A Predictive Model for Slip Resistance Using Artificial Neural Networks

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Why This Paper is Important

Slips and falls are a serious ergonomic problem in industry accounting for between 10 and 20 percent of occupational injuries. Prevention of falls is currently being addressed by measuring the coefficient of friction (COF) of the shoe/floor interface to quantify slip potentials. To date, no models have been developed that predict COF from parameters of the environment. Such a model would be extremely useful in the design of the work environment towards the prevention of occupational falls. The purpose of this research was to develop a unique predictive model which relates the COF to characteristics of the environment. Dynamic COF (DCOF) measurements recorded from a slip resistance testing device were used to develop an artificial neural network model which predicts the DCOF based on the input variables, including flooring type, contaminant and shoe sole material. The neural network approach is validated using an adaptation of the statistical re-sampling method of grouped cross validation. Results are shown to be robust across the range of variables investigated. This predictive model represents a significant step forward in developing useful mathematical models of biomechanical phenomena where no known analytical or parametric model have been identified.

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Abstract

This paper describes the formulation, building and validation of an artificial neural network model of the dynamic coefficient of friction (DCOF) as measured by a slip resistance testing device. The model predicts the DCOF as a function of six independent variables over a wide range of conditions. A grouped cross validation method is used to show the consistent performance of the model in predicting the DCOF for new values of the independent variables.

1. Introduction

1.1 Slip Resistance and DCOF

Slips and falls continue to be a major ergonomic and safety concern in the work place, accounting for between 10 to 20 percent of all reported occupational injuries [10, 17]. Prevention of slips has focused on designing the flooring environment to be "slip resistant." This becomes a difficult task when various contaminants (such as oil, water, detergent, etc.) are present. Measurement systems have been developed to evaluate the slip resistance of a shoe/floor interface under various conditions. These slip resistance measurements are used by industry, shoe/floor manufacturers and the legal profession as a criterion for determining the slip potential and designing the flooring environment. Static coefficients of friction (COF) of a shoe/floor interface have been traditionally used, however, dynamic COF (DCOF), also known as kinetic friction, measures are believed to be more relevant to the biomechanics of slips and falls [13, 16]. The dynamics of heel contact during walking and carrying are the critical component of slips leading to most falls. Thus, devices which measure the COF, which is representative of this event, are optimal. In previous research, DCOF measures were recorded using a new slip resistance testing device [1, 12]. These measures were recorded for different environmental factors (shoe materials, floor types and contaminants) and biomechanical or experimental factors (heel velocity, vertical force and heel contact angle). The DCOF was shown to be affected by both types of factors and their interactions.

Predictive models that could estimate the DCOF for particular types of shoes and floors under contaminant conditions would be extremely useful in designing the work place or shoes. Such predictive modeling is very difficult due to the non-linear relationships involved between these variables and the DCOF [8]. Experimental factors such as velocity of the heel, contact time between the shoe and floor, heel contact angle, and vertical force also add another level of complexity to the frictional response at the shoe/floor interface [8, 12]. Heel velocity has been shown to have a potentially large effect on DCOF due to the dissimilar material interactions [8, 11]. Thus, developing an empirical model given a limited number of observations of the independent variables and the resulting dependent variable (the DCOF) is possible, but extremely difficult. Along with the non-linear responses, other problems arise. First, the available data from the testing device is noisy in the sense of machine imprecision and human involvement and judgment while performing the testing. Second, no analytic relationship between the environmental and experimental factors and DCOF is known. Third, the relationship between the independent variables and DCOF is not known to adhere to any parametric assumptions. Statistical models generally require assumptions about the parametric nature of the factors, assumptions which may or may not be true. Fourth, the objective of the model is to generalize the relationship to values and combinations of the independent variables that were not used in creating the model. The term novel data will be used to describe this data not used in building the model. Statistical models have limited value for this type of general interpolation, especially when non-linearities are involved.

Artificial neural networks are an alternative model because of their potential in accommodating multivariate, nonparametric and non-linear data. One previous use of neural networks for biomechanical modelling was in [6], where human gait data was used to train a neural network to determine whether the data came from a person with a healthy or a pathological gait. In this paper an artificial neural network approach produces a model relating DCOF to the factors discussed above, using the results of the DCOF measurement device developed by Redfern and Bidanda [1, 12], without any analytical or parametric assumptions or requirements. The model described in this paper is quite different than that of [6]; rather than a two class classification problem, it is a function approximation model, which is much more complex and difficult to achieve.

