Random Keys Genetic Algorithm with Adaptive Penalty Function for Optimization of Constrained Facility Layout Problems

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Abstract

This paper presents an extended formulation of the unequal area facilities block layout problem which explicitly considers uncertainty in material handling costs by use of expected value and standard deviations of product forecasts. This formulation is solved using a random keys genetic algorithm (RKGA) to circumvent the need for repair operators after crossover and mutation. Because this problem can be highly constrained depending on the maximum allowable aspect ratios of the facility departments, an adaptive penalty function is used to guide the search to feasible, but not suboptimal, regions. The RKGA is shown to be a robust optimizor which allows a user to make an explicit characterization of the cost and uncertainty trade-offs involved in a particular block layout problem.

I. INTRODUCTION TO THE FACILITY LAYOUT PROBLEM

Facility Layout Problems are a family of design problems involving the partition of a planar region into departments or work areas of known area, so as to minimize the costs associated with projected interactions between these departments. These costs may reflect material handling costs or preferences regarding adjacencies among departments. There are problems which are strongly related to the facility layout problem that arise in other engineering design contexts such as VLSI placement and routing. All of these combinatorial problems are known to be NP-hard [5]. The problem primarily studied in the literature has been "block layout" that only specifies the placement of the departments, without regard for aisle structure and material handling system, machine placement within departments or input/output locations. Block layout is usually a precursor to these subsequent design steps, termed "detailed layout."

The problem was originally formulated by Armour and Buffa [1] as follows. There is a rectangular region, R, with fixed dimensions H and W, and a collection of n required departments, each of specified area a_j and dimensions (if rectangular) of h_j and w_j, whose total area, $\sum_j a_j$ = A = H×W. There is a material flow F(j,k) associated with each pair of departments (j,k) which generally includes a traffic volume in addition to a unit cost to transport that volume. The objective is to partition R into n subregions representing each of the n departments, of appropriate area, in order to:

$$\min \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \mathrm{F}(j,k)\,\mathrm{d}(j,k,\Pi) \qquad (1)$$

where d(j,k,Π) is the distance (using a pre-specified metric) between the centroid of department j and the centroid of department k in the partition Π.

A. NON-INTERCHANGEABLE DEPARTMENTS

The locations of the department centroids depend on the exact configuration selected, making formulations of the unequal area problem less tractable, but also much more realistic, than their equal area counterparts. The best known large test problem for the unequal area facility layout problem is that of Armour and Buffa [1], who devised a 20 department problem with a symmetrical flow matrix using the rectilinear (Manhattan) distance metric. They approached this problem by requiring all departments to be made up of contiguous rectangular "unit blocks," and then applied departmental adjacent pairwise exchange.

A related, but more restrictive formulation than slicing trees [13, 14], is the flexible bay structure used by the authors [15]. This structure first allows slices in a single direction, creating bays, which are then sub-divided into departments by perpendicular slices. Although the flexible bay formulation is slightly more restrictive than the slicing tree formulation, it does allow a natural aisle structure to be inherently created in the layout design (see Figure 1) and strictly enforces departmental areas and shapes.

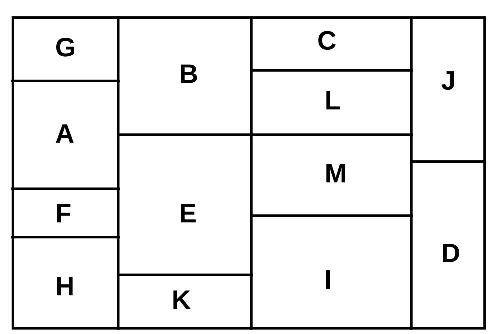

Figure 1. Typical Flexible Bay Layout.

B. PAST WORK CONSIDERING UNCERTAINTY

Most work in facility layout has assumed that projected material handling costs are known with certainty. This is an unrealistic assumption given that a layout will probably be operational for years if not decades. The first work considering stochastic parameters in the layout design problem was that of Shore and Tompkins [12] who studied four possible scenarios based on product demand. They optimized each scenario separately using the unit block approach to unequal area problems, and then selected the layout which had the lowest penalty when considering the likelihood of each scenario. The idea of multiple discrete scenarios caused by uncertainty is central to the research on stochastic plant layout. Other papers using uncertainty in product forecasts include Rosenblatt and Lee [11], Rosenblatt and Kropp [10], Kouvelis et al. [6] and Cheng et al. [3].

