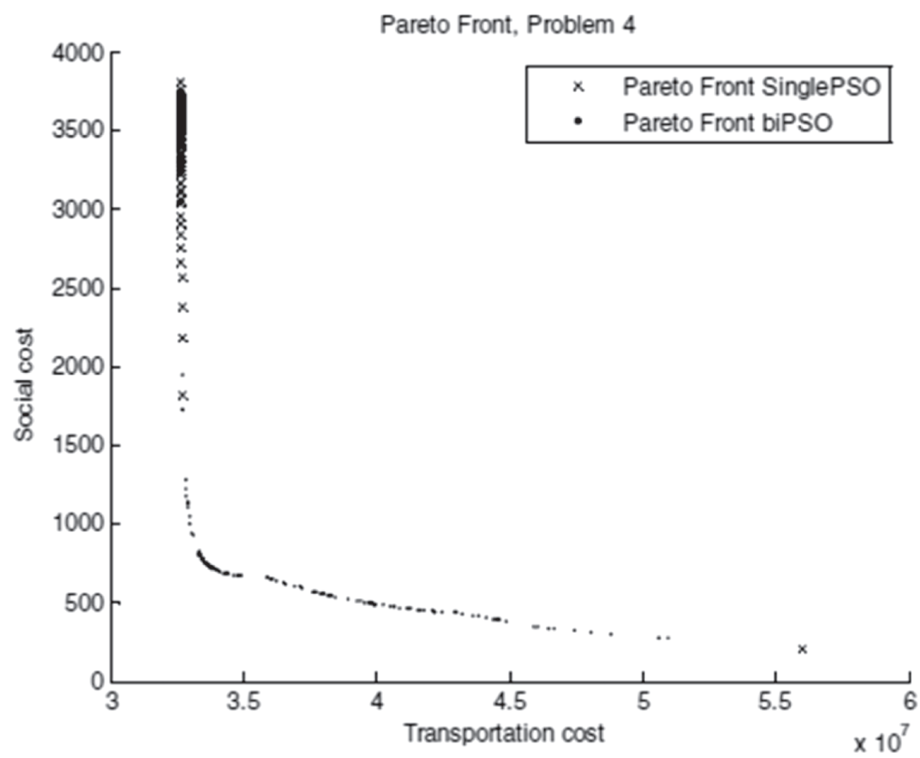
4.2.2.1. Auburn fire stations problem. The city of Auburn currently has five fire stations. The actual locations are shown in Fig. 6.

Both the simultaneous and the sequential strategies produced solutions that were far better than the actual, real-world placement. For the sequential approach, the total number of facilities that met demand without exceeding capacities ranged between 5 and 8. For the simultaneous strategy, the number of facilities went from 5 to 9. The solutions found using the simultaneous placement strategy dominated all the solutions obtained using the sequential strategy, and it found solutions that were more spread out, as can be seen in Fig. 9. However, the sequential placement strategy found many more non-dominated solutions, but all are very similar.

An analysis of the solutions found using both strategies reveals why those generated using the simultaneous strategy dominate those obtained using the sequential strategy. Under the sequential strategy, the first facility is consistently placed in a central position (the downtown area). With a single facility non-social costs far outweigh the social costs, so that the pull objective dominates, drawing the facility close to the demand sites. As additional facilities are added, the social costs begin to have a major impact, leading to a pushing of the facilities towards the peripheries. However, since previously added facilities are fixed, their negative impact on the social cost remain. Improvements are due to the newly added facilities reducing the non-social costs. Fig. 7 shows typical facility distributions using both strategies. The



a) ES



b) PSO

Fig. 5. Pareto Front Problem 4 Comparison.

Table 3
PSO vs. ES Performance by Number of Non-Dominated Solutions Found and CPU Runtime.

