CONSIDERING UNCERTAINTY IN UNEQUAL AREA BLOCK LAYOUT DESIGN

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Abstract - This paper presents an extended formulation of the unequal area facilities block layout problem which explicitly considers uncertainty in material handling costs by use of expected value and standard deviations of product forecasts. This formulation is solved using a random keys genetic algorithm (RKGA) heuristic with a flexible bay construct of the departments and total facility area. It is shown that depending on the attitude towards uncertainty of the decision maker, the optimal layout can change significantly. The seminal 20 department problem of Armour and Buffa is studied with varying numbers of products and amount of forecast uncertainty. The RKGA is shown to be a robust optimizor that allows a user to make an explicit characterization of the cost and uncertainty trade-offs involved in a particular block layout problem.

Keywords - facilities layout, block layout, genetic algorithm, production uncertainty

1. INTRODUCTION TO THE FACILITY LAYOUT PROBLEM

Facility Layout Problems are a family of design problems involving the partition of a planar region into departments or work areas of known area, so as to minimize the costs associated with projected interactions between these departments. These costs may reflect material handling costs or preferences regarding adjacencies among departments. There are problems which are strongly related to the facility layout problem that arise in other engineering design contexts such as VLSI placement and routing. All of these combinatorial problems are known to be NP-hard [6]. The problem primarily studied in the literature has been "block layout" that only specifies the placement of the departments, without regard for aisle structure and material handling system, machine placement within departments or input/output locations. Block layout is usually a precursor to these subsequent design steps, termed "detailed layout." Two recent survey articles on the facility layout problem are Kusiak and Heragu (1987) [11] and Meller and Gau (1996) [12].

The problem was originally formulated by Armour and Buffa [1] as follows. There is a rectangular region, R, with fixed dimensions H and W, and a collection of n required departments, each of specified area a_j and dimensions (if rectangular) of h_j and w_j, whose total area, $\sum_j a_j$ = A = H×W. There is a material flow F(j,k) associated with each pair of departments (j,k) which generally includes a traffic volume in addition to a unit cost to transport that volume. The objective is to partition R into n subregions representing each of the n departments, of appropriate area, in order to:

$$\min \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \mathrm{F}(j,k)\mathrm{d}(j,k,\Pi) \qquad (1)$$

where d(j,k,Π) is the distance (using a pre-specified metric) between the centroid of department j and the centroid of department k in the partition Π.

1.1 Non-interchangeable Departments

A block layout where departments may have different areas and/or different shapes precludes assigning *n* departments to *m* distinct locations as is done in the popular quadratic assignment problem (QAP) formulation of block layout, which requires all departments be of identical shape and size. Instead, the locations of the department centroids will depend on the exact configuration selected, making formulations of the unequal area problem less tractable, but also much more realistic, than their QAP counterparts. The best known large test problem for the unequal area facility layout problem is that of Armour and Buffa [1], who devised a 20 department problem with a symmetrical flow matrix using the rectilinear (Manhattan) distance metric. They approached this problem by requiring all departments to be made up of contiguous rectangular "unit blocks," and then applied departmental adjacent pairwise exchange.

There are other fundamental formulations for unequal area layout. One is slicing trees (see Figure 1), where departments are required to be rectangular and the layout is represented by alternating vertical and horizontal slices. This approach was pioneered for the placement problem [4] and was developed in detail for facility layout by Tam [20, 21]. A drawback of this approach is the computational effort involved and the inability to maintain departmental aspect ratio constraints (at least in Tam's work). Because of the centroid to centroid distance in the objective function, without strict aspect ratio constraints, departments tend to become unrealistically elongated.

A related, but more restrictive formulation than slicing trees, is the flexible bay structure used by the authors [22]. This structure first allows slices in a single direction, creating bays, which are then sub-divided into departments by perpendicular slices. Although the flexible bay formulation is slightly more restrictive than the slicing tree formulation, it does allow a natural aisle structure to be inherently created in the layout design (see Figure 1) and strictly enforces departmental areas and shapes.

