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Layout optimization considering production uncertainty and routing flexibility

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The unequal area facility layout problem is studied considering both production uncertainty and routing flexibility. Stochastic production quantities are introduced under the assumption of dependent demands and incorporated into the usual objective of material handling cost minimization. Concurrently, alternative routings are considered based on production quantity. An efficient simulation approach is used to estimate the resulting department pairwise flows, both their expected values and variances. Using these estimates, a tabu search heuristic procedure performs the layout design optimization.

1. Introduction

The static version of the facility layout problem (FLP), where a given planar region is divided into departments assuming that the material flows between departments and the unit cost of transportation are given and will not change during the planning horizon, has been widely studied (e.g. Tate and Smith 1995, Coit *et al.* 1996, Bozer and Meller 1997, Kulturel-Konak *et al.* 2004). Kusiak and Heragu (1987) and Meller and Gau (1996) provide comprehensive reviews of approaches to the static FLP. The unequal-area version of the static FLP was originally formulated by Armour and Buffa (1963) as follows: a given rectangular region with fixed dimensions $H \times W$, where H and W are the height and width of the rectangular region, respectively, is divided into N departments with known areas. Each pair of departments (i,j) is associated with a traffic flow $F(i,j)$ (the sum of flow from i to j and from j to i). The objective is to partition the region into N subregions (i.e. departments) to minimize the material handling cost that is expressed as the following sum:

$$\min \sum_{i=1}^{N} \sum_{\substack{j=1 \\ (i \neq j)}}^{N} F(i,j) \times d(i,j,\Pi), \tag{1}$$

where $d(i,j,\Pi)$ is the distance (using a prespecified metric) between the centres of departments i and j in partition Π. Manufacturing companies, however, often operate in a dynamic and uncertain environment. Therefore, future expectations, risks and opportunities arising from uncertainties should be considered in the design

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phase of a facility. This paper considers two important notions, robustness to uncertainty and flexibility for future changes, and how to integrate them in design optimization. A robust facility design means that it performs well over a variety of scenarios and outcomes. On the other hand, a flexible facility design means that it can readily adapt to changes without significantly affecting performance. In this paper, a framework is defined to consider production uncertainty and routing flexibility together in facility design.

Sethi and Sethi (1990) made an extensive survey of flexibility in manufacturing. Their research is devoted to understanding the concept of flexibility in manufacturing and defining the various types of flexibility found in the literature. According to their survey, there are two types of uncertainties. The first type is due to internal disturbances, such as equipment breakdowns, variable task times, queuing delays, rejects and rework. The second type is caused by external forces such as uncertainties in the level of demand, product prices or product mix. We focus on the second type of uncertainty, termed the fundamental uncertainty of the competitive environment, and how to design facilities robust to manifestations of this uncertainty.

In the facility design literature, uncertainty has been studied as either the dynamic FLP and/or the stochastic FLP. As first studied by Rosenblatt (1986) and then by others (Balakrishnan *et al.* 1992, Urban 1992, 1993, 1998, Yang and Peters 1998, Kochhar and Heragu 1999), the dynamic FLP primarily addresses minimizing the combination of material handling costs and rearrangement costs over all production periods. For a survey of dynamic FLP algorithms, see Balakrishnan and Cheng (1998). On the other hand, the stochastic FLP uses the idea of probabilistically changing demand patterns in the same production period and/or from one period to another (Rosenblatt and Lee 1987, Kouvelis and Kiran 1991, Kouvelis *et al.* 1992, Palekar *et al.* 1992).

The stochastic dynamic FLP is the most complex of all cases and all other models can be viewed as special cases of this problem. The present paper will use the concept by Smith and Norman (2000) that models production uncertainty on a continuous scale under the assumption that product demands are independent. Using that idea, we relax product independence and allow correlated product demands. Correlation does not permit a closed form expression, as was developed by Smith and Norman for independent demands, therefore we develop and use an alternative method of evaluation.

Production uncertainty and the need for product routing flexibility have been commonly studied when machine breakdowns occur (Gerwin 1982, Browne *et al.* 1984). Due to the nature and purpose of flexible manufacturing systems (FMSs), the concept of routing flexibility has also been studied in these systems. Lin and Solberg (1991) considered the effectiveness of flexible routing control in FMSs. They compared the different alternatives of no routing flexibility, fixed sequencing, flexible sequencing and flexible processing. Although implementation is difficult, flexible processing has a significant positive impact on the control and scheduling of FMSs. Chan (2001) studied the effects of different levels of routing flexibility on the performance of FMSs with and without the factor of machine breakdowns. Routing flexibility was defined as ‘a measure of the average number of choices of machine that an individual product can choose’. Five levels of routing flexibility were studied. To measure system performance, three criteria were considered: makespan, lead time and machine utilization. It was found that increasing routing flexibility does not always improve system performance.

The routing flexibility considered in this paper results from changes in the design or the demand of products, not from machine breakdowns. That is, routing flexibility of a layout, which is its ability to produce a part by alternative routes through the system, is investigated. When the demand of each product is known with a probability distribution, the probabilities of following different routes can be calculated. We consider both production uncertainty and routing flexibility concurrently. This cannot be accomplished with a closed form expression, therefore we use a simulation approach to model the uncertainty. One other study uses simulation of flow matrices. Gupta (1986) considered the FLP by obtaining the material flow matrices using simulation randomly to generate the flow between all pairs of departments. He used equal area, square-shaped departments and assumed that individual flow volumes are independent and Normally distributed. In our simulation, the mean, variance and covariance of interdepartmental flows are estimated, then these estimations are used in the design process. This approach is computationally efficient since the simulation is performed only once before starting the optimization process.

