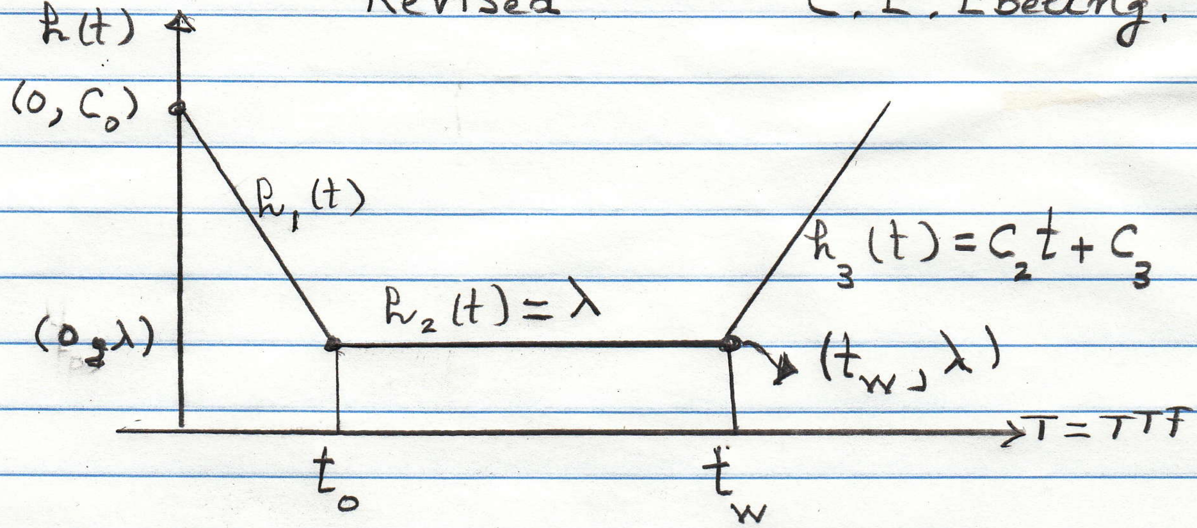
INSY 7380-6 M2014 S. Maghsoodloo

The Bathtub Curve (BTC) on pp. 31-32 of Revised C. E. Ebeling.



t_0 = The end of burn-in ; $c_0, c_1, c_2 > 0$.

t_w = The beginning of wearout period

$-c_1$ = The slope during burn-in

c_2 = " " " wearout.

Thus, $h_1(t) = -c_1 t + c_0$; At $t = t_0$, $h_1 = \lambda$

$\rightarrow \lambda = -c_1 t_0 + c_0 \rightarrow c_0 = \lambda + c_1 t_0 \rightarrow$

$h_1(t) = -c_1 t + \lambda + c_1 t_0 = \lambda - c_1 (t - t_0)$, $0 \le t \le t_0$.

$h_2(t) = \lambda$; $h_3(t) = c_2 t + c_3$; At $t = t_w$,

$h_3(t) = \lambda \rightarrow \lambda = c_2 t_w + c_3 \rightarrow c_3 = \lambda - c_2 t_w$.

Thus, $h_3(t) = c_2 t + \lambda - c_2 t_w = \lambda + c_2(t - t_w)$,

$$\text{Hence, } h(t) = \begin{cases} \lambda - c_1(t - t_0), & 0 \le t \le t_0 \\ \lambda, & t_0 \le t \le t_w \\ \lambda + c_2(t - t_w), & t_w \le t < \infty. \end{cases}$$

As an example, suppose the maximum FR during burn-in is $C_0 = 0.001$ failures/hr, and the end of burn in is $t_0 = 100$ hours; further, the burn-in has reduced FR to $\lambda = 0.0005$/hr. Then, $h_1(t) = \lambda - c_1(t - t_0)$ shows that $0.001 =$

$0.0005 - c_1(0 - 100) \rightarrow 0.0005 = 100\,C_1 \rightarrow C_1 = 0.000005$

$h_1(t) = 0.0005 - 5\times10^{-6}(t - 100) = 0.001 - 5t\times10^{-6}$

Next, assume that $t_w = 2000$ hrs while the the slope of the wearout is $C_2 = 0.0020$ failures/hr^2.

$h_3(t) = 0.0005 + 0.002(t - 2000)$, $2000 \le t < \infty$

The BTC (Cont'd)

$$h(t) = \begin{cases} 0.001 - 5\times10^{-6}\,t, & 0 \le t \le 100 \text{ hours} \\ 0.0005, & 100 \le t \le 2000 \text{ hours} \\ 0.0005 + 0.002(t - 2000), & 2000 \le t < \infty. \end{cases}$$

The CHZF is

$$H(t) = \begin{cases} 0.001t - 2.5\times10^{-6}\,t^2, & 0 \le t \le 100 \\ 0.075 + \int_{100}^{t} 0.0005\,dx = 0.025 + 0.0005t, & 100 \le t \le 2000 \\ 1.025 + \int_{2000}^{t} h_3(x)\,dx = 4000.025 - 3.9995\,t + 0.001t^2, & 2000 \le t < \infty \end{cases}$$

The RE function is

$$R(t) = e^{-H(t)} = \begin{cases} e^{-.001t + 2.5\times10^{-6}\,t^2}, & 0 \le t \le 100 \\ e^{-(0.025 + .0005t)}, & 100 \le t \le 2000 \\ \exp[-0.001t^2 + 3.9995t - 4000.025], & 2000 \le t < \infty \end{cases}$$

$T_W = T_{Replace} = 2000$ hours $\longrightarrow$ Do maintenance

such that $C_2 = 0.002/hr^2$.

$R(t = 100\text{ hrs}) = 0.9277435$

Unconditional $R(t=1000) = e^{-0.525} = 0.5915554$

$$R(T=1100 \mid T>100) = \frac{R(1100)}{R(100)} = \frac{0.562705}{0.927735}$$

$$= 0.6065307.$$

The above represents the RE at 1000 hours of service-life after burn-in, i.e., all units that fail during $0 \le t \le 100$ hrs must be repaired (or renewed to perfect condition), or they must be discarded if they are irreparable.

The unconditional B-10 (or L-10) life is given by setting $Exp(-.025 - 0.0005\, t_{0.10}) = 0.90 \rightarrow$

Uncond. B-10 life $= 160.72103$ hours.

To obtain the conditional B-10 life, set

$$R(t_{0.10} + 100 \mid T>100) = e^{-.025 - 0.0005(t_{0.10}+100)} / R(100)$$

$$= 0.90 \rightarrow$$

B-10 service life $= 210.72103$ hours