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Retail space design considering revenue and adjacencies using a racetrack aisle network

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In this article, a model and solution approach for the design of the block layout of a single-story department store is presented. The approach consists of placing departments in a racetrack configuration within the store subject to area and shape constraints. The objective function considers the area allocated to each department, contiguity of the departments to the aisle network, adjacency requirements among departments, and department revenues. The revenue generated by a department is defined as a function of its area and its exposure to the aisle network. The aisle network is comprised of two components: the racetrack, which serves as the main travel path for the customers, and the entry/exit aisle. The racetrack aisle itself is treated as a department with area allocation and corresponding revenue generation. A general tabu search optimization framework for the model with variable department areas and an aisle network with non-zero area is devised and tested.

Keywords: Store design, racetrack aisle, tabu search, hybrid optimization

1. Introduction

Broadly, the facility layout problem is defined as locating a number of departments within a given area to optimize a performance metric. The two most widely used performance measures are minimizing the total traffic cost among departments and maximizing adjacency among departments. Although there is a rich body of literature addressing facility layout problems, papers addressing service facilities are very few and quite problem specific, such as Elshafei's hospital layout paper (Elshafei, 1977). However, the retail trade sector has the third-largest impact on U.S. Gross Domestic Product (GDP) according to the 2007 Economic Census. General merchandise stores provide job opportunities for 5000 000 people, creating $900 billion in sales with 260 000 establishments (Economic Census, 2007).

The most basic distinction between manufacturing facilities and retail facilities is that in the latter the traffic is mostly human. Hence, the traditional performance measure of traffic cost minimization is not appropriate. In this article, the design of the block layout of single-story retail stores is addressed, where the retail establishment is a department-type store (the approach would also work for stores with distinct merchandise departments such as book stores, sporting goods stores, and electronics stores). The objective is twofold: (i) revenue maximization and (ii) adjacency satisfaction. Constraints are placed on department sizes and shapes and the configuration is a single racetrack with variable aisle area. To solve the model, a special solution representation and decoding/encoding mechanisms are developed and utilized in a tabu search framework.

2. Background

There are two previous studies that addressed the retail facility layout problem. Peters *et al.* (2004) considered three retail layout settings, aisle-based, hub and spoke, and serpentine layouts, and calculated expected tour lengths for given shopping lists. Botsali and Peters (2005) built on this initial study by proposing a network-based model for the serpentine layout that aimed to maximize revenue by increasing the customers' exposure to impulse purchase items. This model assumed that the customer shopping lists are known. Three significant differences between this article and the existing literature are (i) *a priori* shopping lists are not used; (ii) a racetrack construct with variable area is assumed; and (iii) the revenue relationship is very different.

In today's retailing world, most companies use one of three layout types: (i) the grid; (ii) the free form; and (iii) the racetrack (Levy and Weitz, 2001). Among these

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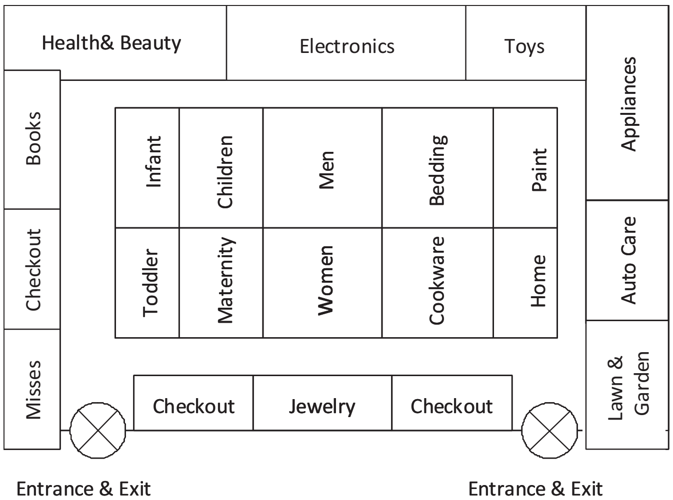

Fig. 1. An example racetrack layout.

three, the main advantage of the racetrack layout is that it allows shoppers to move past all of a store's merchandise in an easy and flowing manner. To illustrate, an example racetrack layout is provided in Fig. 1. Departments are lined up around the outer rim and are grouped within the center space. A racetrack aisle separates the outer rim from the inner space and provides one or more entrances. Larry Montgomery, the CEO of Kohl's, believed that customers could spend less time in his stores but still buy more items when products were arranged in a simple and logical manner in an environment that was attractive to the eye (http://www.answers.com/topic/larry-montgomery). His assertion was validated when other retail stores began to adopt Kohl's store layouts (Tatge, 2003).

2.1. *The shelf space allocation problem*

Regardless of the physical layout, a model of revenue must be devised. While the literature on retail layout design is sparse, there is much on a related topic, shelf space allocation. The relationship between the shelf space allotted to merchandise and profit and/or revenue generated has drawn the attention of researchers for almost 50 years. Brown and Tucker (1961) asserted that the law of diminishing marginal returns applies to the space/revenue relationship in retail settings. They identified three different product groupings based on the effect of increasing shelf space in sales. Lee (1961) also stated that the effect of increasing shelf space must be of diminishing returns. However, neither of these studies specified a functional form. The assumption of diminishing marginal returns was validated by the field studies of Frank and Massy (1970), Curhan (1973), and Dreze *et al.* (1994). Corstjens and Doyle (1981) formulated demand (q) for product i based on the space elasticity as follows:

$$q_i = r_i s_i^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{K} s_j^{\delta_{ij}}, \tag{1}$$

where β_i is the space elasticity for product i, s_i is the shelf space allocated to product i, δ_{ij} is the cross-elasticity between product i and product j, and r_i is the demand multiplier for product i.

In this formulation, the demand for product i is defined as a function of the shelf space, and the space elasticity of the product reflects the law of marginal diminishing returns. The model also considers the effect of the shelf space allocated to complementary and substitute products on the demand for product i by incorporating the cross-elasticities to the formulation.

