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A continuous approach to considering uncertainty in facility design

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Abstract

This paper presents a formulation of the facilities block layout problem which explicitly considers uncertainty in material handling costs on a continuous scale by use of expected values and standard deviations of product forecasts. This formulation is solved using a genetic algorithm meta-heuristic with a flexible bay construct of the departments and total facility area. It is shown that depending on the attitude of the decision-maker towards uncertainty, the optimal design can change significantly. Furthermore, designs can be optimized directly for robustness over a range of uncertainty that is pre-specified by the user. This formulation offers a computationally tractable and intuitively appealing alternative to previous stochastic layout formulations that are based on discrete scenario probabilities.
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1. Introduction

Facility design problems generally involve partitioning a planar region into departments (work centers or cells) along with an aisle structure and a material handling system to link the departments. The primary objective of the design problem is to minimize the costs associated with production and materials movement over the lifetime of the facility. Such problems occur in many organizations, including manufacturing cell layout, hospital layout, semiconductor manufacturing and service center layout. They also

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occur in environmental management of forests, wetlands, etc. (e.g. [1]). For US manufacturers, between 20% and 50% of total operating expenses are spent on material handling and an appropriate facilities design can reduce these costs by at least 10%–30% [2]. Altering facility designs due to incorrect decisions, forecasts or assumptions usually involves considerable cost, time and disruption of activities. On the other hand, good design decisions can reap economic and operational benefits for a long-time period. Therefore, the critical aspects are designs that translate readily into physical reality and designs that are "robust" to departures from assumptions.

The problem primarily studied in the literature has been "block layout" which specifies the placement of the departments, without specifying aisle structure and material handling system, machine placement within departments or input/output locations. Block layout is usually a precursor to these subsequent design steps, termed "detailed layout". Two recent survey articles on the block layout problem are [2,3].

1.1. Formulation of the unequal area block layout problem

A block layout where departments may have different areas and/or different shapes precludes assigning n departments to m distinct locations as is done in the popular quadratic assignment problem (QAP) formulation of block layout, which requires all departments be of identical shape and size. For unequal area departments, the first formulation appeared in [4] as follows. There is a rectangular region with fixed dimensions H and W, and a collection of n required departments, each of specified area a_j and dimensions (if rectangular) of h_j and w_j, whose total area $= A = H \times W$[1]. There is a material flow $F(j,k)$ associated with each pair of departments (j,k) which generally includes a traffic volume in addition to a unit cost to transport that volume. There may also be fixed costs between departments j and k. The mathematical objective is to partition the region into n subregions representing each of the n departments, of appropriate area, in order to

$$\text{Min}\, C \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} F(j,k)d(j,k,\Pi), \tag{1}$$

where C is the cost to transport one unit of flow for one unit of distance[2] and $d(j,k,\Pi)$ is the distance (using a pre-specified metric, most commonly shortest rectilinear between centroids) between department j and department k in the layout Π.

Armour and Buffa [4] approached this problem by requiring all departments to be made up of contiguous rectangular unit blocks, and then applied departmental adjacent pairwise exchange. A general linear programming approach was formulated by Montreuil and his co-authors [5–7] to avoid departmental overlap and minimize interdepartmental flow costs, when first given a "design skeleton". Another approach is to use graph theory to develop the optimal relative locations of the department as initiated by Foulds and his co-authors [8–10].

There are other basic formulations for unequal area block layout. One is slicing trees, where the departments and the bounding facility are required to be rectangular and the layout is represented by alternating vertical and horizontal slices [11–13]. A related formulation is the flexible bay structure

[1] This is relaxed in some formulations so that $A > \sum_j a_j$.

[2] Without loss of generality, C is assumed to be constant. However it might be a variable that depends on the type of material transported or route of transportation.

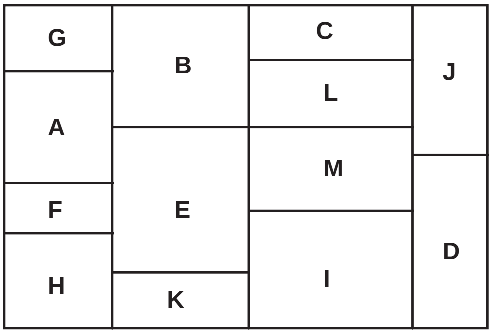

Fig. 1. A flexible bay block layout.

(Fig. 1) [14]. This structure first allows slices in a single direction, creating bays, which are then subdivided into departments by perpendicular slices. Although the flexible bay formulation is slightly more restrictive than the slicing tree formulation, it does allow a natural aisle structure to be inherently created in the layout design, a property that may be useful.

For both the unit and non-unit block approaches mentioned above, the objective of minimizing distance between centroids (Eq. (1)) causes departments to be shaped as unrealistically elongated rectangles if an aspect ratio[3] or a minimum side length constraint on each department is not imposed. This constraint can be handled by making layouts that violate the constraint infeasible or by using an exterior penalty function.