Section 1.2 contains an overview of artificial neural networks as they relate to this research. Section 2 describes the neural network model, its training and validation. Sections 3.1 and 3.2 give results of the performance of the neural network for predicting DCOF using a variation of the statistical re-sampling technique of grouped cross validation, with a 20 percent holdout each time. The DCOFs predicted by the neural network are compared to the measured DCOF. Section 3.3 also contains examples of sensitivity analysis of the effects of input parameters on DCOF over the parameter's range when all other variables are held constant.

1.2 Overview of Artificial Neural Network Models

Artificial neural networks are massively parallel computing mechanisms emulating the biological brain. Neural networks store model knowledge in their many interconnecting weights, which move from an initially random state to a stable state during training. These variable weights hierarchically connect nodes (neurons) both in parallel and in sequence. The entire mechanism processes vector input through the network of nodes and weights, arriving at a vector output. There are generally an input layer, an output layer and one or more hidden layers in between.

The fundamental building block of every neural network is a neuron with input connections and a single output value. The signal flow of the neuron's inputs, x_i, and output, o, are considered to be unidirectional, as generically shown in Figure 1.

INSERT FIGURE 1 HERE

A general description of how a neural network processes data follows. The neuron output signal is given by the following relationship:

$$o = f\left(\sum_{i=1}^{n} w_i x_i\right) = f(\mathbf{w}^t\mathbf{x})$$

where **w** is weight vector:

$$\mathbf{w} \equiv [w_1\ w_2\ ...\ w_n]^t$$

and **x** is the input vector:

$$\mathbf{x} \equiv [x_1\ x_2\ ...\ x_n]^t$$

The function $f(\mathbf{w}^t\mathbf{x})$ is the transfer function with its domain being the set of activation values, or *net*, of the neural network paradigm, where *net* is the scalar product computation:

$$net \equiv \mathbf{w}^t\mathbf{x}$$

The most typical continuous activation function is sigmoidal (i.e. logistic or squashing):

$$f(net) \equiv \frac{1}{1 + e^{(-\,net)}}$$

This produces output between 0 and 1 for all input values.

The weight matrices used in the above discussion are trained prior to the use of the neural network model, i.e. **w** transitions from a random state to a fixed equilibrium state, storing the values of the model parameters. The most well known of supervised techniques is backpropagation, which adjusts initially randomized weights during training according to the steepest gradient along the error surface [14, 18]. Weights are adjusted in proportion to their contribution to the output by recycling the squared error signal back through the layers of weights. For a sigmoidal transfer function, error signals are calculated for the output neurons 1 to K as:

$$\delta_{ok} = (d_k - o_k)(1 - o_k)\,o_k \quad \text{for } k = 1, 2, ..., K$$

and the hidden layer neurons 1 to J as:

$$\delta_{yi} = y_j(1 - y_j) \sum_{k=1}^{K} \delta_{ok}\, w_{kj} \qquad \text{for } j = 1, 2, ..., J$$

where $y_j = f(\mathbf{w}^t_j\, \mathbf{z})$, i.e. the output of the first hidden layer of neurons, and $\mathbf{z}$ is the input vector processes in this iteration.

Weights between the hidden neurons 1 to J and the output neurons 1 to K are adjusted by:

$$w_{kj} \leftarrow w_{kj} + \eta\, \delta_{ok}\, y_j \qquad \text{for } k = 1, 2, ..., K \text{ and } j = 1, 2, ..., J$$

and weights between the input neurons 1 to I and the hidden neurons 1 to J are adjusted by:

$$w_{ji} \leftarrow w_{ji} + \eta\, \delta_{yj}\, z_i \qquad \text{for } j = 1, 2, ..., J \text{ and } i = 1, 2, ..., I$$

Neural network models are built using a training set, or in more familiar terminology, using a sample from the population. Thus, they are empirical models. In fact the resulting trained weights of a network is a vector-valued statistic, and training is the process of computing that statistic. Regarding neural networks and their acquisition of model knowledge within a statistical framework will help clarify their applicability to DCOF. A more complete discussion on this subject can be found in [5, 19].

The goal during training is to minimize $\lambda(w)$, e.g. for backpropagation training where the error measurement function π is the squared error:

$$\min \lambda(\mathbf{w}) = E([y - g(x)]^2) + E([g(x) - f(x, \mathbf{w})]^2)$$

The weights determined by training, $\mathbf{w}^*$, therefore "...generalize optimality by construction in the sense that given a random drawing from the probability law ν governing x and y, network output $f(x, \mathbf{w}^*)$ has the best average performance $\lambda(\mathbf{w}^*)$." [19] Since ν is usually unknown, $\mathbf{w}$ is approximated through neural network training algorithms and a sample (training set) from x and y. This approximation is arbitrarily close to $\mathbf{w}$ under general conditions for large samples with probability of 1.