Using the definition above, Corstjens and Doyle (1981) also performed a field study that suggested that the space elasticities for different products fluctuate between 0.06 and 0.25 and cross-elasticities between −0.01 and −0.05. The Corstjens–Doyle formulation has been used by Yang and Chen (1999), Irion *et al.* (2004), and Martínez-de-Albéniz and Roels (2011), among others. The approach of Yang and Chen (1999) is especially relevant because their model starts by allocating the available space to departments, and for each department a sub-problem is generated for product categories. After the shelf space allocated to each product category is determined, individual products are considered as yet another sub-problem. Considering the cross-elasticities to be of secondary importance, Irion *et al.* (2004) simplified the Corstjens–Doyle model by removing the cross-elasticities from the formulation. They formulated the demand function as

$$q_i = r_i s_i^{\beta_i}. \tag{2}$$

The simplified model proposed by Irion *et al.* (2004) has also been widely accepted by researchers. Using the Brown–Tucker classification, Irion *et al.* (2004) suggested [0.06, 0.1], [0.16, 0.20], and [0.21, 0.25] as the intervals of space elasticity for unresponsive, moderately responsive, and responsive products, respectively, to an increase in shelf space.

While Lee (1961) defined space in two dimensions rather than a one-dimensional shelf length, his and Dreze *et al.*'s (1994) studies are the only two that do so. Dreze *et al.* empirically determined that the area allocated to a product impacts sales according to the Gompertz model:

$$q_i = r_i e^{\beta_i \exp(\omega_i s_i)}, \tag{3}$$

where q_i is the demand for product i; r_i, β_i, and ω_i are the parameters of the Gompertz model; and s_i is the area occupied by product i. More specifically, r_i is referred to as the horizontal asymptote where $q_i \leq r_i$, and $\beta_i < 0$ and $\omega_i < 0$ together control the increase in demand as a function of the increase in area.

2.2. *The effect of impulse purchases and customer traffic density*

The sales made by a store broadly fall into two categories: planned purchases and unplanned purchases. Planned purchases constitute the portion of the sales that the customers come to the store with the intention of buying. Impulse purchases can be defined as those purchases that are not planned before the shopping event occurs (Kollat and Willet, 1967). Decisions leading to unplanned purchases are made within the store and are affected by in-store stimuli (Wilkinson *et al.*, 1982). The notion of impulse purchase has been studied from various perspectives, including descriptive studies (Bellenger *et al.*, 1978); effects of space allocation, promotion, price; and advertising (Wilkinson *et al.*, 1982); customer characteristics and customer activities (Kollat and Willet, 1967); and the effects of product display and environmental variables (Fiore *et al.*, 2000). The marketing literature suggests that different lines of products have different impulse purchase rates. For example, Bellenger *et al.* (1978) found that the percentages of impulse purchase are 27% for women's lingerie and 62% for costume jewelry.

Customer traffic within a store is another issue investigated by marketing researchers. Traffic density within a retail store can be defined as the number of visits to a certain zone of the store by customers. The customer traffic density throughout the store is not uniform, and certain zones of a store have denser traffic than others. Empirical studies (Larson *et al.*, 2005), stochastic models (Farley and Ring, 1966), and agent-based modeling applications (Batty, 2003) support this observation.

3. Model development

The model presented considers two criteria in evaluating a layout. These are the revenue generated by departments and adjacency satisfaction. The revenue generated by a department depends on the area allocated to the department (area effect) and the location of the department within the store (location effect). Definitions and mathematical formulations for the revenue and the layout adjacency are discussed in the following sections.

3.1. *The effect of area on revenue*

The main motivation behind the use of the Gompertz model in Dreze *et al.* (1994) was the assumption of "bounded unit sales" for a given product. This assumption implies that there is a limit to the sales of a product, and once that limit (r_i) is reached, increasing the allotted area, s_i, does not increase the sales, q_i. This assumption, however, is not valid for the model herein. A department defines a set of different merchandise lines and products that are grouped together, rather than being composed of a single item. For example, the shoe department of a retail store might include men's shoes and women's shoes. When the space allotted to the shoe department is increased, the additional space can be used for sports shoes, children's shoes, or another brand of women's shoes. Each of these alternatives can increase sales so that the asymptotic behavior of the Gompertz function is unsuitable. A power model ($q_i = r_i s_i^{\beta_i}$) is more suitable to define the relationship between the area and the revenue for a department store. The area effect on the revenue of a department is formulated as

$$\bar{R}_i = r_i A_i^{\beta_i}, \tag{4}$$

where r_i, A_i, and β_i are the revenue multiplier, the area, and the space elasticity for department i, respectively. The total expected revenue for the store is

$$\bar{R} = \sum_{i=1}^{n} \bar{R}_i + R_a \tag{5}$$

where $R_a = r_a A_a^{\beta_a}$. The total expected revenue is the sum of the revenues of individual departments and the revenue generated by the aisle space, R_a. R_a stems from the use of aisle space as a promotional display area. As mentioned in Section 2.1, the power model was used by Yang and Chen (1999) to allocate available shelf space to departments, not to products individually, and this approach is used here to represent the revenue generated by the departments. The power model parameters can be estimated using sales data for different product categories through experimental design (as used by Corstjens and Doyle (1981)) or with regression analysis (suggested by Yang and Chen (1999)).

3.2. *The effect of location on revenue*

Considering impulse purchasing along with uneven customer traffic within a store, a mechanism that locates departments with high impulse purchase likelihoods to more frequently visited parts of the store is needed. Two sets of ratings are defined: one for the different zones of the store denoted by $\mathbf{z}$ with respect to traffic density; the other is for the departments denoted by $\mathbf{d}$ with respect to the likelihood of impulse purchases. Using these classifications, the store area is divided into three ordinal zones, namely, high-traffic zones ($z_k = 1$), medium-traffic zones ($z_k = 2$), and low-traffic zones ($z_k = 3$). The partitioning used is depicted in Fig. 2. In the same manner, departments are grouped into three classes, high impulse purchase departments ($d_i = 1$), medium impulse purchase departments ($d_i = 2$), and low impulse purchase departments ($d_i = 3$). These two sets of rankings can then be used to measure the deviation of the departments' actual locations from their ideal locations. If department i would benefit from a high-traffic zone ($d_i = 1$) and is placed in a low-traffic zone ($z_k = 3$), the deviation would be positive, which indicates a missed revenue opportunity. On the other hand, if department i would not benefit from a high-traffic zone ($d_i = 3$) and is placed in a