1.2 Past Work Considering Uncertainty

Probably the first work considering stochastic parameters in the layout design problem was that of Shore and Tompkins [19] who studied four possible scenarios based on product demand. They optimized each scenario separately using the unit block approach to unequal area problems, and then selected the layout which had the lowest penalty when considering the likelihood of each scenario. The idea of multiple discrete scenarios motivated by uncertainty is central to the research on stochastic plant layout. Other papers using this notion of uncertainty in product forecasts approach include Rosenblatt and Lee [18], Rosenblatt and Kropp [17], Kouvelis et al. [10] and Cheng et al. [3]. The first three studied the QAP formulation while the last studied unequal area departments using a slicing tree encoding.

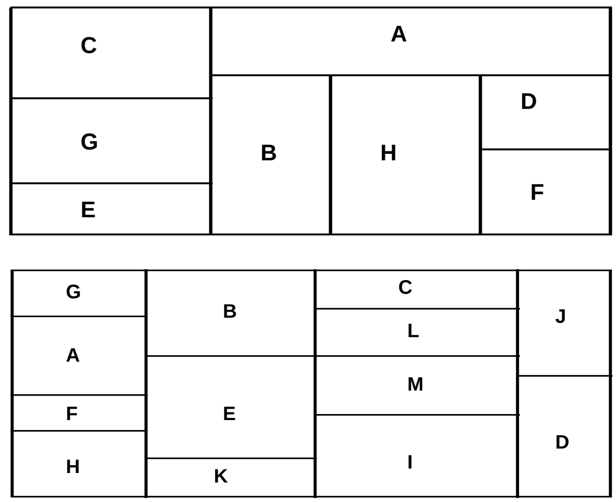

Figure 1. Typical Slicing Tree (top) and Flexible Bay (bottom) Layouts.

2. APPROACH AND FORMULATION

The main considerations of this paper are as follows. *One*, to accommodate unequal area departments and facilities with realistic shapes, typically rectangular, where both the area and shape of the rectangles can be enforced in the final solution, but the exact shape does not have to be prespecified and there is little or no "dead space" ($A \approx \sum_j a_j$). This is achieved using the flexible bay formulation previously cited. *Two*, any distance metric and/or adjacency criteria can be accommodated through the use of a flexible objective function that can be easily altered. *Three*, severe constraints can be handled, such as forced placements and small aspect ratios. *Four*, the search can yield near-optimal solutions in a reasonable computation time that does not explode with the size of the problem. Items two through four are accomplished through use of a genetic algorithm (GA) heuristic. Genetic algorithms are a class of meta-heuristic that are inspired by the biological process of evolution. They were introduced by Holland [9] and popularized by Goldberg [7] for optimization of continuous functions. The main distinguishing characteristics of GA are a population of candidate solutions, a selection mechanism for breeding that gives preference to better solutions, a breeding mechanism to create child solutions by recombination (crossover) of parent solutions, a perturbation method (mutation) to avoid convergence to a local optima, and a culling operator to remove old or inferior solutions in the population. A GA works iteratively by improving the initially random solutions creating subsequent generations of (generally) improving solutions. The GA of Tate and Smith [22] which has been enhanced through use of the random keys (RK) encoding of Norman and Bean [2, 13-15] and the adaptive penalty approach of Coit, Smith and Tate [5] was used. The random keys encoding eliminates the need for special purpose crossover and mutation operators to maintain encoding integrity for permutations. Each block layout is encoded as a string of floating point numbers indicating the sequence of department placements and the breaks between the vertical bays. There is a one to one correspondence between each possible layout and each encoded solution in the RKGA. More details on the RKGA for block layout can be found in [16].

The basic objective is the minimization of a statistical bound of total material handling costs (see Figure 2) subject to constraints on departmental shapes given a fixed total rectangular area A with fixed H and W, and fixed departmental areas, a_j. There are p independent products each with an expected demand or production volume and a standard deviation per unit of time (e.g., day, week or month). Invoking the central limit theorem of sums, the probability distribution of the total material handling costs is Normal, even when only a few products are involved. (The central limit of sums has been invoked for sample sizes as small as two [8].)