Most previous work has considered material quantities and costs to be fixed and known with certainty. This is an idealistic view and results in an objective function to minimize the deterministic material handling costs. There is a body of work, however, discussed in the next section, that relaxes these assumptions and considers alternative material handling scenarios or probabilistic material handling costs.

1.2. Stochastic plant layout

Both consideration of multiple study periods and stochastic parameters are discussed in this section because they involve the same central idea of generating a block layout based on more than one (certain or uncertain) scenario. These aspects make the layout problem more reflective of actual situations, although they complicate the solution approach.

The main idea of multi-period layout (also termed dynamic layout) formulations is to consider any expansion and reconfiguration costs in the determination of the initial layout. The assumptions necessary for these formulations include the ability to specify, at the onset of the study, all relevant time periods, projected departmental interactions for each time period, and rearrangement costs for each department for each period. Rosenblatt [15] first discussed dynamic layout in 1986 and demonstrated a dynamic programming approach on a QAP problem with six departments and five time periods. Montreuil and Laforge [5] also studied the multiple period problem using a linear programming approach which assumes a pre-specified design skeleton for each period. Lacksonen [16] included unequal area departments in

[3] Aspect ratio is the ratio of the longer side to the shorter side.

his multi-period study. The QAP formulation is first optimized to find the skeleton, followed by a mixed integer linear program to flesh out the block layout, this second optimization step being much like [5]. In a subsequent work, he developed a heuristic to allow solving larger problems [17]. Conway and Venkataramanan [18] used a genetic algorithm to optimize the QAP version of block layout over different time periods.

The idea of multiple scenarios motivated by uncertainty is central to the research on stochastic plant layout. Shore and Tompkins [19] studied four possible scenarios based on product demand. They optimized each scenario separately using the unit block approach to unequal area problems, and then selected the layout that had the lowest penalty when considering the likelihood of each scenario. This was called the most "flexible" layout. Rosenblatt and Lee [20] first developed the idea of "robustness". They considered a single period problem where product demand (and thus production) could be represented as a three-point random variable. Robustness is defined as the number of times a layout falls within a pre-specified percentage of optimal when enumerating the optimal layouts for each production scenario. This approach was demonstrated on a small QAP problem. Rosenblatt and Kropp [21] studied the stochastic single-period QAP formulation using an expected value objective function, which depends on the projected demand/cost scenarios assumed along with their probability of occurrence. Instead of optimizing over each scenario, they proved that a single optimization can be done on the expected value flow matrix and that this results in finding the layout that minimizes the expected material handling costs. Taking a different approach, Cheng et al. [22] considered material flow a random variable by assuming it to be a fuzzy number.

A common theme of the work cited in this section is that uncertainty is handled by identifying a few, mutually exclusive production scenarios then assigning a probability mass to each or assigning a continuous distribution to total demand. This paper offers an alternative formulation where uncertainty is expressed by product and is fully characterized by an expected value and a standard deviation (or variance). This can be viewed as a logical progression from deterministic layout (a single scenario, the expected value) to stochastic layout (a few mutually exclusive scenarios, each with a probability mass) to the formulation herein (material handling costs as a continuous random variable). The data needed are not necessarily more voluminous or more difficult to gather than that required for the discrete scenario approach. For some production environments, it might be more natural to estimate product by product variability rather than aggregate production variability, and similarly, it might be straightforward to measure uncertainty by an expected value and variance instead of a set of values each with an associated probability. As noted in prior research in stochastic plant layout, the methodology of using a few discrete demand scenarios with associated probabilities can be appropriate when a company's demand comes from a relatively small number of major contracts and each has a certain probability of being accepted. However, if demand is comprised of many small orders then it may be easier to estimate the expected value and variation of the anticipated demand rather than assigning acceptance probabilities to each order. A second common theme of many of the papers cited in this section is the implicit assumption of risk neutrality, that is, decisions on optimal layouts depend on expected values. While this is often the stance taken, it may enhance decision making to consider statistical percentiles other than 50% or regions of uncertainty, not necessarily symmetric about the expected value. This paper develops a new robustness measure that can be optimized directly which identifies the best block layout for a given range of uncertainty. Therefore the two motivations for the formulation here are: (a) specifying production uncertainty on a continuous scale by product rather than on a discrete scale by aggregate production and (b) offering the user alternatives to the implicit assumptions of risk neutrality and expected value.

2. Formulation using expected values and variances of products

The basic idea of this paper is to characterize uncertainty by assuming an expected value and a variance for the forecasted amount of each product to be handled in the facility. In an industrial setting, a company could estimate the expected value and variance of their product demands using data mining methods. For example, the company may examine past production volumes and use data analysis and curve-fitting methods in conjunction with knowledge about market conditions to develop production forecast distributions. Given the product demand and routing information, the optimization problem, then, can take one of two forms. The first involves the minimization of a statistical percentile of total material handling costs subject to constraints on departmental shapes given a fixed total rectangular area A with fixed H and W, and fixed departmental areas, a_j. The second involves the minimization of the area under the total material handling cost curve for a pre-specified range of uncertainty. This gives the layout that is most robust over that range of uncertainty.

