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A novel approach for modeling order picking paths

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Abstract

We introduce the visibility graph as an alternative way to estimate the length of a route traveled by order pickers in a warehouse. Heretofore it has been assumed that workers travel along a network of travel paths corresponding to centers of aisles, including along the right angles formed where picking aisles join cross aisles. A visibility graph forms travel paths that correspond to more direct and, we believe, more appropriate "travel by sight." We compare distance estimations of the visibility graph and the aisle-centers method analytically for a common traditional warehouse design. We conduct a range of computational experiments for both traditional and fishbone warehouse layouts to assess the impact of this change in distance metric. Distance estimations using aisle-centers calculates a length of a picking tour on average 10–20% longer compared to distance estimations based on the visibility graph. The visibility graph metric also has implications for warehouse design: when comparing three traditional layouts, the distance model using a visibility graph resulted in choosing a different best layout in 13.3% of the cases.

KEYWORDS

computational geometry, routing, shortest path problem, visibility graph, warehouse design

1 | INTRODUCTION

An important part of warehouse modeling is the task of determining the distance between two points. Virtually every warehousing paper in the literature uses the rectilinear or Manhattan metric to estimate the distance between two points. However, in practice often workers "cut corners" and do not make turns at exact right angles (see Figure 1). The effects of this imprecision when modeling depend on the size of the warehouse, the width of aisles, the number of corners a worker could cut, and most importantly, the question one seeks to answer with the model. As we will show, the existing distance metric almost certainly over-estimates the distance traveled by workers, and therefore labor costs are most likely over-estimated and throughput rates are under-estimated. However, if the question addresses different operating policies within the same layout the results are "relatively correct," or at least mostly so. However, for different warehouse layouts this relative effect may not hold. In summary, the choice of distance metric can affect which layout design is most suitable and if the visibility graph models worker travel with more fidelity, using it during warehouse design will result in truly superior layouts. The purpose of this paper is to explain this effect and to offer an alternative distance metric, which we argue is more appropriate for modeling travel paths of workers in a warehouse especially in manual order picking operations. In automated environments, the effects are much reduced such as in the case where forklifts are used where the use of predetermined lanes reduces the possibility of cutting corners. Automated Guided Vehicles generally also follow a fixed path, which is well represented by the aisle-centers metric.

In their model of a warehouse, Gue and Meller (2009) assumed a network in which nodes represent picking points (e.g., in front of pallet locations), the intersections of cross aisles and picking aisles, and the pick up and deposit point (P&D). Without explicitly saying so, they defined a graph network using the "aisle-centers method" in which the path between points in different picking aisles or between a pick location and the P&D point passes through the single point of intersection between the picking aisle(s) and the cross aisle (see Figure 2). This assumption has a limited effect on their

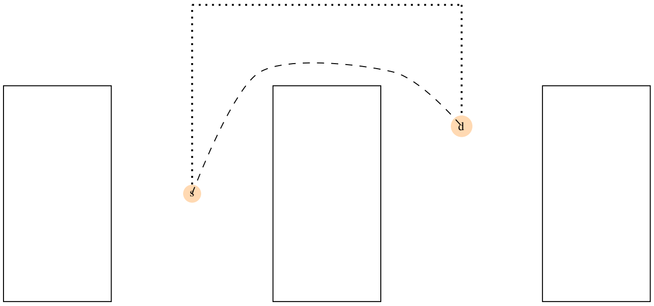


FIGURE 1 Example of a shortest path using a rectilinear distance (short dashes) versus a path that cuts the corners (long dashes) [Colour figure can be viewed at wileyonlinelibrary.com]

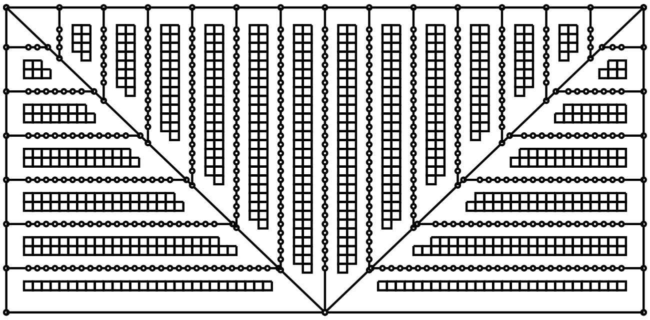


FIGURE 2 A graph network using the aisle-centers method defined by Gue and Meller (2009)

problem because they are only interested in paths between locations and the P&D point, and the paths generated by their models are similar to those that one would suppose being taken by a typical worker.

One of the most important statistics in an order picking warehouse is the (average) pick tour length. Every favorable angle has a corresponding unfavorable angle that poorly represents what might be expected of ordinary worker travel patterns. By any reasonable estimate of worker behavior, using the network model of Gue and Meller (2009) overestimates the distance of tours that require turns of greater than 90 degrees (see Figure 3).

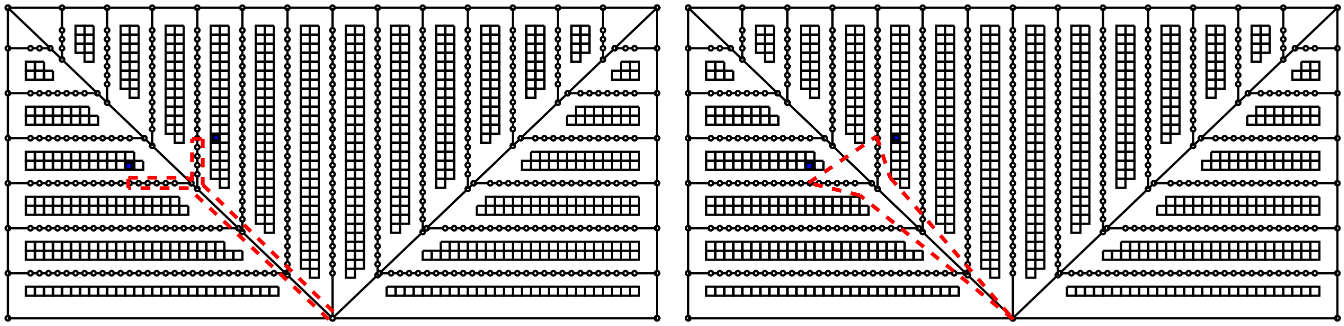
We propose the alternative approach of using visibility graph based distances between points in a warehouse to represent reasonable levels of corner cutting that one would expect of productive order pickers and compare them with the distances based on aisle-centers. The visibility graph is a special graph whose nodes are the vertices of obstacles (or points of attraction) and whose edges are pairs of mutually visible nodes. Figure 3b shows an example of a pick path using a visibility graph, where obstacles are the storage locations (racks or pallets). These proposed paths also serve as a lower bound on travel distances because no shorter path exists without passing through obstacles. However, the example in Figure 3b has its own issue: because storage locations are themselves obstacles, paths intersect storage racks along their perimeters. To address this difficulty, we create a buffer distance around storage locations and present the effects of the size of the buffer distance in Section 3.

