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A new mixed integer programming formulation for facility layout design using flexible bays

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Abstract

This paper presents a mixed-integer programming formulation to find optimal solutions for the block layout problem with unequal departmental areas arranged in flexible bays. The nonlinear department area constraints are modeled in a continuous plane without using any surrogate constraints. The formulation is extensively tested on problems from the literature.
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1. Introduction

In this paper, a new mixed-integer programming (MIP) formulation is proposed to solve the facility layout problem (FLP) with unequal departmental areas which is encountered in many manufacturing and service facilities. In its most general form, the FLP is defined as follows: Given N departments with known area requirements and interdepartmental material flow requirements, partition a planar area of size $W \times H$ into departments in order to minimize the total material handling cost. In general, the total material handling cost is expressed as

$$Z = \sum_{i=1}^{N} \sum_{j=i+1}^{N} c_{ij} f_{ij} d_{ij}, \qquad (1)$$

where d_{ij} is the distance between departments i and j for a specified distance metric, f_{ij} is the amount of material flow, and c_{ij} is the material handling cost per unit flow per unit distance traveled between departments i and j. The constraints of the problem include satisfying the area requirements of the departments and the boundaries of the layout. In addition, restrictions on the shapes and locations of departments might be enforced because of practical concerns.

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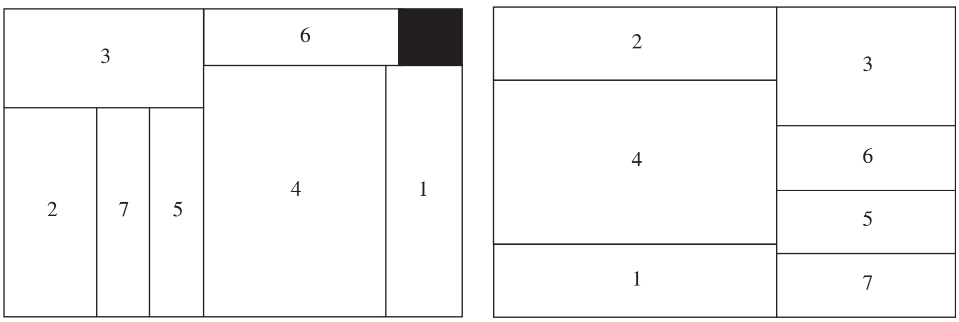
The FLP has attracted extensive attention from industry and academia. Survey papers by Kusiak and Heragu [16] and Meller and Gau [21] summarize different modeling and solution approaches to the FLP. Due to the difficulty of the problem, however, the majority of work on the FLP has focused on heuristic approaches to find good solutions.

The formulation developed in this paper is based on the flexible bay structure (FBS), which is a continuous layout representation allowing the departments to be located only in parallel bays with varying widths (see Fig. 1(b)). The width of each bay depends on the total area of the departments in the bay. Bays are bounded by straight aisles on both sides, and departments are not allowed to span over multiple bays. Therefore, the FBS restricts possible layout configurations. However, it also has some desirable features. The bay boundaries form the basis of an aisle structure that facilitates the user transferring the block design into an actual facility design [1]. In addition, many manufacturing facility designs follow an implicit bay structure [19,27,31]. The FBS representation has been used as a design scheme in several heuristic approaches to the FLP [1,2,15,29]. Facility layout software such as BLOCKPLAN, developed by Donaghey and Pire [8], and SPIRAL, developed by Goetschalckx [10], also generate layouts based on the FBS. However, no exact solution approach or mathematical formulation has been reported in the literature for the FBS representation. Meller, in his related work [19] on layouts with bays, proposed a two-stage optimization approach where in the first stage, departments are assigned to bays to minimize the inter-bay material handling cost, and then in the second stage, departments are arranged within the bays to minimize the within-bay material handling cost. However, this two-stage approach cannot guarantee an optimal FBS solution for the overall facility. The formulation developed herein simultaneously assigns the departments to the bays and determines layouts within the bays considering departmental shape constraints; therefore, it is the first exact approach to find optimal FBS solutions.

2. MIP approaches to the FLP

As mentioned earlier, only a limited number of papers have addressed exact solution methods for the FLP. Exact solution methods based on MIP to date cannot solve large problems (more than nine departments) and/or they make assumptions, such as equal-sized departments and departments with fixed shapes and orientations, which are difficult to justify in practical cases. The quadratic assignment problem (QAP) introduced by Koopmans and Beckman [14] is the first mathematical formulation to optimally solve the FLP when all departments have the same area. In the QAP formulation, the facility is divided into N equal-sized grids, and each equal-sized department is assigned to exactly one grid and vice versa. An obvious drawback of the QAP formulation is the assumption of equal-sized departments. It is possible to model unequal area departments in the QAP formulation by breaking the departments into small grids with equal areas and not allowing the separation of grids of the same



(a) (b)

Fig. 1. Comparison of the optimal solutions for O71 found by: (a) Sherali et al. [28]; and (b) our formulation.

department by assigning large artificial flows between them [13,16]. However, this significantly increases the number of integer variables in the formulation so that solving even small problems becomes difficult. Another disadvantage of this approach is that unintended department shape constraints are implicitly enforced by the penalty induced by high artificial flows between the grids of the same department. Therefore, the optimal solution to the QAP with high artificial flows is likely to be a poor solution to the FLP (see [6]). Bazaraa [5] presented a quadratic set covering problem (QSCP) formulation to the FLP where the shapes of the departments are selected from a finite set determined in advance by the layout designers. Therefore, the QSCP is more scaleable than the QAP. Both the QAP and the QSCP are NP-complete. Therefore, little work has been focused on solving these problems optimally.