To more formally state the objective functions, there are N independent products each with an expected demand or production volume $(\bar{v})$ and a standard deviation (σ). The expected value and standard deviation are volume per unit of time (e.g., day, week or month). Invoking the central limit theorem of sums [23], the probability distribution of the total material handling costs is normal when there are either many products or many steps for one (or a few) products. For each product, it must be known which departments will be included in the product manufacture, assembly or handling. For example, product 1 could be routed through departments a, c, d and g while product 2 is routed through departments c, d, e, f and g. With this formulation, the variability of the forecasts of each product can be considered separately. An established product might have low variability of forecast while a new or future product may have high variability. The product volumes, variability and routings along with unit material handling costs and departmental areas and shape constraints are the required information prior to the design phase. Fixed costs of locating a department or of building a materials transport system could also be included.

Mathematically, the cost of material handling at a statistical percentile of production is

$$L(\Pi, z_p) = E(\Pi) + z_p s(\Pi), \tag{2}$$

where $E(\Pi)$ is the expected value of material handling costs and $s(\Pi)$ is the standard deviation of material handling costs for a given layout, Π, and z_p is the standard normal z value for percentile p ($z < 0$ when $p < 0.5$, $z = 0$ at $p = 0.5$, and $z > 0$ when $p > 0.5$). Following this, the first objective function is

Min $L(\Pi)$

and specifically

$$L(\Pi) = C\left[\sum_{i=1}^{N} \bar{v}_i \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} d_{jk}\right) + z_p \sqrt{\sum_{i=1}^{N} \sigma_i^2 \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} d_{jk}\right)^2}\,\right], \tag{3}$$

where $\bar{v}_i$ is the expected volume for product i per unit time, where $i = \{1, 2, \ldots, N\}$, σ_i^2 the variance of the volume of product i per unit time, $\delta_{ijk} = \begin{cases} 1 \text{ if product } i \text{ is transported from dept } j \text{ to dept } k, \\ 0 \text{ if product } i \text{ is not transported from dept } j \text{ to dept } k, \end{cases}$ d_{jk} the distance between departments j and k along material handling route, and z_p the standard normal z value for percentile p.

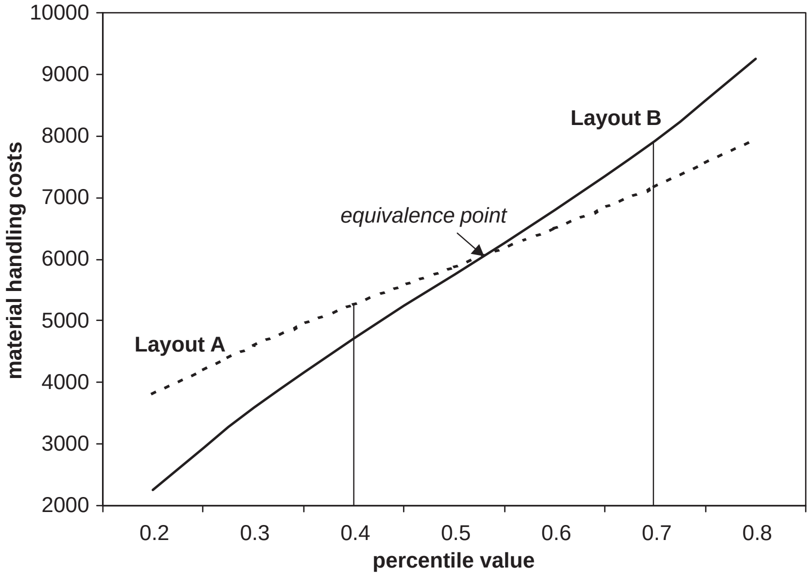

Fig. 2. Comparison of two layouts.

The problem data included in the above formulation are the expected volumes and variances (or standard deviations) of the volumes of each product, each product's routing through the facility (that is, the departments visited in sequence), the departmental areas and the unit cost, C, per unit transport distance. The user would also specify a z_p value, a distance metric (typically Euclidean or rectilinear) and any constraints on the shape (squareness) of the departments. The decision variables are the distances of transport, which depend on the layout, both shaping and relative positioning of the departments. This formulation allows for the explicit consideration of production uncertainty on a continuous scale rather than a few, mutually exclusive production scenarios, while averting specifying probabilities or random variable distributions. It takes into account individual product variability, as well as contribution to the entire production mix. Although it is not shown here, it could straightforwardly be extended to consider non-independent products through specification of a covariance matrix.

This formulation provides a method for the analyst to consider alternative layouts identified as a result of optimizing Eq. (3) with different values of z_p. A graphic view as shown in Fig. 2 can be used to quickly ascertain the cost/uncertainty trade-offs of any particular layout design problem. The optimal layouts from different z_p values are compared by plotting their material handling costs using Eq. (3). The y-axis is the value of Eq. (3) (cost to be minimized) and the x-axis is the value of p that corresponds to the z_p value used in Eq. (3). The regions and magnitude of each layout's dominance are clearly shown, with the equivalence points easily identifiable.

