An Efficient Algorithm for Using a Perimeter Distance Metric in Unequal Area Facility Layout

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Abstract

The unequal area facility layout problem has been studied using several distance metrics in the objective function, most commonly rectilinear distance and Euclidean distance between departmental centroids. This paper modifies the distance metric to consider material travel along the perimeter of the departments to and from input/output locations. This perimeter distance metric is used in the objective function to produce facility layouts that optimize simultaneous design of departmental shapes, departmental placement and location of the input/output points. The perimeter distance metric is more reflective of actual material flow than is the centroid metric, and is calculated using a network formulation that is quite computationally efficient. The new distance metric and its use in facility design are illustrated using several test problems from the literature.

Keywords
Facility design, facility layout, distance metric, material handling, network optimization

1. Introduction

Facility layout problems are a family of design problems involving the partition of a planar region into departments or work centers of given area, so as to minimize the costs associated with projected interactions between departments. These costs usually reflect material handling costs among departments. Such problems occur in many organizations, including manufacturing cell layout, hospital layout, and service center layout. The problem primarily studied in the literature has been "block layout" which only specifies the placement of the departments, without regard for aisle structure and material handling system, machine placement within departments or input/output (I/O) locations. Block layout is usually a precursor to these subsequent design steps, termed "detailed layout." Two recent survey articles on the facility layout problem can be found in [10] and [11].

The problem was originally formalized by Armour and Buffa [2] as follows. There is a rectangular region, R, with fixed dimensions H and W, and a collection of n required departments, each of specified area a_j and dimensions (if rectangular) of h_j and w_j, whose total area, $\sum_j a_j$ = A = H×W. There is a material flow F(j,k) associated with each pair of departments (j,k) which generally includes a traffic volume in addition to a unit cost to transport that volume. There may also be fixed costs between departments j and k. F(j,k) might also include inter-floor costs. The objective is to partition R into n subregions representing each of the n departments, of appropriate area, in order to:

$$\min \qquad Z = \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} F(j,k) d(j,k,\Pi) \qquad (1)$$

where d(j,k,Π) is the distance (using a pre-specified metric) between the centroid of department j and the centroid of department k in the partition Π. This centroidal distance is easy to calculate and it is intuitive in that the mass of material is considered to move from center to center of departments along the shortest rectilinear (Manhattan) or Euclidean distance. However, the centroid distance metric is not realistic in that ignores the aisle structure that will be present in all facilities, where the aisles are normally located along the departmental perimeters and connect I/O points in each department. Limited work has been done to improve upon the centroid to centroid distance metric; distance along aisles [5, 18] and expected distance using integration [6]. The recent work of Benson and Foote [5] in particular, considers the placement of aisles and I/O points *after* the relative location of the departments and the general aisle structure have been selected. Related work on integrated facility layout that considers machine placement includes papers by Nagi and others [7, 9]. This work uses predefined departmental shapes set on a grid covering the facility space. In [7], Dijkstra's shortest path algorithm is used to calculate the rectilinear distance to and from pre-specified I/O points. In [9], I/O points are placed during the optimization and a corner constraint is imposed to encourage paths that are straight. Both papers use a simulated annealing heuristic to alter departmental placement. Another strongly related work is by Banerjee et al. [3], where a genetic algorithm searches over design skeletons that are then optimized using a subordinate mathematical programming routine. The number of I/O's per department is pre-specified and then optimally located with the departmental placement. Rectilinear distance (but not necessarily along departmental perimeters) is calculated between I/O points.

This paper seeks to improve the centroid to centroid distance metric during facility design by substituting a perimeter distance metric. If aisles have negligible area compared to the plant area and aisle capacity and direction of flow are not considered (i.e., two way flow through each aisle is allowed), I/O points can be placed concurrently with block layout, producing a one stage optimization procedure that considers material flow from I/O to I/O along departmental perimeters. This still does not achieve the ideal situation where a true aisle structure will be also be optimally designed concurrently. This simplification, instead, assumes that all perimeters are legitimate aisles.