	Bi-PSO$_{cc}$		Bi-PSO$_{wocc}$		Bi-PSO$_{hyb}$		ES	
Problem	Sols.	Runtime	Sols.	Runtime	Sols.	Runtime	Sols.	Runtime
Problem 1.1	148	20.633	126	19.853	143	20.181	7211	160.32
Problem 4	148	54.976	114	53.268	142	52.533	5687	175.42

Fig. 6. Auburn City Fire station locations.

a) Sequential placement b) Simultaneous placement

Fig. 7. Sequential vs. Simultaneous facility distribution.

simultaneous strategy, on the other hand, allows the search algorithm to move previously added facilities so that both the pull and push objectives have a more balanced influence in the search. Hence, poor initial placements can be corrected as the search progresses, so that both the social and non-social costs can be improved. This results in a distribution of facilities that is less centralized than its sequential counterpart.

Despite it yielding worse results, the sequential placement strategy better reflects the reality of how the facilities were placed over time. The location of the different facilities using the sequential strategy is similar to the actual placement, as shown in Fig. 8, for the case of five facilities. The differences can be explained by the fact that our model

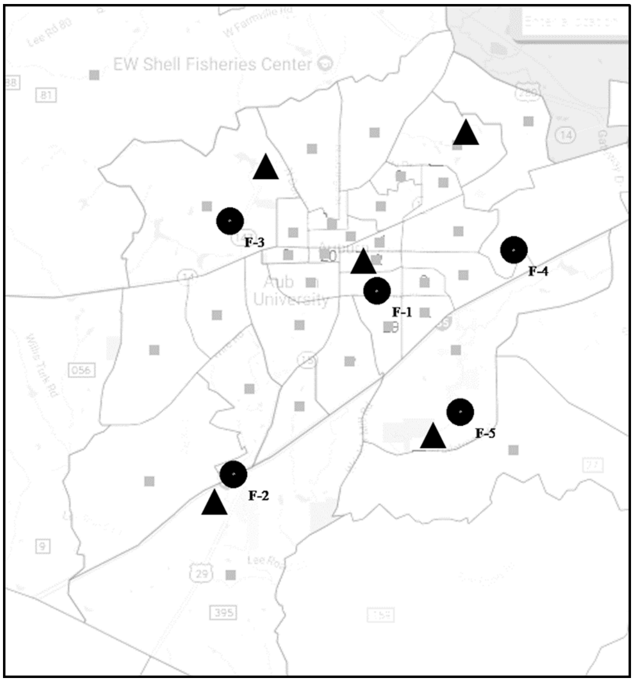


Fig. 8. Sequential placement locations vs. actual locations (triangles represent the actual locations).

does not consider the road network or the availability of locations, whereas the actual fire stations are all placed along important roads. That said, the continuous-space solutions found by the proposed method provide a decision maker with valuable information about the relative location of the facilities within the city. With these solutions in hand, the decision maker can then pick among the available locations found nearby.

An evaluation of the solutions that form the Pareto front reveal some additional insights into the effect of different distribution patterns. Solutions found in region A of the front (see Fig. 9) tend to have two traits: (1) they have a higher number of facilities, and (2) they concentrate the facilities in the center. By doing so, they reduce the total non-social costs at the expense of a much higher social cost. Solutions on the other extreme of the front (area B), tend to have less facilities and place them in the surrounding areas of the city, thus reducing social cost while increasing non-social costs. Due to the behavior of the sequential strategy which places facilities in the central areas first, before extending towards the outside, its solutions tend to be in region A. Hence, the Pareto Front for the sequential strategy is skewed towards higher social costs and does not spread out as far as the one for the simultaneous strategy.

4.2.2.2. Mexico city transfer stations problem. Mexico City is divided into 16 municipalities and, as of 2016, had 12 solid waste transfer stations located throughout the city. The solid waste picked up at dumpsters from businesses and private homes is first gathered in a "collection point". There are close to 8000 points throughout the city.