Pohl et al. (2009b) used the aisle-centers metric to compare a fishbone layout to a traditional layout for dual command operations and found that the fishbone layout can reduce travel distance by 10–15%. Çelik and Süral (2014) also used the aisle-centers metric to compare a fishbone layout to a traditional layout for general order picking operations. They found that the fishbone layout can perform up to 30% worse than an equivalent traditional layout as the number of picks increases. Hall (1993) developed distance approximations for a one block warehouse by using the aisle-centers metric and compared different picking strategies for routing a manual order picker. Petersen (1999), Dijkstra and Roodbergen (2017), and Altarazi and Ammouri (2018) used the aisle-centers metric to compare various storage policies and routing heuristics simultaneously. Roodbergen et al. (2008) used the aisle-centers metric to optimize a traditional layout by finding the optimal number of pick and cross aisles and the length of a pick aisle, excluding the width of the cross aisles. Theys et al. (2010) used the aisle-centers metric to compare well known routing heuristics such as S-shape to dedicated traveling salesman problem (TSP) heuristics and found up to 47% average savings in route distance. Çelik and Süral (2019) considered the order picking problem as a special case of the Steiner TSP and proposed a routing heuristic that uses the Steiner TSP's graph theoretic properties. Like all the previous papers that used the Steiner TSP for order picking problems such as Ratliff and Rosenthal (1983) and Roodbergen and De Koster (2001b), their distance model was based on the aisle centers-method. Our focus is to model and use an alternative metric for estimating distances of order picking tours and to determine if these distances, which we believe are more reasonable estimates of real worker behavior, make a difference in warehouse design, such as investigated by Roodbergen and De Koster (2001a), Roodbergen and Vis (2006), and Gue and Meller (2009). Our experiments address the following questions:

- By how much does expected travel distance differ between the two distance estimation metrics for a particular layout?
- Using a visibility graph, does the best layout design change from the best design assessed with aisle-centers when comparing traditional layouts A, B, and C (Figure 4)?
- How does the size of the warehouse, the number of picks, and demand skewness affect the difference between the two distance estimation metrics?

The aim of using visibility graphs to model travel distances is not to find shorter paths than those of aisle-centers, but rather to model more closely the distances that typical workers would travel. We postulate that paths produced by the visibility graph with suitable buffers are closer to the actual distance traveled, however validation of this assertion must be left to an empirical study of real worker behavior.

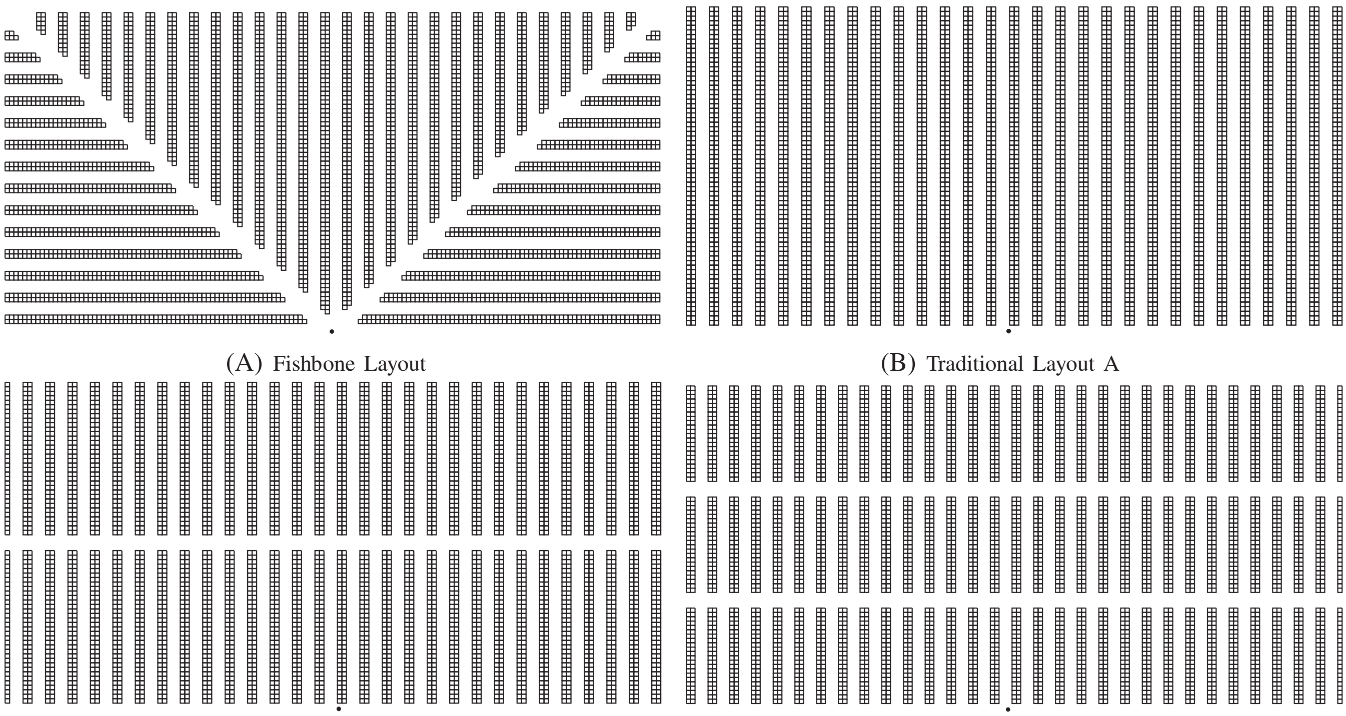
The rest of this paper is organized as follows. In the next section, the literature review related to the shortest path problem is presented. In Section 3, the model to calculate the



(A) A pick path by following aisle-centers

(B) A pick path by using a visibility graph

FIGURE 3 Pick path examples for picking two items from the storage locations shown in bold and returning to the depot (at bottom center) [Colour figure can be viewed at wileyonlinelibrary.com]



(A) Fishbone Layout

(B) Traditional Layout A

(C) Traditional Layout B

(D) Traditional Layout C

FIGURE 4 Four warehouse layouts used throughout this paper (dimensions of the warehouse might change depending on the number of SKUs based on the experiment, i.e., number of aisles and storage locations are not constant)

average distance of a tour for a given warehouse layout and set of pick lists is developed by using the visibility graph distance metric. In Section 4, we address the questions described in Section 1 with three experiments. Conclusions are presented in Section 5.

2 | THE SHORTEST PATH PROBLEM

The shortest path problem (SPP) seeks a path from a specific origin to a specific destination in a network such that the total cost is minimized. The SPP has widespread applications including vehicle routing in transportation systems (Zhan & Noon, 1998), telecommunications (Moy, 1994), VLSI design (Peyer et al., 2009), and path planning in robotics (Soueres & Laumond, 1996). There are well-known algorithms for solving the SPP, including Dijkstra's algorithm for solving the single source SPP with nonnegative weights (Dijkstra, 1959), the Bellman–Ford–Moore algorithm for solving the single source SPP with possible negative weights (Bellman, 1958; Ford, 1956; Moore, 1959), the A* search algorithm for solving the single source SPP using heuristics for fast evaluation (Hart et al., 1968), the Floyd–Warshall algorithm for solving the all-pairs SPP (Floyd, 1962), and Johnson's algorithm for solving the all-pairs SPP on sparse graphs (Johnson, 1977). The details of these algorithms can be found in Gallo and Pallottino (1986). The all-pairs SPP has been also studied on massively parallel computer architectures using parallel computing (Habbal et al., 1994).

The Euclidean shortest path problem (ESPP) seeks the shortest path between two points in Euclidean space that does not intersect any of a given set of obstacles. There exist efficient exact algorithms with $\mathcal{O}(n \log n)$ time complexity for problems in the plane, where n is the number of nodes in the graph (Hershberger & Suri, 1999). In three or more dimensions, the Euclidean shortest path problem is NP-hard (Canny & Reif, 1987). In this paper, we are interested in exact algorithms that work in two dimensional space with multiple obstacles. Hershberger and Suri (1999) state that there have been two fundamentally different approaches to this problem: the visibility graph method and the shortest path map method.