This formulation also enables a direct mathematical construction of a *robustness metric* for competing designs. The robustness measure examines the cost of the layout over a user-specified range of statistical percentiles and weighs the percentiles by their likelihood. If a decision maker selects a range over which the layout is to be maximally cost effective (p_L to p_U), the second objective, R, is a measure of a given layout's performance and is given by Min $R(\Pi)$ where

$$R(\Pi) = \int_{p_L}^{p_U} (L(\Pi, z))\,dp, \tag{4}$$

where $L(\Pi, z)$ is the value of Eq. (3) when evaluated with the corresponding z_p value. The layout with the minimum value of $R(\Pi)$ is the most robust over the specified range of p values. This can be seen in Fig. 2 where the integration will take place between $p_L = 0.4$ and $p_U = 0.7$ for layouts A and B. While layout A performs better at higher p values and layout B performs better at lower p values, it is not clear which layout is superior over the specified range of 0.4–0.7. Integration, however, will yield this answer directly. The smaller area under the curve will be the more cost effective, or more robust, layout for that range of uncertainty.

Because the relationship between z and α is defined by the cumulative normal probability function, Φ, Eq. (4) cannot be calculated directly. However, it can be rearranged as shown below, so that the objective function during optimization can be calculated in a closed form manner. Expanding Eq. (4):

$$R(\Pi) = \int_{p_L}^{p_U} [E(\Pi) + z_p s(\Pi)]\,dp,$$

then using the identity, $z_p = \Phi^{-1}(p)$, yields

$$R(\Pi) = E(\Pi)(p_U - p_L) + s(\Pi) \int_{p_L}^{p_U} \Phi^{-1}(p)\,dp$$

letting $p = \Phi(x)$, $x = \Phi^{-1}(p)$ and therefore

$$R(\Pi) = E(\Pi)(p_U - p_L) + s(\Pi) \int_{\Phi^{-1}(p_L)}^{\Phi^{-1}(p_U)} x \frac{1}{\sqrt{2\pi\pi}} e^{-(x^2/2)}\,dx.$$

Carrying out the integration yields the final objective function:

$$\min R(\Pi) = \min \left(E(\Pi)(p_U - p_L) + \frac{s(\Pi)}{\sqrt{2\pi}} \left[\frac{1}{e^{(\Phi^{-1}(p_L))^2/2}} - \frac{1}{e^{(\Phi^{-1}(p_U))^2/2}} \right] \right), \tag{5}$$

where the inverse cumulative normal calculation, Φ^{-1}, is done numerically for p_L and p_U at the onset of the optimization.

3. Optimization methodology

While the formulations shown in the previous section could be optimized by many methods, a genetic algorithm meta-heuristic was chosen based on the authors' previous computational experience. In particular, genetic search has proven to be an effective approach for unequal area facility layout problems as demonstrated in [12,14,24,25]. The main distinguishing characteristics of GA are a set (*population*) of candidate solutions, a breeding mechanism to create new solutions (*children*) by recombination (*crossover*) of existing (*parent*) solutions, a perturbation method (*mutation*) to avoid convergence to a local optimum, and a culling operator to remove old or inferior solutions in the population. A GA works iteratively by improving the initially random solutions creating subsequent *generations* of (generally) improving solutions.

This paper modifies the GA of Tate and Smith [14] where the encoding determines two things: a departmental sequence and the location of bay divisions. For example, the department sequence

G A F H B E K C L M I J D

with bay divisions after departments 4, 7 and 11 in the ordering would generate the flexible bay layout in Fig. 1. The width of each bay is determined by considering the sum of the area for all of the departments in the bay. There are two potential difficulties with this representation. First, since the department sequence represents a permutation vector there is the potential for the crossover or mutation operations to produce infeasible sequences. Second, depending on the location of the bay divisions the resulting bay structure may create department shapes that violate the aspect ratio constraints.

To overcome potential feasibility problems in the department ordering problem, the representation in this paper uses the random keys encoding of Norman and Bean [26–29]. This encoding assigns a random $U(0,1)$ variate, or random key, to each department in the layout and these random keys are sorted to determine the department sequence. Consider the thirteen department example of Fig. 1. The chromosome of random keys given below, when sorted in ascending order, would create the sequence depicted in Fig. 1:

A	B	C	D	E	F	G	H	I	J	K	L	M
0.16	0.28	0.49	0.93	0.37	0.19	0.07	0.24	0.74	0.81	0.43	0.55	0.66

The random keys encoding eliminates the need for special purpose crossover and mutation operators or repair mechanisms to maintain encoding integrity for permutations because crossover or mutation always results in a set of random keys which can be sorted to determine a feasible permutation.

