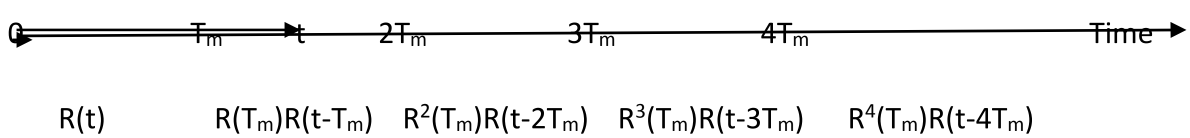
Section 9.5 of Ebeling (The Reliability function under preventive maintenance)

Ebeling (2nd Ed. pp. 237 to 241) has an excellent discussion as how preventive maintenance (PM) can improve system RE iff the hazard function h(t) is increasing. Suppose PM is performed periodically every T_m days. We further assume that once preventive maintenance (PM) is performed on a system, then the system is practically as-good-as-new. There are two MUEX (mutually exclusive) possibilities:

(1) The system fails by T_m, (2) the system survives beyond at least one cycle of length T_m. These 2 possibilities lead to the following RE function with PM:

$$R_m(t) = \begin{cases} \mathbf{R(t),\ 0 \le t < T_m} \\ \mathbf{R(T_m)^n R(t - nT_m),\ nT_m \le t < (n+1)T_m} \end{cases} . \qquad \text{(9.26 of Ebeling)}$$

Note that Ebeling uses T for T_m in his Eq. (9.26) to represent the length of one PM cycle. The argument for the above RE function is depicted in the following figure.



The MTTF of a PM system is given by (m standing for with maintenance)

$$\text{MTTF}_m = \int_0^\infty R_m(t)dt = \sum_{n=0}^{\infty} \int_{nT_m}^{(n+1)T_m} R(T_m)^n R(t - nT_m)dt = \sum_{n=0}^{\infty} R(T_m)^n \int_{nT_m}^{(n+1)T_m} R(t - nT_m)dt$$

In the above integral, put x = t– nT_m. This leads to

$$\text{MTTF}_m = \sum_{n=0}^{\infty} R(T_m)^n \int_0^{T_m} R(x)dx = \frac{\int_0^{T_m} R(t)dt}{1 - R(T_m)} \qquad \text{(Eq. 9.27 of Ebeling)}$$

Ebeling provides a good example of an exponential lifetime with PM in his Example 9.17 on p. 238 in which he illustrates that PM does not alter R(t) iff h(t) = λ is a constant. Note that if we compute the $MTTF_m$ using Eq. (9.27) under constant h(t), we obtain MTTF = $\frac{\int_0^{T_m} e^{-\lambda t}dt}{1-e^{-\lambda T_m}}$ =

$\frac{\frac{-1}{\lambda}\left[e^{-\lambda t}\right]_0^{T_m}}{1-e^{-\lambda T_m}}$ = $\frac{\frac{-1}{\lambda}\left[e^{-\lambda T_m}-1\right]}{1-e^{-\lambda T_m}}$ = $1/\lambda$, i.e., for an exponential TTF the MTTF with and without PM are identically equal to $1/\lambda$. For another excellent example, see the Example 9.18 on p. 238 of Ebeling. For this Example, I am changing the value of t_c for the compressors from 100 days to 120 days so that now TTF ~ W(0, θ =120 days, shape= β =2); further, I am changing the maintenance cycle to one month = 30 days (25% of θ). Thus, the RE function with PM is given

by $R_m(t) = \begin{cases} \mathbf{R(t),\ 0 \le t < 30\ days} \\ \mathbf{R(30)^n R(t-30n),\ \ 30n \le t < 30(n+1)} \end{cases}$, n = 0, 1, 2, 3, 4, …

The value of RE at 160 days from time zero is computed first by recognizing that n = 5 PM cycles and that 165 days lies within the interval (5×30, 6×30), and hence t = 10 days; thus,

$R_m(160)$ = $\mathbf{R(30)^5 R(160-150),\ \ 150 \le t < 180\ days}$. That is, PM improves RE by 329.88%.

The MTTF(W/O PM) = 106.347 days versus MTTF(with PM) =$MTTF_m$= $\frac{\int_0^{T_m} R(x)dx}{1-R(T_m)}$ =

$\frac{\int_0^{30} e^{-(x/120)^2}dx}{1-0.9394131}$ = $\frac{\int_0^{\sqrt{2}/4} e^{-z^2/2}(120dz/\sqrt{2})}{0.0605869}$, where x/120

= z/$\sqrt{2}$. Thus, $MTTF_m$ = $\frac{120\sqrt{\pi}\int_0^{\sqrt{2}/4} e^{-z^2/2}(dz/\sqrt{2\pi})}{0.0605869}$ = $\frac{120\sqrt{\pi}[\Phi(\sqrt{2}/4)-0.50]}{0.0605869}$ =

$\frac{120\sqrt{\pi}(0.1381632)}{0.0605869}$ = 485.03136, which is an improvement of 356.083% in MTTF due to PM.

Ebeling also discusses the case when PM induces failure into the system with a Pr, p, and the RE function with PM is given atop page 240 in his Eq. (9.28). See his Example 9.19 on p.