The FLP formulations based on the QAP and QSCP use a discrete representation since the departments can only be assigned to predefined grid locations. An alternative to the discrete representation is the continuous representation where department locations are not restricted to a two-dimensional grid but are allowed to be anywhere within the facility. Using the continuous representation, the FLP can be formulated as a MIP. A pioneering MIP formulation for the continuous representation was given by Montreuil [23]. In Montreuil's model, the relative location of a department with respect to another department is expressed using two binary decision variables, one for north–south and the other for east–west relationships. The model includes disjunctive constraints to prevent departmental overlaps and constraints to satisfy specified departmental area and shape requirements. Unfortunately, the problem instances that can be solved to optimality by Montreuil's model in its original form are very limited in size. Therefore, this formulation has been used by researchers as a basis to develop heuristics to find good feasible solutions [17,18,24,25]. Currently, the largest problem instance optimally solved by Montreuil's original model includes 6 departments. Tightening Montreuil's model and using valid inequalities, Meller et al. [22] were able to optimally solve problems with 8 departments. Improving the work of Meller et al. [22] by using a polyhedral outer-approximation to department areas, a new symmetry-breaking approach, elaborate valid inequalities, and branching priorities, Sherali et al. [28] reported the optimal solution for a nine-department problem, which could not be optimally solved in a 24-h CPU limit by Meller et al. [22].

Under the assumption of rectangular departmental shapes, the models given in [22,23,28] can find the global optimal solution to the FLP. On the other hand, the model given in this paper solves a restricted version of the FLP based on the FBS representation which limits the number of possible layout configurations. Therefore, optimal FBS solutions for larger sized problems can be found. In this study, we were able to find optimal FBS layouts for up to 14 departments.

Formulating the nonlinear department area constraints is an important concern in the MIP approach for a continuous layout representation. For each department, the department area constraint enforces the nonlinear relationship $a_i = w_i h_i$, where a_i is the area requirement, w_i is the width, and h_i is the length of department i. Existing formulations use various surrogate constraints to model the nonlinear department area constraint. For example, in Montreuil's original model [23], the shape and area constraints are enforced by using upper and lower limits on the department perimeters as follows:

$$4\sqrt{a_i} \leqslant 2(h_i + w_i) \leqslant 2(1 + \alpha_i)\sqrt{a_i/\alpha_i} \quad \forall i, \tag{2}$$

where α_i is the maximum permissible aspect ratio (the ratio of the longer side to the shorter) of department i. However, the departmental areas enforced by the surrogate perimeter constraint (2) tend to be smaller than the original areas. Meller et al. [22] reported that the difference between the original and surrogate areas can be as much as 11%, 25%, 36%, and 44% for $\alpha_i = 2$, 3, 4, and 5, respectively. In the same paper, the authors also propose an improved surrogate perimeter constraint. In a recent paper [28], Sherali et al. managed to reduce the error in department areas significantly by using a polyhedral outer-approximation of the area constraints.

Another approach to avoid the non-linear area constraint was proposed by Heragu and Kusiak [12] by assuming that the department shapes and orientations are specified a priori, which completely eliminates the area and shape constraints; however, this is a very restrictive assumption. Lacksonen [17,18] used a piecewise linear approximation for modeling the area constraints. Although this approximation can force the actual areas to be very close to the specified areas, it requires additional binary variables.

Using a set of linear equations due to the special properties of the FBS, in our formulation, the department area constraints are exactly modeled in a continuous plane without requiring any surrogate constraints, linearization, or specifying the department shape and orientation a priori. Therefore, they are enforced 100%.

3. Model description

The problem parameters and assumptions are summarized as follows:

N	number of departments
W	width of the facility along the x-axis
H	length of the facility along the y-axis
B	maximum number of parallel bays
a_i	area requirement of department i
α_i	aspect ratio of department i
$l_i^{\max}$	maximum permissible side length of department i
$l_i^{\min}$	minimum permissible side length of department i
f_{ij}	amount of material flow between departments i and j

The decision variables of the formulation are as follows:

$$z_{ik} = \begin{cases} 1 & \text{if department } i \text{ is assigned to bay } k \\ 0 & \text{otherwise} \end{cases}$$

$$r_{ij} = \begin{cases} 1 & \text{if department } i \text{ is above department } j \text{ in the same bay} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_k = \begin{cases} 1 & \text{if bay } k \text{ is occupied} \\ 0 & \text{otherwise} \end{cases}$$

w_k	width (the length in the x-axis direction) of bay k
l_i^y	height (the length in the y-axis direction) of department i
h_{ik}	height of department i in bay k
(x_i, y_i)	coordinates of the centroid of department i
d_{ij}^x	distance between the centroids of departments i and j in the x-axis direction
d_{ij}^y	distance between the centroids of departments i and j in the y-axis direction

Assumptions:

- The coordinates of the southwest corner of the facility are (0, 0).
- In the model description, without loss of generality, the long side of the facility is along the x-axis direction, and bays are assumed to run vertically.
- If a department is assigned to a bay, the bay must be completely filled. The formulation can solve problems with $\sum_i^N a_i \leqslant (W \times H)$ by allowing empty space in the far west and/or east sides of the facility.
- If the aspect ratio is specified to control departmental shapes, then $l_i^{\min} = \sqrt{a_i/\alpha_i}$ and $l_i^{\max} = \min\{H, \sqrt{\alpha_i a_i}\}$.