The visibility graph method produces a graph whose nodes are the vertices of the obstacles (or attractions) and whose edges are pairs of mutually visible nodes. In other words, for any pair of nodes, if the line segment that connects them does not pass through an obstacle, an edge is created between them. Once the graph is defined, Dijkstra's algorithm produces the shortest path between any two nodes. The run time complexity of this approach is $O(n \log n + E)$ for sparse graphs, where E is the number of edges in the graph (Ghosh & Mount, 1991). However, the visibility graph can have n^2 edges in the worst case; therefore algorithms that depend on the visibility graph will have $\mathcal{O}(n^2)$ worst-case run time to find the shortest path between two points. The algorithm we implement in this paper is naïve in that it compares every pair of points in the set of locations and determines whether a line between them intersects any edges of obstacles. All pairs have to be compared with n edges (a set of obstacles with n vertices has n edges) which makes the overall time complexity of the algorithm $\mathcal{O}(n^3)$.

The shortest path map method decomposes the plane into regions such that all points d in a region have the same sequence of obstacle vertices in their shortest path to s. The last obstacle vertex along the shortest $s-t$ path is the root r of the cell containing d. The root r can see all points within its region. Figure 5 shows an example of the shortest path map. Point s reaches d via obstacle vertices v and r. In this example, vertex r is the root of the region containing d. Hershberger's algorithm computes shortest paths in the presence of polygonal obstacles in $\mathcal{O}(n \log n)$ time and space, where n is the number of nodes in the graph. For a detailed review of algorithms that use the visibility graph method or the shortest path map method, see Toth et al. (2004).

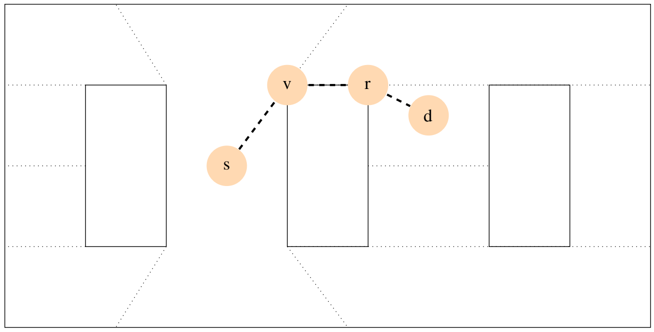


FIGURE 5 A shortest path map with respect to source point s within a polygonal domain with three obstacles. The heavy dashed path indicates the shortest $s-d$ path, which reaches d via the root r of its cell. Extension segments are shown thin and dotted [Colour figure can be viewed at wileyonlinelibrary.com]

3 | METHODOLOGY

There are seven steps to calculate the average distance of a tour for a given warehouse layout and set of pick lists. Steps 4 and 5 are the tasks related to the visibility graph method.

Step 1: Create exterior aisles for a given width and depth of the warehouse. These exterior aisles serve as the boundaries of the rectangle-shaped warehouse.

Step 2: Create cross aisles using exterior and interior points. An exterior point lies on the boundaries of the warehouse (i.e., exterior aisles), and an interior point lies inside the boundaries of the warehouse. Cross aisles divide the warehouse into regions.

Step 3: Create pick aisles, pick locations, and storage locations for each region. We assume that items on both sides of a pick aisle may be accessed with negligible lateral movement. Each region may have different angled pick aisles. Figure 6 illustrates such a warehouse.

Step 4: Calculate polygons that will be used in the visibility graph calculation. In this step, we create the polygons considering a buffer region (Lozano-Pérez & Wesley, 1979). The corners of the polygons are also included as nodes in the visibility graph.

Step 5: Let B be the set of polygons which are constructed for each storage location including a buffer region of size β. We define the visibility graph as G = (V, E), where V is the set of nodes containing all pick locations and the corners of each polygon in B and where E is the set of edges between the nodes that are in line of sight of each other. Then, the set of edges is defined as:

$$E = \left\{ (n_1, n_2) | n_1, n_2 \in V \text{ with } n_1 \neq n_2, \text{ and } \bigcup_{B_j \in B} L_{n_1,n_2} \cap B_j = \emptyset \right\}, \quad (1)$$

where L_{n_1,n_2} is the set of points on the line segment that connect node n_1 and n_2 and B_j is the set of points in polygon j. If for any polygon j, $L_{n_1,n_2} \cap B_j \neq \emptyset$, the line between node n_1 and n_2 and polygon j intersect. If no polygon intersects with the line between node n_1 and n_2, the nodes are in each other's line of sight. Furthermore, let d_{n_1,n_2} be the Euclidean distance between nodes n_1 and n_2, for $(n_1, n_2) \in E$. Finally, the distance between any nodes in V which are not in line of sight can be obtained by applying Dijkstra's algorithm (Dijkstra, 1959).

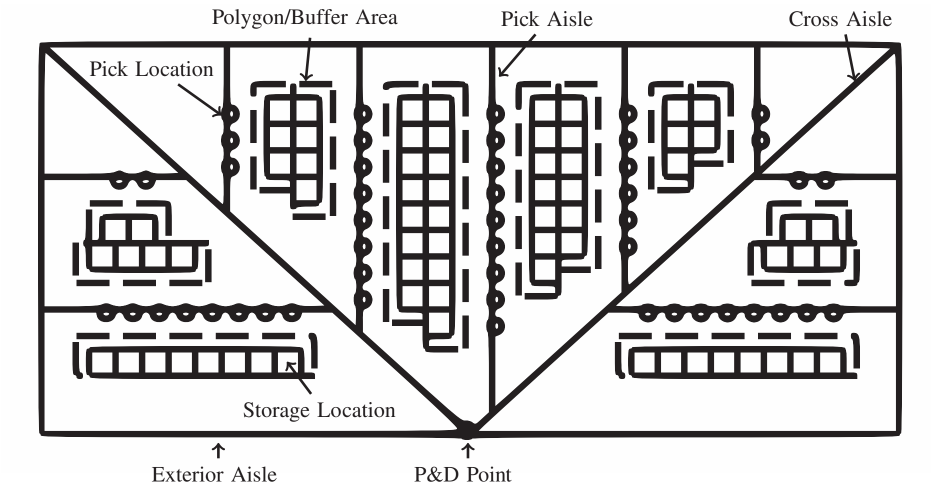


FIGURE 6 Representation of a warehouse. This particular fishbone layout has a single P&D point, 67 storage locations, 51 pick locations, 9 pick aisles, 2 cross aisles, and 4 exterior aisles. The lines that represent the aisle-centers are both used for building the warehouse structure (i.e., storage locations and pick locations) and finding the shortest path distances with the aisle-centers method

Step 6: Once the all-pairs shortest path distances are calculated and stored in the dist matrix, allocate products to storage locations. We use a dedicated storage policy to keep the locations of the products identical between the two distance metrics so storage will not affect comparisons. We assign the SKU with the smallest ID number to the storage location with the smallest ID number and generate a distance matrix for each order by using the dist matrix. In this way, the relative order and position of each SKU is maintained in any warehouse layout design, making the pick distances comparisons as fair as possible. This matrix contains the shortest path distances between the P&D point, or depot, and every storage location to be visited. Finding the shortest tour distance in a warehouse for a pick list is an example of a TSP. In the our experiments, we calculate optimal travel distances using the Concorde TSP solver (Applegate et al., 2007), but any TSP solver can be used.

Step 7: Calculate the average of all TSP distances.

3.1 | Adding a buffer

As mentioned before, it is not practical to assume an order picker will cut corners so sharply as to brush up along side of the storage infrastructure. Therefore, we devise the concept of the buffer which artificially increases the size of the storage obstacles so the path better mimics the way a typical human would walk. When the size of the buffer increases, the polygons (buffer areas) become larger and visibility decreases. Figure 7 shows an example of a visibility graph for a fishbone layout with a 2 ft buffer size. Figure 8a,b are two examples of visibility between pick locations. If the buffer size is 1 ft, then pick location 1 is visible to both pick locations 2 and 3. However, visibility between pick locations 1 and 2 is lost when the buffer size increases to 2 ft.