Bay divisions are encoded by adding an integer to each random key. The integer indicates the bay number for the department. Consider the chromosome presented below which would decode to the layout shown in Fig. 1:

A	B	C	D	E	F	G	H	I	J	K	L	M
1.16	2.28	3.49	4.93	2.37	1.19	1.07	1.24	3.74	4.81	2.43	3.55	3.66

Using the flexible bay construct with a constraint on maximum aspect ratio of each department, a rectilinear distance metric between departmental centroids, and the unit material handling cost, C, equal to 1, Eq. (3) becomes

Min $L(\Pi)$ where:

$$L(\Pi) = C \sum_{i=1}^{N} \bar{v}_i \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(|m_{xj} - m_{xk}| + |m_{yj} - m_{yk}| \right) \right) + C z_p \sqrt{ \sum_{i=1}^{N} \sigma_i^2 \left(\sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} \delta_{ijk} \left(|m_{xj} - m_{xk}| + |m_{yj} - m_{yk}| \right) \right)^2 } \quad (6)$$

$$\text{s.t.} \quad r_j \leqslant R_j \quad \forall j,$$

where r_j is the aspect ratio of dept j, R_j the maximum allowable aspect ratio of dept j, m_{xj} the x coordinate of dept j centroid, and, m_{yj} the y coordinate of dept j centroid.

The problem of infeasibility due to violation of the aspect ratio constraints is handled using the adaptive penalty approach of Coit et al. [30]. The penalty imposed on infeasible layouts is a function of both the infeasibility of the layout and the relative fitness of the best feasible solution ($L(\Pi)_{\text{feas}}$) and best unpenalized solution ($L(\Pi)_{\text{all}}$) yet found. Eq. (6) is modified to

$$L(\Pi)_{\text{p}} = L(\Pi) + (L(\Pi)_{\text{feas}} - L(\Pi)_{\text{all}})(n_{\text{i}})^{\kappa}, \tag{7}$$

where n_{i} is the number of infeasible departments in the layout, κ the severity parameter, $L(\Pi)$ the unpenalized objective function value from Eq. (6), and $L(\Pi)_{\text{p}}$ the penalized objective function value. The objective function used when optimizing for robustness (Eq. (5)) is the same except that $R(\Pi)$ is substituted for $L(\Pi)$. κ was set to 3 for all work in this paper.

4. Computational experience

Test problems were developed based on the 10 department problem of van Camp et al. [31] and the 14 department problem of Bazaraa [32]. Note that the 14th department of the Bazaraa problem is a "dummy" department with no interactions with any other department. The department areas and the facility area were taken directly from these published test problems and a set of products with their routings and forecasted means and standard deviations were added. The test problem data are given in Appendix A. A significant amount of routing overlap between products was included and the product mix was a diverse set of expected values and coefficients of variance ($\sigma/\bar{v}$) for each test problem. For all of the test problems the value of C, the cost to transport one unit of flow for one unit of distance, was assumed to be 1.0. The idea of using product routings to construct the from-to chart for flow in the facility has been used in other layout studies including [33,34]. For optimizing a specific percentile (Eq. (3)), six p values were used: 0.16, 0.31, 0.50, 0.69, 0.84, 0.93. For optimizing the robustness metric (Eq. (5)), four ranges were used: (0.40–0.60), (0.25–0.50), (0.50–0.75), (0.25–0.60). The first range is symmetric about the 50th percentile and therefore is equivalent to optimizing the expected value of material handling costs only. The second range is a somewhat pessimistic view of the forecast while the third range is a somewhat optimistic view of the forecast. The last range is more pessimistic than optimistic.

Tables 1 and 2 present a sampling of the results. Table 1 contains the van Camp et al. problem with an aspect ratio constraint of 5 (somewhat constrained) while Table 2 is a very constrained version of the Bazaraa problem with a maximum aspect ratio of 2. The block layouts associated with most of these layouts are given in Appendix B.

First, it can be seen that as the p value or range changes, the relative contribution of expected value (mean costs) and variance (standard deviation of costs) changes. By remembering that these are the mean and standard deviation of costs over the entire facility, it is apparent why this happens. At $p = 0.50$ (or a symmetric p range), the variance is irrelevant and the costs depend only on expected values. On the optimistic side of the forecast, the standard deviation will probably be reduced although this may result in a slight increase in mean costs. This is because it becomes more important to place departments nearer to each other that have either a high expected flow value or a high variance (or both). This serves to reduce the overall variance of the layout because distance is part of that calculation. On the pessimistic side, this logic is reversed. Departments for which flows have high variance can be placed further apart.

Table 1
Results of van Camp et al. problem with maximum aspect ratio = 5

p Range	Eq. (5) value	Mean costs	Std. dev. costs	Notes
0.40–0.60	1130.43	5652.17	3223.34	Same as $p = 0.50$
0.25–0.50	1100.37	5754.17	4165.54	Better than any $p < 0.50$
0.50–0.75	1720.92	5876.97	2469.25	Same as $p > 0.50$
0.25–0.60	1643.59	5754.17	4165.54	2nd best at $p = 0.50$

Table 2
Results of Bazaraa problem with maximum aspect ratio = 2

p Range	Eq. (5) value	Mean costs	Std. dev. costs	Notes
0.40–0.60	1328.62	6643.12	1829.12	Same as $p = 0.50$
0.25–0.50	1510.44	6715.19	2073.85	Same as $p < 0.50$
0.50–0.75	1797.82	6655.12	1651.05	Same as $p > 0.50$
0.25–0.60	2199.64	6643.12	1829.12	Same as $p = 0.50$