The MIP formulation for the flexible bay structure problem (FBSP) is given as follows:

$$\textit{FBSP}: \min \sum_i \sum_{i<j: f_{ij}>0} f_{ij} \left(d_{ij}^x + d_{ij}^y \right) \tag{3}$$

subject to:

$$d_{ij}^x \geqslant x_i - x_j \quad \forall i < j, \tag{4}$$

$$d_{ij}^x \geqslant x_j - x_i \quad \forall i < j, \tag{5}$$

$$d_{ij}^y \geqslant y_i - y_j \quad \forall i < j, \tag{6}$$

$$d_{ij}^y \geqslant y_j - y_i \quad \forall i < j, \tag{7}$$

$$\sum_k z_{ik} = 1 \quad \forall i, \tag{8}$$

$$w_k = \frac{1}{H} \sum_i z_{ik} a_i \quad \forall k, \tag{9}$$

$$l_i^{\min} z_{ik} \leqslant w_k \leqslant l_i^{\max} + W(1 - z_{ik}) \quad \forall i, k, \tag{10}$$

$$\begin{aligned} x_i &\geqslant \sum_{j \leqslant k} w_j - 0.5 w_k - (W - l_i^{\min})(1 - z_{ik}) \\ x_i &\leqslant \sum_{j \leqslant k} w_j - 0.5 w_k + (W - l_i^{\min})(1 - z_{ik}) \\ &\forall i, k, \end{aligned} \tag{11}$$

$$\frac{h_{ik}}{a_i} - \frac{h_{jk}}{a_j} - \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\}(2 - z_{ik} - z_{jk}) \leqslant 0$$
$$\frac{h_{ik}}{a_i} - \frac{h_{jk}}{a_j} + \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\}(2 - z_{ik} - z_{jk}) \geqslant 0$$
$$\forall i < j, \quad \forall k, \tag{12}$$

$$\sum_i h_{ik} = H\delta_k \quad \forall k, \tag{13}$$

$$l_i^{\min} z_{ik} \leqslant h_{ik} \leqslant l_i^{\max} z_{ik} \quad \forall i, k, \tag{14}$$

$$\sum_k h_{ik} = l_i^y \quad \forall i, \tag{15}$$

$$y_i - 0.5 l_i^y \geqslant y_j + 0.5 l_j^y - H(1 - r_{ij}) \quad \forall i \neq j, \tag{16}$$

$$r_{ij} + r_{ji} \leqslant 1 \quad \forall i < j, \tag{17}$$

$$r_{ij} + r_{ji} \geqslant z_{ik} + z_{jk} - 1 \quad \forall i < j, \quad \forall k, \tag{18}$$

$$0.5 l_i^y \leqslant y_i \leqslant H - 0.5 l_i^y \quad \forall i. \tag{19}$$

The objective function (3) is the summation of the interdepartmental flow times the rectilinear distance between department centroids for all pairs of departments with positive interdepartmental flow. Constraints (4)–(7) linearize the absolute value term in the rectilinear distance function. Constraint (8) ensures that each department is assigned to a single bay. In constraint (9), the width of each bay is calculated as the total area of the departments assigned to that bay divided by the length of the facility in the direction of the y-axis. Constraint (10) imposes upper and lower bounds on the widths of the bays based on the departments assigned to each bay. Constraint set (11) determines the locations of the department centroids along the x-axis. In the FBS representation, notice that the centroid of a department along the x-axis is located at the middle of the bay in which the department resides. Therefore, if department i is assigned to bay k, then x_i can be calculated as follows:

$$x_i = \sum_{j=1}^{k} w_j - \frac{1}{2} w_k. \tag{20}$$

In constraint (11), if $z_{ik} = 1$ for any combination of i and k, then the inequalities are reduced to Eq. (20); otherwise, if $z_{ik} = 0$, constraint set (11) is inactive and always satisfied. Constraints (9) and (11) also ensure that the departments will be located inside the boundaries of the facility along the x-axis.

Constraints (12)–(14) are the most important contribution of the FBSP since they enable calculating the length of the departments along the y-axis without using any quadratic terms. Constraint (13) sets the summation of the heights of the departments within a bay equal to either H if the bay is used, or zero if the bay is empty. Constraint (14) restricts the height of the departments between the maximum and minimum permissible side lengths, and, in addition, it enforces $h_{ik} = 0$ when department i is not located in bay k. The following two propositions show how department heights can be calculated by the FBSP without using quadratic terms.

Proposition 1. *Constraint* (12) *is valid for* $\forall i < j, \forall k$.

Proof. *Case* 1: $z_{ik} = 0$ and $z_{jk} = 0$. Notice that $h_{ik} = 0$ and $h_{jk} = 0$ due to constraint (14); therefore, constraint (12) is always satisfied for this case.

Case 2: $z_{ik} = 1$ and $z_{jk} = 0$. In this case, constraint (12) reduces to

$$\frac{h_{ik}}{a_i} - \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\} \leqslant 0 \quad \text{and}$$
$$\frac{h_{ik}}{a_i} + \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\} \geqslant 0.$$

From constraint (14), we have $h_{ik} \leqslant l_i^{\max}$; therefore, the reduced constraints are feasible.

Case 3: $z_{ik} = 0$ and $z_{jk} = 1$. This case is very similar to the previous case. For this case, constraint (12) reduces to

$$-\frac{h_{jk}}{a_j} - \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\} \leqslant 0 \quad \text{and}$$
$$-\frac{h_{jk}}{a_j} + \max\left\{\frac{l_i^{\max}}{a_i}, \frac{l_j^{\max}}{a_j}\right\} \geqslant 0.$$

Due to constraint (14), we have $h_{jk} \leqslant l_j^{\max}$ which makes the reduced constraints feasible.

Case 4: $z_{ik} = 1$ and $z_{jk} = 1$. In this case, the constraint reduces to $h_{ik}/a_i - h_{jk}/a_j \leqslant 0$ and $h_{ik}/a_i - h_{jk}/a_j \geqslant 0$, which enforces $h_{ik}/a_i = h_{jk}/a_j$ in the formulation. □

Proposition 2. *Constraints* (12)–(14) *yield exact heights of the departments in the y-axis direction without using quadratic terms in the formulation.*

Proof. *Case* 1: Assume that bay k is empty, that is, $z_{ik} = 0, \forall i$. From constraint (14), therefore, we have $h_{ik} = 0, \forall i$. Then, δ_k must be equal to zero due to constraint (13).