II. RANDOM KEYS GENETIC ALGORITHM

We use an encoding that is based on the flexible bay representation and is similar to that found in Tate and Smith [15]. For this representation the GA determines two things: a sequence for the departments and where the bay divisions will occur. For example, the department sequence

G A F H B E K C L M I J D

with bay divisions at 4, 7, and 11 would generate the flexible bay layout in Figure 1. The width of each bay is determined by considering the sum of the area for all of the departments in the bay. There are two potential difficulties with this representation. First, since the department sequence represents a permutation vector there is the potential for the crossover operation to produce infeasible sequences. Second, depending on the location of the bay divisions the resulting bay structure may create departments shapes that violate the aspect ratio constraints. Tate and Smith used a repair operator to fix strings that were not valid permutations as a result of crossover or mutation.

To overcome potential feasibility problems in the department ordering problem, we have enhanced the representation through use of the random keys (RK) encoding of Norman and Bean [2, 7-9]. This encoding assigns a random $U(0,1)$ variate, or random key, to each department in the layout and these random keys are sorted to determine the department sequence. Consider the thirteen department example of Figure 1. The chromosome of random keys given below, when sorted in ascending order, would create the sequence depicted in Figure 1.

A	B	C	D	E	F	G	H	I	J	K	L	M
.16	.28	.49	.93	.37	.19	.07	.24	.74	.81	.43	.55	.66

The random keys encoding eliminates the need for special purpose crossover and mutation operators to maintain encoding integrity for permutations because crossover always results in a set of random keys which can be sorted to determine a feasible permutation. It also adds no computational overhead to the GA search.

Bay divisions can be determined on a separate chromosome as in Tate and Smith [15] or included in the random keys encoding by adding an integer to each random key. The integer indicates the bay number for the department (a similar idea was used for resource allocation in [9]). Consider the chromosome presented below which would decode to the layout shown in Figure 1.

A	B	C	D	E	F	G	H	I	J	K	L	M
1.16	2.28	3.49	4.93	2.37	1.19	1.07	1.24	3.74	4.81	2.43	3.55	3.66

The feasibility of the crossover mechanism for the random keys encoding is demonstrated by the following example. Consider two chromosomes that will serve as the parents:

Parent 1

A	B	C	D	E	F	G	H	I	J	K	L	M
1.16	2.28	3.49	4.93	2.37	1.19	1.07	1.24	3.74	4.81	2.43	3.55	3.66

Parent 2

A	B	C	D	E	F	G	H	I	J	K	L	M
2.87	3.12	1.19	1.91	2.23	4.32	1.67	3.96	4.87	1.02	2.29	3.71	2.56

If single point crossover is performed on these two parents between the genes for departments E and F the following offspring result:

Offspring 1

A	B	C	D	E	F	G	H	I	J	K	L	M
1.16	2.28	3.49	4.93	2.37	4.32	1.67	3.96	4.87	1.02	2.29	3.71	2.56

Offspring 2

A	B	C	D	E	F	G	H	I	J	K	L	M
2.87	3.12	1.19	1.91	2.23	1.19	1.07	1.24	3.74	4.81	2.43	3.55	3.66

The random keys for these offspring can be sorted and the resulting bay assignments and ordering within the bays readily determined. The chromosomes of the offspring will always maintain encoding integrity for permutations.

The problem of infeasibility due to violation of the aspect ratio constraints is handled using the adaptive penalty approach of Coit, Smith and Tate [4]. This approach permits infeasible solutions initially but penalizes infeasibility more as the search continues. The penalty imposed on infeasible layouts is a function of both the number of generations and the relative fitness of the best feasible and infeasible solutions yet found. This permits a broad range of search paths initially but drives the RKGA to find feasible solutions at the conclusion of the search process.

The proposed random keys encoding has been compared with the encoding of Tate and Smith [15] for problems with deterministic flows and found to perform better on average.

III. OPTIMIZATION APPROACH

The basic objective is the minimization of a statistical bound of total material handling costs (see Figure 2 for example of an upper bound) subject to constraints on departmental shapes given a fixed total rectangular area A with fixed H and W, and fixed departmental areas, a_j. There are p independent products each with an expected demand or production volume and a standard deviation per unit of time (e.g., day, week or month). Invoking the central limit theorem of sums, the probability distribution of the total material handling costs is Gaussian, even when only a few products are involved. α can be in the optimistic range ($\alpha > 0.50$), or at the expected value (α = 0.50) or in the pessimistic range (α > 0.50). To clarify this, a layout which is optimized for a small α value will have a low cost even if the quantity actually produced of products is on the high side of the forecast (where there is variability in the forecast). A layout for a large α value would perform the best when production was on the low side of the expected value of the forecast. A user would probably select a layout which performed well for production both in excess and less than the expected value of the forecast.