It is important to note that the density of the visibility graph decreases with greater buffer size. For example, when there is no buffer distance between storage locations and picker paths there are 1092 arcs created for the visibility graph in this example. The number of arcs for 0.5, 1.0, and 2.0 ft buffer distances are 1046, 1014, and 930, respectively. When the buffer size becomes sufficiently large, the visibility graph becomes similar to the aisle-centers graph.

3.2 | Comparison of visibility graph and aisle-centers methods

While our approach for the paper is one of practical significance of the differences between the aisle-centers method and the visibility graph method, a baseline can be established analytically. In this section, we analyze the differences between the visibility graph and aisle-centers methods using closed form expressions. To examine this difference in more detail analytically we must look at a specific layout design. We do so by choosing a very common design - Traditional Layout A.

For Traditional Layout A we assume random storage policy which is commonly used in unit-load storage to maximize space utilization. Traditional Layout A, as depicted in Figure 9, has parallel picking aisles that are perpendicular to the front wall, but has no cross aisle. A single depot is optimally located in the middle of the bottom cross aisle Roodbergen and Vis (2006). Example travel paths between two locations by using aisle-centers and the visibility graph are depicted as solid lines and dashed lines, respectively. We do not consider a buffer distance here. The location on the left is a distance x from the bottom of aisle i and the location on the right is a distance y from the bottom of aisle j. The length of each picking aisle is L and the half width of the cross aisle is v. The warehouse has n picking aisles, where n can be odd or even.

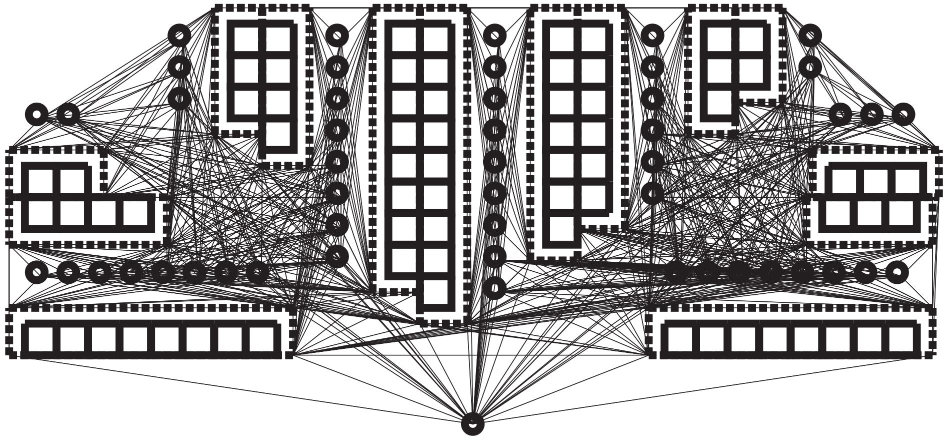
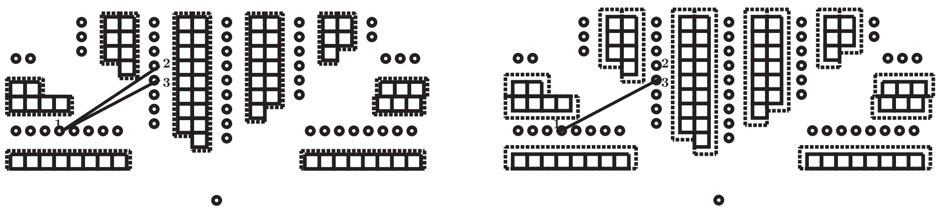


FIGURE 7 Visibility graph of a fishbone layout with buffer size of 2 ft



(A) Buffer Size = 1 ft., more visibility (B) Buffer Size = 2 ft., less visibility

FIGURE 8 Increasing the buffer size decreases the visibility between pick locations

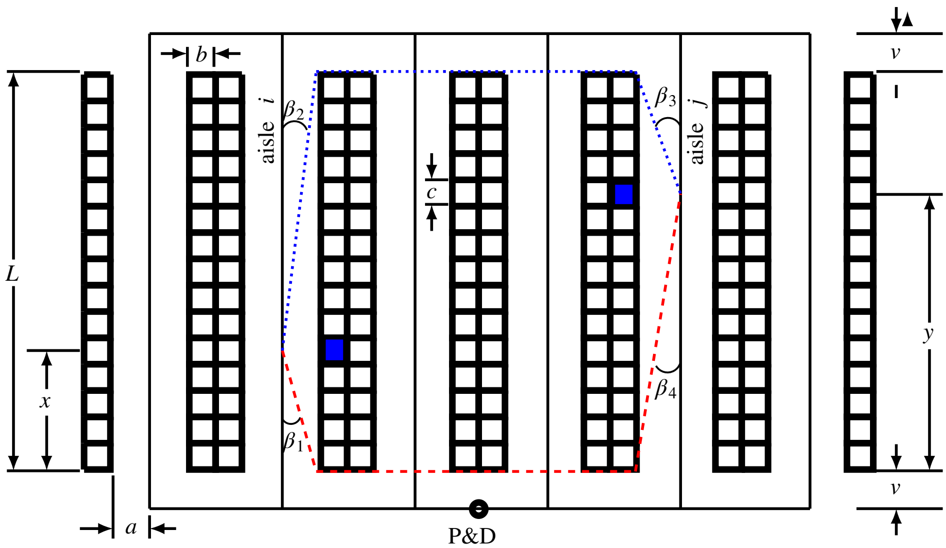


FIGURE 9 Traditional layout A [Colour figure can be viewed at wileyonlinelibrary.com]

There is no distance difference between the two methods if two pick locations are on the same picking aisle (travel between distance, TB, will be $|x-y|$). If two items are on different picking aisles, the maximum difference occurs when aisle i and aisle j are adjacent, i.e., $|i-j|=1$. In other cases, the picker travels parallel to the cross aisle which will not perform the shortcut leveraged by the visibility graph approach. There are n possible ways that $i=j$, and these incur no cross aisle travel. Therefore, the probability that $|i-j|=0$ is n/n^2. Similarly, there are $2(n-1)$ combinations for the case $|i-j|=1$. Therefore, $Pr(|i-j|=1)=2(n-1)/n^2$. In general,

$$Pr(i-j|=k)=\frac{2(n-k)}{n^2} \tag{2}$$

Since $\lim_{n\to\infty}\frac{2(n-1)}{n^2}=0$, we can conclude that the number of aisles, n, for the maximum distance difference is 2. To fully calculate the expected paths for the visibility graph method, the values of the pick aisle half width, a, the storage location depth, b, the storage location width c, the cross aisle half width, v, the pick location's distance to cross aisle in aisle i, x, and the pick location's distance to the cross aisle in aisle j must be included. There are two possible ways of picking: exiting either from bottom or from top, and both are equally likely because of the uniform distribution of picking. The expected cross aisle travel distance for the visibility graph is

$$\frac{2(n-1)(a(n-2)+b(n-1))}{3n} \tag{3}$$

and the expected picking aisle travel distance for the visibility graph is

$$\frac{1}{n}\left(\frac{L}{3}\right)+\frac{n-1}{n}f_1(a,L) \tag{4}$$

The derivations of (3) and (4) including the definition of $f_1(a, L)$ are found in the Appendix. Given (3) and (4), the expected travel distance between locations for Traditional Layout A using the visibility method is

$$E[TB_{VG}]=\frac{1}{n}\left[\frac{L}{3}+(n-1)f_1(a,L)\right] + \frac{2(n-1)(a(n-2)+b(n-1))}{3n} \tag{5}$$