Case 2: Assume that only department i resides in bay k ($l_i^{\max} \geqslant H$ must hold), that is, $z_{ik}=1$, and $z_{jk}=0$, $\forall j \neq i$. From constraint (14), $h_{ik} \geqslant l_i^{\min}$ and $h_{jk} = 0$, $\forall j \neq i$. In this case, constraint (13) is only satisfied when $\delta_k = 1$, which also sets $h_{ik} = H$.

Case 3: Assume that only two departments, i and j, reside in bay k ($l_i^{\max} + l_j^{\max} \geqslant H$ must hold), that is, $z_{ik} = 1$ and $z_{jk} = 1$ for departments i and j, and $z_{mk} = 0, \forall m \neq i, j$. From constraint (14), $h_{ik} \geqslant l_i^{\min}$, $h_{jk} \geqslant l_j^{\min}$, and $h_{mk} = 0, \forall m \neq i, j$. In this case, constraint (13) is only satisfied when $\delta_k = 1$, which also sets $h_{ik} + h_{jk} = H$. In addition, from constraint (12) we have $h_{ik}/a_i = h_{jk}/a_j$ (Proposition 1: Case 4). Therefore, constraints (12) and (13) define a linear system as follows:

$$h_{ik} + h_{jk} = H,$$

$$a_j h_{ik} - a_i h_{jk} = 0,$$

which has the unique solution of $h_{ik} = a_i/(a_i + a_j)H$ and $h_{jk} = a_j/(a_i + a_j)H$. This case can be repeated for three or more departments in the same bay. □

The functions of the remaining constraints are as follows. Constraint (15) calculates the length of the departments in the y-axis directions to be used in other constraints. Constraints (14)–(18) are used to determine the location of the department centroids along the y-axis. These constraints also prevent departments from overlapping in the y-axis direction. For each pair of departments i and j which are located in the same bay, constraints (17) and (18) enforce either department i to be above department j or below department j without overlapping. Constraint (16) uses these above/below relationships of the departments in the same bay to determine their positions along the y-axis. Constraint (19) ensures that the departments will be located inside the boundaries of the facility along the y-axis.

4. Tightening and enhancing the formulation

In this section, we present several enhancements in terms of valid inequalities and extensions to the basic FBSP model in order to improve its computational performance. We discuss the effect of using the proposed valid inequalities within FBSP on several test problems ranging in size from seven to nine departments. These problems were used in [22,28] to test their proposed model enhancements. Some properties of these problems are given in Table 1, and more specific information can be found in the corresponding references. All problem instances were solved by using CPLEX/AMPL 8.0 on a PC with 2.6 GHz CPU, 1.5 GB memory, and the LINUX operating system. The computational results are summarized in Table 2. In the first row of this table, the solution effort in terms of solution time in CPU seconds and the number of nodes searched by the CPLEX branch-and-bound procedure is given for the basic FBSP model. In the following rows, the percent computational improvements $(100((\textit{value in row}1) - (\textit{row value}))/(\textit{value in row}1))$ obtained using valid inequalities within the basic FBSP model are presented. In the table, the model FBSP + (#) represents the basic FBSP model with valid inequality (#) included. As seen in the table, we also studied two and three way interactions of promising valid inequalities in order to find the best possible combination.

4.1. Tightening the linear programming (LP) lower bound

In the LP-relaxation of the FBSP, the centroids of the departments overlap at the same point making distance variables d_{ij}^x and d_{ij}^y equal to zero. Therefore, the LP lower bound of the FBSP is zero. The minimum permissible side lengths of the departments can be used to improve the LP lower bound. The rectilinear distance between the centroids of any two departments i and j must be at least $0.5(l_i^{\min} + l_j^{\min})$. Hence,

$$d_{ij}^x + d_{ij}^y \geqslant 0.5(l_i^{\min} + l_j^{\min}) \quad \forall i < j. \tag{21}$$

In [22,28], several valid inequalities were proposed to force the distance variables to be nonzero by separating the centroids of departments for the general MIP formulation. Although those valid inequalities are

Table 1
Summary of test problems

Problem name	Problem data reference	Number of departments	Common shape constraint	Best solution reported	Best solution reference
FO7	[22]	7	$\alpha=5$	20.95	[28]
FO71	[22]	7	$\alpha=5$	20.25	[28]
FO72	[22]	7	$\alpha=5$	17.75	[28]
O71	[22]	7	$\alpha=5$	121.07	[28]
O72	[22]	7	$\alpha=5$	116.96	[28]
FO8	[22]	8	$\alpha=5$	22.39	[28]
O9	[22]	9	$\alpha=5$	235.95	[28]
VC10R-s	[30]	10	$l_{\min}=5$	22,395.00	[4]
VC10R-a	[9]	10	$\alpha=5$	21,926.40	[9]
MB12	[6]	12	$\alpha=4$	148.50	[6]
MB11-s	[7]	11	$l_{\min}=1$	1372.70	[20]
MB11-a	[7]	11	$\alpha=5$	1185.20	[9]
MB15-s	[7]	15	$l_{\min}=1$	31,936.30	[20]
MB15-a	[7]	15	$\alpha=5$	29,157.60	[9]
Nug12	[26]	11	$\alpha=4$	279.02	[10][a]
Nug15	[26]	15	$\alpha=4$	562.05	[10][a]
Ba12	[5]	12	$l_{\min}=1$	11,140.00	[11][a]
Ba12TS	[29]	16	$l_{\min}=1$	8630.00	[15][a]
Ba14TS	[29]	14	$l_{\min}=1$	5077.00	[29][a]
Ba14	[29]	14	$l_{\min}=1$	5004.55[b]	
AB20	[3]	20	$\alpha=1.75$	7205.40	[29][a]