As a solution technique in this paper, a tabu search-based heuristic for the stochastic FLP is presented. Tabu search (TS), suggested by Glover (1989, 1990) in its current form, is a metaheuristic optimization method. It is an effective heuristic method for many combinatorial optimization problems with large and complex search spaces, such as vehicle routing (Gendreau *et al.* 1994), flowshop scheduling (Norman 1999), constraint satisfaction (Nonobe and Ibaraki 1998) and redundancy allocation (Kulturel-Konak *et al.* 2003).

The methodology presented in this paper differs from previous research efforts because it proposes an efficient solution methodology for the FLP considering both routing flexibility and volume uncertainty. Moreover, the volume uncertainty can follow any general form and is not limited to certain classes of distributions. Additionally, we permit the departments to have different areas and rectangular shapes. The paper is designed as follows. Section 2 defines the problem. In section 3, the simulation approach for parameter estimation and the TS metaheuristic are introduced. In section 4, the effectiveness of the proposed methodology is demonstrated on two well-known problems from the literature. Concluding remarks and future research avenues are in section 5.

2. Problem definition

This section will define the two aspects of uncertainty considered. First, production quantity uncertainty will be discussed and an objective function that incorporates this uncertainty will be derived. Second, routing changes resulting from changes in production quantity will be integrated into the objective function.

2.1. *Incorporating production uncertainty with dependent products*

In real life, product demands are likely to be dependent. For example, many products are positively correlated, such as doors and tires for automotive assembly. On the other hand, competing products are negatively correlated, such as DVD players and VCRs. Consider how equation (1) must be altered to accommodate the notion of correlated product production. There are M dependent products each with an expected demand or a production volume (μ_m) and a standard deviation (σ_m). The covariance (cov_{ml}) defines the magnitude of dependency between demand for products m and l. The expected value and standard deviation are each given as volume per unit of time while covariance is given as volume-squared per unit of time. For each product, the route that it follows through the facility (i.e. the sequence of the departments visited by each product) is known in advance.

Equation (1) is modified as follows:

$$C(\Pi) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} X_{ij} d_{ij}(\Pi), \tag{2}$$

where X_{ij} is a random variable representing the material flow between departments i and j. Therefore, $C(\Pi)$ is a random variable representing the material handling cost of partition Π. Mathematically, X_{ij} is expressed as follows:

$$X_{ij} = \sum_{m=1}^{M} V_m \delta_{mij} \quad \forall i, j, \tag{3}$$

where V_m is a random variable representing the quantity of product m and:

$$\delta_{mij} = \begin{cases} 1 & \text{if product } m \text{ is transported between departments } i \text{ and } j \\ 0 & \text{if product } m \text{ is not transported between departments } i \text{ and } j \end{cases}.$$

Since many real-world data fall naturally within the framework of Normality theory, in this paper, $V_1, V_2, \ldots, V_M$ are assumed to be Normally distributed dependent variables with known expected value μ_m and variance σ_m^2. The joint probability distribution function of $V = (V_1, V_2, \ldots, V_M)$ follows the multivariate Normal distribution.

If equation (2) is rewritten by using the X_{ij} definition in equation (3), it is as follows:

$$C(\Pi) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \left(\sum_{m=1}^{M} V_m \delta_{mij} \right) d_{ij}(\Pi)$$

$$C(\Pi) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sum_{m=1}^{M} V_m \delta_{mij} d_{ij}(\Pi)$$

$$C(\Pi) = \sum_{m=1}^{M} V_m \sum_{i=1}^{N} \sum_{j=i+1}^{N} \delta_{mij} d_{ij}(\Pi). \tag{4}$$

In equation (4), $\Sigma_{i=1}^{N} \Sigma_{j=i+1}^{N} \delta_{mij} d_{ij}(\Pi)$ is the total distance that product m travels for a given partition Π, and if this constant term is called $D_m(\Pi)$, then:

$$C(\Pi) = \sum_{m=1}^{M} V_m D_m(\Pi). \tag{5}$$

Therefore, $C(\Pi)$ is a linear combination of M dependent Normally distributed variables, V, which is also Normally distributed. It is denoted by $C(\Pi) \sim \mathrm{N}(\mu_{C(\Pi)}, \sigma_{C(\Pi)}^2)$, where

$$\mu_{C(\Pi)} = \sum_{m=1}^{M} D_m(\Pi) \mu_m \tag{6}$$

and

$$\sigma_{C(\Pi)}^2 = \sum_{m=1}^{M} D_m(\Pi)^2 \sigma_m^2 + 2 \sum_{m=1}^{M} \sum_{l=m+1}^{M} D_m(\Pi) D_l(\Pi) \operatorname{cov}(V_m, V_l). \tag{7}$$

If a user selected statistical percentile (p) of the material handling cost, $L(\Pi, p)$, is minimized during the design optimization:

$$\Pr(C(\Pi) \leq L(\Pi, p)) = p$$
$$\Pr\left(\frac{C(\Pi) - \mu_{C(\Pi)}}{\sigma_{C(\Pi)}} \leq \frac{L(\Pi, p) - \mu_{C(\Pi)}}{\sigma_{C(\Pi)}}\right) = p$$
$$\Pr\left(z_p \leq \frac{L(\Pi, p) - \mu_{C(\Pi)}}{\sigma_{C(\Pi)}}\right) = p$$
$$\Phi\left(\frac{L(\Pi, p) - \mu_{C(\Pi)}}{\sigma_{C(\Pi)}}\right) = p \Rightarrow \Phi^{-1}(p) = \frac{L(\Pi, p) - \mu_{C(\Pi)}}{\sigma_{C(\Pi)}}.$$

Since $\Phi^{-1}(p) = z_p$,

$$L(\Pi, p) = \mu_{C(\Pi)} + z_p \sigma_{C(\Pi)}. \tag{8}$$

To summarize, the objective of minimizing a statistical percentile (p) of material handling cost for layout Π can be expressed as:

$$\min L(\Pi, p) = \mu_{C(\Pi)} + z_p \sigma_{C(\Pi)} \tag{9}$$

subject to

$$\alpha_i(\Pi) \leq A_i, \quad \forall i,$$

where $\alpha_i(\Pi)$ is the aspect ratio of department i for a given partition Π, and A_i is the maximum allowable aspect ratio of department i. (This constraint prevents unrealistically shaped departments.) Thus, a user could determine which layout is optimal for the 20th, 80th or some other percentile of material handling costs depending on whether the user wants to consider an optimistic or pessimistic production scenario.