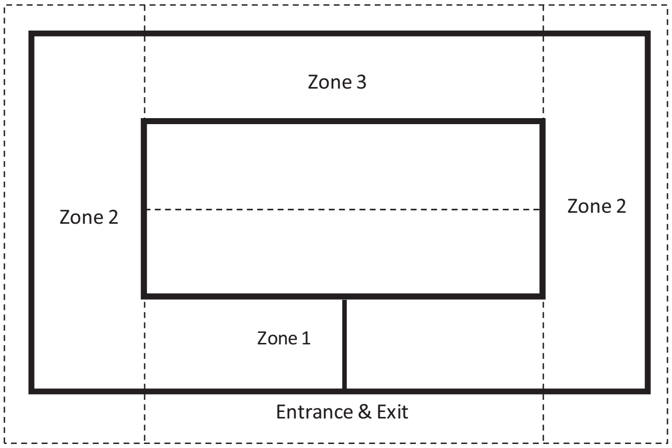

Fig. 2. Partition of the store area into zones with respect to traffic density.

high-traffic zone ($z_k = 1$), there is no missed revenue opportunity. Thus, for department i, the deviation from its desired location can be represented as $\max(0, z_k - d_i)$. If a department spans more than one zone, it is considered as being in the most advantageous zone (that is, the zone with the lowest classification).

The revenue of a department is formulated by taking the location into account as follows:

$$R_i = \frac{r_i A_i^{\beta_i}}{1 + \sum_{k=1}^{K} y_{ik} [\max(0, \quad z_k - d_i)]}, \tag{6}$$

where

$$y_{ik} = \begin{cases} 1 & \text{if department } i \text{ is in zone } k, \\ 0 & \text{otherwise,} \end{cases}$$

and the total revenue generated by the store is defined as

$$R = \sum_{i=1}^{n} R_i + R_a. \tag{7}$$

3.3. *Layout efficiency*

The layout efficiency, denoted by ε, defines the extent to which desired adjacencies are satisfied by a given layout and is formulated as

$$\varepsilon = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (c_{ij}^{+} x_{ij}) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (c_{ij}^{-}(1 - x_{ij}))}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij}^{+} - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij}^{-}}, \tag{8}$$

where

$$x_{ij} = \begin{cases} 1 & \text{if department } i \text{ is adjacent to department } j \\ & \qquad \forall i, j; \quad i > j. \\ 0 & \text{otherwise} \end{cases}$$

In measuring the layout efficiency, the well-known closeness ratings concept from the manufacturing facilities layout literature is used. Traditionally, closeness ratings reflect the level of interaction among departments and six closeness ratings are defined as depicted in Table 1. The denominator represents the largest possible value of the adjacency score for a given matrix of adjacency ratings (i.e., a layout such that all department pairs with c_{ij}^{+} are adjacent and all department pairs with c_{ij}^{-} are not). Two departments are considered adjacent if they share a common edge. Departments separated only by an aisle are also considered to be adjacent. For instance, in Fig. 3, department L is adjacent to departments A, B, and C, in addition to departments K and M.

Table 1. Adjacency ratings

Rating	*Definition*	c_{ij}
A	Absolutely necessary	125
E	Especially important	25
I	Important	5
O, U	Ordinary closeness	1
X	Undesirable	−25
XX	Prohibited	−125

3.4. *Overall model*

Given the definition of performance metrics, the complete model is

$$\max z = R\varepsilon, \tag{9}$$

subject to

$$\frac{P_i}{4\sqrt{A_i}} \le \overline{\alpha_i}, \quad \forall i, \tag{10}$$

$$A_i^L \le A_i, \quad \forall i, \tag{11}$$

$$A_a^L \le A_a, \tag{12}$$

$$\sum_{i=1}^{n} A_i + A_a = A, \tag{13}$$

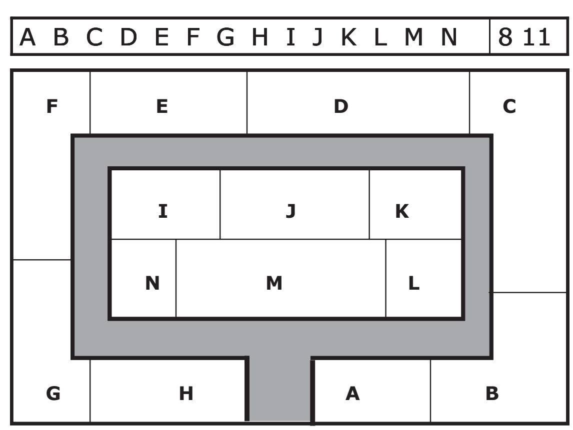

Fig. 3. Solution representation and the corresponding layout.

$$\sum_{i=1}^{n} A_i y_i = A^O, \quad (14)$$

$$\sum_{i=1}^{n} A_i \ (1 - y_i) = A^I, \quad (15)$$

$$w^L \leq w_a \leq w^U, \quad (16)$$

$$x_{ij} = \begin{cases} 1 & \text{if department } i \text{ is adjacent to department } j \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j; \quad i > j,$$

$$y_i = \begin{cases} 1 & \text{if department } i \text{ is in the outer rim} \\ 0 & \text{otherwise} \end{cases} \quad \forall i.$$

Adjacency maximization is treated as a damper to the revenue to avoid scaling problems of the two (often conflicting) objectives. An ideal layout, insofar as adjacency is concerned, has no effect on revenue, whereas as departments adhere less to desired adjacencies, revenue is decreased accordingly. Constraints (10) impose aspect ratio requirements for each department where P_i denotes the perimeter of department i. These ensure that departments are of reasonable squareness. Constraints (11) and (12) ensure that the areas allocated to the departments and to the aisle are larger than their lower bounds. Constraint (13) ensures that the total department and aisle areas cannot exceed the area of the store. Constraints (14) and (15) ensure that the areas allocated to the departments in the outer rim and in the inner region do not exceed the area available in those regions. Constraint (16) guarantees that the width of the aisle is within some pre-specified limits. Note that the racetrack aisle is treated as a department in this formulation.