[a]The best-known FBS solutions.
[b]The objective function value was provided by the referee.

not directly applicable to FBSP, the underlying ideas could be used. The following constraints are valid for FBSP:

$$d_{ij}^x \geqslant 0.5(l_i^{\min}+l_j^{\min}) + W(z_{ik}+z_{jm}-2) \quad \forall i<j,\ \forall k\neq m, \tag{22}$$

$$d_{ij}^y \geqslant 0.5(l_i^{\min}+l_j^{\min}) + H(z_{ik}+z_{jk}-2) \quad \forall i<j,\ \forall k. \tag{23}$$

Constraint (22) states that d_{ij}^x must be greater than $0.5(l_i^{\min}+l_j^{\min})$ if departments i and j are assigned to two different bays. We further tighten this constraint by not allowing empty bays between any two filled bays as discussed in the next section. Constraint (23) imposes a similar lower bound on d_{ij}^y if the departments are located in the same bay. As seen in Table 2, on average the model FBSP + (21) reduced the solution time by 21% and the number of nodes searched by 34%. On the other hand, constraint (23) appeared to be more promising yielding on average a 43% reduction in both solution time and the number of nodes searched.

4.2. Reducing model degeneracy

The FBSP does not require the number of bays to be specified a priori. It is possible to search all FBSP solutions with from one to B bays in a single run by allowing bays with zero width when no department is assigned to them. Consider a feasible solution with b filled bays and B-b empty bays. Since the bays can be selected arbitrarily, there are C_b^B possible ways to select the bays to be filled, which means that C_b^B different combinations of z_{ik} decision variables correspond to the same layout. Therefore, the basic FBSP model is highly degenerate. To remedy the degeneracy of the basic model, we propose a constraint to fill bays sequentially as follows:

$$N\sum_i z_{ik} \geqslant \sum_i z_{i,k+1} \quad \forall k<B. \tag{24}$$

This constraint states that if bay $(k+1)$ is filled, then bay k must also be filled. Therefore, no empty bay is allowed between any two filled bays by forcing all empty bays toward the east end of the layout. The

Table 2
Computational results for valid inequalities

No	Model	FO7		FO71		FO72		O71		O72		FO8		O9	
		Time	B&B	Time	B&B	Time	B&B	Time	B&B	Time	B&B	Time	B&B	Time	B&B
1	FBSP	587.0	20292.0	722.0	23347.0	643.0	18386.0	1553.0	52603.0	1500.0	51083.0	1903.0	38205.0	93259.0	1441620.0
2	FBSP + (21)	26.8	37.2	31.8	93.8	44.2	46.3	6.1	14.1	13.3	1.6	18.0	31.7	9.0	15.8
3	FBSP + (23)	59.1	54.8	49.4	49.7	69.5	67.1	17.6	20.6	−10.6	2.4	64.8	63.0	46.8	43.6
4	FBSPp	29.8	42.0	63.5	63.2	45.7	50.9	26.5	38.1	43.5	48.4	27.7	35.4	23.7	25.5
5	FBSPpq	56.6	54.0	51.1	51.0	43.5	44.3	38.8	44.9	30.7	39.3	43.9	44.9	57.8	54.8
6	FBSP + (24)	85.5	88.8	79.6	81.3	45.0	57.2	89.4	89.6	61.8	67.4	78.8	75.9	71.4	73.0
7	FBSP + (24) + (25)	84.1	87.7	90.8	88.2	83.2	84.8	92.6	92.6	77.9	81.0	85.2	86.0	88.2	87.1
8	FBSPp + (21)	69.2	73.5	77.6	80.5	92.9	85.2	56.1	65.0	49.2	49.4	75.9	77.1	71.8	72.6
9	FBSPpq + (21)	44.7	56.7	39.2	59.5	64.9	68.5	60.4	58.5	58.6	61.2	75.9	77.2	63.8	62.8
10	FBSPp + (23)	71.1	73.8	68.6	65.7	83.4	82.3	45.2	53.3	40.8	50.6	73.1	73.6	79.5	74.8
11	FBSPpq + (23)	64.0	66.1	78.4	72.7	77.9	74.7	61.3	63.0	48.0	53.0	76.8	76.0	68.4	69.5
12	FBSPp + (24)	87.0	89.4	95.5	95.5	87.8	87.1	96.3	95.7	88.3	87.9	77.5	80.4	92.8	92.0
13	FBSPpq + (24)	95.7	94.5	91.9	93.4	75.4	76.0	90.5	90.9	82.5	84.6	86.8	87.6	91.1	89.6
14	FBSP + (21) + (24)	85.9	90.2	70.5	79.2	78.6	81.1	88.7	91.0	77.5	77.8	78.1	81.7	90.7	82.5
15	FBSP + (23) + (24) + (25)	87.6	91.2	89.5	91.7	88.6	89.6	87.8	89.7	79.9	83.9	91.0	90.9	89.5	89.3
16	FBSP + (21) + (23) + (24) + (25)	78.2	87.1	91.5	94.3	79.2	84.6	89.9	92.9	74.4	82.7	84.3	88.3	88.7	88.9
17	FBSPp + (23) + (24) + (25)	96.0	97.2	97.4	97.8	87.4	90.2	92.9	94.5	89.2	91.9	93.4	93.5	95.1	94.9
18	FBSPpq + (23) + (24) + (25)	89.6	92.6	96.3	97.2	90.8	91.9	93.0	94.4	88.8	91.2	94.1	94.7	94.9	95.0
19	FBSPp + (21) + (23) + (24) + (25)	94.2	96.8	92.8	95.4	89.8	92.8	88.4	92.5	85.3	90.4	91.1	93.7	94.3	94.5
20	FBSPp + (21) + (23) + (24) + (25)	93.3	96.2	89.5	93.6	88.9	91.9	95.2	96.8	83.8	90.1	91.2	93.8	94.4	95.0

effect of using constraint (24) within the formulation is very significant as shown in Table 2. On average, the model FBSP + (24) provided a 73% reduction in solution time and a 76% reduction in the number of nodes searched with respect to the FBSP.