The choice of bound is determined by the attitude towards uncertainty of the user. A user with a undefined attitude will want to establish the trade-offs of different layouts under different bounds, thus identifying a robust layout. While this paper assumes that α will be an upper bound ($\alpha < 0.50$), it could be risk neutral ($\alpha = 0.50$) or a lower bound ($\alpha > 0.50$). To clarify the interpretation of risk, a layout which is optimized for a small α value will have a low cost if the quantity actually produced of products is on the high side of the forecast (where there is variability in the forecast). This is the risk averse (*optimistic*) user. A risk seeking (*pessimistic*) user would select a layout which is minimum cost when products produced are on the low side of the forecast. A risk neutral (*indifferent*) user would only be concerned with the expected value of the production forecast.

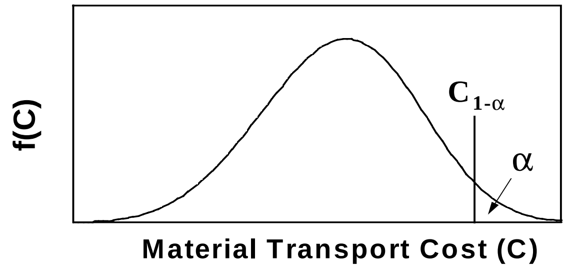

Figure 2. The Objective Function is an Upper Bound on Material Transport Costs.

For each product, it must also be known which departments will be included in the product manufacture, assembly or handling. For example product 1 could be routed through departments a, c, d and g while product 2 is routed through departments c, d, e, f and g. With this formulation, the variability of the forecasts of each product can be considered separately. An established product might have low variability of forecast while a new or future product may have high variability. The product volumes, variability and routings along with unit material handling costs and departmental areas and constraints are the required information prior to the block layout design phase. This formulation using products and their individual characteristics is very natural for managers and engineers, and averts specifying probabilities or random variable distributions. Although they are not included here, fixed costs of locating a department or of transport could also be included.

Mathematically the problem formulation, using a rectilinear distance metric, is:

$$\min \sum_{i=1}^{p} \bar{v}_i \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(\left| c_{xj} - c_{xk} \right| + \left| c_{yj} - c_{yk} \right| \right) \right) + z_{1-\alpha} \sqrt{ \sum_{i=1}^{p} \sigma_{vi}^2 \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(\left| c_{xj} - c_{xk} \right| + \left| c_{yj} - c_{yk} \right| \right) \right) } \qquad (2)$$

s.t. $r_j \leq R_j \ \forall j$

where

r_j maximum aspect ratio of dept *j*

R_j maximum allowable aspect ratio of dept *j*

$\bar{V}_i$ expected volume for product *i* per unit time

where $i = \{1, 2, \ldots, p\}$

σ_{vi}^2 variance of volume of product *i* per unit time

$$\delta_{ijk} = \begin{cases} 1 \text{ if product } i \text{ is transported from dept } j \text{ to dept } k \\ 0 \text{ if product } i \text{ is not transported from dept } j \text{ to dept } k \end{cases}$$

c_{xj} x coordinate of dept j centroid

c_{yj} y coordinate of dept j centroid

$z_{1-\alpha}$ standard Normal z value for bound $1-\alpha$

Since this formulation is unique, four test problems were developed using from 2 to 16 products and the 20 department areas of the Armour and Buffa problem [1]. A significant amount of routing overlap between products was included and the product mix was a diverse set of expected values and coefficients of variance (σ/μ) for each test problem. A full factorial design of experiments was conducted using the four problems to test the performance of the methodology considering alterations in the following parameters: population size (10, 25, 50), mutation rate (0.25, 0.50), random number seed (5 seeds), maximum aspect ratio (3, 5, 10), and bound ($z_{1-\alpha}$ = 0, 1, 2, 3). The first two items are GA specific parameters, the third tests pure stochastic sensitivity and the last two change the problem being solved. The maximum aspect ratio varies from very constrained (3) to hardly constrained (10) and the uncertainty attitude ranges from indifferent (0) to very optimistic (3). This full factorial experiment totaled 1440 design procedures.