The expected travel distance between locations for Traditional Layout A using the aisle-centers method (using the equation from Pohl et al. (2009a)) is

$$E[TB_{ACM}]=\frac{1}{n}\left[\frac{L}{3}+(n-1)\left(\frac{2}{3}L+2v\right)\right] + \frac{2(a+b)(n^2-1)}{3n} \tag{6}$$

Note that the aisle width term in Pohl et al. (2009a) is replaced with $2a+2b$ in our version. The expected overestimation between the aisle-centers method (using the equation from Pohl et al. (2009a) for the expected aisle-centers method distance) and visibility graph method can be shown as

$$\frac{E[TB_{ACM}]-E[TB_{VG}]}{E[TB_{ACM}]}=\frac{(n-1)\begin{pmatrix}16a^2\sqrt{a^2+L^2}-8L^2\sqrt{a^2+L^2}+15a^2L\log\left(\frac{a}{\sqrt{a^2+L^2}+L}\right)\\ -6a^2L\log\left(\frac{\sqrt{a^2+L^2}+L}{a}\right)-3a^2L\tanh^{-1}\left(\frac{L}{\sqrt{a^2+L^2}}\right)\\ -16a^3-8an^3+40an^2+24an-8bn^3\\ +32bn^2+24bn+8L+24v\end{pmatrix}}{4(6(n-1)(an(n+1)+bn(n+1)+v)+L(2n-1))} \tag{7}$$

If β_1, β_2, β_3, and β_4 are equal to 45° the maximum difference between the two distance metrics occurs. This only happens with a single pick location per pick aisle (i.e., L = c) and $a = c/2$.

Note that an increase in v will increase the distance for the aisle-centers method but does not affect the visibility graph method for this type of layout. For any constant values of x, y, L, a, b, the $\lim_{v\to\infty}\frac{Min[x+y+2v+2b,(L-x)+(L-a)+2v+2b]}{Min[\sqrt{x^2+a^2}+2b,\sqrt{(L-x)^2+(L-a)^2}+2b]}=\infty$. In other words, warehouses with wider cross aisles will have a greater distance difference between the two metrics. To evaluate visibility graph distances relative to aisle-centers distances, we calculated the expected travel distances between two locations for various values of L,v,b,and n. Our calculations on overestimation by the aisle-centers distance metric for a variety of warehouse sizes and parameters show a range of 0–47%, over a variety of warehouse shapes and sizes (see the Appendix, Table A1), with the maximum overestimation occurring with two picking aisles with only a few storage locations.

4 | COMPUTATIONAL RESULTS

To make specific comparisons we devised computational experiments with a variety of warehouse layouts, sizes, and pick characterization. For the results that follow, we assume warehouses between 200 and 1000 pick locations, where a location is a stopping point that gives access to storage slots on both sides of the aisle. We assume a uniform demand and random storage policy in order not to create "hot zones" that are visited more often. We assume picking and cross aisles are 12 ft wide and that storage locations are 4 × 4 ft. We use a 2.5 ft buffer around all obstacles to estimate the paths with small size pickers such as human pickers with carts. We also used a 3.5 ft buffer around all obstacles to estimate the paths with large size pickers such as forklifts or automated guided vehicles. These are all reasonable assumptions in current warehousing (Goetschalckx & Ratliff, 1988).

Picking tours vary between 1 and 30 locations. We use the order generation method of Çelik and Süral (2014), but instead of evaluating 100 orders for each pick list size, we vary the number of orders. In general, tours with few picks have a higher variance in travel distance, so we increase the sample size for small pick lists to decrease the variability. Table 1 shows the number of orders sampled for each pick list size. The number of orders sampled for each pick list size is sufficient to guarantee a relative error of estimating mean travel distance of at most 1% with a probability 95%.

TABLE 1 Number of orders evaluated for each pick list size

Pick list size	Number of orders
1	10,000
2	5000
3	3333
5	2000
10	1000
30	333

TABLE 2 Average tour distance overestimations for traditional A, B, and C designs with the aisle-centers method (ACM) compared to the visibility graph method (VGM) with 2.5 and 3.5 ft buffer sizes

		% difference					
		Traditional A		Traditional B		Traditional C	
Picks	SKUs	2.5 ft	3.5 ft	2.5 ft	3.5 ft	2.5 ft	3.5 ft
1	200	9.17	6.79	13.57	10.63	17.00	13.86
1	400	7.51	5.81	11.48	8.90	15.03	12.11
1	600	6.35	4.88	11.55	9.44	12.91	10.20
1	800	6.66	5.42	10.99	9.12	11.77	9.13
1	1000	4.50	3.31	9.15	7.25	12.48	10.20
2	200	10.89	8.20	13.90	10.79	16.92	13.72
2	400	8.92	6.92	11.50	8.84	14.48	11.63
2	600	7.57	5.86	11.42	9.27	12.82	10.15
2	800	7.76	6.28	10.89	8.97	11.29	8.75
2	1000	5.63	4.21	9.06	7.17	11.79	9.57
3	200	12.32	9.39	14.12	10.97	16.90	13.68
3	400	10.13	7.89	11.68	9.00	14.30	11.46
3	600	8.65	6.72	11.54	9.35	12.70	10.05
3	800	8.65	6.96	10.97	9.01	11.08	8.57
3	1000	6.59	4.99	9.17	7.25	11.55	9.37
5	200	14.26	10.95	15.38	11.96	17.50	14.16
5	400	11.90	9.29	12.82	9.90	14.69	11.75
5	600	10.23	7.96	12.69	10.28	13.08	10.35
5	800	10.15	8.14	12.09	9.91	11.41	8.84
5	1000	7.94	6.04	10.13	8.02	11.90	9.64
10	200	16.13	12.40	17.87	13.93	19.33	15.57
10	400	13.89	10.83	15.30	11.86	16.68	13.35
10	600	12.17	9.47	15.38	12.45	14.65	11.52
10	800	12.21	9.79	14.74	12.06	13.23	10.28
10	1000	9.74	7.43	12.51	9.93	13.85	11.23
30	200	15.35	11.84	20.92	16.30	22.43	18.01
30	400	13.97	10.88	18.30	14.18	20.30	16.23
30	600	12.78	9.92	18.83	15.17	17.69	13.80
30	800	13.23	10.58	18.41	14.98	16.91	13.17
30	1000	10.76	8.20	15.91	12.60	17.94	14.55

The percentage difference between average travel distances in two layouts T_1 and T_2 is

$$\frac{T_1 - T_2}{T_1} \times 100. \tag{8}$$

4.1 | Comparing distance models in a typical warehouse

For traditional layouts (Figure 4b–d) distances produced by the visibility graph method are less than those produced by the aisle-centers method, but how significant is the difference? Table 2 shows the percentage overestimation of average travel distances for traditional layouts A, B, and C, which is as high as 22.43%. The overestimation is smaller for larger warehouses and smaller pick lists. Even the smallest overestimation case is noteworthy (3.31%).

It should be mentioned that the estimated differences between the aisle-centers method and the visibility graph method are upper bounds in some sense, as one would not always expect every picker to always take the shortest possible path while moving from one location to another.

Careful investigation of individual picking tours revealed that paths created by a visibility graph are especially shorter around corners at the ends of aisles, as when a picker exits a picking aisle and begins travel in a cross aisle. The aisle-centers method assumes the worker walks straight ahead until he or she reaches the center of the cross aisle, then makes a 90 degree turn and continues. The visibility graph assumes the worker walks directly to the corner of the buffer region and then "hugs" the edge if the next pick is on the same side of the cross aisle. This sort of corner cutting results in a sizeable reduction in distance.