Since no empty bay is allowed between any two filled bays when constraint (24) is used, constraint (22) can be further tightened as follows:

$$d_{ij}^{x} \geqslant 0.5(l_i^{\min} + l_j^{\min}) + \min_{p \neq i,j}\{l_p^{\min}\}(|m-k|-1) + B \times W(z_{ik} + z_{jm} - 2) \quad \forall i < j,\ \forall k \neq m. \tag{25}$$

Like the minimum side length distance in constraint (22), this constraint also considers the minimum possible total width of the $(|m-k|-1)$ bays between two departments i and j which are located in two different bays, k and m. On the average, the model FBSP+(24)+(25) provided an 86% reduction in both the solution time and the number of nodes searched. When constraint (23) was also used along with these two constraints (FBSP+(23)+(24)+(25)), 87% and 89% reductions were obtained in solution time and the number of nodes searched, respectively. However, when constraint (21) was added to FBSP + (23) + (24)+(25), the respective numbers obtained were 83% and 88%. As may be seen in Table 2, constraints (23) and (25) were more effective than constraint (21) and constraint (21) increased solution times when it was used along with these constraints.

4.3. Removing symmetric solutions

Existence of symmetric solutions in the FLP is also a source of degeneracy and increases the number of nodes searched by the branch-and-bound procedure to prove optimality. A solution is called symmetric if it is the mirror image of another solution on any axis. To prevent the mirror effect, Meller et al. [22] proposed symmetry-breaking constraints, and we consider similar constraints as follows:

$$x_p \leqslant \frac{1}{2W}\sum_i a_i, \tag{26}$$

$$y_p \geqslant 0.5H, \tag{27}$$

where department p was selected as the department with the maximum number of flows. These constraints efficiently eliminate three-fourths of all possible solutions for a problem. This symmetry-breaking strategy is called a p-position strategy [28]. Notice that in the right-hand side of constraint (26), we use the half width of the layout instead of the facility since the total departmental areas can be smaller than the area of the facility.

Sherali et al. [28] proposed different symmetry-breaking constraints as follows:

$$x_p \leqslant x_q, \tag{28}$$

$$y_p \leqslant y_q, \tag{29}$$

where p and q are critical departments which can be selected based on departmental flows and areas. This symmetry-breaking strategy is called a pq-position strategy. In Table 2, the FBSP models with the p-position and pq-position strategies are denoted by FBSPp and FBSPpq, respectively. Both symmetry breaking strategies provided very significant reductions in solution effort. For most problems, model FBSPpq was proved to be more useful than model FBSPp. On average, the former reduced the solution time by 46.1% and the number of the nodes by 47.6% whereas the latter provided 37.2% and 43.4% reduction, respectively. When these strategies were used together with other valid inequalities, however, the p-strategy is slightly more effective than the pq-strategy in most cases. For example, FBSPp+(24) provided an 89.3% reduction in solution time whereas FBSPpq + (24) provides an 87.7% reduction. As seen in Table 2, in most cases, the best computational results were obtained by the combination of the p-position strategy, the sequential bay filling constraint (24), and the distance lower bounding constraints (23) and (25) (i.e., the model FBSPp + (23) + (24) + (25)). On average, this combination reduced solution time by 93% and the number of nodes searched by 94%.

5. The formulation for a pre-specified fixed number of bays

The basic FBSP model can be easily modified to work with a fixed number of bays. A constraint ensuring that each bay is filled by at least the minimum number of departments must be added to the model

Table 3
Summary of results

Problem name	Best FBS solution	Percent difference	Opt. gap (%)	Solution
FO7	23.12	−9.4[a]	0.0	4 | 5 − 3 | 6 − 2 | 7 − 1
FO71	22.65	−10.6[a]	0.0	4 | 3 − 5 | 2 − 6 | 1 − 7
FO72	19.02	−6.7[a]	0.0	1 | 2 | 3 | 4 | 5 − 6 − 7
O71	134.63	−10.1[a]	0.0	1 − 4 − 2 | 3 − 6 − 5 − 7
O72	121.07	−3.4[a]	0.0	2 − 1 − 4 | 5 − 7 − 6 | 3
FO8	22.39	0.0[a]	0.0	4 − 3 − 2 − 1 | 5 − 6 − 7 − 8
O9	241.06	−2.1[a]	0.0	7 − 8 | 4 − 1 − 2 | 3 − 6 − 9 − 5
VC10R-s	22,899.65	−2.2[a]	0.0	5 − 3 | 8 − 10 − 9 | | 4 − 2 | 7 − 6 | 1
VC10R-a	21,463.07	2.2[a]	0.0	10 − 8 − 5 − 3 | 7 − 4 − 9 | 6 − 2 | 1
MB12	145.28	2.2[a]	0.0	12 | 9 − 1 − 5 − 10 | 6 − 4 − 3 | 8 − 2 − 7 | 11
MB11-s	1317.79	4.2[a]	0.0	4 − 7 − 8 | 2 − 5 − 9 − 10 | 3 − 6 − 1 − 11
MB11-a	1225.00	−3.2[a]	0.0	3 − 4 − 8 | 2 − 9 − 10 | 7 − 6 − 5 − 1 − 11
MB15-s	27,781.95	15.0[a]	47.3	3 − 4 − 10 − 9 − 12 − 14 − 5 | 13 − 11 | 7 | 8 | 2 − 1 − 6 − 15
MB15-a	31,779.09	−8.2[a]	0.0	3 − 4 − 10 − 12 − 14 − 5 | 1 − 9 − 7 − 13 − 11 | 2 − 6 − 8 − 15
Nug12	265.50	5.1[b]	9.2	8 − 4 − 7 − 11 − 12 − 9 | 1 − 5 − 6 − 10 − 2 − 3
Nug15	526.75	6.7[b]	59.4	12 − 5 − 6 − 15 − 10 | 7 − 11 − 8 − 9 − 13 − 14 − 2 − 3 − 4 − 1
Ba12	8801.33	26.6[a]	0.0	7 − 6 | 1 − 2 − 3 | 12 − 8 − 11 | 9 − 5 − 10 | 4
Ba12TS	8600.38	0.3[b]	48.0	14 − 7 − 13 | 12 − 6 − 11 | 1 | 2 | 3 | 10 − 8 − 9 | 4 | 16 − 5 − 15
Ba14TS	4927.69	3.0[b]	19.6	11 − 5 − 10 | 1 | 3 | 9 − 6 − 12 − 13 | 2 | 4 | 7 − 8 − 14
Ba14	5004.55	0.0[b]	0.0	10 − 5 − 11 | 1 | 3 | 13 − 12 − 6 − 9 | 2 | 4 | 14 − 8 − 7
AB20	6890.82	4.6[b]	61.1	20 − 18 − 13 | 6 − 2 − 4 − 5 − 1 | 9 − 8 − 7 − 10 − 14 | 12 − 19 − 3 − 15 | 17 − 16 − 11