2. Basic Formulation and Solution Methodology

The basic assumption is of rectangular departments of specified area within a rectangular bounding facility that is equal to, or larger than, the sum of the departmental areas. The formulation used is "flexbay" by Tate and Smith [12, 15-17] that is a more restrictive version of a slicing tree formulation [13, 14] (see Figure 1). Flexbay makes cuts in a single direction to establish a set of bays that can vary in area. The bays are then subdivided into departments. The flexbay encoding can enforce both departmental areas and departmental shapes, through use of a maximum aspect ratio constraint or a minimum departmental side length constraint for a stated departmental area.

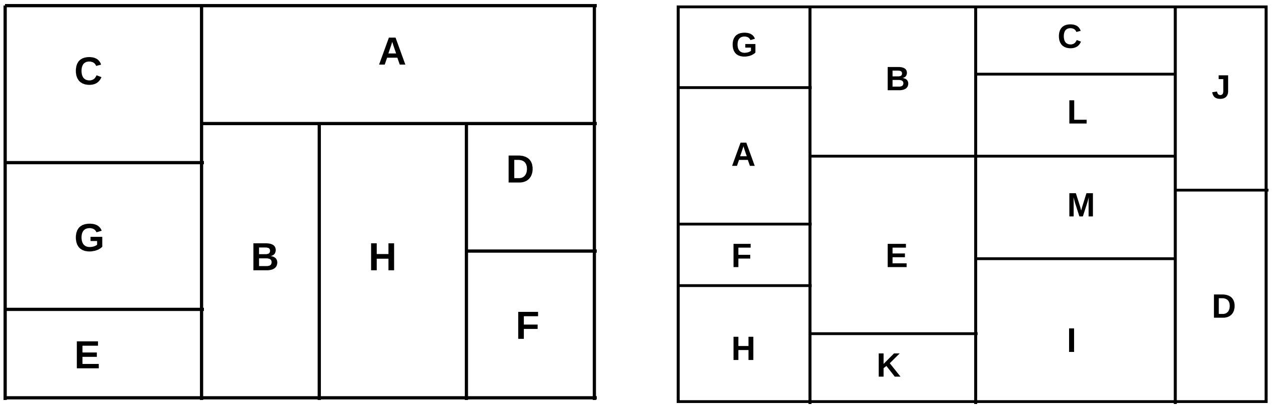

Figure 1. Typical slicing tree (left) and flexbay (right) layouts.

2.1 Optimization Methodology

To find the optimal or near-optimal block layout, a genetic algorithm (GA) is used with the flexbay formulation. The genetic algorithm works with an encoding of the layout where there is a one to one correspondence between each encoding and each layout. The encoding is a permutation of departments that specifies their order within the layout, with a concatenated string indicating where the bay breaks within the permutation occur. For example, the flexbay layout of Figure 1 would be represented with the following encoding:

G A F H B E K C L M I J D H K I

Where the last three characters indicate bay breaks after departments H, K and I. It is easy to see that there is a one to one correspondence between all possible layouts and the encodings.

The GA begins with a randomly generated set of solutions (the *population*) and modifies them through recombination (*crossover*) and perturbation (*mutation*). Crossover is accomplished through a variant of uniform crossover and mutation consists of permutation altering, or adding or deleting a bay. The original Tate and Smith paper [16] includes the details of these. Solutions are selected for crossover using a rank-based quadratic method (see [16]) and the worst solutions are deleted during each generation to maintain a constant population size.

2.2 I/O Location and Distance Metric in the Objective Function

The version of I/O location that is considered in this paper is where unlimited I/O's per department are allowed. This might seem unrealistic, but due to the perimeter metric, it can be readily verified that the set of candidate I/O points for a department can be limited to the locations where that department intersects the corner of any of its adjacent departments. This set of I/O points represents a dominant set and therefore the algorithm can be limited to consider only these points as potential I/O locations. Using the example of Figure 1, the candidate I/O points would be as shown in Figure 2 on the left. To clarify the perimeter distance metric, if the I/O's used were as shown in Figure 2 on the right, the material will traverse over the perimeters shown in the dashed line.