The end-of-aisle effect also explains why the difference in expected distances between the two metrics increases with the number of picks: more picks means more aisles visited, and each transition to a new picking aisle incurs an end-of-aisle difference. Moreover, the difference in expected distances

TABLE 3 Average tour distance overestimations for traditional A, B, and C designs with the aisle-centers method (ACM) versus the visibility graph method (VGM) with 2.5 and 3.5 ft buffer sizes

		Average tour distance (ft)													
		Traditional A			Traditional B			Traditional C			Best to worst			Penalty cost (%)	
Picks	SKUs	ACM	2.5	3.5	ACM	2.5	3.5	ACM	2.5	3.5	ACM	2.5 ft	3.5 ft	2.5 ft	3.5 ft
1	200	141	128	132	162	140	145	175	145	151	A, B, C	A, B, C	A, B, C	0.0	0.0
1	400	194	179	183	210	186	191	229	194	201	A, B, C	A, B, C	A, B, C	0.0	0.0
1	600	234	219	222	255	225	231	266	231	238	A, B, C	A, B, C	A, B, C	0.0	0.0
1	800	274	255	259	293	261	267	297	262	270	A, B, C	A, B, C	A, B, C	0.0	0.0
1	1000	297	283	287	317	288	294	338	295	303	A, B, C	A, B, C	A, B, C	0.0	0.0
2	200	236	210	216	248	214	221	270	224	233	A, B, C	A, B, C	A, B, C	0.0	0.0
2	400	325	296	302	324	286	295	349	298	308	B, A, C	B, A, C	B, A, C	0.0	0.0
2	600	391	362	369	395	350	358	407	355	366	A, B, C	B, C, A*	B, C, A*	**3.4**	**2.9**
2	800	458	423	430	453	404	412	454	403	414	B, C, A	C, B, A*	B, C, A	**0.3**	0.0
2	1000	498	470	477	491	447	456	514	454	465	B, A, C	B, C, A*	B, C, A*	0.0	0.0
3	200	302	264	273	303	260	270	330	274	285	A, B, C	B, A, C*	B, A, C*	**1.6**	**1.3**
3	400	417	375	384	395	349	360	427	366	378	B, A, C	B, C, A*	B, C, A*	0.0	0.0
3	600	504	460	470	483	428	438	498	434	448	B, C, A	B, C, A	B, C, A	0.0	0.0
3	800	588	537	547	556	495	506	555	493	507	C, B, A	C, B, A	B, C, A*	0.0	**0.3**
3	1000	640	598	608	603	548	559	628	556	569	B, C, A	B, C, A	B, C, A	0.0	0.0
5	200	393	337	350	384	325	338	413	341	355	B, A, C	B, A, C	B, A, C	0.0	0.0
5	400	551	485	500	504	440	454	531	453	469	B, C, A	B, C, A	B, C, A	0.0	0.0
5	600	668	599	615	616	538	552	621	540	557	B, C, A	B, C, A	B, C, A	0.0	0.0
5	800	782	702	718	707	622	637	691	612	630	C, B, A	C, B, A	C, B, A	0.0	0.0
5	1000	851	784	800	767	689	706	783	690	707	B, C, A	B, C, A	B, C, A	0.0	0.0
10	200	542	455	475	523	430	451	553	446	467	B, A, C	B, C, A*	B, C, A*	0.0	0.0
10	400	779	671	695	690	584	608	711	593	616	B, C, A	B, C, A	B, C, A	0.0	0.0
10	600	958	841	867	851	720	745	835	713	739	C, B, A	C, B, A	C, B, A	0.0	0.0
10	800	1135	996	1024	978	834	860	923	801	828	C, B, A	C, B, A	C, B, A	0.0	0.0
10	1000	1238	1117	1146	1060	928	955	1050	905	932	C, B, A	C, B, A	C, B, A	0.0	0.0
30	200	758	642	669	797	630	667	872	677	715	A, B, C	B, A, C*	B, A, C*	**1.8**	**0.2**
30	400	1181	1016	1052	1105	903	948	1148	915	961	B, C, A	B, C, A	B, C, A	0.0	0.0
30	600	1538	1342	1386	1409	1144	1196	1370	1128	1181	C, B, A	C, B, A	C, B, A	0.0	0.0
30	800	1887	1637	1687	1654	1349	1406	1524	1266	1323	C, B, A	C, B, A	C, B, A	0.0	0.0
30	1000	2114	1887	1941	1818	1529	1589	1754	1440	1499	C, B, A	C, B, A	C, B, A	0.0	0.0

increases with a smaller buffer size (2.5 ft vs. 3.5 ft) because of greater flexibility of taking shortcuts. In other words, a human worker with a cart can take a shorter path because of his or her size compared to a forklift or automated guided vehicle. Finally, percent differences tend to go down as the size of the warehouse increases: as aisles get longer, more of each tour is within aisles and corner cutting comprises less of the overall distance.

4.2 | Traditional design using the visibility graph versus aisle-centers with fixed pick list size

Choosing the number and orientation of cross aisles in a layout requires finding a design that minimizes expected travel distance. With a model based on visibility graphs instead of aisle-centers, should we expect different design decisions?

We consider this question in the context of layouts with parallel picking aisles and orthogonal cross aisles (so-called "traditional designs"). Table 3 shows best to worst ranking of traditional layouts A, B, and C with each method ("*" indicates that best to worst layout ranking order has changed). Smaller warehouses have a larger percentage gap in average tour distance between the aisle-centers method and the visibility graph method. 7 out of 30 best to worst rankings change for both 2.5 and 3.5 ft buffer sizes. More importantly, in 13.3% of the cases the best layout with respect to average tour distance changes for both 2.5 and 3.5 ft buffer sizes. For these cases, the difference in estimated travel distance is between 0.2 and 3.4%. The penalty of choosing a design using aisle-centers is nonzero only when the best layout changes between the two distance estimation methods.

As an added analysis, we considered a small example to see if and how the ordering of the picks changes by using the visibility graph method. We considered a small warehouse of traditional B design with 200 SKUs and 10 pick lists with each pick list of 5 items. This warehouse has 12 ft cross aisle and

TABLE 4 Three sets of actual pick list (PL) data

PL set#	Number of SKUs	Number of PLs	Average number of SKUs per PL
1	597	4625	7.93
2	510	4957	7.40
3	267	98	10.21

TABLE 5 Warehouse parameters used for the computational experiments for variable pick list data

PL set#	Layout	Number of locations	Area	Aspect ratio	Pick aisle angles
1	Traditional A	598	33,811	0.50	90
1	Traditional B	600	38,662	0.50	90, 90
1	Traditional C	600	41,800	0.54	90, 90, 90
2	Traditional A	520	30,746	0.55	90
2	Traditional B	528	34,322	0.50	90, 90
2	Traditional C	525	38,662	0.50	90, 90, 90
3	Traditional A	272	17,681	0.50	90
3	Traditional B	272	20,812	0.50	90, 90
3	Traditional C	285	24,994	0.52	90, 90, 90

pick aisles and 4 ft wide and deep storage locations. The pick lists were generated using Benders Model and all items have uniform demand (i.e., each item has an equal probability of being picked). All items were allocated to the same locations in both the visibility graph and aisle-centers methods. From the solution files of Concorde TSP solver for each of 10 pick lists and found that 8 out of 10 pick lists had a different optimal sequence to pick the items on the pick list.

4.3 | Traditional design using the visibility graph versus aisle-centers with variable pick list size

In previous section, we have used fixed size generated pick lists for comparison. Even though it is useful analysis on the comparative efficiency of these layouts, the design of the warehouse is not determined based on a single pick list size unless it is a single-command or a dual-command system. Therefore, we extended the analysis of the previous section by using actual pick list data which has combinations of different pick list sizes (see Table 4). We compared the same three traditional designs from previous section. We used three different set of pick lists from real data. Table 5 shows the warehouse design parameters for each set of pick lists.