Figure 1 shows the effect of considering dependent product demands on material handling cost. For the same layout, the material handling costs are computed by

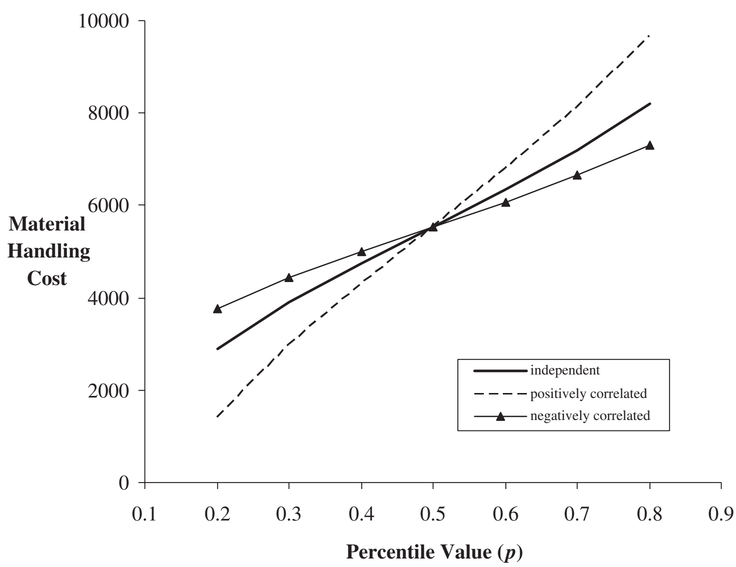

Figure 1. Effects of dependent versus independent production quantities on material handling cost.

using independent (solid line), positively correlated (dashed line) and negatively correlated (solid line with triangles) dependent quantities and are plotted for different p percentile levels (costs is on the y-axis and p of the uncertainty in equation (9) is on the x-axis). As shown in figure 1, the costs are different for the independent and dependent cases over the range of uncertainty. Positive correlation between product quantities increases the variance of material handling cost, while negative correlation decreases it. Therefore, considering dependent production quantities can lead to very different layout decisions.

Following Smith and Norman (2000), if a decision-maker wants to find the layout that is to be maximally cost effective over a range (from p_l to p_u), a robustness measure of a given partition Π, $R(\Pi)$, can be calculated as:

$$R(\Pi) = \int_{p_l}^{p_u} (L(\Pi, p))\,\mathrm{d}p. \tag{10}$$

The layout with the lowest $R(\Pi)$ (i.e. smallest area under the cost curve) is the most robust over the specified range of p values. Therefore the optimization problem is:

$$\min R(\Pi) = \int_{p_l}^{p_u} (L(\Pi, p))\,\mathrm{d}p$$

subject to

$$\alpha_i(\Pi) \leq A_i \quad \forall i.$$

Minimizing a statistical percentile of the material handling cost function, i.e. equation (8), solves the problem for a single scenario. However, the notion of robustness requires considering alternative production scenarios and choosing a layout performing well over these scenarios. In the robustness measure given in equation (10), each unique p between p_l and p_u corresponds to a different scenario. Therefore, $R(\Pi)$ is a measure of how well a layout performs over a continuous range of scenarios defined by the p values between p_l and p_u. Unlike many studies in the literature, which try to optimize for a few discrete scenarios, we optimize for all possible scenarios in the predefined continuous range.

2.2 *Incorporating routing flexibility*

An example of the relationship between product routing and production quantity is given in figure 2. If product demand is less than level a, then it is not economical to produce that product. If product demand is between levels a and b, the product

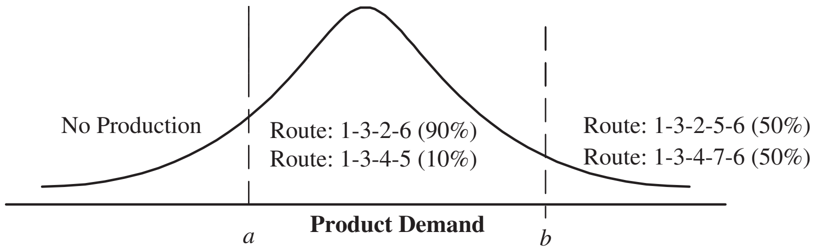

Figure 2. Example of the relationship between product demand and routings.

is produced and its production sequence is 1-3-2-6 in 90% of the cases. Otherwise, due to machine breakdown or maintenance (for example), the sequence is 1-3-4-5. If product demand is greater than level b, there are two alternative routes with equal probabilities that are followed. The alternative routes are 1-3-2-5-6 and 1-3-4-7-6. However, note that the assumption that routes depend on demand is not a restriction of the approach defined here. Other routing relationships can be defined.