There are limitations on the designs possible with this model. First, there is a single racetrack aisle with the same aspect ratio as the bounding facility. There are upper and lower bounds imposed on the width of that aisle. Second, in the inner region of the racetrack, departments can only be split into two rows; that is, only split into two bays. Third, in the outer region of the racetrack, departments are only one layer deep and those spanning a corner do so by use of a continuous representation. Since the area is not discretized, corners are spanned using the area of the department and the width of the outer region. Fourth, an aspect ratio constraint using a perimeter measure is set for each department to ensure reasonable shapes. Finally, there is a single entrance aisle located midway across the bottom of the facility, and the area of this entry aisle is treated identically to that of the racetrack aisle. While these are restrictions, they are reasonable considering the layout of most racetrack retail enterprises, and they make the optimization more tractable.

3.5. *Solution approach*

Due to the non-linear nature of the revenue function, a two-stage optimization procedure is devised. In the first stage, the area allotments including the aisle area are determined by solving the following model:

$$\max \ \bar{R} = \sum_{i=1}^{n} \bar{R}_i + R_a, \quad (17)$$

subject to

$$\sum_{i=1}^{n} A_i + A_a \leq A, \quad (18)$$

$$A_i^L \leq A_i, \quad \forall i. \quad (19)$$

The objective function is denoted by $\bar{R}$ since these equations disregard the zone requirements of departments. A non-linear optimization package can be used to solve this problem. Then, using the area allotments, tabu search is performed to optimize the block layout of the store by assigning departments to the outer rim or to the inner area and specifying their sequence. This assignment determines the satisfaction of adjacency and aspect ratio constraints along with revenue generated by the region placement. The details of the tabu search are presented in Section 3.6.

3.6. *Tabu search for retail spatial design with flexible department areas*

Tabu search (TS) was introduced in the combinatorial optimization literature by Glover (Glover, 1986, 1989, 1990; Reeves, 1995; Glover and Laguna, 1997). The procedure is a neighborhood search algorithm with a mechanism that prohibits certain moves within the neighborhood to avoid cycling. TS has been successfully implemented in many combinatorial, NP-hard optimization problems, including quadratic assignment (Taillard, 1991), unequal area facility layout (Kulturel-Konak *et al.*, 2004), vehicle routing (Taillard *et al.*, 1997), redundancy allocation (Kulturel-Konak *et al.*, 2003), job shop scheduling (Barnes and Chambers, 1995), and weighted maximal planar graphs (Osman, 2006).

In following sections, the overall TS scheme is discussed, starting with the solution representation.

3.6.1. *Solution representation and decoding*

The solution representation is depicted in Fig. 3. It is an extension of the flexible bay structure (FLEXBAY) representation that was introduced by Tong (1991) and popularized by Tate and Smith (1995).The encoding is composed of the sequence of departments, **p**, followed by two baybreaks. The departments from the beginning of the permutation up to the first baybreak, b_1, are assigned to the outer bay. The departments starting from the first baybreak up to the second baybreak are assigned to the upper inner bay, while the remaining departments are assigned to the lower inner bay.

A	B	C	D	E

(a)

2	4

(b)

B	A	C	D	E
C	B	A	D	E
D	B	C	A	E
E	B	C	D	A
A	C	B	D	E
A	D	C	B	E
A	E	C	D	B
A	B	D	C	E
A	B	E	D	C
A	B	C	E	D

(c)

1	2
1	3
1	4
2	3
2	4
3	4

(d)

Fig. 4. The initial solution: (a) permutation; (b) partitioning; (c) permutation neighborhood; and (d) partitioning neighborhood.

Assignment of departments to the outer bay starts from the lower midpoint of the rectangular store area (the entry aisle) and continues counterclockwise. The placements inside the racetrack are done using Boustrophedon ordering starting from the upper left corner. The initial permutation and its corresponding baybreaks are generated randomly.

3.6.2. *Move operator*

Two move operators are used: one for the sequence and one for the baybreaks.

Swap: The swap operator exchanges the places of the two departments located in the ith and jth positions in the permutation $\mathbf{p}$ where $i = 1, \ldots, n-1$ and $j = i + 1, \ldots, n$.

Re-partite: The re-partite operator is used to change the number of departments allocated to each bay.

3.6.3. *Tabu list entries*

In the dynamically sized tabu list, the most recent department pairs that are swapped are kept.

3.6.4. *Neighborhood definition*

First, partitioning is kept constant and all other permutations that can be reached by a single swap operation are considered. Then, with the fixed new permutation, the search is performed in the partition neighborhood. An example solution representation and the corresponding neighborhood are provided in Fig. 4.

3.6.5. *Fitness function*

These fitness functions (Equations (20) to (22)) are the adjacency score only (F_a), revenue only (F_r), and the combination of adjacency score and revenue (F_f). The fitness functions F_a and F_r are used to gauge the extent the revenue and the adjacency components can be maximized individually. All fitness functions include a penalty function that penalizes layouts with departments violating the aspect ratio, constraint. The penalty term is $(n - s/n)^\kappa$, where n is the number of departments, s is the number of departments violating the aspect ratio, and κ is the exponent controlling the severity of the penalty function:

$$F_a = \varepsilon \left(\frac{n-s}{n}\right)^\kappa, \tag{20}$$

$$F_r = R \left(\frac{n-s}{n}\right)^\kappa, \tag{21}$$

$$F_f = R\,\varepsilon \left(\frac{n-s}{n}\right)^\kappa. \tag{22}$$

3.6.6. *Aspiration criterion*

If a solution within the neighborhood has a better fitness function than the best solution encountered so far, a move to that solution is allowed even if the move is tabu.