Two important points can be drawn from the discussion above. First, neural network training results, in theory, to an optimal model of the stochastic relationship between *x* and *y* for the given error measurement. Second, neural network training algorithms produce weights which converge to optimal weights in the limit. The DCOF measurements are not completely deterministic because of measurement imperfections and the absence of potentially significant independent variables (e.g. gait, heel shape, area of contact). Using neural networks for modeling DCOF will theoretically result in performance at least equal to, and with the potential to be much better than, parametric statistical analysis.

2. Experimental Methods

2.1 Description of Variables

The environmental variables included in the experimental testing were shoe material, tread, contaminant and flooring. There were four types of flooring - washed and dried vinyl tile, waxed and buffed vinyl tile, smooth stainless steel plate and concrete. There were four contaminant conditions - dry, wet, oiled with SAE 10 oil and oiled with SAE 30 oil. Three heel materials were used - PVC, blown urethane and rubber. One shoe had a tread while the other two did not. The experimental variables were heel contact angle, vertical force and velocity. There were two heel contact angles (5 and 15 degrees), two vertical forces (40 and 80 N) and three velocities (1, 5 and 15 cm/s) tested. The DCOF was sampled over 5 seconds of shoe movement at 200 Hz. The recorded DCOF was the mean of these sampled values when they achieved stability (generally 200 to 300 sampled values shortly after heel contact). (See Redfern and Bidanda [12] for a detailed description of the DCOF measurement technique.) The variables and their representation scheme are shown in Table 1. Due to measurement limitation, flooring and contaminant were represented as categorical measures, using a binary encoding.

Table 1. Variables and Their Representations

Variable	Representation	Variable	Representation
Heel Material	Durometer Hardness	Heel Velocity	Speed
Heal Tread	Present or Not Present	Heel Contact Angle	Angle
Contaminant	Categorical	Vertical Force	Force
Flooring	Categorical	DCOF	Mean of Observed

2.2 Neural Network Architecture and Training

The multi-layered perceptron paradigm trained by backpropagation was selected because of its documented ability to model any function [4, 7]. This is a feedforward, fully connected network as shown in Figure 2. After brief experimentation, a two hidden layer network with 25 neurons in each hidden layer was selected. Two hidden layers generally perform better for continuous data [9, 15]. Using 25 neurons in each hidden layer was the smallest network with the capacity to learn the relationship, which facilitates good generalization ability [5]. Each network was trained to a maximum error of 0.1 on a normalized 0 to 1 scale for each training vector, or to 10,000 epochs (iterations through the training set), whichever was achieved first.

INSERT FIGURE 2 HERE

2.3 Validation Procedure

A total of 1625 observations were available from the slip resistance testing device. This represented two or three repetitions of each of 576 combinations of the independent variables. To train and validate the neural network approach, a variation of the grouped cross validation method, a statistical re-sampling procedure [2, 3], was adapted. The procedure works as follows:

1. Randomly selected 20% of the 576 combinations, and remove those repetitions from the training set.
2. Train the neural network on the remaining 80% of the combinations and their repetitions.

3. Create a test set of the expected (mean) DCOF values of the repetitions of the 20% removed combinations.
4. Test the trained neural network on the novel combinations and their expected DCOF obtained in step 3.
5. Repeat steps 1 through 4 four more times, using a different 20% removed each time. This results in 100% of the data tested as novel over 5 neural networks.

This grouped cross validation method, although intensive in effort, gives a nearly unbiased estimate of performance on novel data. It also leverages all the available data by using all observations for both training and testing. Validation was of great importance due the originality of the approach described in this paper and the lack of confidence interval methods for neural models. Therefore, this statistical estimation procedure was adapted for use with a neural model.

3. Results and Discussion

3.1 DCOF Prediction Results

Figures 3 through 7 show graphically the predictions of the neural network versus the actual observed DCOF for each of the 20 percent hold back test sets. The graphed results are the predictions for the expected (mean) value of the DCOF given previously unseen values of independent variables, i.e. factorial combinations not used in the training set. In examining Figures 3 through 7, it can be seen that the neural network approach performed well across all the available data (576 factorial combinations). The results indicate that the neural approach is capable of unbiased generalization to all combinations and values of the independent variables used in this study.