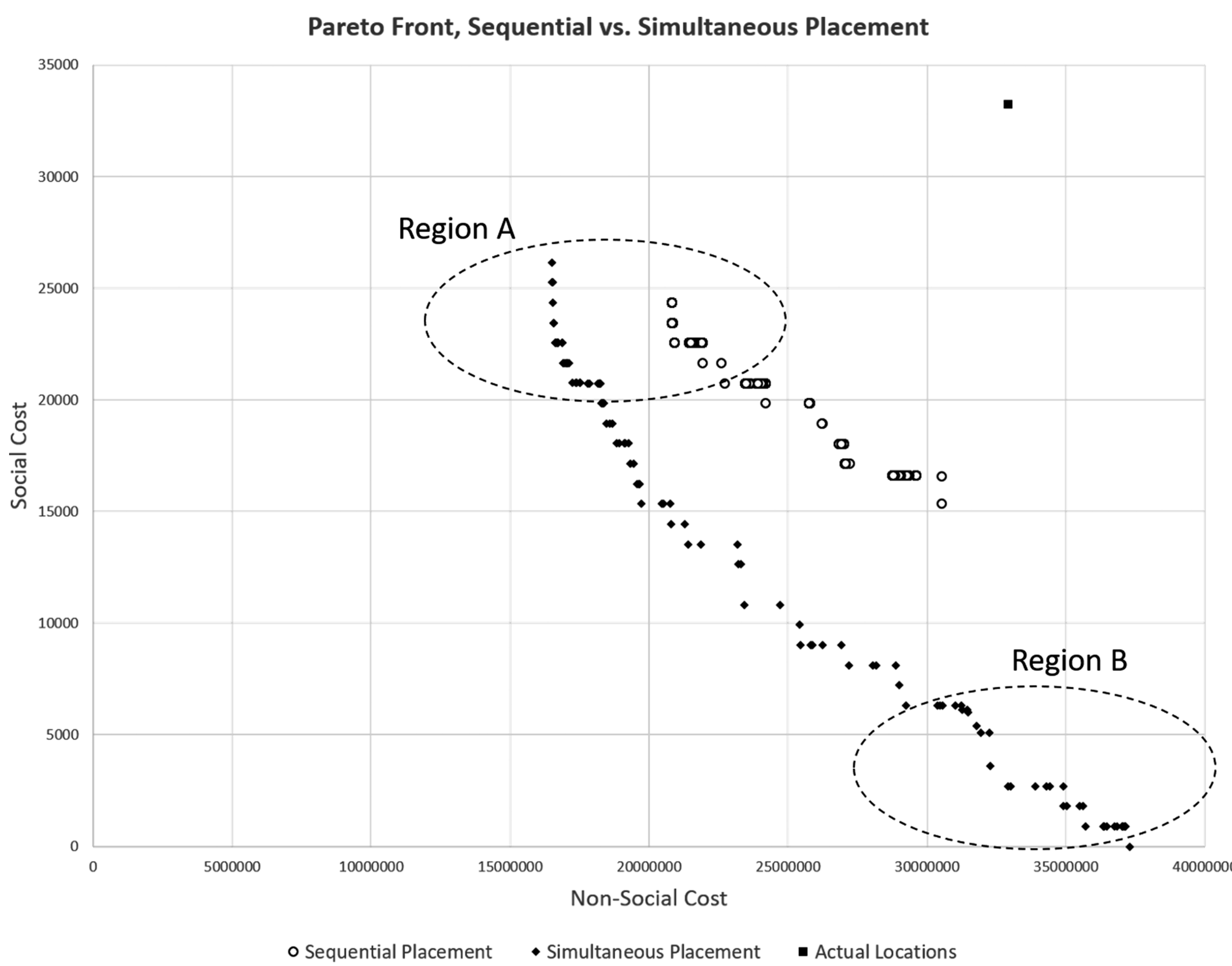


Fig. 9. Pareto Front, Auburn Fire station problem.