3.6.7. *Candidate list strategies*

For this problem, $n\,(n-1)^2(n-2)/4$ neighboring solutions can be reached from any given solution. Of these solutions, $n\,(n-1)/2$ are due to permutations, and for each permutation there are $(n-1)\,(n-2)/2$ baybreak combinations. Recall that the location of the first baybreak, b_1, determines the number of departments inside and outside the racetrack and thus determines the width of the racetrack. As the value of b_1 increases, the number of departments outside the racetrack increases, and the racetrack is located closer to the store center, making the aisle shorter. The reverse is also true: as the value of b_1 decreases, the number of departments outside the racetrack decreases, making the aisle longer. Therefore, there are natural limits on b_1 that are imposed by the lower and upper bounds on the aisle width. Given the total area of the departments inside the racetrack and the aisle area, the width of the aisle can be calculated from geometry:

$$w_a = \frac{1}{2}\left(\sqrt{(A^I + A_a)/\alpha^f} - \sqrt{A^I/\alpha^f}\right), \tag{23}$$

where A^I is the total area of the departments inside the racetrack, A_a is the aisle area, and α^f is the facility's aspect ratio.

Then, the set of feasible first baybreaks for the sequence of departments, $\mathbf{p}$, is defined as

$$\mathbf{b}_1 = \left\{k | w_a \geq w_a^L \wedge w_a \leq w_a^U\right\}. \tag{24}$$

Finally, for each $k \in \mathbf{b}_1$, the set of second baybreaks is defined as

$$\mathbf{b}_2 = \{m | m > k \wedge m < n,\ k \in \mathbf{b}_1, b_1 < b_2\}, \tag{25}$$

where n is the number of departments in the permutation. With Equations (24) and (25), the number of baybreak combinations is reduced substantially.

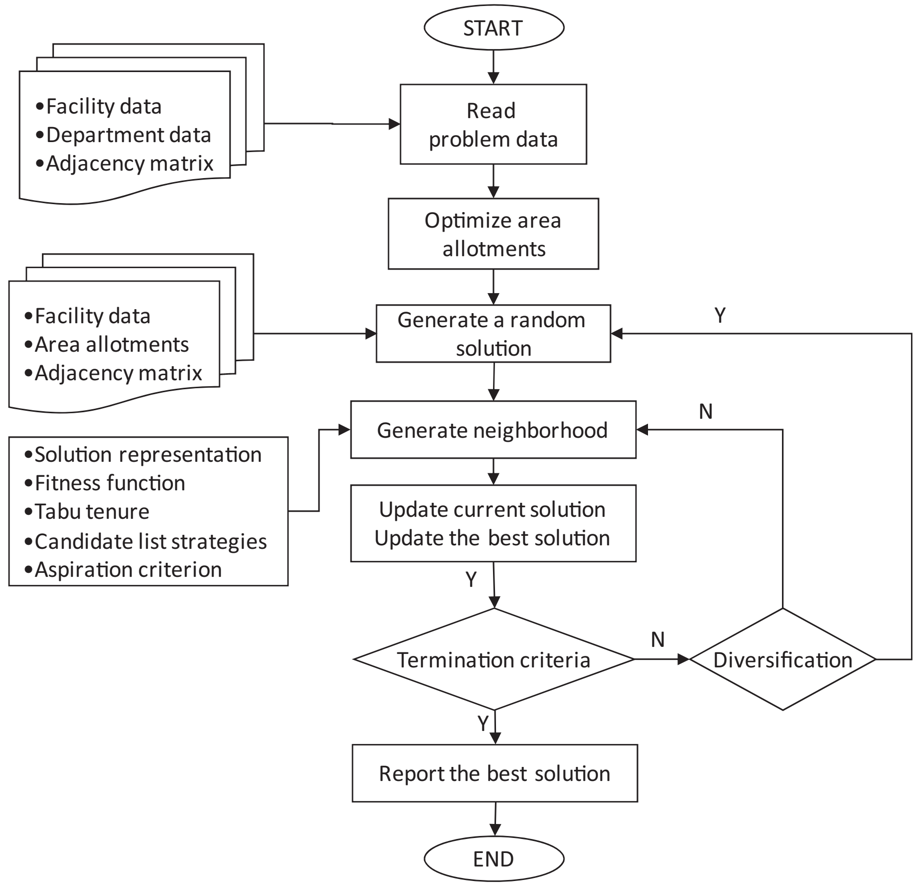

Fig. 5. Overall optimization framework.

3.6.8. *Diversification strategy*

The search restarts from a randomly generated solution and the tabu list is reset after 50 consecutive non-improving moves.

3.6.9. *Termination criterion*

If the best available solution has not been improved for a certain number of consecutive moves, the search terminates.

3.7. *Overall optimization algorithm*

Given the area allotment model, neighborhood generation, and the TS algorithm, the overall optimization procedure can be depicted as in Fig. 5.

4. Computational experience

Because of the novelty of this research, no test problems from the literature were identified. Two problem cases with $n = 12$ and $n = 20$ were devised. For both of these problems, adjacency matrices were generated randomly and other parameters such as β, $\mathbf{r}$, $\mathbf{d}$, and $\mathbf{A}^L$ were chosen randomly using a pre-specified range for each parameter. The problem data are available from the authors.

For each case, three available store areas (24 × 16, 25.5 × 17, 27 × 18), three fitness functions (F_r, F_a, F_f), and two modes for penalization of aspect ratio violations ($\kappa > 0$, $\kappa = 0$) were used. For $\kappa > 0$ in the 12-department problem, $\kappa = 3$ and for the 20-department problem $\kappa = 1$.

First, area allotments were determined using in a nonlinear program (NLP) written in Lingo. Figures 6(a) and 6(b) show the percentage increases in department areas with respect to the departments' minimal area requirements.