When product routings are considered to be random variables, it becomes very complicated to calculate the material handling cost, $C(\Pi)$, using equation (5) since both $D_m(\Pi)$ and V_m are random variables. Instead, equation (2) will be used to calculate $C(\Pi)$. In equation (2), the only random variable is X_{ij}, which is the flow between departments i and j, and a relatively large number of random variables are summed (e.g. for 10 departments, there might be 45 interdepartmental flows). Therefore, using the central limit theorem, $C(\Pi)$ approaches a Normal distribution with parameters

$$\mu_{C(\Pi)} = E\left(\sum_{i=1}^{N}\sum_{j=i+1}^{N} X_{ij} d_{ij}(\Pi)\right) \tag{11}$$

$$\sigma^2_{C(\Pi)} = \mathrm{Var}\left(\sum_{i=1}^{N}\sum_{j=i+1}^{N} X_{ij} d_{ij}(\Pi)\right). \tag{12}$$

In equations (11) and (12), the distances between departments are constants for a given partition. Therefore, equation (11) can be written as:

$$\mu_{C(\Pi)} = \sum_{i=1}^{N}\sum_{j=i+1}^{N} E(X_{ij}) d_{ij}(\Pi). \tag{13}$$

In equation (12), the interdepartmental flows are not independent even if individual product demands are assumed to be independent. Therefore, equation (12) is rewritten as:

$$\sigma^2_{C(\Pi)} = \left(\sum_{i=1}^{N}\sum_{j=i+1}^{N} \mathrm{Var}(X_{ij})\, d_{ij}(\Pi)^2\right) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\sum_{k=1}^{N}\sum_{\substack{l=k+1\\ l>j}}^{N} \mathrm{Cov}(X_{ij}, Y_{kl})\, d_{ij}(\Pi)\, d_{kl}(\Pi). \tag{14}$$

3. Methodology

3.1. *Parameter estimation using simulation*

In equations (13) and (14), the expected value, variance and covariance terms of the interdepartmental flows do not depend on the layout. Once these values are known, they can be used to calculate the expected value and variance of the cost of any given layout. However, for the general case in which demand and routing uncertainties are combined, it is extremely difficult to calculate the variance and covariance of interdepartmental flows with a closed form expression. In this paper, a simulation approach is used to estimate the expected value, variance and covariance terms in equations (13) and (14). Since the simulation can be run once before the design optimization step (i.e. before the tabu search approach), computational time is not a concern, and a large number of replications can be performed to estimate the parameters accurately, The simulation includes the following three steps:

Step 1. Randomly generate K demand vectors $(V_1^k, V_2^k, \dots, V_m^k)$ for $k = 1, \dots, K$.
Step 2. Using the demand vectors from Step 1 and the routing information, calculate the interdepartmental flows, $(X_{12}^k, X_{13}^k, \dots, X_{n,n-1}^k)$ for $k = 1, \dots, K$.
Step 3. Estimate the expected value and variance–covariance of the interdepartmental flows by using the K flow vectors generated in Step 2.

The simulation method is very versatile in terms of handling general demand and routing patterns. For example, the elements of the demand vector in Step 1 might come from any probability distribution. They can be defined as independent or dependent. Product routes in Step 2 can be similarly defined. A disadvantage of the simulation method is that the expected value and covariance of the flows are estimates, which can be different from the exact values. To remedy this disadvantage and ensure an accurate estimation, a large number of replications, $K = 7500$, is used in this paper.

3.2. *Optimization approach: tabu search*

The method described in the preceding sections does not depend on the manner in which the planar region is formulated nor the optimization algorithm used. Different formulations such as the quadratic assignment or the slicing tree might be used to depict departments. Similarly, alternative heuristic methods might be used such as simulated annealing or genetic algorithms; in this paper a flexible bay (Tate and Smith 1995) departmental construct is used with a tabu search heuristic. The basic elements of the TS heuristic method developed in this paper are described below.

3.3. *Fitness and penalty functions*

The material handling cost of the layout is minimized subject to a single constraint of maximum department aspect ratio. Following Tate and Smith (1995), a penalized objective function, i.e. a fitness function of a layout, is in the following form:

$$F_p(\Pi) = F(\Pi) + (F_{\text{feas}} - F_{\text{all}}) \times \left(\frac{\sum_{i=1}^{N} \max(0, \alpha_i(\Pi) - A_i)}{\text{NFT}} \right)^{\kappa}, \tag{15}$$

where $F(\Pi)$ and $F_p(\Pi)$ are the unpenalized and penalized objective function values, respectively, for partition Π, F_{all} is the unpenalized objective function value of the best solution found, F_{feas} is the value of the best feasible solution found, $\alpha_i(\Pi)$ is the aspect ratio of department i for a given partition Π, and A_i is the maximum allowable aspect ratio of department i. The exponent κ, which is a user-defined severity parameter amplifying the behaviour of the ratio in parenthesis, is set to 2.

3.4. *Neighbourhood and move operator*

To represent the solutions, the flexible bay structure (Tate and Smith 1995) is used with a variable length string encoding which concatenates a permutation of the department order (using a boustrophedon filling) and the bay break position. The entire planar area is vertically divided into bays of varying width, and there is no limit on the number of bays. This representation is called ‘flexible bay’ since the number, width and content of the bays are not restricted. For a layout design with a certain number of bays and ordering of departments within each bay, the width of each bay is computed using departmental areas (which are fixed) and then the length

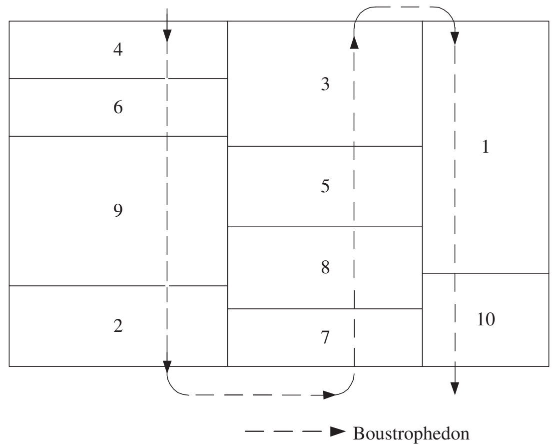

Figure 3. Example of van Camp layout with Boustrophedon ordering of the encoding string 4 6 9 2 7 8 5 3 1 10 | 4 8.

of each department is found. An illustration of the encoding and boustrophedon filling of the 10-department van Camp *et al.* (1991) problem can be seen in figure 3.