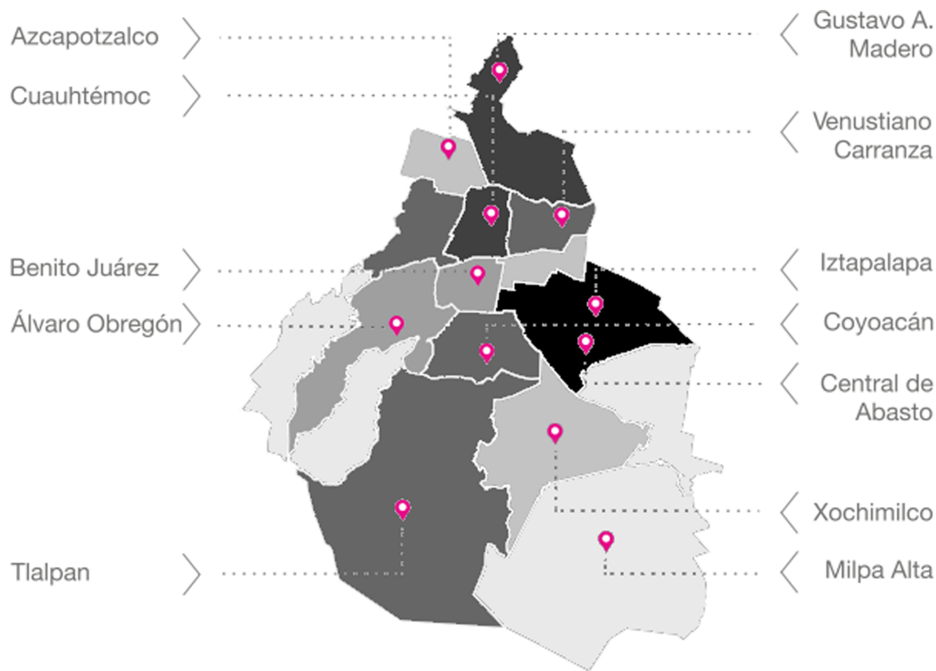
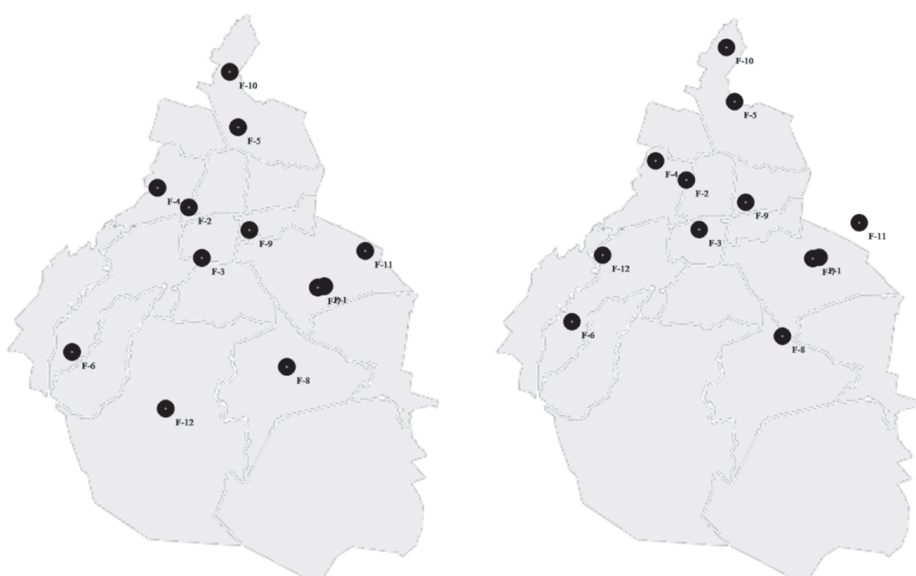


Fig. 10. Location of transfer stations in Mexico City with demand levels. Darker areas are those of higher demand. (Secretaría del Medio Ambiente de la Ciudad de México, 2016).



