## STAT 3611 Lab 5 S2015 Maghsoodloo

<u>**1(a, 3 points).**</u> Use Minitab to work Exercise 58 of J. L. Devore (8e) on his page 338, describing the exact relationship between his value of 1.616 & 0.29504 on my Excel Data-file Before proceeding, change Devore's first sentence to: A random sample of "**size n = 30**" soil specimen was obtained ..... **"LOS = Level Of Significance"** 

Do the above part by first, i.e., testing  $H_0:\mu = 3.00$  recognizing that  $\sigma$  is truly unknown and using William Sealy Gosset's t-statistic and then the normal (or the Z as an approximate test) using the sample  $S_x = 1.61564$  as a rough approximate value of process-parameter  $\sigma$ .

<u>**1(b, 3 points).**</u> Use Minitab to estimate the power of the nominal statistical 5%-level test (i.e., the LOS  $\alpha = 0.05$ ) if the true process mean organic matter  $\mu$  were equal to 3. 40%. This represents an upward shift of 0.40 in the process mean  $\mu$ . Go to Stat  $\rightarrow$  Power and Sample Size; first select 1-Sample t followed by 1-sample Z, and compare your power results.

J.L. Devore (8e) seems to imply at the end of his Exercise-58 on p. 338-statement that the assumption of underlying normality is almost tenable. Do you agree? Why, or why not. Provide precise statistical answer using Minitab's Stat  $\rightarrow$  Basic Statistics  $\rightarrow$  Graphical Summary, and the resulting AD-Statistic *P*-value to draw conclusion about the normality assumption of the data-underlying distribution (the smaller a *P*-value is, the stronger a null-hypothesis of Normality must be rejected). Secondly, on your data-sheet, approximately (& roughly) compute the Pr(X  $\leq$  0), where X = % Organic Matter, using the normality assumption. Use this last left-tail Pr (below zero) to ascertain if you agree with J. L. Devore's (8e) assertion of acceptable normality pattern.

**Definition.** In the field of Statistics [where QC (Quality Control) is strictly an application area], an OC (Operating Characteristic) curve is always the graph of "type II error, or error of" (= "accepting a false null hypothesis H<sub>0</sub>") Pr, denoted  $\beta$ , as a function of the parameter under the null hypothesis H<sub>0</sub>. For example, in the above Problem 1, the null hypothesis is H<sub>0</sub>:  $\mu = 3.00\%$ , i.e.,  $\mu_0 \equiv 3.00\%$ , then the OC curve will graph  $\beta$  = Type II error Pr = P<sub>a</sub>(at a given  $\mu$ ) as a function of  $\mu$  (where  $\mu$  is the abscissa), and  $\beta$  (at a specified  $\mu$ ) = P<sub>a</sub>( $\mu$ )

will be the ordinate (or the so-called y-axis of the OC curve). See Table A.17 on p. Appendix A-28 of Devore (8e), but please change his title from  $\beta$  Curves to OC Curves. Please also note that Statistical-Literature (specially QC-literature) always Studentdizes the OC-Curve-abscissa to construct OC curves on mean(s), i.e., uses the Non- central t-distribution. However, if for example null hypothesis were H<sub>0</sub>: $\sigma$  = 0.25 cm versus H<sub>1</sub>:  $\sigma$  < 0.25 (a left-tail test), then the corresponding OC-Curve will graph  $\beta$  = Error Pr of 2<sup>nd</sup>-kind = P<sub>a</sub>(at a given  $\sigma$ ) versus the unitless abscissa Lambda =  $\sigma/\sigma_0$ .

**<u>2(4 points).</u>** Use MS Excel to draw the (0.05-Level, i.e. the LOS of the test is set nominally at  $\alpha = 0.05$ ) OC curve of the above problem 1 by assuming the true process  $\sigma$  were equal to 1.4790 ( $\rightarrow$  the Z-test), and computing the acceptance probability  $\beta = P_a(H_0: \mu = 3.00\%)$ VS the 2-sided alternative hypothesis H<sub>1</sub>:  $\mu \neq 3.00\%$ . Thus, I am making the assumption that the parent-variable X = % Organic Matter ~ N(unknown  $\mu$ ,  $\sigma^2 = 2.187441$ ), which you will find out is not quite tenable. In order to draw this OC (Operating Characteristic) curve, compute  $\beta$  at  $\mu = 3.00 \pm 1.2\sigma x$ ,  $\delta = \pm 1\sigma x$ ,  $\pm 0.80\sigma x$ ,  $\pm 0.6\sigma x$ ,  $\pm 0.4\sigma x$ ,  $\mu_0 \pm 0.20\sigma x$ , starting your  $\mu$ -Column in Excel at  $\mu = 3.00 - 1.20 \times 1.4790$ . However, before the  $\mu$ -Column provide another column headed by  $\delta$  whose first value will be  $-1.20 \times 1.4790$ ; the adjacent columns are described in my Excel-SOLN file on the Lab-screen. The quantity  $\delta$  represents the amount of shift-in- $\mu$  in terms of  $\sigma_x$  from the null-value of  $\mu_0 \equiv 3.00\%$ . A negative-value of  $\delta$  always implies a downward-shift  $\mu$ , and vice a versa.

Next use the above information to also draw the power curve  $(1-\beta \text{ versus }\mu)$  for testing H<sub>0</sub>:  $\mu = 3.00$  at the 5% LOS, where power of a Statistical test graphs the Pr of rejecting a false hypothesis for a specified parameter-value. By Statistical power we mean the apriory Pr of rejecting a false H<sub>0</sub>.

2