In this TS, a swap move is used to generate the neighbourhood of a given solution. The swap operator exchanges the positions of department pair $[i,j]$ in the department permutation array for $i=1,\ldots,N$ and $j=i+1,\ldots,N$. The swap move operates only on the department permutation array and does not perturb the bay structure of a solution. It is impossible to visit all possible solutions of the problem by using only the swap move without altering the bay structure of a solution. Therefore, following the swap of departments, the best bay structure for that permutation is found by enumerating all bay combinations that have one more, one less, and the same number of bays.

3.5. *Tabu list and termination criterion*

The tabu list includes the pair of departments swapped and the bay structure of the related solution. The same department pair cannot be swapped whilst on the tabu list unless the bay structure is different. An exception occurs only if it improves upon the best feasible solution found so far (a standard aspiration criterion). A dynamic length tabu list is used that varies every 20 iterations according to a uniform integer random number between eight and 15. The termination criterion is 1000 moves without improvement in the objective function of the best feasible solution.

Algorithm steps:

Step 0. Initialization. Generate a random initial solution. Assign it to the BEST SO FAR and the CURRENT CANDIDATE solutions. The initial value of the fitness of the BEST CANDIDATE solution is assigned a big number. If the initial solution is feasible, assign it to the BEST FEASIBLE SO FAR solution; otherwise, the initial fitness value of the BEST FEASIBLE SO FAR solution is assigned the value of $2 \times H \times W \times p$ (percentile of the total flow).

Step 1. Neighbourhood search for departments. Search the neighbourhood of all possible swap moves for the department permutation. The non-tabu candidate solution with the best objective function (or if it is tabu, but improves upon the BEST FEASIBLE SO FAR solution) is assigned to the BEST CANDIDATE solution. Compare it with the BEST SO FAR and the BEST FEASIBLE SO FAR solutions and make the necessary updates.

Step 2. Neighbourhood search for bays. For the departmental sequence of the BEST CANDIDATE solution found in *Step* 1, investigate all possible bay break positions using the same, one less, and one more than the number of bay breaks in the BEST CANDIDATE solution and choose the bay arrangement providing the best fitness. If the BEST CANDIDATE solution is feasible, only bay break positions that provide a feasible solution are investigated. Since many arrangements of bay break points yield infeasible solutions, considering only feasible arrangements eliminates unnecessary computational effort. To increase efficiency, Step 2 is performed once in every 10 swap moves. We did experimentation that performed step 2 at every swap move and results were virtually identical to those obtained with doing the bay search every 10 moves.

Step 3. Tabu list update. Enter the solution selected by Steps 1 and 2 on the tabu list, and delete the oldest tabu list entry if the tabu list is full.

Step 4. Stopping criterion check. Check the stopping criterion, and if it is not satisfied, return to *Step* 1.

4. Computational study

First, a well-known test problem, that of van Camp *et al.* (1991) (hereafter, this problem will be called the van Camp problem), is used to develop a new problem that includes demand uncertainty and routing flexibility. This problem divides a planar region into 10 unequal-size departments. The total facility area and areas of the departments are taken directly from the original published problem. A set of products with their possible routings and the means and a variance-covariance matrix for product demand are defined. (Additional van Camp problem data along with the original problem data are given in appendix A.) Without losing generality, the problem is solved by considering positively correlated product demands since this case is typical and the effect of the variance on the material handling cost is readily apparent. The relationship between demand levels and routings is as explained in section 2.2.

For the purposes of this study, we have assumed that the department areas remain the same for different production quantities. While this assumption may not be valid in some cases, in many instances equipment is not fully utilized at expected demand levels, so production can be increased without additional equipment. Or it may be possible to add shifts to increase production using the same floorspace. Changing departmental areas with a change in production quantity is worthy of inclusion in our method and will be the subject of a further study.

Four p ranges, (0.40–0.60), (0.25–0.50), (0.50–0.75) and (0.25–0.60) and three p values, 0.31, 0.50 and 0.69, are chosen for optimization, comprising seven distinct objective functions. The scenario ranges have the following properties. The first range is symmetric around the 50th percentile, and, therefore, optimizing for this range is the same as optimizing for the expected value of material handling costs. The second range is somewhat pessimistic while the third range is somewhat optimistic in terms of the expected demand pattern. Finally, the last one is more pessimistic than optimistic.

Each instance is run 10 times using different random number seeds. The TS algorithm is coded in C and run using a SUN Ultra 2 Model 2170 workstation with a 168-MHz dual-processor and 128 MB RAM.

Table 1 shows the results when only considering the effect of changes in the production quantity and no routing alternatives are considered. First, note that the TS is consistent across the 10 random number seeds. Next, for the seven objectives, six distinct layouts are identified with (as expected) the symmetric p range (0.40–0.60) and the expected value ($p = 0.50$) finding the same layout. The problem is then solved by considering alternative routings along with production quantity uncertainty. (Table 2 contains the results.) Similar to the results in table 1, the TS exhibits little variability to random number seed. It can be seen that considering alternative routings increases the expected material handling costs. For example, the layout for the p range of (0.40–0.60) has an expected material handling cost of 8393.69 and 5652.17 with and without alternative routings, respectively. This result may seem unexpected since demand is unchanged. However, this can be explained as follows. If there is only one possible routing, a layout that minimizes travel distance for the routing would be identified. However, with multiple routings, the layout will tend to optimize for the dominant routing and the secondary routings will incur extra distance, thus increasing the overall material handling cost. CPU time (s) ranges from 9.74 to 32.26