The additional area allotments are generally made in decreasing order of space elasticities of departments, β_i, but they are also affected by the revenue multiplier, r_i. For instance, for the smallest available store area (24 × 16 = 384 sq. units), the excess space is shared among departments E, G, and K for $n = 12$. Departments E and K have the highest space elasticities for this set of departments, and department G's unit revenue is the highest among all r_i values. By the same token, departments C, K, and P share the excess space for the available store area of 384 sq. units when $n = 20$. Even though departments E and Q have higher space elasticities than C, they do not receive additional area until the available store space increases to 433.5 sq. units. The increased space returns less for departments E and Q than for

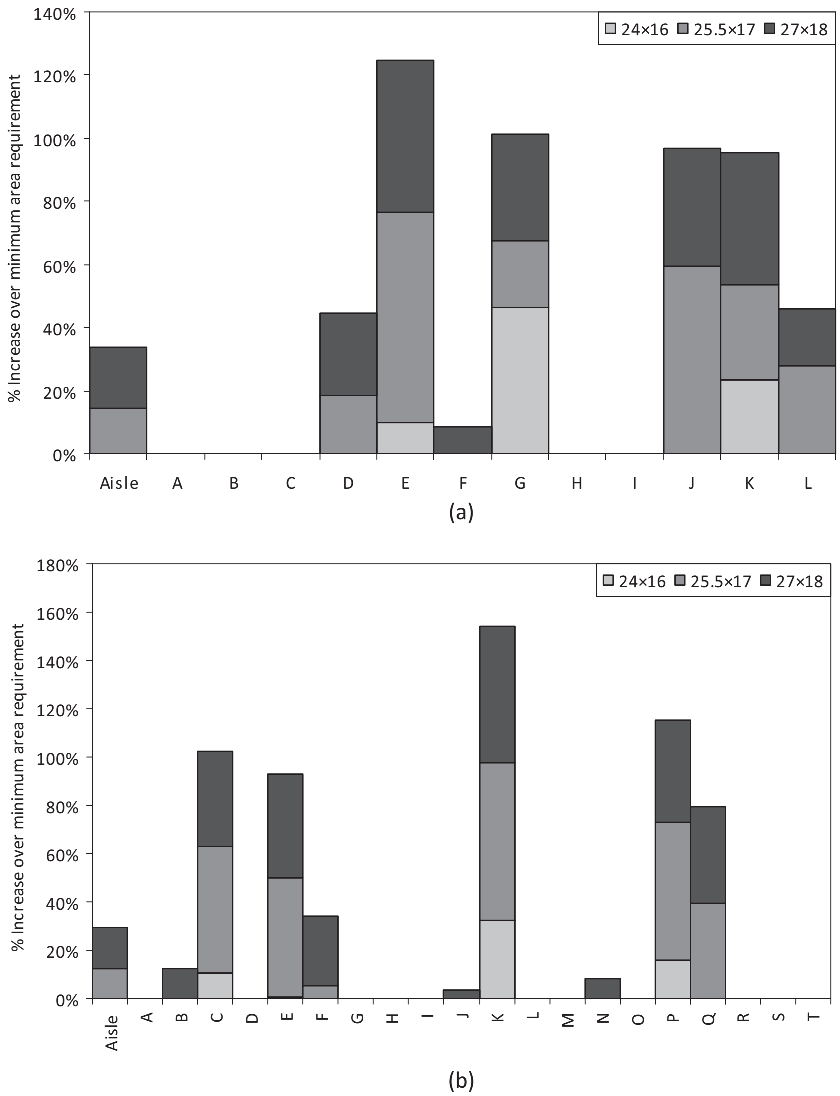

Fig. 6. Increase in area allotments as a percentage of minimum required area: (a) $n = 12$ and (b) $n = 20$.

department C due to the larger minimal area requirements of E and Q. Under all scenarios, departments with a space elasticity less than 0.1 do not receive additional space. As given in the model by Equations (16) to (18), the aisle also competes for additional space. The racetrack aisle does not receive additional area allotment when the store area is 24 $\times$ 16. Thereafter the aisle area increases as more store space becomes available. The excess area allotted to the aisle is subject to the same limitations and considerations as for the departments (e.g., space elasticity and unit revenue).

Once the area allocations were made to departments, the sequence and baybreaks were found by TS. For $n =$ 12, the termination criteria was set to 1000 non-improving moves. For $n = 20$, the termination criteria for F_r, F_a, and F_f were 1000, 5000, and 10 000 non-improving moves, respectively. Maximizing F_{r} is the equivalent of assigning three groups of departments as defined by their zone requirements to their respective regions within the facility, a straightforward optimization (while adhering to aspect ratio constraints). However, F_a is more complex with interactions among departments as governed by the adjacency matrix. F_f considers both F_r and F_a simultaneously and requires more search time to converge.

For each case, 10 trials were conducted. The results for all of these cases are summarized in Tables 2 and 3 for $n =$ 12 and $n = 20$, respectively.

Solutions found by the TS were compared to the upper bounds denoted by $\overline{F_r}$, $\overline{F_a}$, and $\overline{F_f}$. $\overline{F_r}$ was calculated using Equation (5). $\overline{F_a}$ is at most $3n - 6$ departments that can be adjacent to each other (Askin and Standridge, 1993). $\overline{F_f}$ is the multiplication of $\overline{F_r}$ and $\overline{F_a}$.

Considering revenue only, the search identified a layout where F_r equals the upper bound when $\kappa = 0$. When $\kappa >$ 0 the layout for the 20-department problem was also at the upper bound, while the layout for the 12-department problem had a slightly lower revenue.

Table 2. Summary of results for $n = 12$

$n = 12$	24 × 16 $\kappa > 0$	24 × 16 $\kappa = 0$	25.5 × 17 $\kappa > 0$	25.5 × 17 $\kappa = 0$	27 × 18 $\kappa > 0$	27 × 18 $\kappa = 0$
$\overline{F_r}$	12 782	12 782	13 117	13 117	13 404	13 404
$\overline{F_a}$	0.948	0.948	0.948	0.948	0.948	0.948
$\overline{F_f}$	12 117	12 117	12 435	12 435	12 707	12 707
F_r	12 056	12 782	11 804	13 117	12 691	13 404
Deviation (%)	6	0	10	0	5	0
F_a	0.756	0.905	0.697	0.906	0.72	0.906
Deviation (%)	20	5	26	4	24	4
F_f	8364	10 905	7804	11 254	8177	11 885
Deviation (%)	31	10	37	9	36	6

Deviation is the deviation from the upper bound for that revenue function.
Average is over 10 runs.

Considering F_a only, the upper bound cannot be reached when the aspect ratio constraint is enforced. In fact, maximization of F_a is the equivalent of the weighted maximal planar graph problem, which is a very difficult problem even without the additional constraints imposed by this layout model.

Considering the dual-objective F_f, the amount of deviation from the upper bound increases because of the partially conflicting objectives. Enforcing the aspect ratio constraint increases the deviation from the upper bound.