Table 6 shows best to worst ranking of traditional layouts A, B, and C with each method ("*" indicates that best to worst layout ranking order has changed). Similar to the previous experiment with uniform pick list sizes, smaller warehouses have a larger percentage gap in average tour distance between the aisle-centers method and the visibility graph method. In one out of three cases, the preferred layout with respect to average tour distance changes for both 2.5 and 3.5 ft buffer sizes changes. For these cases, the difference in estimated travel distance is between 3.7 and 4.6%.

4.4 | Fishbone design using the visibility graph versus aisle-centers

Even a cursory look at the geometry of visibility graphs suggests important differences from the aisle-centers method when picking and cross aisles meet at other than right angles. Specifically, when a worker must turn more than 90 degrees, the aisle-centers method assumes the worker travels an unreasonable path, whereas a visibility graph represents a more reasonable path. Conversely, if a worker turns into an aisle at, say, 45 degrees, then the aisle-centers and visibility graph methods produce similar, though not identical, distances.

For single-command, unit-load operations in a fishbone design, pickers make turns only at 45 degrees, and so there is less advantage to cutting a corner. This observation suggests that the difference between the aisle-centers and visibility graph methods should be less for a fishbone design than for a traditional design. For tours with more than one pick, workers will sometimes have to turn more than 90 degrees in a fishbone design, so we expect the difference in average distance between the visibility graph and aisle-centers methods to increase.

To investigate these questions, we consider a fishbone and traditional design B with the parameters in Table 7. Aspect ratio is the ratio of depth to the width of the warehouse. A 0.5 aspect ratio means that the warehouse is twice as wide as deep. Traditional Layout B has two regions and each region has 90 degree pick aisles. The fishbone layout has three regions. The left and right regions have 0 degree pick aisles and the middle region has 90 degree pick aisles. For many warehouses, the customer demand distribution is skewed. Therefore, in this section we also consider cases in which demand is not uniform. Figure 10 shows the results for uniform demand and for 20 of 40, 20 of 60, and 20 of 80 demand skewness. For single-command operations, the fishbone layout's performance improvement over traditional layout B decreases from 18.17 to 15.12% (2.5 ft buffer) and 15.87% (3.5 ft buffer) when using the visibility graph method, as expected. As the size of the pick list increases, the aisle-centers method produces longer distances relative to the visibility graph, so the percent improvement of a fishbone design is lower, and eventually negative. The results are similar for all values of demand skewness.

4.5 | The effect of the warehouse size in percent improvement of fishbone layouts

In a final set of experiments, we analyze the percent improvement of the fishbone layout over traditional layout A (see Figure 4b) for unit-load (i.e., single command) operations and different warehouse sizes. We use traditional layout A instead of traditional layout B because addition of the middle aisle of traditional layout B has no benefit for unit-load operations (Gue & Meller, 2009). This set of experiments is similar to the discrete model analysis performed by Öztürkoğlu et al. (2012)

TABLE 6 Average tour distance overestimations for traditional A, B, and C designs with the aisle-centers method (ACM) versus the visibility graph method (VGM) with 2.5 and 3.5 ft buffer sizes with variable pick list data sets

	Average tour distance (ft)													
	Traditional A			Traditional B			Traditional C			Best to worst			Penalty cost (%)	
PL set	ACM	2.5	3.5	ACM	2.5	3.5	ACM	2.5	3.5	ACM	2.5 ft	3.5 ft	2.5 ft	3.5 ft
1	451	411	420	544	470	486	542	463	481	A, C, B	A, C, B	A, C, B	0.0	0.0
2	434	396	405	478	407	422	491	410	427	A, B, C	A, B, C	A, B, C	0.0	0.0
3	390.9	345	356	391	329	343	477	393	411	A, B, C	B, A, C	B, A, C	**4.6**	**3.7**

TABLE 7 Warehouse parameters used for the fishbone experiments

Layout	Number of locations	Area	Aspect ratio	Pick aisle angles
Traditional B	4012	191,260	0.51	90, 90
Fishbone	4000	196,109	0.50	0, 90, 0

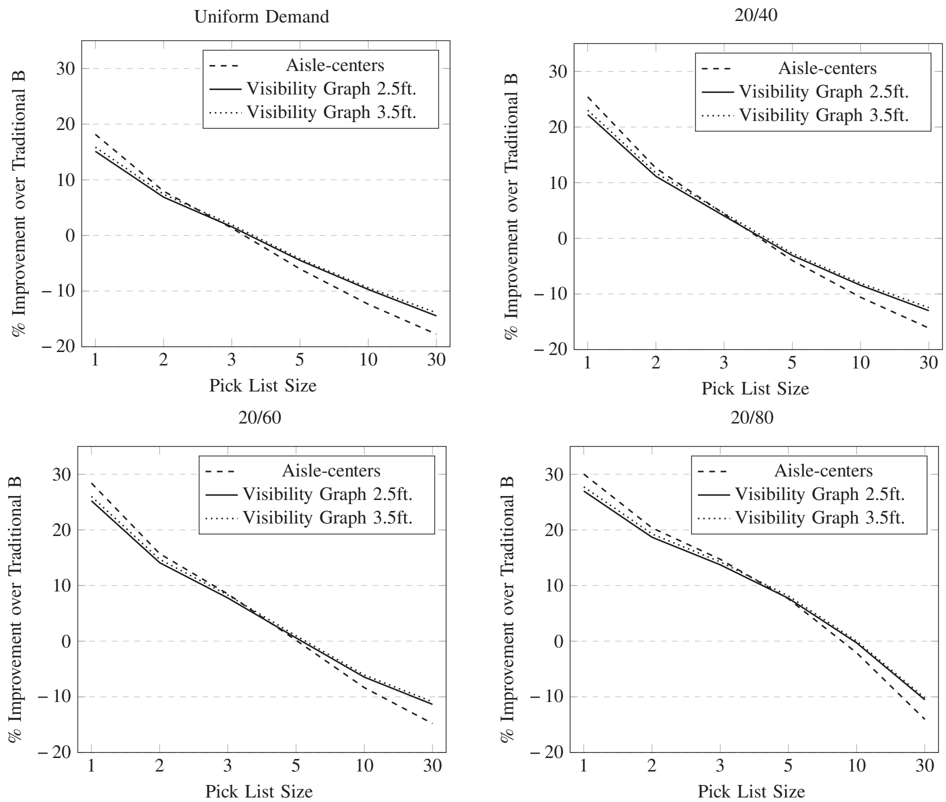


FIGURE 10 The change in relative fishbone percent improvement over traditional layout B as pick list size increases

and Pohl et al. (2011). Figure 11 shows the results. The curves in Figure 11 are not smooth because as warehouse size (number of SKUs) increases, the number of picking aisles in the fishbone and traditional layout A warehouses change discretely and independently of one another. For small warehouses, the difference between the two distance models is as high as 5.89%. As the size of the warehouse increases, the difference between the two metrics decreases because a greater percentage of travel is within aisles and not between them.

5 | CONCLUSIONS

Although we have not made the case with observational data, we contend that a properly established visibility graph better represents travel paths associated with reasonable worker behavior in order picking warehouses. The traditional model of distances based on aisle-centers assumes that workers walk to the center of picking and cross aisles and then (assuming a traditional design) make right angle turns to continue. The visibility graph assumes workers will make reasonable efforts to cut corners and therefore take shorter paths to their destinations. We have shown analytically and empirically that using the aisle-centers method overestimates the distance that a rational worker would walk.