	Equation (8) or (10) value		Best MHC*														
p	Best	Worst	Mean	Std. Dev.	Encoding												
0.40–0.60	1130.43	1130.43	5652.17	4724.97	4	6	9	2	10	5	8	7	1	3	4	8	9
0.25–0.50	949.77	973.93	6754.73	9101.86	1	7	8	5	6	4	9	10	2	3	1	6	7
0.50–0.75	1789.56	1789.56	5876.97	3945.59	4	6	9	2	3	7	8	5	10	1	4	5	9
0.25–0.60	1621.46	1654.21	5754.17	5723.08	3	2	9	6	4	7	8	5	10	1	1	5	9
0.31	2147.79	2241.30	7897.13	11594.08	10	2	3	9	1	4	6	5	8	7	3	4	5
0.50	5652.17	5652.17	5652.17	4724.97	4	6	9	2	10	5	8	7	1	3	4	8	9
0.69	7833.53	7833.53	5876.97	3945.59	4	6	9	2	3	7	8	5	10	1	4	5	9

*Material handling cost.

Table 1. Results of the van Camp problem without considering routing flexibility and the corresponding layout of the best feasible solutions over 10 seeds.

	Equation (8) or (10) value		Best MHC														
p	Best	Worst	Mean	Std. Dev.	Encoding												
0.40–0.60	1678.74	1678.74	8393.69	5186.05	3	7	8	5	10	1	9	2	6	4	1	5	6
0.25–0.50	1677.41	1677.41	8393.69	5186.05	4	6	2	9	1	10	5	8	7	3	4	5	9
0.50–0.75	2507.66	2519.44	8554.52	4545.70	4	6	2	9	7	8	5	3	1	10	4	8	
0.25–0.60	2582.12	2582.12	8393.69	5186.05	3	10	5	8	7	1	4	6	2	9	1	5	6
0.31	5775.59	5775.59	8551.11	5597.10	4	6	9	2	1	7	8	5	10	3	4	5	9
0.50	8393.69	8393.69	8393.69	5186.05	3	10	5	8	7	1	4	6	2	9	1	5	6
0.69	10808.67	10808.67	8554.52	4545.70	10	1	2	5	8	7	9	2	6	4	2	6	

Table 2. Results of the van Camp problem when considering routing flexibility and the corresponding layout of the best feasible solutions over 10 seeds.

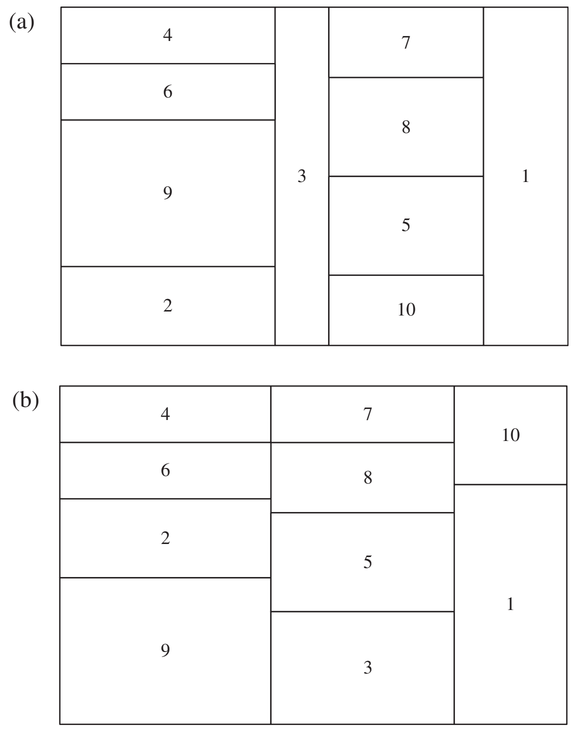

Figure 4. Best layouts found for a p range of (0.50–0.75): (a) without considering routing alternatives and (b) considering routing alternatives.

and from 10.38 to 36.02 for tables 1 and 2, respectively, and averages 15.08 and 15.05, respectively, over the 10 replications.

To demonstrate how considering routing flexibility in facility design may result in different layouts, for a p range of (0.50–0.75), the best layouts found with and without considering routing flexibility are given in figure 4. The best layout when considering routing flexibility is significantly different than that identified without considering routing alternatives. Furthermore, in general, considering routing flexibility during design results in better operating layouts. For example, using a p range of (0.40–0.60), if the new flows due to routing changes are used to calculate the material handling cost for the layout without considering routing flexibility, the mean and standard deviation of the material handling cost are 14 534.12 and 7477.08, respectively. These are much larger (much worst, that is) than the corresponding 8393.69 and 5186.05 obtained when routing flexibility is considered.

In terms of the effect of alternative routings on the variance of the material handling cost, the results are mixed. In most cases, variance increases with more routings; however, in some cases, the converse is true. A detailed analysis of the results shows that when alternative routings are considered, negative covariances between interdepartmental flows occur since only one route is chosen at a time. For instance, as shown in table A.4, a negative correlation exists between interdepartmental flows of department pairs 5 and 8 and those of department pairs 5 and 9 for product 1 since if the route includes 5–8, the flow between 5–9 will be zero, and visa versa. In summary, when alternative routings are allowed, although the distances that the products travel may increase, inflating the material handling cost variance, the negative correlation induced by alternative routings reduces (and, sometimes dominates) the effect of longer travel distances.