The best layouts obtained for $n = 12$ and $n = 20$ using F_f and $\kappa > 0$ are provided in Figs. 7 and 8, respectively. Departments in white are those with high-impulse purchase likelihoods ($d_i = 1$), gray departments are those with $d_i = 2$, and black departments are those with $d_i = 3$. In Fig. 7, many of the locations of departments are not in agreement with their ideal locations. This is because in the 12-department problem, adhering to the aspect ratio constraint and attending to the adjacency objective reduces the flexibility to place departments in their desired zone. This effect lessens in the larger problem as there are more possible department arrangements.

All experiments were performed using $w_a^L = 0.75$ and $w_a^U = 1.00$. The average aisle widths with different store areas under different fitness configurations are provided in Table 4. For $n = 12$, the aisle width is significantly larger when the aspect ratio penalty is enforced because in order to maintain the aspect ratio requirements, the TS clusters smaller departments inside the racetrack, leading to a wider aisle. For the 20-department problem, aisle width is between 0.76 and 0.82, independent of whether or not the aspect ratio is enforced. When the aisle is narrower there

Table 3. Summary of results for $n = 20$

	$n = 20$	24 × 16 $\kappa > 0$	24 × 16 $\kappa = 0$	25.5 × 17 $\kappa > 0$	25.5 × 17 $\kappa = 0$	27 × 18 $\kappa > 0$	27 × 18 $\kappa = 0$
	$\overline{F_r}$	15 989	15 989	16 494	16 494	16 915	16 915
	$\overline{F_a}$	0.935	0.935	0.935	0.935	0.935	0.935
	$\overline{F_f}$	14 950	14 950	15 421	15 421	15 815	15 815
Best	F_r	15 989	15 989	16 494	16 494	16 915	16 915
	Deviation (%)	0	0	0	0	0	0
	F_a	0.809	0.834	0.819	0.839	0.82	0.843
	Deviation (%)	13	11	12	10	12	10
	F_f	12 061	12 770	12 786	13 274	13 263	13 795
	Deviation (%)	19	15	17	14	16	13
Average	F_r	15 989	15 989	16 494	16 494	16 915	16 915
	Deviation (%)	0	0	0	0	0	0
	F_a	0.799	0.826	0.810	0.829	0.81	0.836
	Deviation (%)	15	12	13	11	13	11
	F_f	11 872	12 710	12 631	13 185	13 065	13 619
	Deviation (%)	21	15	18	15	17	14

Deviation is the deviation from the upper bound for that revenue function.
Average is over 10 runs.

Table 4. Average aisle widths (over 10 runs)

	$n = 12$			$n = 20$		
Config.	24×16	25.5×17	27×18	24×16	25.5×17	27×18
F_r, $\kappa > 0$	0.85	0.98	0.92	0.76	0.76	0.81
F_a, $\kappa > 0$	0.90	0.96	0.99	0.76	0.77	0.78
F_f, $\kappa > 0$	0.90	0.96	0.89	0.76	0.77	0.79
F_r, $\kappa = 0$	0.79	0.76	0.82	0.82	0.79	0.80
F_a, $\kappa = 0$	0.81	0.80	0.81	0.82	0.79	0.80
F_f, $\kappa = 0$	0.86	0.78	0.77	0.78	0.79	0.79

are more departments inside the racetrack. Allowing more departments inside the racetrack increases the chances for those departments to be adjacent to the other departments, leading to better adjacency scores.

The average execution times of the TS in seconds for the 12 configurations are listed in Table 5. For the 12-department problem, the average execution time was around 2 minutes, and the difference between the execution times using different fitness functions was negligible. However, with the 20-department problem, the differences between execution times for the different fitness functions are more apparent and, overall, more than 16 times longer than the 12-department problem on average. Even with these increases in average execution times, the number of solutions evaluated is less than one-trillionth of all possible solutions.

5. Discussion and future research

Retail facility layout is inherently distinct from manufacturing facility layout, and to approach this problem new ideas must be devised and formulated. This article proposes a model and solution approach for the department store facility layout problem with a racetrack layout with a single entrance. The quality of a layout is evaluated considering two criteria: (i) the degree of adjacency satisfaction among the departments and (ii) the revenue generated. The latter is affected by department area and department location within the store. The aisle (racetrack and entry) is treated as a department for revenue purposes where width engenders display area and shopping pleasure. This is a new approach in the facility layout literature as before aisles were assumed to not exist, to be of zero area, or to be of fixed area. Besides the objectives considered, the formulation enforces reasonable departmental shapes and assigns excess store area optimally to departments.

Table 5. Average execution times in seconds (over 10 runs)

Config.	$n = 12$	$n = 20$
F_r, $\kappa > 0$	112	211
F_a, $\kappa > 0$	129	2205
F_f, $\kappa > 0$	139	4498
F_r, $\kappa = 0$	118	217
F_a, $\kappa = 0$	148	1769
F_f, $\kappa = 0$	127	3906

Because a standard mixed integer programming approach could not capture the aspects desired and proved computationally impractical, a hybrid optimization approach was applied. The solution approach involves a two-stage optimization where the areas are assigned to departments first by an NLP. Then, with the areas fixed, the permutation of departments and the location of the bay-breaks defining the racetrack structure are chosen by TS. A penalty approach is used to enforce aspect ratio constraints on the departments. This solution approach is tractable and effective as demonstrated by the computational experience with several test problems.

Evidence is provided that often the revenue objective is in conflict with the adjacency objective. While these were weighted to be essentially equal in this article (through the multiplier approach), it may be that certain establishments place stronger desire on one versus the other. Furthermore, clearly the adjacency objective is only there to account for revenue generated by proper department relative locations. If an appropriate formula that maps adjacencies to revenue could be devised, the objective function reduces to a single objective—that of revenue. When allocating store space, beyond the minimums needed for each department, an interaction between a department's revenue multiplier (margin per unit area) and its elasticity to increased area takes place. Some departments would never increase in size beyond their minimums. Note that in the optimization this increase is determined prior to the department placement. During the layout phase, there is competition among departments for the prime locations in the store, and the ultimate winners of that competition depend on the revenue elasticity for impulse purchases, the size of the department, and the margin per unit area (revenue multiplier). The optimization takes all these factors into account when seeking the best store design.