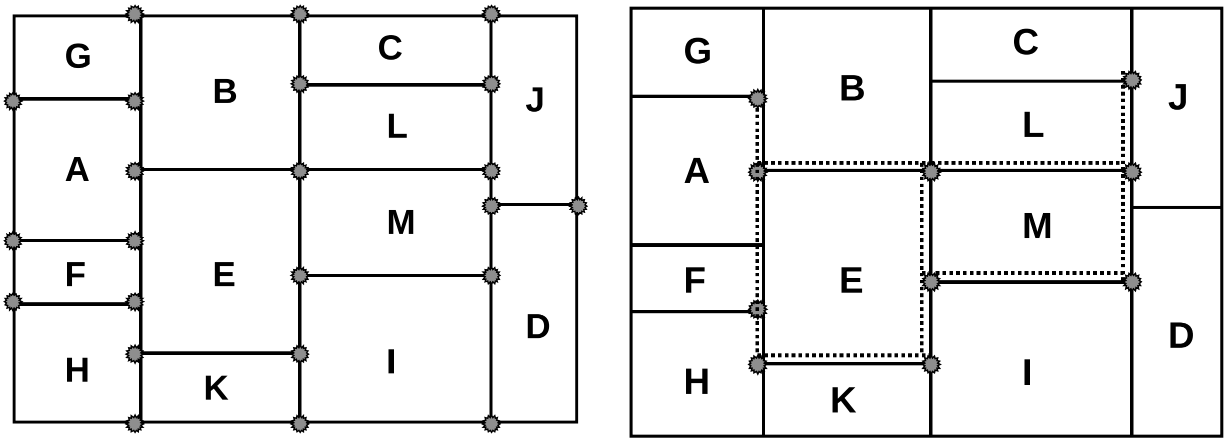

Figure 2. Possible I/O points on a flexbay layout.

If each department can have an unconstrained number of I/O stations then the interdepartmental aisle travel distances can be found by formulating this problem as one of finding the shortest path on a network. The flexbay representation facilitates the development of this model due to the inherent bay structure imposed on the layout. All of the arc lengths in the resulting shortest path problem will be positive since they represent physical distances. The shortest path problem with positive arc lengths has been well studied in the network optimization literature and efficient algorithms exist for solving this problem [1]. This makes it possible to quickly evaluate the actual aisle travel distance for each layout that is generated during the search process. The objective function is:

$$Z(\Pi) = \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} F_{j,k} d_{j,k} + m^3 \left(Z_{feas} - Z_{all} \right) \tag{2}$$

where m is the number of departments in layout Π which violate the aspect ratio constraint, Z_{feas} is the objective function value for the best feasible solution found so far, and Z_{all} is the unpenalized objective function value for the best solution found so far. In this case $d_{j,k}$ is defined as the shortest rectilinear distance along departmental perimeters between the I/O stations of departments j and k.

The flow of the algorithm is shown below in pseudocode:

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1. Initialize the population of chromosomes

2. For $j = 1$ to the maximum number of generations
    i. Select two parents based on fitness and perform crossover
    ii. Randomly select chromosomes for mutation and perform mutation
    iii. Evaluate the new offspring from crossover and mutation
        1) Determine the current layout and bay structure from the chromosome
        2) Identify infeasible departments
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        3) Construct a network of nodes corresponding to each department in the layout and its candidate I/O locations
        4) Find the shortest path between each pair of nodes in the network
        5) Using these shortest path distances, calculate the objective value using equation 2
    iv. Cull out the solutions with the worst objective function values

3. Print the final solution

3. Test Problems and Results

Several unequal area problems from the literature were solved in the manner described in Section 2. While the material flows, departmental areas and constraints are identical to those previously studied, results cannot be compared directly as the distance metric used previously was the centroid to centroid. The problems are from Bazaraa [4, 8] (14 departments) and Armour and Buffa [2] (20 departments). The ten department problem of van Camp et al. [19] was also studied, however there is a solution (optimal) where equation 2 is zero. In this layout, all departments are located adjacent to the departments with which they have material flow, and since there is no limitation on the number of I/Os, the distance traveled over the entire facility is zero. The GA settings were the same as in [16]: population size of 10, mutation rate of 50%, number of solutions generated = 600,000, and number of random seeds = 10.