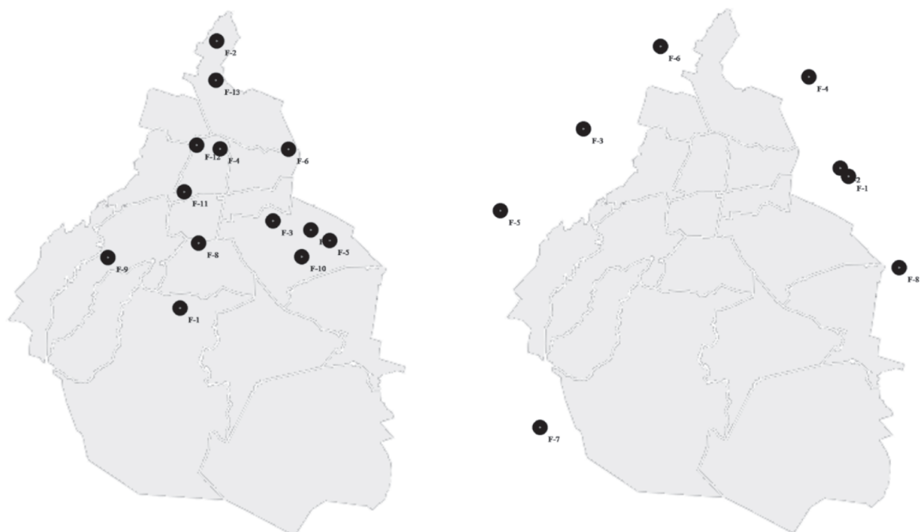
a) Sequential strategy b) Simultaneous strategy

Fig. 11. Example solutions with 12 facilities.

The waste is then moved to one of the 12 transfer stations, where it is separated, processed, and then sent to one of five different final disposal sites located outside the city. The current locations of the 12 transfer stations are shown in Fig. 10. The number of collection points per municipality is publicly available (Secretaría del Medio Ambiente de la Ciudad de México, 2016), but their exact location is not, so it was generated randomly when constructing the problem.

As with the fire station case, both placement strategies found better solutions than the actual current placement. The simultaneous placement strategy also yielded solutions dominating all the ones from the sequential placement, as seen in Fig. 13. And like in the previous problem, the Pareto front is more widespread, even when it found less non-dominated solutions that the sequential case. Like with the fire station case, the initial placements using the sequential strategy tend to be in areas where non-social costs are minimized. Since the demand in the city is skewed heavily towards the east and north-east, these locations tend to be in that general direction. Once placed, they are not moved, leading to lesser improvements than with the simultaneous strategy. 12 facility solutions found under both strategies are shown in Fig. 11.

For the simultaneous placement strategy, the minimum number of facilities required to satisfy demand without exceeding capacity was 11, and the highest number of facilities placed was 16. For the sequential strategy, the minimum number of facilities was also 11, but the highest found had 13.



a) Solution with least non-social cost

b) Solution with least social cost

Fig. 12. Extreme solutions.