The implications of this new approach are significant. First, it suggests that travel distances between locations in a warehouse are not as long as commonly assumed and therefore that associated labor costs are not as high. Said another way, throughput for a given number of pickers should be higher

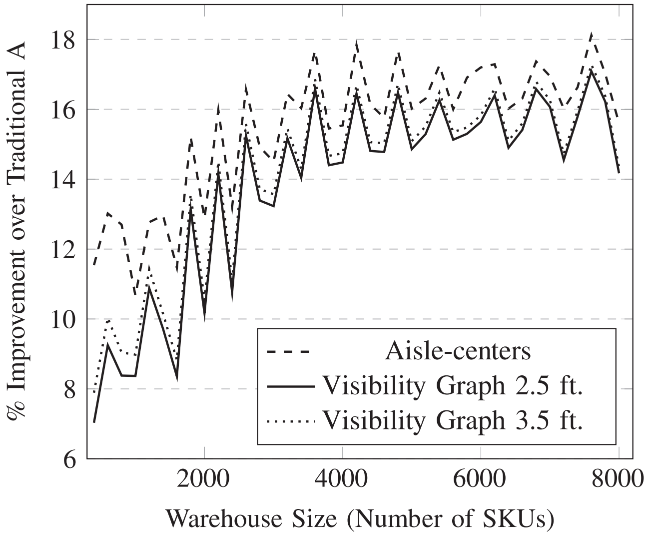


FIGURE 11 Percent improvement of a fishbone design over traditional layout A for single-command unit-load operations and different warehouse sizes

than predicted by the traditional model because workers travel shorter paths than the model assumes.

Second, preferred warehouse designs based on the new distance estimation model can be different that those based on the aisle-centers method. In 13.3% of the cases we considered, the selected traditional layout was different when using a visibility graph distance metric. An interesting follow-on study could consider nontraditional designs with diagonal aisles using the visibility graph during the warehouse design phase.

Third, our results suggest that the previous results of Gue and Meller (2009) and Çelik and Süral (2014) over-estimate the potential benefit of diagonal aisles versus traditional layouts for unit-load and order picking operations. This is not to say that these designs do not offer benefit, only that the level of benefit is probably not as high as their original models suggest.

As future work, we can ‘smooth’ the corners of the paths using a Bezier interpolation which takes turning radii into account and provide more accurate distance measures. Another major step forward would be a human study conducted in warehouse environments to ascertain the degree of natural corner cutting by workers.

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APPENDIX A

In this appendix we derive closed forms expressions for the expected travel distance between two random locations for the visibility graph method and compare this to the same value for the aisle-centers method from the work of Pohl et al. (2009a). Then, we calculate the expected difference between them, and thus the overestimation of the aisle-centers method.

We divide the travel distance into two components: cross aisle travel and picking aisle travel (as does Pohl et al. (2009a)). We assume the locations are uniformly distributed among the aisles. All aisles are equally likely to contain one of the two locations because of the uniform distribution. There are n^2 possible combinations of i and j for n number of aisles (all locations are equally likely to be picked). There are n possible ways that $i = j$, for which no cross aisle travel is required. The probability that $|i = j| = 0$ is n/n^2. Similarly, there are $2(n-1)$ combinations of $|i-j|$, so $Pr(|i-j| = 1) = 2(n-1)/n^2$. Unlike Pohl et al. (2009a), we have a special case where the cross aisle distance is $2b$ when $|i-j| = 1$ instead of $2a+2b$ as in Pohl et al. (2009a). Therefore we directly calculate the expected cross aisle travel distance instead of first calculating the expected number of aisle widths between i and j (which is not uniform for all i and j values). Considering the special case, the expected cross aisle travel distance for the visibility graph (for $n > 0$) is

$$E[TB_{crossaisle}] = \sum_{k=1}^{n-1} \frac{2(n-k)}{n^2}(2b + (k-1)(2a+2b)) = \frac{2(n-1)(a(n-2)+b(n+1))}{3n} \quad \text{(A1)}$$

For picking aisle travel using the visibility graph, there are two cases: two locations are in the same picking aisle and two locations are in different picking aisles. Since we assume a continuous uniform distribution within each aisle, we let X_i and Y_j be uniform random variables that represent the positions of the locations in aisles i and j, respectively, where $X_j \sim U(0, L)$ and Y_j $U(0, L)$. From probability theory, we know the expected distance between two locations on the same aisle of length L is $E[|x-y|] = L/3$. For the second case where locations are on different aisles, the picking aisle travel distance is $min[\sqrt{x^2+a^2} + \sqrt{y^2+a^2}, \sqrt{(L-x)^2+(L-y)^2}]$. Then,

$$\begin{aligned} f_1(a, L) &= E\left[min\left[\sqrt{y^2+a^2}, \sqrt{(L-x)^2+(L-y)^2}\right]\right] \\ &= \frac{1}{12L^2}\Bigg(16a^3 - 16a^2\sqrt{a^2+L^2} + 8L^2\sqrt{a^2+L^2} \\ &\quad + 3a^2 L ArchTanh\left[\frac{L}{\sqrt{a^2+L^2}}\right] \\ &\quad - 15a^2 L Log\left[\frac{a}{L+\sqrt{a^2+L^2}}\right] \\ &\quad + 6a^2 L Log\left[\frac{L+\sqrt{a^2+L^2}}{a}\right]\Bigg) \end{aligned} \quad \text{(A2)}$$

TABLE A1 Expected travel between distances by visibility graph (VG) and aisle-centers method (ACM), simulation results and absolute percentage difference between $E[TB_{VG}]$ and simulation, and the overestimation of the ACM for various warehouse parameters

a	*L*	*n*	*v*	*b*	$E[TB_{VG}]$	Simulation	% Diff	$E[TB_{ACM}]$	Overestimation (%)
6	6	1	6	4	2.00	2.00	0.00	2.00	0.00
6	6	1	6	2	2.00	2.00	0.00	2.00	0.00
6	6	1	6	1	2.00	2.00	0.00	2.00	0.00
6	6	2	6	4	11.46	11.46	0.02	19.00	39.69
6	6	2	6	2	9.46	9.47	0.13	17.00	44.35
6	6	2	6	1	8.46	8.46	0.05	16.00	47.13
6	6	50	6	4	334.14	334.88	0.22	348.92	4.24
6	6	50	6	2	267.50	266.54	0.36	282.28	5.24
6	6	50	6	1	234.18	234.32	0.06	248.96	5.94
6	240	1	6	4	80.00	80.00	0.00	80.00	0.00
6	240	1	6	2	80.00	80.00	0.00	80.00	0.00
6	240	1	6	1	80.00	80.00	0.00	80.00	0.00
6	240	2	6	4	124.58	124.64	0.05	136.00	8.39
6	240	2	6	2	122.58	122.71	0.10	134.00	8.52
6	240	2	6	1	121.58	122.02	0.35	133.00	8.58
6	240	50	6	4	480.99	480.38	0.13	503.36	4.44
6	240	50	6	2	414.35	413.48	0.21	436.72	5.12
6	240	50	6	1	381.03	381.73	0.18	403.40	5.55

The probability the second location will be in the same aisle as the first location (i.e, $|i-j|=0$) is $1/n$, and the probability the second location will be in a different aisle $(n-1)/n$. The expected picking aisle travel distance is then

$$\frac{1}{n}\left(\frac{L}{3}\right)+\frac{n-1}{n}f_1(a,L) \tag{A3}$$

To compare this quantity for different warehouse sizes we calculate the values in Table A1 and also verify the analytical results for the visibility graph with simulation. Simulation experiments are done based on 100,000 trials. The results range from no overestimation to nearly 50%. The simulation results provide nearly total agreement with the analytic equations derived in this appendix.