[a] 100(the best known solution − solution in this paper)/(solution in this paper).
[b] 100(the best known FBS solution − solution in this paper)/(solution in this paper).

as follows:

$$\sum_i z_{ik} \geqslant \left\lceil \frac{H}{\max_j\{l_j^{\max}\}} \right\rceil \quad \forall k. \tag{30}$$

In (30), $k = 1, \ldots, B$ where B denotes the number of the bays to be completely filled as opposed to the maximum number of bays in the basic FBSP model. In addition, constraint (13) must be changed to make sure that the sum of the department heights in each bay is equal to H as follows:

$$\sum_{i=1}^{N} h_{ik} = H \quad \forall k. \tag{31}$$

Notice that an indicator variable δ_k for each bay is not needed in constraint (31). After these modifications, we will denote the resulting formulation by F-FBSP for fixed number of bays FBSP. The previously discussed symmetry breaking strategies and all of the distance lower bounding constraints are also valid for F-FBSP. Clearly, constraint (24) is not necessary since all bays must be used. In fact, the F-FBSP is a smaller and tighter model than the basic FBSP model. Therefore, one may prefer to run F-FBSP multiple times with a different number of bays in each run to find the same optimal solution obtained in a single run of the FBSP.

6. Numerical examples from the literature

In this section, several problems from the literature are studied to illustrate the capabilities and performance of the proposed formulations. A brief description of the problems and some of their properties are given in Table 1. In the literature, some of the well known FLP test cases were solved with different problem parameters (e.g., with different shape constraints) or their data were slightly modified. We also considered the modified versions of the problems, and the reader may refer to the references given in the table for more specific problem properties. The results are summarized in Table 3. Our best solutions are coded

Table 4
Summary of computational effort for large problems

Problem	B^a	CPU time[b]	Number of nodes searched
VC10-s	5	581	98,634
VC10-a	4	1022	158,888
MB11-s	3,4,5	221	3832
MB11-a	3,4,5	1372	28,421
MB15-s	3,4,5	110,039	13,947,135
MB15-a	3,4	33,986	6,010,918
Nug12	4	86,380	11,866,440
Nug15	2	86,370	39,829,571
MB12	7	4859	555,008
BA12	7	58,722	1,989,373
BA12TS	8	85,905	294,212
BA14TS	7	86,380	3,453,259
BA14	5	435,657	18,021,866

[a] A single value indicates that the FBSP$p+(23)+(24)+(25)$ was used to solve problem, and multiple values indicate the number of bays searched by F-FBSP$p+(23)+(25)$. For the latter case, the CPU times and the number nodes searched are summed over runs.
[b] Using CPLEX/AMPL 8.0 on a PC with 2.6 GHz CPU, 1.5 GB memory, and LINUX operating system.

in the last column of the table such that bays are separated by bar symbols and the order of the departments within the same bay is given from top to bottom. In Table 3, the percent difference column gives the percent difference in solution quality between the solutions in this paper and previously reported best FBS solutions for the problems. If no FBS solution has been previously reported for a problem, the comparison is based on its best known solution. Table 4 presents the computational effort to solve large sized problems.

Test problems FO7, FO71, FO72, O71, O72, FO8, and O9 were optimally solved in [28] using the general formulation enhanced by a polyhedral outer approximation and valid inequalities. Therefore, these problems provide good benchmarks to compare the FBS representation and our formulation in that respect. The model FBSP$p+(23)+(24)+(25)$ was able to solve all problems to optimality in relatively short times, with the solution time for the largest problem, O9, being a little over 1 h of CPU time. As seen in Table 3, the optimal FBS solutions found in this paper are worse than the optimal solutions reported by Sherali et al. [28]. Since the flexible bay construct restricts the format of the design, it is not possible to achieve objective function results better than those optimal solutions obtained by the general model, which has an unrestricted solution space for rectangular departmental shapes. Therefore, the 3–10% gaps between our results and the solutions reported in [28] are due to the FBS, not the formulation. However, note that the flexible bay construct does result in a natural aisle structure. For example, the optimal solutions for problem O71 obtained in [28] and this paper are given in Fig. 1(a) and (b), respectively. Clearly, it is easier to form aisles for the optimal FBS solution (b) than for the optimal solution (a). In addition, some manufacturing technologies require or follow an implicit bay structure [19,27,31]. In solution (a), the departments have smaller areas than the targeted areas due to polyhedral outer-approximation of the area constraints in the formulation. Therefore, the facility is not completely filled by the departments although the sum of the department areas is equal to the facility area in the problem data. The differences between the obtained and targeted department areas result in empty space in the north–west corner of the facility. On the other hand, in solution (b) obtained by our formulation, the facility can be completely filled since the targeted department areas are fully satisfied.