A future direction will be to enlist the cooperation of one or more retail enterprises so that actual data can be used in the model and results compared with current layouts and with those crafted by a subject-matter expert. This is challenging as retailers regard sales data as highly proprietary.

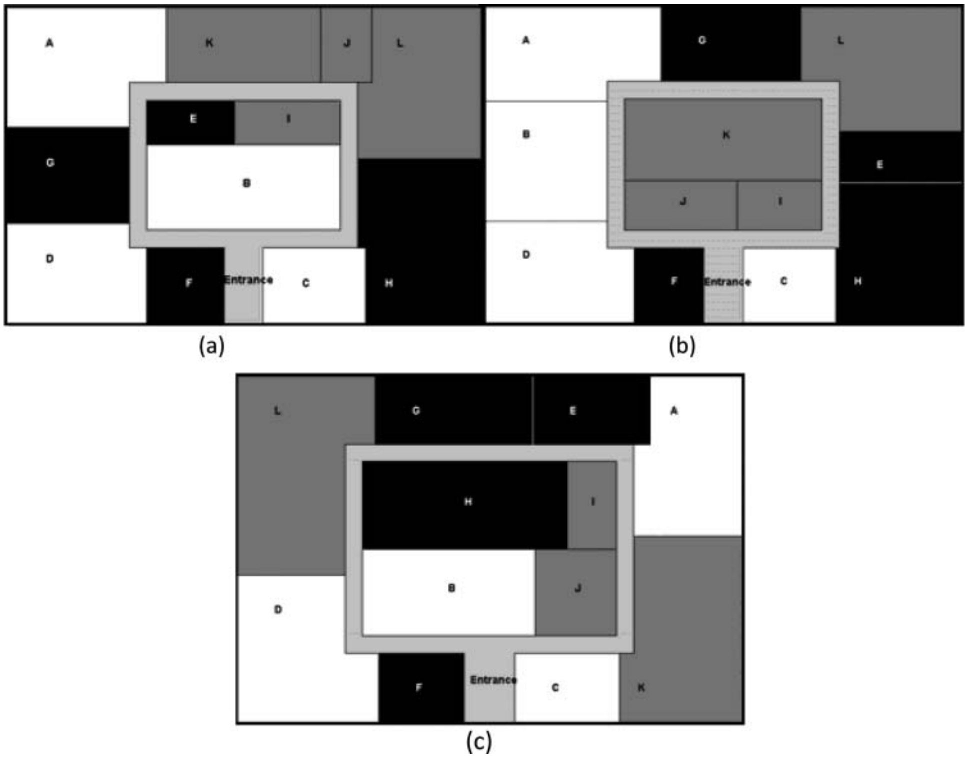

Fig. 7. The best layouts obtained for $n = 12$, $F_f, \kappa > 0$ for available areas of: (a) 24×16; (b) 25.5×17; and (c) 27×18.

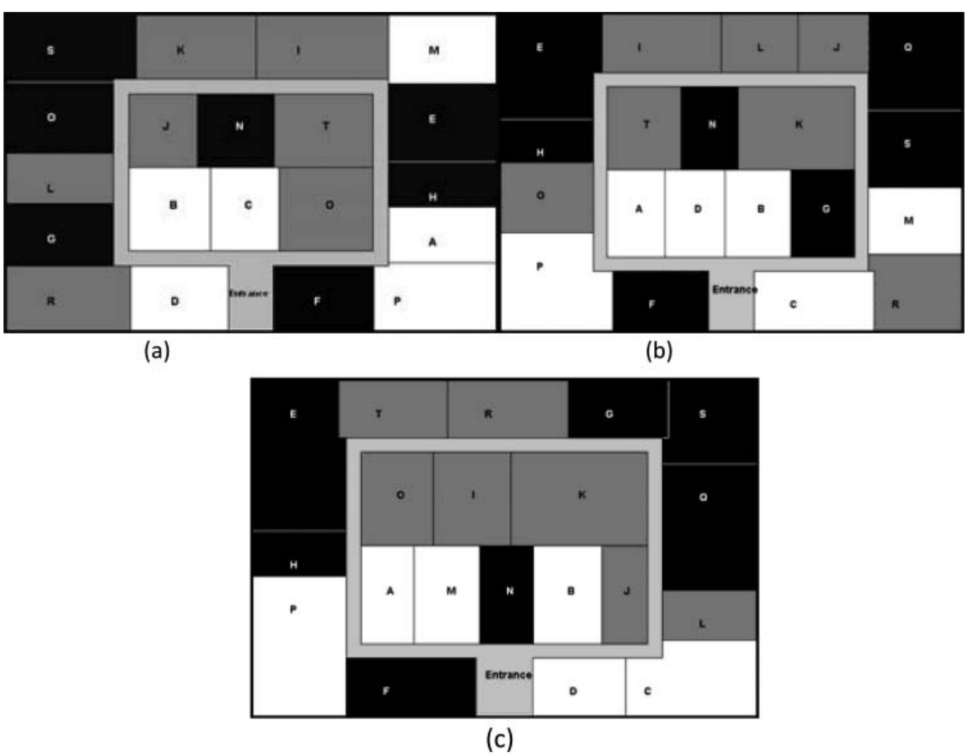

Fig. 8. The best layouts obtained for $n = 20$, $F_f, \kappa > 0$ for available areas of: (a) 24×16; (b) 25.5×17; and (c) 27×18.

Data that would be useful include traffic patterns and density in the store, product elasticities with regard to impulse purchases, product revenue margins, and relationship of increased merchandise within a department to increased sales (the area to revenue aspect). Data that would be interesting but very difficult for even a retailer to ascertain would be quantifying the effect of desired adjacencies on sales.

Another step forward would be to treat the adjacency objective separate from the revenue objective and perform bi-objective optimization. This is readily achievable within the TS framework. This would create a Pareto set of alternative designs from which a human could choose. Another algorithmic extension would be to assign area to the other aisles (the entry/exit aisles) and to consider multiple points of entry/exit. Similarly, considering the double racetrack construct would be important, as this is closer to the actual structure used by Kohl's and some other retailers.

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