Objective function values from the perimeter metric are in Table 1, where the best, median, worst and standard deviation over ten runs are shown. The twenty department Armour and Buffa (A&B) was studied with maximum aspect ratios of 10, 7, 5, 4, 3 and 2. The Bazaraa problem used a maximum side length of one as the shape constraint. For comparison using the Bazaraa 14 problem, the best layout using the perimeter metric is shown compared to the best layout from [16] using the rectilinear centroid to centroid distance metric in Figure 3. Also shown are the assumed I/O points and the assumed material flow paths inherent in each formulation. Note that department 14 is a “dummy” department with no flows, hence the lack of an I/O. It appears that the perimeter metric with I/O location on the boundaries of the departments is a better reflection of the physical movement of material for most manufacturing and service scenarios. The centroid method not only traverses through the interior of intervening departments, it assumes the minimum rectilinear distance between pairs of departments, creating nearby parallel paths as seen in departments 5 and 6. Layouts where the centroids were not located along the same line, as in departments 1 through 4, would create even more paths.

Table 1. Comparisons of Results over Ten Seeds.

Problem	**Best**	**Median**	**Worst**	**Standard Deviation**
Bazaraa 14	1343.2	1459.2	1607.9	92.8
A&B 20/10	757.1	862.9	1221.0	147.2
A&B 20/7	785.8	1079.9	1267.4	155.2
A&B 20/5	949.4	1319.6	2176.4	343.0
A&B 20/4	1025.7	1189.2	1758.1	236.3
A&B 20/3	942.6	1478.1	2298.5	354.0
A&B 20/2*	1244.1	1873.6	3359.8	787.2

* Over the six of the ten runs that found feasible layouts.

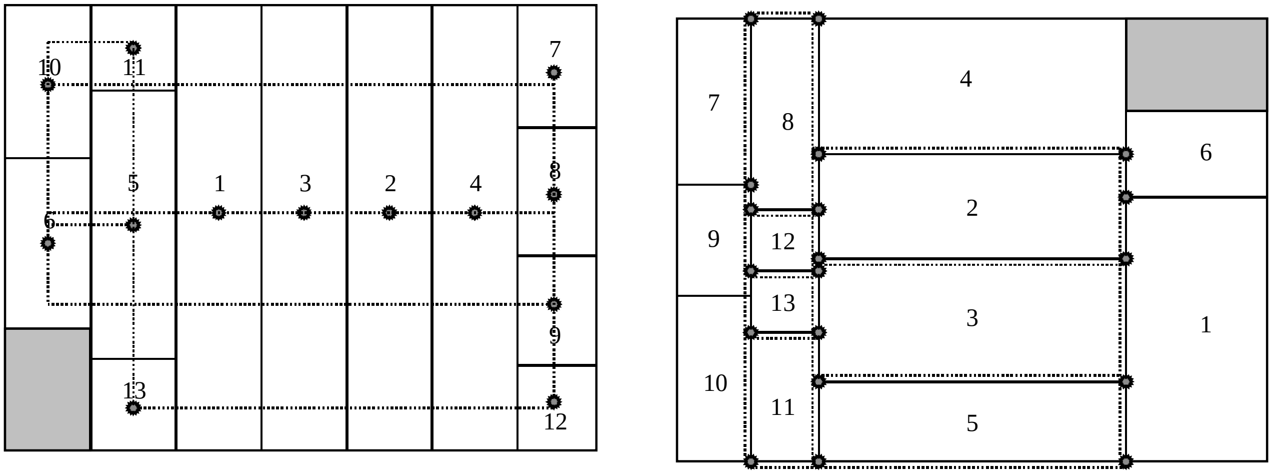

Figure 3. I/O points and flow paths (dashed) for the centroid distance metric (left) and the perimeter distance metric (right) for the Bazaraa 14 problem.

4. Conclusions

This paper introduced a new distance metric that can be incorporated into many versions of the facility layout problem that more accurately reflects material handling costs than does the popular departmental centroid to centroid distance metric. The perimeter distance metric is coupled with the location of the input and output locations of each department. This makes it possible to concurrently optimize four facets of the facility design problem: departmental location within the facility, departmental shape within certain constraints, I/O placement and travel paths along the departmental perimeters. This last item may be regarded as a precursor to development of a true aisle structure, where perimeter travel is further constrained. The solution methodology is based on a network algorithm that exploits the flexible bay structure within a genetic algorithm meta-heuristic framework.

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