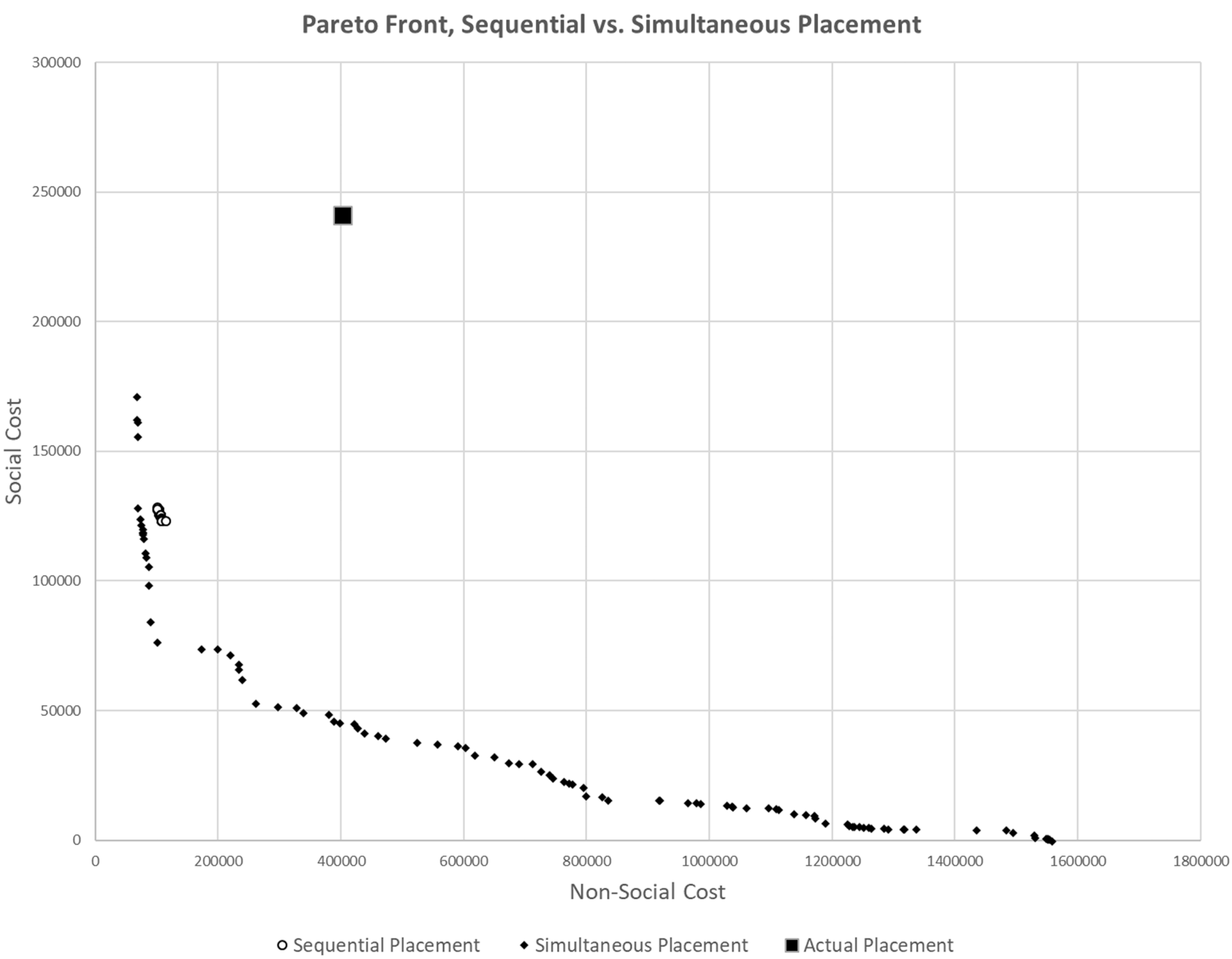
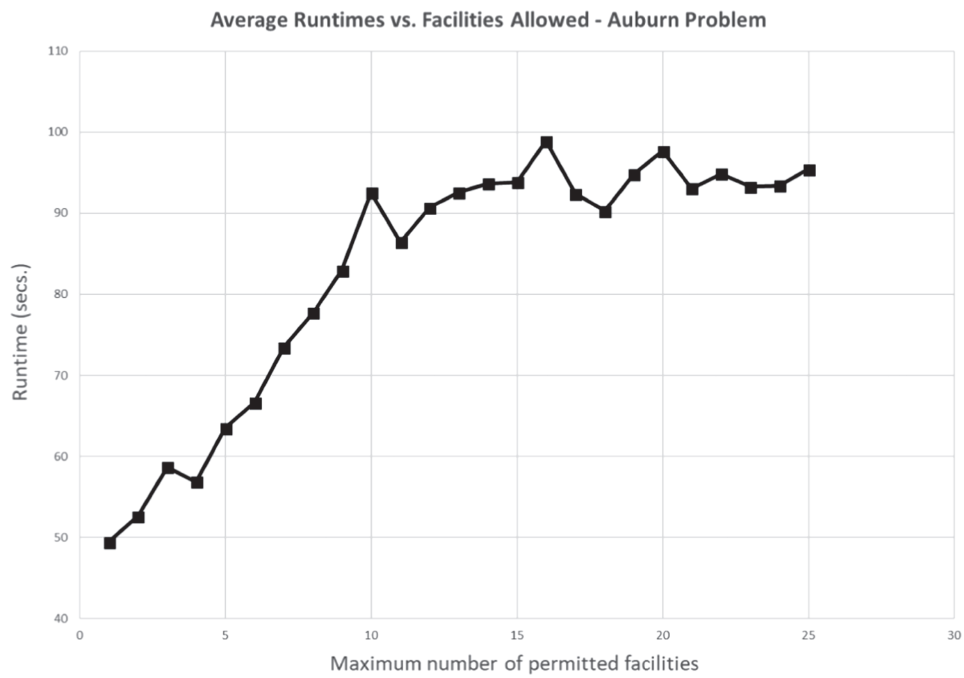