3. RESULTS

The results have demonstrated the effectiveness of the RKGA heuristic. Its performance is relatively insensitive to the GA parameter settings, random number seed and problem instance. More importantly, the research shows the effect of the explicit consideration of uncertainty. Figure 3 shows the layout for the most constrained (r_j = 3) version of the largest (16 product) problem when an implicit assumption of neutrality is made ($z_{1-\alpha}$ = 0) as opposed to the layout when an explicit consideration of optimism ($z_{1-\alpha}$ = 1, 2, or 3) is made. In practice most decision makers will prefer to over-capacitize rather than under-capacitize, especially in situations with large long term ramifications, as in facility layout design. Identifying superior, over-capacitated layouts that are robust when under-capacity occurs will be a likely objective. Table 1 shows the results of when each $z_{1-\alpha}$ value is used in the objective function. It can be seen that as $z_{1-\alpha}$ changes, the relative contribution of the expected value and the standard deviation of total material handling costs changes. For an expected value ($z_{1-\alpha}$ = 0) objective function, variance is ignored and the standard deviation of costs is large. Furthermore, where the objective function does not properly reflect the risk attitude (i.e., where $z_{1-\alpha}$ of the objective function differs from that of the actual risk attitude), the designs are uniformly sub-optimal. Figure 4 shows the dominance of each of the four plant layouts as attitude changes from neutrality (α = 0.5) to extreme optimism (α = 0.001). It can easily be seen that the design resulting from implicitly assumed risk neutrality is clearly sub-optimal when almost any degree of upper bound is considered. A similar analysis can be done for pessimistic attitudes (α > 0.5). This type of graph is extremely useful for an analyst to quickly ascertain the cost/uncertainty trade-offs of any particular layout design problem. Recall that these results are for the most constrained problem — as constraint lessens, even greater disparity in optimal plant designs will be observed over the studied uncertainty levels.

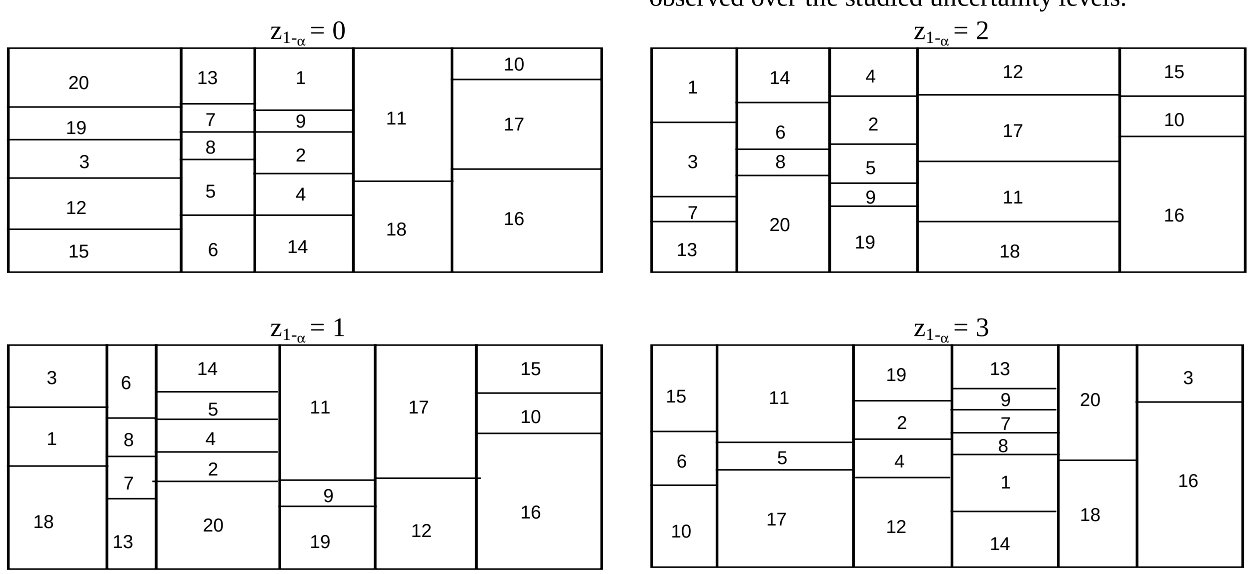

Figure 3. Even Highly Constrained Plant Layout Design is Dependent on Attitude Towards Uncertainty.