Therefore, considering alternative routings cannot reduce expected material handling cost; however, it can reduce the uncertainty by reducing the coefficient of variation of layouts. For the seven objectives, while six layouts were identified in table 1 for the case without routing flexibility, only three layouts were identified in table 2 for the case with routing flexibility. One of the layouts is robust across three of the four p ranges and one of the p values. This is more clearly shown in figures 5 and 6. A layout is optimal where its cost (y-axis value) is lowest among the layouts. In figure 5 (without

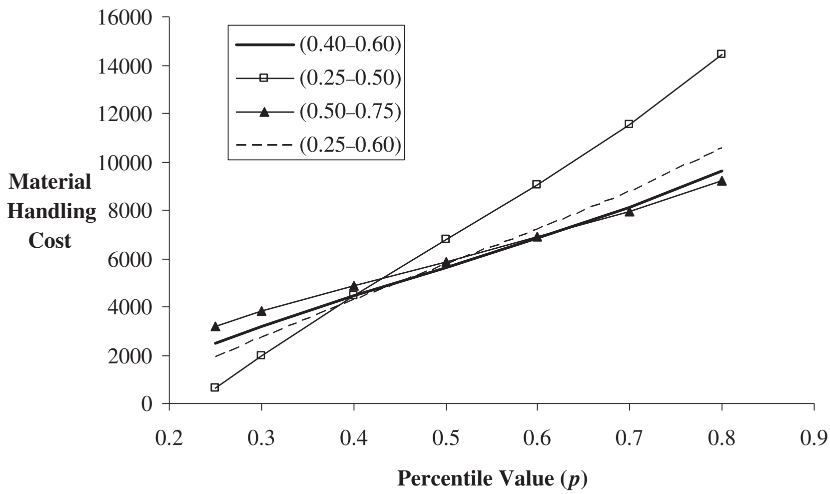

Figure 5. Material handling costs of the best layouts for different p ranges without considering routing alternatives.

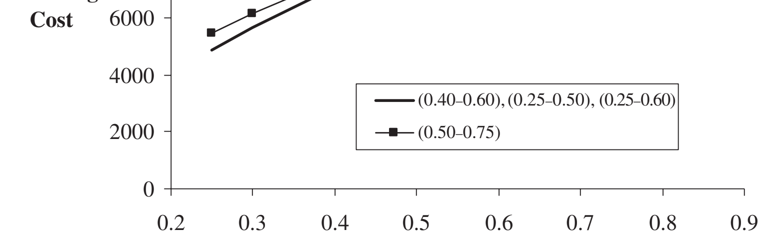

Figure 6. Material handling costs of the best layouts for different p ranges when considering routing alternatives.

routing flexibility), four distinct layouts are best when considering all possible ranges of uncertainty whilst in figure 6 (with routing flexibility) one layout is best over most possible operating scenarios and is very close to the layout that is best for (0.50–0.75) even in that range of p values. We can conclude that allowing alternative routings engenders more robust facility layouts.

To gauge the effectiveness of our methodology on a larger problem, another well-known test problem (Bazaraa 1975) is used to develop a new problem that includes demand uncertainty and routing flexibility. This problem divides a planar region into 14 unequally sized departments. The total facility area and areas of the departments are taken directly from the original published problem. A set of products with their possible routings and the means and a variance–covariance matrix for product demand are defined. (Additional Bazaraa problem data along with the original problem data are given in appendix B.) As before, without losing generality, the problem is solved by considering positively correlated product demands.

The results from the Bazaraa problem echo those of the van Camp problem. Table 3 shows the results when no routing alternatives are considered. The problem is then solved by considering alternative routings along with production quantity uncertainty, and table 4 displays the results. The CPU time (s) is 44.84 and 75.72 for the average of 10 replications, and it ranges from 33.56 to 111.76 and from 42.74 to 140.57 for the results shown in tables 3 and 4, respectively. Similar to the van Camp problem, it is observed that considering alternative routings cannot reduce the

p	Equation (8) or (10) value: Best	Equation (8) or (10) value: Worst	Best MHC: Mean	Best MHC: Std. Dev.	Encoding
0.40–0.60	1328.62	1532.62	6643.12	3109.83	1 3 7 8 10 6 13 12 11 9 14 2 5 4 | 2 6 11
0.25–0.50	1406.91	1628.34	6715.20	3349.04	1 4 3 9 11 12 13 10 7 6 8 5 2 14 | 3 8 11
0.50–0.75	1899.18	2202.40	6655.12	2899.67	14 1 7 10 6 3 8 9 13 12 11 4 5 2 | 3 6 11
0.25–0.60	2111.81	2439.26	6643.12	3109.83	2 5 4 13 12 11 9 14 7 8 10 6 1 3 | 3 8 12
0.31	5054.45	5907.33	6715.20	3349.04	14 2 5 8 6 7 10 13 12 11 9 3 4 1 | 3 6 11
0.50	6643.12	7663.12	6643.12	3109.83	1 3 7 8 10 6 13 12 11 9 14 2 5 4 | 2 6 11
0.69	8093.02	9404.67	6655.12	2899.67	14 1 7 10 6 3 8 9 13 12 11 4 5 2 | 3 6 11

Table 3. Results of the Bazaraa problem without considering routing flexibility and the corresponding layout of the best feasible solutions over 10 seeds.

p	Equation (8) or (10) value: Best	Equation (8) or (10) value: Worst	Best MHC: Mean	Best MHC: Std. Dev.	Encoding
0.40–0.60	1499.39	1802.13	7496.93	3686.21	2 5 4 13 11 12 9 6 7 8 10 14 1 3 | 3 8 12
0.25–0.50	1574.98	1604.08	7496.93	3686.21	3 1 14 10 8 7 6 9 12 11 13 4 5 2 | 2 6 11
0.50–0.75	2158.76	2587.97	7644.13	3051.46	14 3 8 10 6 1 7 9 13 12 11 4 5 2 | 3 6 11
0.25–0.60	2371.12	2870.46	7496.93	3686.21	3 1 14 10 8 7 6 9 12 11 13 4 5 2 | 2 6 11
0.31	5660.21	6791.35	7521.43	3753.34	1 3 14 8 10 7 13 11 12 9 6 2 5 4 | 2 6 11
0.50	7496.93	9010.65	7496.93	3686.21	2 5 4 13 11 12 9 6 7 8 10 14 1 3 | 3 8 12
0.69	9027.83	9838.43	7477.15	3127.10	1 4 3 8 11 13 7 6 9 12 10 14 5 2 | 3 7 11

Table 4. Results of the Bazaraa problem when considering routing flexibility and the corresponding layout of the best feasible solutions over 10 seeds.

expected material handling cost; however, it reduces the uncertainty by reducing the coefficient of variation of solutions. For example, the same layout is robust for three of the four p ranges, (0.40–0.60), (0.25–0.50) and (0.25–0.60) (table 4), since this layout has a low coefficient of variation.