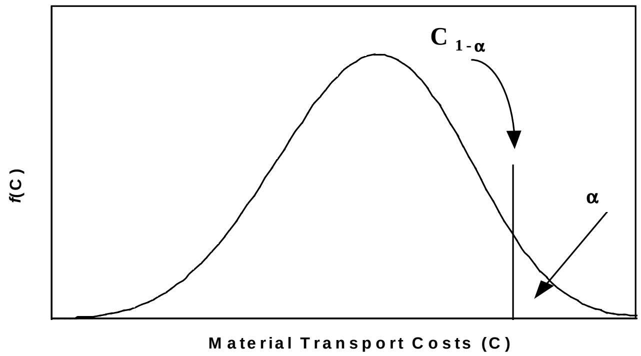

Figure 2. The Objective Function can be an Upper Bound on Material Handling Costs.

For each product, it must also be known which departments will be included in the product manufacture, assembly or handling. For example product 1 could be routed through departments a, c, d and g while product 2 is routed through departments c, d, e, f and g. With this formulation, the variability of the forecasts of each product can be considered separately. An established product might have low variability of forecast while a new or future product may have high variability. The product volumes, variability and routings along with unit material handling costs and departmental areas and constraints are the required information prior to the design phase. This formulation using products and their individual characteristics is very natural for managers and engineers, and averts specifying probabilities or random variable distributions. Although they are not included here, fixed costs of locating a department or of transport could also be included.

Mathematically the problem formulation, using a rectilinear distance metric, is:

$$\min \sum_{i=1}^{p} \bar{v}_i \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(\left| c_{xj} - c_{xk} \right| + \left| c_{yj} - c_{yk} \right| \right) \right) + z_{1-\alpha} \sqrt{ \sum_{i=1}^{p} \sigma_{vi}^2 \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(\left| c_{xj} - c_{xk} \right| + \left| c_{yj} - c_{yk} \right| \right) \right)^2 } \qquad (2)$$

s.t. $r_j \leq R_j \ \forall j$

where

r_j maximum aspect ratio of dept j

R_j maximum allowable aspect ratio of dept j

$\bar{v}_i$ expected volume for product i per unit time where $i = \{1, 2, \ldots, p\}$

σ_{vi}^2 variance of volume of product i per unit time

$\delta_{ijk} = \begin{cases} 1 \text{ if product } i \text{ is transported from dept } j \text{ to dept } k \\ 0 \text{ if product } i \text{ is not transported from dept } j \text{ to dept } k \end{cases}$

c_{xj} x coordinate of dept j centroid

c_{yj} y coordinate of dept j centroid

$z_{1-\alpha}$ standard Gaussian z value for level $1-\alpha$

Since this formulation is unique, four test problems were developed using from 2 to 16 products and the 20 department areas of the Armour and Buffa problem [1]. A significant amount of routing overlap between products was included and the product mix was a diverse set of expected values and coefficients of variance (σ/μ) for each test problem. A full factorial design of experiments was conducted using the four problems to test the performance of the methodology considering alterations in the following parameters: population size (10, 25, 50), mutation rate (0.25, 0.50), random number seed (5 seeds), maximum aspect ratio (3, 5, 10), and risk level ($z_{1-\alpha}$ = 0, 1, 2, 3). The first two items are GA specific parameters, the third tests pure stochastic sensitivity and the last two change the problem being solved. The maximum aspect ratio varies from very constrained (3) to hardly constrained (10) and the uncertainty ranges from expected value (0) to very optimistic (3). This full factorial experiment totaled 1440 design procedures.