INSERT FIGURES 3 THROUGH 7 HERE

Table 2 shows a summary of the mean absolute error (MAE), the mean squared error (MSE) of the predictions of each neural network, and the mean errors for all five networks, which is the predictive performance across the whole data set. Since values of

DCOF > 1 are not critical (i.e. there is no propensity to slip), the MAE for DCOF between 0 and 1 was isolated to gauge the error in the critical zone. The standard deviation for the MAE of each network and across the data set are also shown.

Table 2. Summary of Neural Network Predictive Error for DCOF.

Network	Mean Absolute Error (MAE)	Mean Squared Error (MSE)	MAE when DCOF < 1	Standard Deviation of MAE
First Group	0.0956	0.0159	0.0882	0.082
Second Group	0.0899	0.0138	0.0853	0.076
Third Group	0.0952	0.0159	0.0961	0.083
Fourth Group	0.0995	0.0163	0.0937	0.080
Fifth Group	0.0996	0.0172	0.0934	0.085
Overall Data Set	0.0960	0.0158	0.0913	0.081

As can be seen by comparing columns two and four, the MAE for the critical predictions when DCOF < 1 is smaller across all networks with an average error of 0.0913. Since the observations in Figures 3 through 7 and in Tables 2 and 3 are for variable combinations not used in the development of the neural model (novel data), the errors are somewhat overstated from what we would expect in the general case [3]. A general model error rate somewhere in between these reported and the model's error rate for the observations used in building the model (MAE = 0.0874) can be expected.

To further examine the model performance Table 3 shows the MAE and percent correct for the five test sets to three levels of tolerance for five ranges of DCOF: (1) < 0.3, (2) 0.3 < > 0.5, (3) 0.5 < > 0.7, (4) 0.7 < > 1.0, and (5) > 1.0. The three levels of tolerance reflect tight, medium and loose tolerances specific to a given DCOF value, i.e. tolerance generally increases with DCOF value. Note that the tolerance for the categories tight, medium and loose change as DCOF increases. This was done because accuracy of the DCOF values in the range from 0.0 to 0.5 are much more important than for those that are greater than 0.5, in terms of predicting slips and falls. Thus, tighter tolerances are required for lower DCOF levels for useful predictions. The number of observations for each range is also shown. The majority of predictions are correct to a tolerance of 0.10 for DCOF < 1. The model does not achieve reliable predictions for the tight tolerance levels, especially for DCOF < 0.5. While shoe/floor DCOF values can be greater than 1.0 [8, 12], the accuracy of predictions for these high levels are not of great practical importance. The chance of slips occurring is small. We discuss plans to improve the model's precision in Section 4.

Table 3. MAE and Percent Correct for Ranges of DCOF to Three Levels of Tolerance.

Range	# Obs.	MAE	% Correct Tight Tolerance	% Correct Medium Tolerance	% Correct Loose Tolerance
< 0.3	234	0.0772	21.3 to ± 0.03	40.2 to ± 0.05	71.8 to ± 0.10
0.3 < > 0.5	193	0.0942	16.0 to ± 0.03	33.3 to ± 0.05	60.8 to ± 0.10
0.5 <> 0.7	62	0.1156	54.8 to ± 0.10	71.9 to ± 0.15	71.9 to ± 0.20
0.7 < > 1.0	32	0.1148	74.7 to ± 0.15	83.1 to ± 0.20	97.8 to ± 0.30
> 1.0	22	0.1790	59.3 to ± 0.20	75.9 to ± 0.30	96.7 to ± 0.50

3.2 Variance of Prediction Errors

Although overall MAE is less than 0.1, the error variance is relatively high, with a coefficient of variation of close to 100%. This can be seen on Figures 3 through 7, and by the high standard deviations for MAE in Table 2. Many predictions are very close to the measured values; however some are not, particularly those in the extremes of the DCOF range of values. Therefore, the model errors were examined to identify any systematic prediction bias. Figure 8 shows the measured DCOFs ordered from smallest to largest with the corresponding predictions. Except for underestimation at the extreme high end and slight overestimation at the extreme low end, the errors were not systematic. Figure 9 shows a histogram of the residuals relative to a normal distribution. A chi-square test confirmed the normalcy of the residuals.

