Magnetostatics – Magnetic Flux Density

The *magnetic flux density*, **B**, related to the magnetic field intensity in free space by

$$\mathbf{B} = \mu_o \mathbf{H}$$

where μ_o is the *free space permeability*, given in units of *henrys* per meter, or

$$\mu_o = 4\pi x 10^{-7} H/m$$

The units of **B** are therefore $(H)(A)/m^2$, but it is more instructive to write *webers* per meter squared, or Wb/m^2, where Wb=(H)(A).

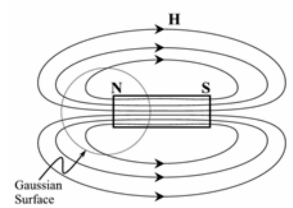
But for brevity, and perhaps to honor a deserving scientist, a tesla , T, equivalent to a Wb/m^2, is the standard unit adopted by the International System of Units.

The amount of magnetic flux, ϕ, in webers, from magnetic field passing through a surface is found in a manner analogous to finding electric flux:

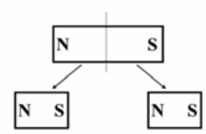
$$\phi = \int \mathbf{B} \bullet d\mathbf{S}$$

Magnetostatics – Gauss's Law

A fundamental feature of magnetic fields that distinguishes them from electric fields is that the field lines form closed loops



We cannot saw the magnet in half to isolate the north and the south poles; as Figure shows, if you saw a magnet in half you get two magnets.



Put another way, *you cannot isolate a magnetic pole*.

From this characteristic of magnetic fields, it is easy to see that the net magnetic flux passing through a Gaussian surface (a closed surface as shown in Figure 3.26) must be zero. What goes into the surface must come back out. Thus we have *Gauss's law for static magnetic fields*

$$\oint \mathbf{B} \bullet d\mathbf{S} = 0$$

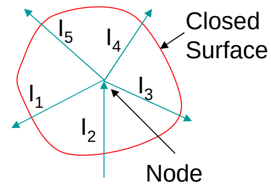
This is also referred to as the law of conservation of magnetic flux.

Gauss's Law and Kirchhoff's Current Law

Gauss's Law: the net magnetic flux passing through a closed surface (Gaussian surface) must be zero

$$\oint \mathbf{B} \bullet d\mathbf{S} = \mu \oint HdS \cos\theta = \mu \cos\theta \oint I = 0$$

$$\oint I = 0$$



Therefore, the algebraic sum of the currents entering any closed surface is zero.

This is analogous to Kirchhoff's Current Law (KCL)!

Kirchhoff's Current Law: The algebraic sum of the currents entering any node is zero.

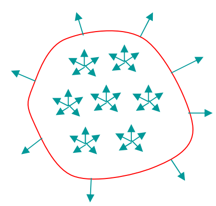
$$\sum_{i=1}^{n} I_i = 0$$

$$\sum_{i=1}^{n} I_i = I_1 - I_2 + I_3 + I_4 + I_5 = 0$$

Point form of Gauss's Law

The divergence theorem states that *the net outflow of flux from a closed surface is equal to the sum of flux outflow (and inflow) from every point inside the volume enclosed by the surface.*

$$\boxed{\oint \mathbf{B} \bullet d\mathbf{S} = \int \nabla \bullet \mathbf{B}\, dv}$$



Applying the divergence theorem, we arrive at the point form of Gauss's Law for static magnetic fields

	Integral Form		Point Form
Gauss's Law:	$\oint \mathbf{B} \bullet d\mathbf{S} = 0$	⇨	$\nabla \bullet \mathbf{B} = 0$

Magnetostatics – Gauss's Law

The differential, or point, form of Maxwell's Equations are easily derived by applying the divergence theorem and stoke's theorem to the integral form of the equations.

Integral form

$$\oint \mathbf{D} \bullet d\mathbf{S} = Q_{enc}$$

$$\oint \mathbf{B} \bullet d\mathbf{S} = 0$$

$$\oint \mathbf{E} \bullet d\mathbf{L} = 0$$

$$\oint \mathbf{H} \bullet d\mathbf{L} = I_{enc}$$

Divergence Theorem

$$\oint \mathbf{F} \bullet d\mathbf{S} = \int \nabla \bullet \mathbf{F} \; dv$$

Stokes Theorem

$$\oint \mathbf{F} \bullet d\mathbf{L} = \int \left(\nabla \times \mathbf{F} \right) \bullet d\mathbf{S}$$

Differential form

$$\nabla \bullet \mathbf{D} = \rho_v$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$