IV. RESULTS

The results have demonstrated the effectiveness of the RKGA optimization. Its performance is relatively insensitive to parameter settings, random number seed and problem instance. More importantly, the research shows the effect of explicit consideration of uncertainty. Figure 3 shows the layout for the most constrained (maximum aspect ratio = 3) version of the 16 product problem when an implicit assumption of certainty is made ($z_{1-\alpha}$ = 0) as opposed to the layout when an explicit consideration of uncertainty ($z_{1-\alpha}$ = 1, 2, or 3) is made. Table 1 shows the results of when each $z_{1-\alpha}$ value is used in the objective function. It can be seen that as $z_{1-\alpha}$ changes, the relative contribution of the expected value and the standard deviation of total material handling costs changes. For an expected value ($z_{1-\alpha}$ = 0) objective function, variance is ignored and the standard deviation of costs is large. Furthermore, where the objective function does not properly reflect the uncertainty (i.e., where $z_{1-\alpha}$ of the objective function differs from that of the actual uncertainty), the designs are uniformly sub-optimal. Figure 4 shows the dominance of each of the four plant layouts as uncertainty (α) changes from expected to the optimistic side of the forecast. It can easily be seen that the design resulting from traditional methods (implicitly assumed certainty) is clearly sub-optimal when

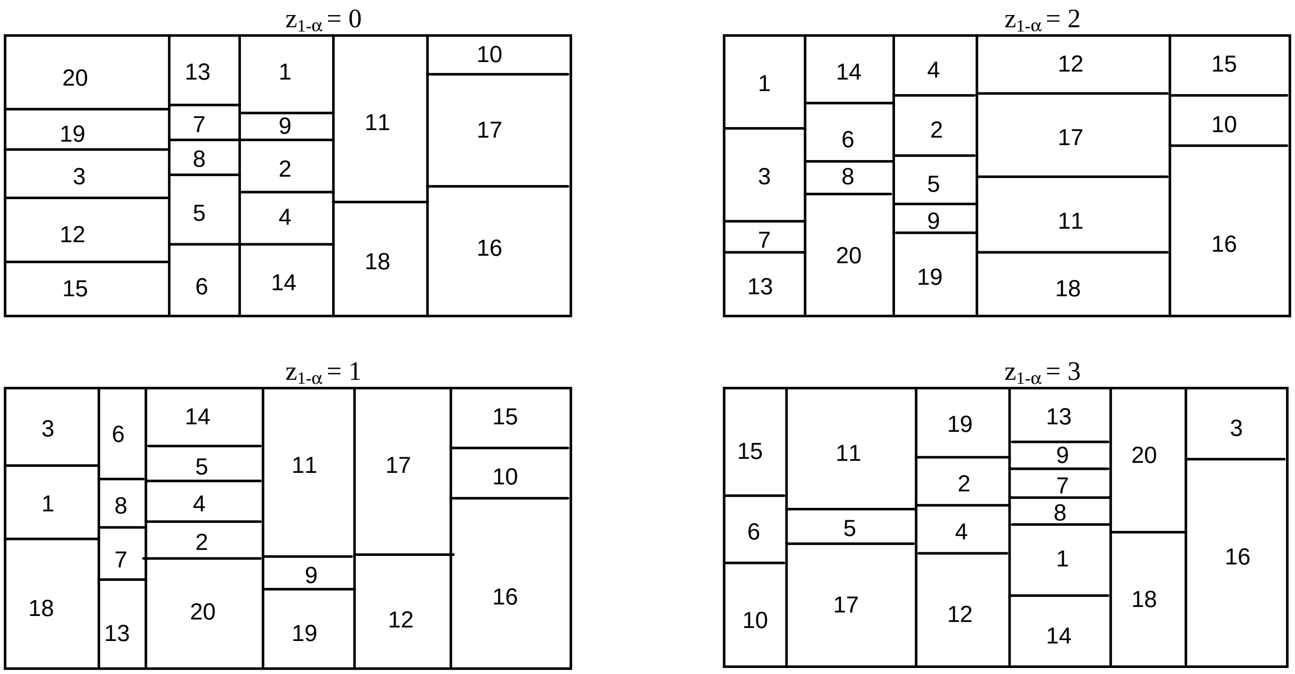

Figure 3. Even Highly Constrained Optimal Plant Layout Design is Dependent on Uncertainty Level.

Table 1. Components of Objective Function of Optimal Solutions as $z_{1-\alpha}$ Changes.

Risk Attitude ($z_{1-\alpha}$)	Mean Costs	Standard Deviation of Costs	Equation 5 Value for $z_{1-\alpha}$=0	Equation 5 Value for $z_{1-\alpha}$=1	Equation 5 Value for $z_{1-\alpha}$=2	Equation 5 Value for $z_{1-\alpha}$=3
0 (risk neutral)	14703	9733	14703	24436	34169	43901
1 (mildly risk averse)	15809	6327	15809	22136	28463	34791
2 (risk averse)	17176	5191	17176	22367	27557	32748
3 (acutely risk averse)	18788	4476	18788	23263	27739	32215

almost any degree of uncertainty is considered. This type of graph is extremely useful for an analyst to quickly ascertain the cost/uncertainty trade-offs of any particular layout design problem. Robust layouts which perform well over a variety of production scenarios can be identified. Recall that these results are for the most constrained problem — as constraint lessens, even greater disparity in optimal plant designs will be observed over the studied uncertainty levels.