Problem VC10 is the 10-department van Camp problem [30], which was originally studied with the Euclidean distance metric. Problem VC10R-a is the version of the problem solved in [9] using a Genetic Algorithm (GA) approach, and problem VC10R-s was solved by a GA based on the Montreuil's MIP formulation with design skeletons [4]. For both instances, the optimal FBS solutions were obtained in relatively short times considering the size of the problem. The optimal FBS solution to problem VC10R-a improved upon the previously reported best solution by 2.2%. On the other hand, the optimal FBS solution to problem VC10R-s is 2.2% worse than the best solution reported in [4].

Similar to problem VC10R-s, problem MB12 was solved using a heuristic approach based on Montreuil's MIP formulation in [6]. For this problem, an optimal FBS solution, which improved on its best known integer solution by 2.2%, was found in less than 2-hour of CPU time.

Both problems MB11 and MB15 [7] include a monument department. Department 11 in problem MB11 is fixed in the south–east corner of the layout, and department 15 in problem MB15 in the south–east corner of the layout. These problems have not been studied before using an approach based on the FBS representation. The previous best solutions reported for problems MB11-s and MB15-s were obtained by a simulating annealing heuristic based on filling curve approach [20]. The modified versions of these problems, problem MB11-a and MB15-a with an aspect ratio of 5, were also solved by using a GA approach based on the slicing tree representation [9]. The F-FBSP model was used to solve these problems due to existence of the monument departments. It should be noted one more time that if a monument department exists, symmetry-breaking constraints (26) and (27) are not needed. The optimality of the solutions could be established for each case excluding problem MB15-s within the 24-h CPU time limit. The best solution of problem MB15-s has an optimality gap of 47%. The CPU times and the number of nodes reported in Table 4 are the total number over the number of bays searched. For problems MB11-s and MB15-s, the FBS solutions improved upon the previous results by 4.2% and 15%, respectively. However, the slicing tree solutions reported for problems MB11-a and MB15-a problems are superior to the optimal FBS solutions. These results are mainly due to the fact that the slicing tree representation, which uses vertical and horizontal slicing cuts to partition the layout space, is less restrictive than the FBS representation, which uses only either of them.

Problems Nug12 and Nug15 were originally published in [26] as QAP test problems and used in [10] without department shape restrictions to test the block layout algorithm SPIRAL which also generates FBS solutions. For both problems, an aspect ratio of four was used in this paper. The optimality of the final solutions of both problems could not be established within the 24-h CPU time limit. The best integer solutions improved 5.1% and 6.7% upon the solutions given in [10] for Nug12 and Nug15, respectively. As reported in [10], however, in both problems the departments have impractical shapes, extremely narrow and long.

Problems Ba12 and Ba14 are the 12- and 14-department Bazaraa problems [5], which have been frequently used in the facility layout literature as benchmark problems. These problems were previously studied by using a GA [29] and a Tabu Search (TS) [15] based on the FBS structure. In the original descriptions of the problems, the total areas of the departments are smaller than the areas of the facilities. Problems Ba12TS and Ba14TS are the versions of the original problems where the total areas of the departments were made equal to the facility areas as described in [29]. As mentioned earlier, FBSP does not require that the total area of the departments be equal to the facility area. Therefore, the original problems Ba12 and Ba14 can also be solved by using FBSP. The best reported FBS solution for problem Ba12TS is 8630 [15]. For this problem, we found a new best FBS solution with a cost of 8600.38. For problem Ba14TS, we also found a new best solution with a cost of 4927.69 and an optimality gap of 19.6%, which improves the previously reported best FBS solution by 2.9%. We ran problems Ba12 and Ba14, the original problem instances, without any CPU time limit. For problem Ba14, the optimality of a solution with a cost of 5004.55 was verified after 121 CPU hours. An interesting observation is that problems Ba12TS and Ba14TS, which were modified by adding dummy departments with no shape restriction and enlarging the area of some departments, yielded solutions with lower costs than the original problems, Ba12 and Ba14. This is mainly due to the fact that using dummy departments without any shape restriction makes the FBS representation less restrictive by enabling some departments to be located in the same bay, which was impossible otherwise due to shape restrictions. The idea of using dummy departments to improve solution quality was also discussed in [9] for the slicing tree representation. However, it should be noted that dummy departments will not degrade solution quality in our formulation as it might happen in the slicing tree representation [9] since if a dummy department is not needed, it will be simply assigned to a bay in the far ends of the facility. On the other hand, adding dummy departments causes a substantial increase in computational effort.

Problem AB20, the 20-department problem given by Armour and Buffa [3], is the largest problem studied in this paper. The search was terminated after the 24 h CPU time limit with an optimality gap of 59.66% for $\alpha = 1.75$. Although this gap could be considered excessive, problem AB20 is a very large problem with 527 binary variables (for $B = 7$). Despite these large optimality gaps, in some cases, the best integer solutions improve upon the existing best FBS solutions which were either found by a GA or TS based on the FBS representation.

7. Conclusions

Future research may include developing more elaborate valid inequalities and cutting plane heuristics to tighten the formulation so that larger problems can be solved.

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