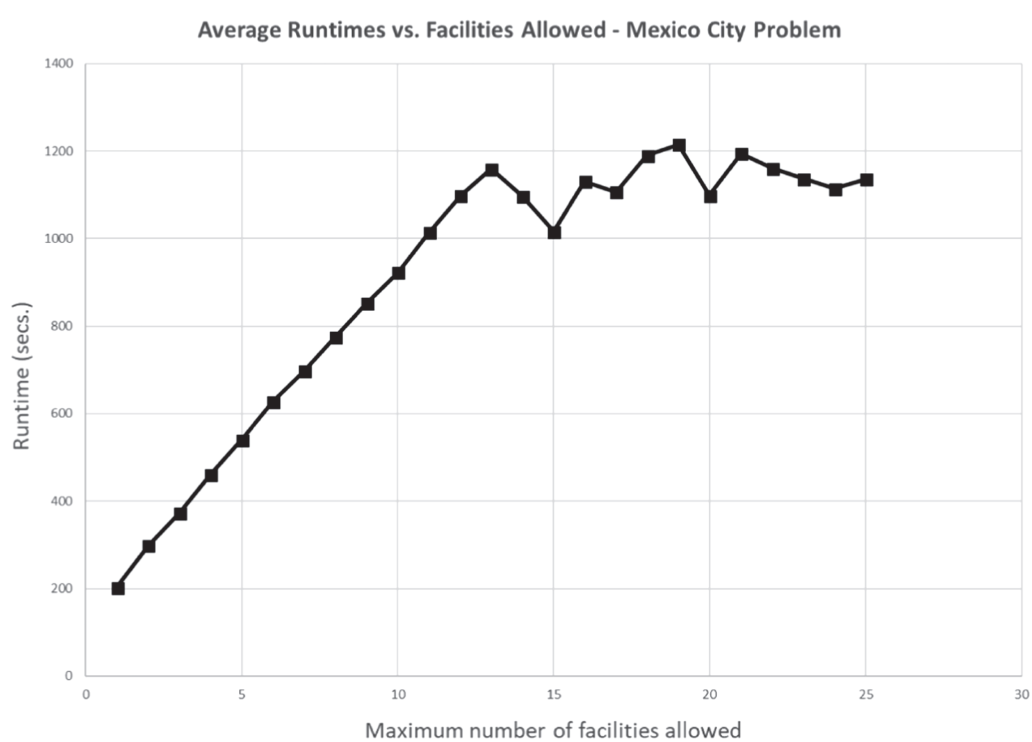