For the less constrained versions of the problem (aspect ratio = 5 or 10), the results were similar. Table 2 shows the optimal solution of the median of five GA runs for each aspect ratio and each value of $z_{1-\alpha}$. Two trends can be easily observed. First, as the maximum allowable aspect ratio is relaxed, the material handling costs become smaller since departments can assume a longer, narrower shape which reduces centroid to centroid distance. Second, as $z_{1-\alpha}$ increases, the mean costs increase while the standard deviations of costs decrease. This is the effect of optimizing an upper bound rather than simply a mean value. Only the layouts for aspect ratio = 3 and $z_{1-\alpha}$ = 2 or 3 depart from this trend, where the solution identified for $z_{1-\alpha}$ = 3 actually dominates that of the solutions for $z_{1-\alpha}$ = 2. Note that the values in Table 2 are somewhat different from those in Table 1. This is because the solutions in Table 2 were the result of increased length runs.

V. CONCLUDING REMARKS

Facility layout design is a problem that when solved properly improves the efficiency, responsiveness and profitability of an organization. Conversely, if the layout is poor, operations suffer daily until the layout is corrected, a step which is costly and time consuming. In most previous approaches, uncertainty in forecasts over the life of the layout design (which can be long) is not considered. The approach described in this paper enables the identification of physically reasonable block layouts which properly and explicitly reflect both product forecast variability and user attitude towards production uncertainty. The identification of robust layouts can be easily made through graphs such as Figure 4.

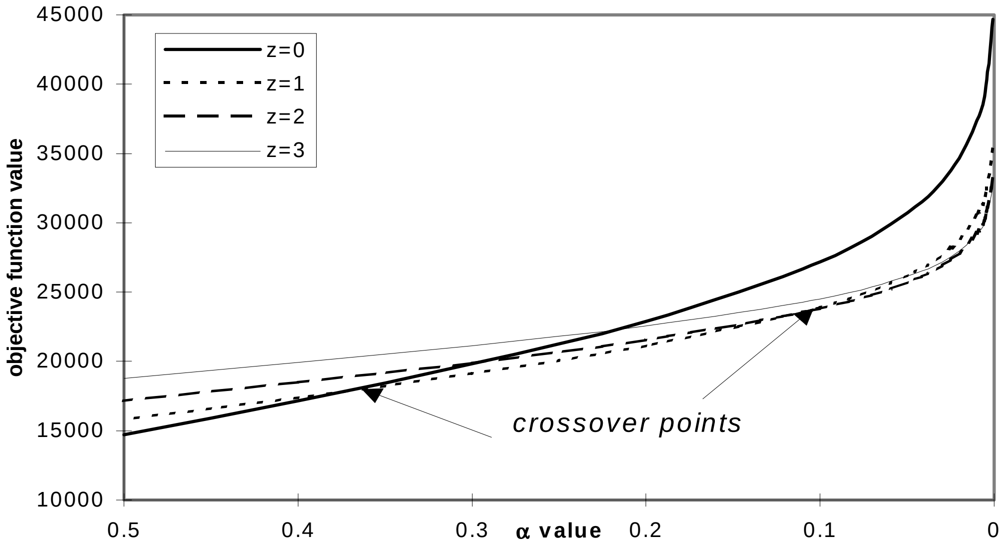

Figure 4. Cost Performance of Optimal Designs as Optimistic Uncertainty Increases.

Table 2. Components of Objective Function as $z_{1-\alpha}$ and Aspect Ratio Change.

	Aspect Ratio = 3		Aspect Ratio = 5		Aspect Ratio = 10	
Uncertainty Level ($z_{1-\alpha}$)	Mean Costs	Standard Deviation	Mean Costs	Standard Deviation	Mean Costs	Standard Deviation
0 (expected value)	13983	8741	12667	7176	10164	7631
1 (mildly optimistic)	14532	5814	14099	5474	10800	5778
2 (fairly optimistic)	15970	4562	15350	4513	12607	4681
3 (very optimistic)	15832	4562	15629	4415	14831	3778

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