This can be demonstrated with a simple example. Consider there is a flow between departments i and j of mean = 100 and standard deviation = 0 and a flow between departments k and l of mean = 100 and standard deviation = 50. For $p = 0.50$, these two flows are identical and it is equally important to locate i close to j and k close to l. If $p > 0.50$, the flows between k and l will be > 100 so it is more important to locate k close to l than i close to j. This will reduce the overall variance of the layout. If $p < 0.50$, the flows between k and l will be < 100 so it is more important to locate i close to j than k close to l. This will increase the overall variance of the layout. This effect can be seen with the layouts from the van Camp et al. problem in Appendix B. Product 1, which routes through departments 3, 5 and 10, has a low mean (10) and a very high standard deviation (100). Layouts optimized for $p < 0.5$ push departments 3 and 5 apart, while layouts optimized for $p > 0.5$ group departments 3, 5 and 10 together. The routing distances for Product 1 are 29.025, 95.790, 39.225 and 19.505, working from top to bottom, which confirms the strong effect of the p range or value on the distance that Product 1 routes through the facility. The Bazaraa problem (Appendix B) also shows this effect. The most variable product, Product 1 with a mean of 100 and a standard deviation of 100, routes through departments 2, 8, 9 and 1. The distance traveled along that routing is 8.570 for $(0.50 < p < 0.75)$, 13.085 for $(0.40 < p < 0.60)$ and 14.225 for $(0.25 < p < 0.50)$.

Returning to Tables 1 and 2, the block layouts identified by using the symmetric robustness range (0.40–0.60) are identical to those when p=0.50, as anticipated. By using the pessimistic range (0.25–0.50), the block layouts are either identical to $p < 0.50$ (i.e. $p = 0.16$, 0.31) if the problem was very constrained (Bazaraa), or superior over that range if the problem was somewhat constrained (van Camp et al.). In a tightly constrained problem there are fewer layouts that are feasible so it is less likely a better layout can be found by small alterations in p. On the optimistic side, results were the same as for $p > 0.50$. On the optimistic side of the forecast, it is harder to identify layouts that reduce the objective function because this depends on reducing the variance of material handling costs over the layout while maintaining a relatively small expected value of costs. For the last range, the Bazaraa problem was solved with the layout that was also found at $p = 0.50$. For the van Camp et al. problem, the layout that performs best over the range (0.25–0.60) is one that was identified as second best for $p = 0.50$.

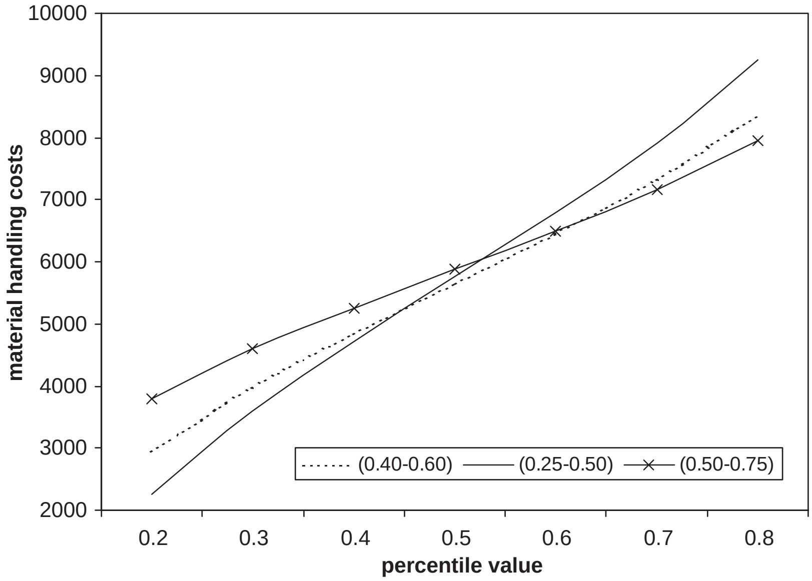

Fig. 3. Comparison of three layouts for the van Camp et al. [31] problem.

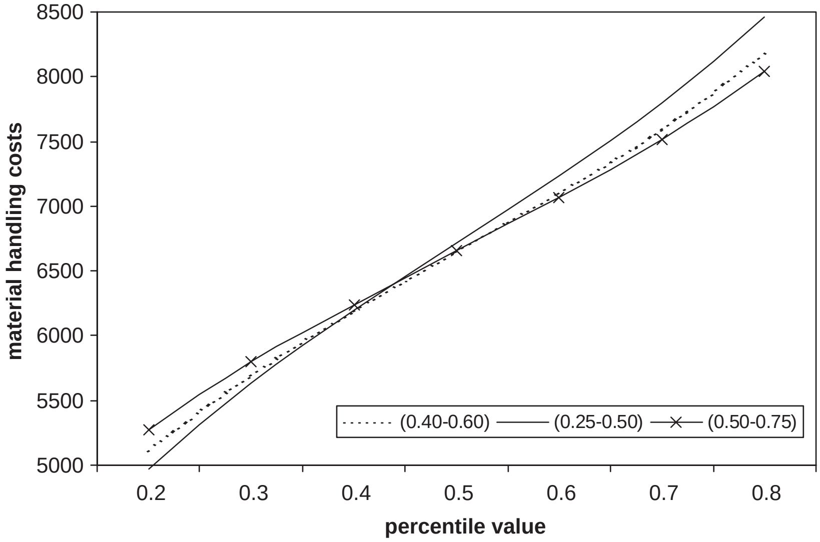

Fig. 4. Comparison of three layouts for the Bazaraa [32] problem.