5. Conclusions

Designing facilities that are robust to variability in production and are flexible to handle different product routings will allow factories to avoid redesigning their layouts each time their operating requirements change. When there is uncertainty in production volume, a method using a robustness measure that minimizes the area under the total material handling cost curve for a predefined range of uncertainty is used. In this method, uncertainty is characterized by the estimated mean and variance–covariance of production quantities. Additionally, a new approach for explicitly accounting for routing flexibility is developed. When volume uncertainty and routing flexibility are considered together, it is extremely difficult to calculate the variance and covariance of interdepartmental flows exactly. Therefore, a simulation approach is used to estimate the expected value, variance and covariance terms, and this approach is versatile in terms of handling different demand distributions. Since the simulation is done off-line before the optimization phase, computational effort is reasonable. While we use a flexible bay construct and a tabu search optimization routine, these are not mandated by the approach. The depiction of production uncertainty and the method for handling routing flexibility are quite independent of problem formulation or solution methodology.

The designs produced when considering routing alternatives contain larger expected material handling costs but are more robust across a range of production scenarios because of the negative correlation induced by the mutually exclusive routings. As one would expect, designs that explicitly consider production uncertainty and/or routing flexibility operate at lower costs than those designs found by traditional layout optimization procedures based on fixed assumptions. Future work includes considering concurrent changes in departmental areas with production quantity changes. This is a fairly complicated undertaking as both shape and area may change (expand or contract) while the enclosing facility boundary remains static. Another important aspect of future work is to apply the method to a current actual facility design problem facing a manufacturer.

Appendix A

Data for the problem of van Camp *et al.* (1991):

Facility:
W: 51 m H: 25 m Area: $25 \times 51 = 1275\,\text{m}^2$.

Distances are rectilinear. Maximum aspect ratio is 5.

Department	1	2	3	4	5	6	7	8	9	10
Area (m^2)	238	112	160	80	120	80	60	85	221	119

Table A.1. Departmental areas.

Product	Mean	1	2	3	4
1	10	10 000	640	4000	1950
2	100		100	400	225
3	50			2500	825
4	50				625

Table A.2. Variance–covariance matrix of products (only positive correlations are considered).

Product	Routing
1	3-5-10
2	1-5-8-7
3	2-9-6-4
4	2-9-5-8-7

Table A.3. Product routings (no flexibility).

Product	Demand	Routing	%
1	< 1	–	
	≥ 1 and ≤ 30	3-5-10	60
		3-5-9	40
	> 30	3-5-8-10	60
		3-5-7-9	40
2	< 80	–	
	≥ 80 and ≤ 125	1-5-8-7	80
		3-5-8-10	20
	> 125	1-5-6-8-7	80
		2-5-8-7	20
3	< 10	–	
	≥ 10 and ≤ 90	1-9-6	50
		1-6-4	50
	> 90	2-9-6-4	30
		2-6-4	70
4	< 25	–	
	≥ 25 and ≤ 75	2-9-5-8-7	50
		2-5-8-7	50
	> 75	4-5-7	50
		4-8-9-7	50

Table A.4. Alternative product routings with corresponding probabilities.

Appendix B

Data for the problem of Bazaraa (1975):

Facility:
W: 9 blocks H: 7 blocks Area: $7 \times 9 = 63$ block^2.

Distances are rectilinear. Maximum aspect ratio is 2.

Department	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Area (block^2)	9	8	9	10	6	3	3	3	2	3	2	1	1	3

Table B.1. Departmental areas.

Product	Mean	1	2	3	4	5	6
1	100	10 000	3500	800	2300	3850	3600
2	150		2500	475	900	2100	1200
3	50			625	450	525	400
4	100				900	1750	1000
5	100					4900	2100
6	120						1600

Table B.2. Variance–covariance matrix of products for the Bazaraa problem (only positive correlations are considered).

Product	Routing
1	2-8-9-1
2	3-8-9-11-12-6
3	7-10-13-6
4	5-9-12-4-1
5	6-8-5-4-11-10
6	4-13-12

Table B.3. Product routings (no flexibility).

Product	Demand	Routing	%
1	< 10	–	
	≥ 10 and ≤ 100	2-8-9-1	50
		2-8-7-1	50
	> 100	2-9-1	60
		2-7-1	40
2	< 25	–	
	≥ 25 and ≤ 125	3-8-9-11-12-6	80
		3-8-7-10-12-5	20
	> 125	3-8-11-12-6	80
		3-8-10-12-5	20
3	< 10	–	
	≥ 10 and ≤ 60	7-10-13-6	50
		7-5-4-6	50
	> 60	7-10-12-9-6	30
		7-5-12-4-9-6	70
4	< 50	–	
	≥ 25 and ≤ 125	5-9-12-4-1	100
	> 125	5-9-12-4-13-1	100
5	< 20	–	
	≥ 20 and ≤ 180	6-8-5-4-11-10	50
		6-9-5-4-12-10	50
	> 180	6-5-4-12-10	30
		6-7-11-10	70
6	> 60	–	
	≥ 60 and < 160	4-13-12	50
		4-13-11-12	50
	> 160	4-5-13-12	50
		4-3-5-13-11-12	50

Table B.4. Alternative product routings with corresponding probabilities.

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