INSERT FIGURES 8 AND 9 HERE

These results indicate that a neural model can predict all ranges of DCOF equally well, and if the information content in the input variables can be improved upon, a neural model with an arbitrary pre-selected degree of precision for all DCOFs can be built. This supposition is further confirmed by the closeness of the MAE for observations used in building the model (0.0874) and the MAE for new inputs (0.0960). Little is lost in the way of accuracy when the neural model is applied to novel data. A neural network model with improved information content of the input variables and a larger range of experimental values should further reduce the prediction errors when applied to novel data.

3.3 Sensitivity Analysis

A sensitivity analysis was used to isolate the effect of each input variable on the DCOF. The neural network model predicted the DCOF over a continuous range of every input variable individually while holding all other variables constant. Figures 10 and 11 show a sample of these sensitivity calculations. The shoe materials were measured for hardness and a continuous scale applied to the sensitivity analysis. Figure 10 shows the response of DCOF as the hardness of the shoe is varied for oily conditions on the three floors of stainless steel, washed vinyl tile and concrete using a speed of 15 cm/s. Figure 11 shows the response of DCOF as speed varies for a hard shoe on the stainless steel floor for dry, wet and oily conditions. For Figures 10 and 11, the vertical force was held constant at 40 N and the heel contact angle was held constant at 5°. A flatter sensitivity graph indicates that a variable does not affect the DCOF prediction for the given condition, i.e. the DCOF prediction is the same regardless of the value of that individual input variable. Note that shoe hardness has differing effects on the three floors. A harder material appears to decrease the DCOF for stainless steel and concrete, but much less for the vinyl floor. Speed during testing increased the DCOF for dry conditions, but not for the wet or oily conditions. Heel velocity has been shown to have a potentially large effect on DCOF due to the dissimilar material interactions [8, 11].

INSERT FIGURES 10 AND 11 HERE

Caution needs to be exercised when drawing conclusions from these sensitivity graphs as most variables had only two or three known values. Interpolation between two or three values can yield an overly smooth curve. When the model is enriched (as described in Section 4) the sensitivity property of the neural model will be more useful. Users will be able to identify significant input variables, and estimate how DCOF responds to them over the entire range of practical values. Interactions can also be estimated in the same manner by varying multiple inputs.

4. Conclusions and Future Research

A neural network model of the DCOF was developed using input data from a slip resistance testing device. This model was shown to have good prediction performance on novel values of the independent variables, and is unbiased for the range of values tested. This approach has the potential for accurate prediction of the DCOF which can then be used as a tool to assist in work place or footwear design.

Several enhancements for future research have been identified to further develop the method. First, the categorical data on the flooring and the contaminant needs to be translated into continuous valued scales of roughness, hardness and viscosity. This will improve the richness of the model, and allow DCOF predictions for interval values of the qualities, i.e. for a floor of intermediate roughness between tile and concrete. Second, the number of independent variables should be expanded to include any other factors of potential significance, such as heel shape, contact area, etc. Third, a series of experiments using the expanded variables and the continuous scales should be designed. Experimentation over the range of each independent variable in a full or partial factorial design will allow greater capacity to create a neural network model that can be used generically to study and predict the DCOF given any combination of shoe material, contaminant, flooring, etc. Sensitivity to the isolated effects of changes in a single independent variable on the propensity to slip can also be better evaluated. Finally, future research aimed at reducing industrial slip and fall injuries will combine slip resistance testing, modelling, and biomechanics. The DCOF prediction models can be combined with biomechanical and ergonomic studies of people in the work place, which will define the frictional requirements for the worker. Slip resistance modelling can then be used in the work place design process to specify appropriate floors and shoes for that environment.

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Figure 1. Generic Components of an Artificial Neuron.

Figure 2. Neural Network Architecture for DCOF Model.

Figure 3. Performance on First 20% Cross Validation: Predicted versus Measured Expected DCOF.

Figure 4. Performance on Second 20% Cross Validation: Predicted versus Measured Expected DCOF.

Figure 5. Performance on Third 20% Cross Validation: Predicted versus Measured Expected DCOF.

Figure 6. Performance on Fourth 20% Cross Validation: Predicted versus Measured Expected DCOF.

Figure 7. Performance on Fifth 20% Cross Validation: Predicted versus Measured Expected DCOF.

Figure 8. Ordered DCOF Measurements and Predictions.

Figure 9. Histogram of Residuals With Normal Distribution.