Visual comparisons are made in Figs. 3 and 4 for the van Camp et al. and Bazaraa problems, respectively. The three distinct layouts for each problem from Tables 1 and 2 are graphed over the percentile range of 0.2–0.8. It is quite clear where each block layout dominates the others, where the equivalence points are, and the relative cost differences among the layouts for each value of *p*. An analyst could easily ascertain the relative performance of each layout. Distinctions are less acute in the Bazaraa problem (Fig. 4).

5. Conclusion

This paper presented an alternative manner to examine uncertainty in the block layout of facilities that is complementary to the existing manner of discrete production scenarios. Uncertainty is characterized by forecasted product volume mean and standard deviation. This formulation allows uncertainty to be considered on a continuous scale without the need to provide probability mass or distribution assumptions. Furthermore, it does not depend on an implicit (and possibly, unrecognized) assumption of risk neutrality on the part of the user.

The robustness metric weighs the quality of different layouts by considering the likelihood of different product volumes and the cost of producing those volumes when using the layout under consideration. This metric can also be used as the objective function during search to identify block layouts that perform well over the specified range of uncertainty, and will be especially useful when the analyst does not have a good idea of what p values are particularly applicable to the production facility under study.

This paper demonstrated these concepts by examining two unequal area block layout problems from the literature using a flexible bay construct and a genetic algorithm meta-heuristic as the optimization method. However, this stochastic formulation and the robustness metric could be used for any block layout problem, including the QAP, slicing tree or unit block approaches, and with other optimization methods.

While this paper considered only independent products, dependence could be handled by a straightforward extension to consider covariance. Another relatively straightforward extension is to multi-period layout, where each product would have a forecasted mean and standard deviation per period. A more complicated and significant extension would be to consider changes in department area coinciding with changes in material flow volume. While some production environments can accommodate increases in production with additional shifts or improved methods and machinery, others may require additional area in some departments. This is a useful area for future research.

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Appendix A. Test problem data

(All flows are bidirectional.)

van Camp et al. problem [31]

Product 1:
mean = 10, standard deviation = 100
routing: departments 3, 5, 10.
Product 2:
mean = 100, standard deviation = 10
routing: departments 1, 5, 8, 7.

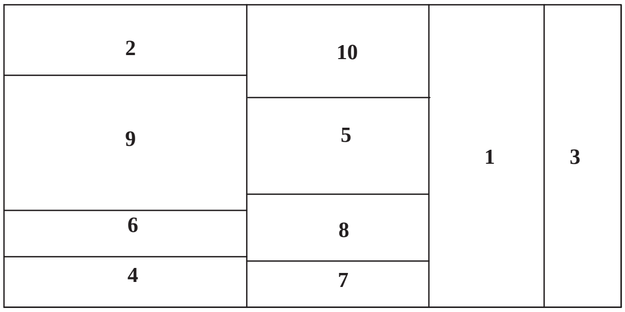
Bazaraa problem [32]

Product 1:
mean = 100, standard deviation = 100
routing: departments 2, 8, 9, 1.
Product 2:
mean = 150, standard deviation = 50
routing: departments 3, 8, 9, 11, 12, 6.

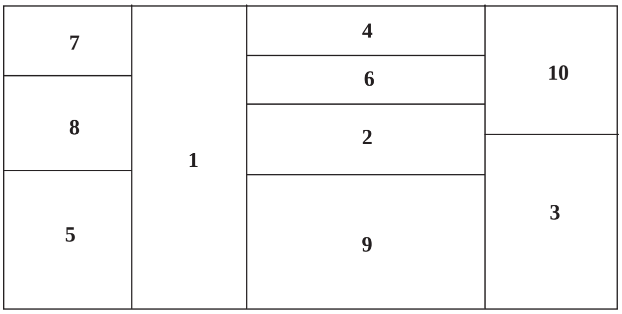
Product 3:
mean = 50, standard deviation = 50
routing: departments 2, 9, 6, 4.
Product 4:
mean = 50, standard deviation = 25
routing: departments 2, 9, 5, 8, 7.

Product 3:
mean = 50, standard deviation = 25
routing: departments 7, 10, 13, 6.
Product 4:
mean = 100, standard deviation = 30
routing: departments 5, 9, 12, 4, 1.
Product 5:
mean = 100, standard deviation = 70
routing: departments 6, 8, 5, 4, 11, 10.
Product 6:
mean = 120, standard deviation = 40
routing: departments 4, 13, 12.