For the less constrained versions of the problem (r_j = 5 or 10), the results were similar. Table 2 shows the optimal solution of the median of five GA runs for each aspect ratio and each value of $z_{1-\alpha}$. Two trends can be easily observed. First, as the maximum allowable aspect ratio is relaxed, the material handling costs become smaller since departments can assume a longer, narrower shape which reduces centroid to centroid distance. Second, as $z_{1-\alpha}$ increases, the mean costs increase while the standard deviations of costs decrease. This is the effect of optimizing an upper bound rather than simply a mean value. Only the layouts for aspect ratio = 3 and $z_{1-\alpha}$ = 2 or 3 depart from this trend, where the solution identified for $z_{1-\alpha}$ = 3 actually dominates that of the solutions for $z_{1-\alpha}$ = 2. Note that the values in Table 2 are somewhat different from those in Table 1. This is because the solutions in Table 2 were the result of increased length runs.

4. CONCLUDING REMARKS

Facility layout design is a problem that when solved properly improves the efficiency, responsiveness and profitability of an organization. Conversely, if the layout is poor, operations suffer daily until the layout is corrected, a step which is costly and time consuming. In most previous approaches, uncertainty in forecasts over the life of the layout design (which can be long) is not considered. The approach described in this paper enables the identification of physically reasonable block layouts which properly and explicitly reflect both product forecast variability and user attitude towards production uncertainty. The method allows the identification of robust layouts over the spectrum of applicable uncertainty bounds. An area of future research is the addition of aisle and input/output structure to the block layout.

Table 1. Components of Objective Function of Optimal Solutions as $z_{1-\alpha}$ Changes.

Uncertainty Attitude ($z_{1-\alpha}$)	Mean Costs	Standard Deviation of Costs	Equation 2 Value for $z_{1-\alpha}$=0	Equation 2 Value for $z_{1-\alpha}$=1	Equation 2 Value for $z_{1-\alpha}$=2	Equation 2 Value for $z_{1-\alpha}$=3
0 (neutral)	14703	9733	14703	24436	34169	43901
1 (mildly optimistic)	15809	6327	15809	22136	28463	34791
2 (optimistic)	17176	5191	17176	22367	27557	32748
3 (acutely optimistic)	18788	4476	18788	23263	27739	32215

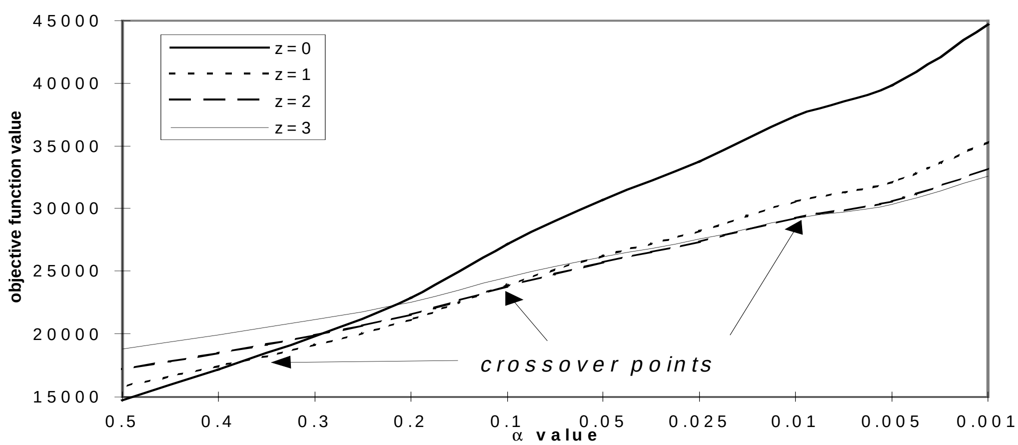

Figure 4. Cost Performance of Optimal Designs as Uncertainty Attitude Alters.

Table 2. Components of Objective Function as $z_{1-\alpha}$ and Aspect Ratio Change.

	Aspect Ratio = 3		Aspect Ratio = 5		Aspect Ratio = 10	
Uncertainty Attitude ($z_{1-\alpha}$)	Mean Costs	Standard Deviation	Mean Costs	Standard Deviation	Mean Costs	Standard Deviation
0 (neutral)	13983	8741	12667	7176	10164	7631
1 (mildly optimistic)	14532	5814	14099	5474	10800	5778
2 (optimistic)	15970	4562	15350	4513	12607	4681

3 (acutely optimistic)	15832	4562	15629	4415	14831	3778

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