Fig. 13. Pareto Front, Mexico City Transfer station problem.



a) Runtime vs. facilities allowed for Auburn problem (fire stations)



b) Runtime vs. facilities allowed for Mexico City problem (transfer stations)

Fig. 14. Runtime as a function of maximum number of facilities.

As with the fire station instance, the solutions located on the left side of the Pareto front result in more centralized distributions, while the ones on the right tend to have more dispersed distributions, often with less facilities. The most extreme solutions are shown in Fig. 12. Note also that the number of facilities placed is lower for solutions with low social cost, while it is higher for those with low non-social costs. In this case, the least social cost solution placed 8 facilities, while the least non-social cost one placed 13.

As opposed to the fire station problem, the location of facilities under both strategies differed significantly from the real locations. Our model did not consider several factors that influence the final placement decision, such as availability of land (which is more significant in the case of a transfer station than a fire station), political boundaries, as well as the road network.

The political component seems to be the most significant factor in this divergence. Unlike the city of Auburn which is governed by a single authority who can make decisions at the scale of the entire town, Mexico City is governed by the city government, as well as by 16 municipal authorities, each with a certain degree of independence. The political impact can be seen in what follows: our model avoids placing transfer stations in the southern part of the city where both population and waste generation are lower and spread over a vast area. In real-life, however, the local governments in those areas would not easily permit their municipality to remain without a station of its own. In accommodating those political realities, the real-life solution must necessarily result in a "less" efficient placement. The model, however, still provides urban planners with important insights into where the best locations for stations should be, and that information can be used as part of the decision of placing new facilities in the future.

4.2.3. Computational effort

The ES runs in a reasonable amount of time for both problems. For the Fire Station problem, an average run of 25,000 generations is completed in 535 s. For the Mexico City problem, an average run of 2500 generations is completed in 6258 s, which is less than two hours.

Total runtime is a linear function of the number of demand points (as shown in Yapicioglu et al., 2007) and, up to a point, of the number of permitted facilities. Fig. 14 shows the runtimes as a function of the maximum number of facilities permitted for each problem. For the Auburn problem, shown in Fig. 14(a), the runtimes increase until around 9 or 10 facilities are permitted, at which point they stabilize. For the Mexico City problem shown in Fig. 14(b), this point is reached at around 11 or 12 facilities. This indicates the point at which constraint (4) ceases to be a tight constraint, that is, once 10 facilities have been added (for the Auburn problem), increasing the number of facilities in a solution does nothing to reduce costs. This finding was unexpected but is significant. The proposed algorithm not only finds solutions to the location problem but can also help decision makers determine an upper bound on the total number of facilities that are necessary, given the current level of demand.

5. Conclusions

This paper presents a new model for the multiple, capacitated, semi-obnoxious facility location problem on a Euclidean plane, a problem that arises in practice but has not been addressed in the literature before. The model developed in this paper is general enough to fit varying types of location problems, with cost functions that are not limited to transportation costs. Two real-world problems are solved using the approach described, one for placing fire stations, and one for solid waste transfer stations. The cost functions used to solve these two problems differ from those in the literature in that they are not exclusively transportation cost functions, but also include other factors such as setup cost and the effects of overloading the facility. In the case of the fire station location problem, the cost function is an expected damage cost dependent on response times.

The solution method proposed is a bi-objective evolutionary strategy algorithm. The results found show that this method is effective at finding non-dominated solutions for both problems. The fire station problem is a medium-sized problem (compared to problems found in the literature), with 33 demand points and up to 9 facilities. The transfer station problem is a very large one, with close to 8000 demand points and up to 20 facilities. This problem instance is significantly larger than any problem found in the literature. The algorithm solved both problems in a reasonable amount of computational time, proving its effectiveness at addressing real-world location problems. This approach could be extended to three dimensions easily and could accommodate a variety of relationships describing the effects of the obnoxious aspects of the facility. Despite the limitations inherent in using a planar model, the results obtained show that this method can provide valuable insights and can be a useful tool for urban planners, and that, given certain conditions, the resulting locations are similar to those found in the real-world.

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