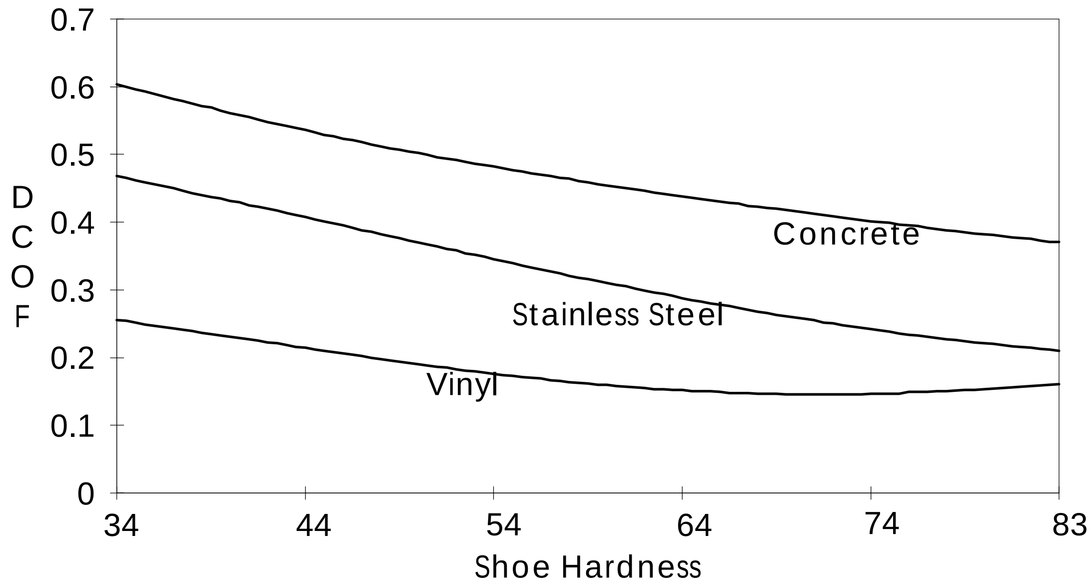

Figure 10. Response of DCOF as Shoe Hardness Changes for Concrete, Vinyl Tile and Stainless Steel Floors Contaminated by Oil When Force = 40N and Heel Contact Angle = 5°.

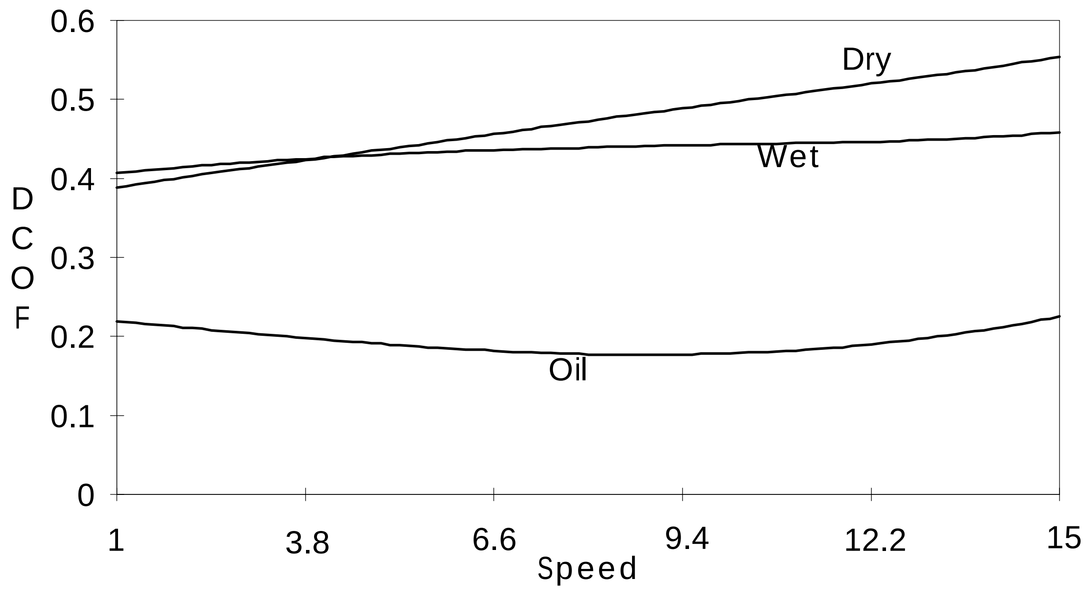

Figure 11. Response of DCOF as Speed Changes for the Hard Shoe on Stainless Steel under Dry, Wet and Oily Conditions When Force = 40N and Heel Contact Angle = 5°.

Figure Captions

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