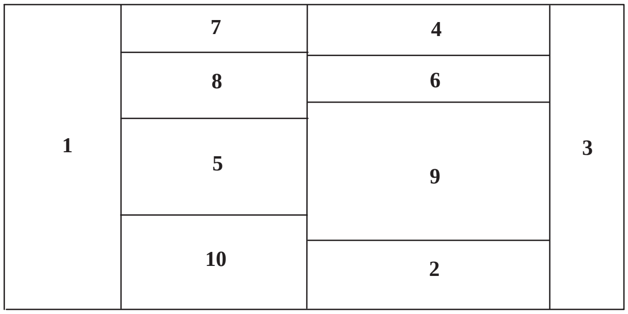
Appendix B. Best block layouts of selected problems



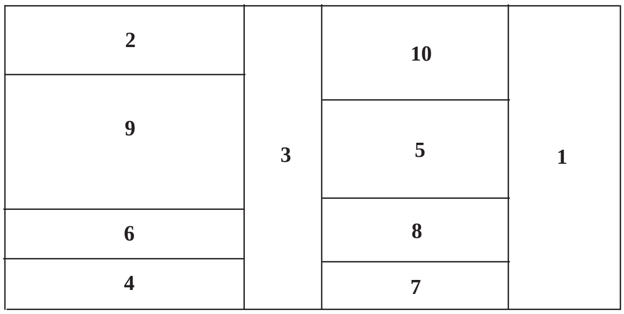
van Camp et al. (0.40–0.60), $p = 0.50$



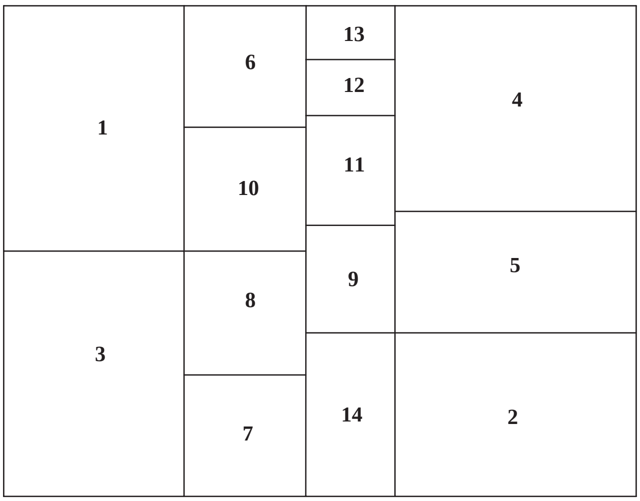
van Camp $p = 0.31$



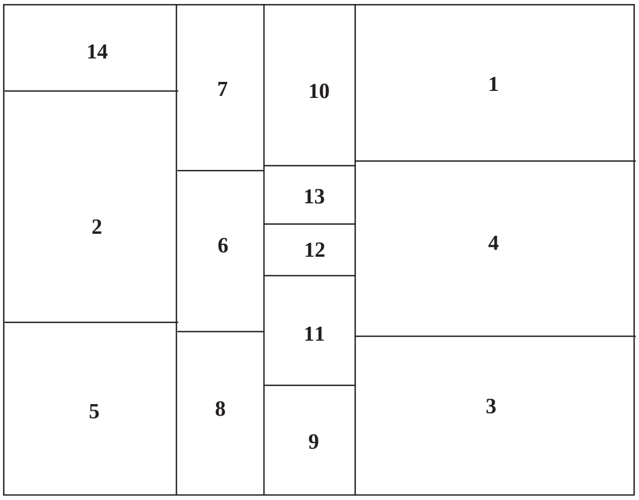
van Camp et al. (0.25–0.50)



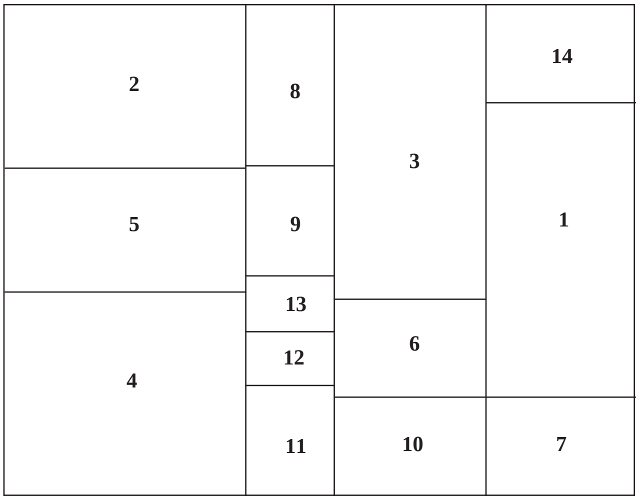
van Camp et al. (0.50–0.75)



Bazaraa (0.40–0.60), (0.25–0.60)



Bazaraa (0.25–0.50)



Bazaraa (0.50–0.75)

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