In most instances component reliability is so high that placing even n = 100 units on test may not yield any failures for a test duration, of say, more than 5000 hours. If a new product is being developed, such long testing times cannot be tolerated because the new product has to get to the global market in due time, or else lack of market share may occur. In such cases, the experimenter has no choice but to use accelerated testing procedures to induce failures in order to estimate component TTF (or reliability).

Accelerated life testing (ALT), in combination with DOE (design of experiments), is conducted by subjecting n identical units to stresses well beyond what the units on test will experience under normal operating conditions. Such high stresses for ALT that accelerate failure mechanism may be applied in many forms: very high, or very low temperatures, humidity levels well beyond normal operating conditions, excessive usage, very high levels of voltage, extreme cycling between low and high levels beyond what is considered normal operating conditions, excessive force, excessive vibrations, ten times more units on test than needed, etc, etc.

As Elsayed (Reliability Engineering, E.A. Elsayed, Chapter 6, Wiley INC. (2012) points out in the beginning of section 6.4, the underlying assumption is that the failure mechanism under ALT conditions is, except for a multiplicative factor, similar to failure mechanism under normal operating conditions. Put differently, ALT is based on the principle that a unit under accelerated test will exhibit the same behavioral statistical pattern in a short testing time under very high stresses as it will exhibit in a much longer time at normal operating stresses. For example, if the underlying failure distribution is $W(\delta, \theta, \beta)$, then under ALT the change in the shape parameter β will be much smaller than the changes in minimum life δ and scale parameter $\theta - \delta$. That is to say, under ALT the change in β (or overall process variability, or CV) will be negligible compared to changes in δ and the characteristic life θ as compared to normal operating conditions. There are 3 different physical models that have been developed in the past 115 years that can be used to estimate the MTTF under normal operating conditions (o = normal operating conditions) from ALT data, where subscript "s" will be used to designate statistics computed under high stressed conditions.

(1) The Arrhenius Model

This is the most commonly used model relating TTF to high thermal stresses. Thermal stresses occur in solid state diffusion, chemical reactions, many semiconductor failure mechanism, battery life, etc. The underlying distribution of TTF_o (TTF under normal operating conditions) in almost all these cases is exponential, Weibull, or lognormal (i.e., all positively skewed pdfs). The Arrhenius rate law that describes the (failure) rate, r, at which reaction to temperature of the test unit occurs is given below.

$$r = C' \mathbf{e}^{-E_a/(kT)} = C' \mathbf{e}^{-B/T} \qquad (102)$$

where C' is a constant which is characteristic of the failure mechanism of the item under test, E_a = the activation energy needed to induce failure measured in eV (= electron volts; close to vaporization energy for metals and chemical bond energies for polymers), k = the Boltzman's constant = 8.6171×10^{-5} eV/Kelvin (Note that Google.com gives $k = 8.616\times10^{-5}$, while L. W. Condra, p. 232, gives $k = 8.617\times10^{-5}$) and T = the temperature in Kelvin = Centigrade + 273.15, and $B = E_a/k$. In RE engineering, the Arrhenius model is also used to measure the impact of temperature on reliability because we make the assumption that the TTF is inversely proportional to the reaction (failure) rate, r, given in equation (102), i.e.,

$$TTF = C\,\mathbf{e}^{E_a/(kT)} = C\,\mathbf{e}^{B/T} \qquad (103)$$

where $C \neq 1/C'$ is the constant of proportionality characteristic of the product under test. The Arrhenius model is applicable when the product $r_1\times TTF_1 = r_2\times TTF_2$, where r_1 and r_2 are reaction rates at testing temperatures T_1 and T_2, respectively. The relationship $Rate_1\times TTF_1 = Rate_2\times TTF_2$ implies that $r\times TTF$ will practically stay constant over the range of temperature applicability, and as a result $r_o\times TTF_o \cong r_s\times TTF_s$, where TTF_o represents TTF_o under normal operating conditions and TTF_s represent component life under (accelerated) stressed conditions. Thus,

$$\frac{\mathbf{TTF_o}}{\mathbf{TTF_s}} = \frac{r_s}{r_o} = \frac{C'\,\mathbf{e}^{-E_a/(kT_s)}}{C'\,\mathbf{e}^{-E_a/(kT_o)}} = \mathbf{e}^{E_a\left(\frac{1}{T_o}-\frac{1}{T_s}\right)/k} \rightarrow TTF_o = TTF_s\,\mathbf{e}^{E_a\left(\frac{1}{T_o}-\frac{1}{T_s}\right)/k}$$

(104a)

Eq. (125a) shows that the acceleration factor for the Arrhenius model is given by

$$A_f = TTF_o / TTF_s = \mathbf{e}^{E_a(\frac{1}{T_o} - \frac{1}{T_s})/k} = \mathbf{e}^{B(\frac{1}{T_o} - \frac{1}{T_s})} \qquad (104b)$$

Note that the smaller the required activation energy E_a is, the more rapidly the unit on test will fail resulting in smaller A_f value.

<u>The Example 6.11 on page 405 of E. A. Elsayed</u>. In this example n microelectronic devices (the value of n not specified by the author) are put on accelerated test at T_s = 200 Celsius = 200 + 273.15 = 473.15 Kelvin and the $MTTf_s$ of the n units was approximately equal to 4000 hours. The operating temperature T_o = 50 °C = 323.15K, and the required activation energy was 0.191 eV. Thus, the sample $MTTf_o$ = $MTTf_s\, \mathbf{e}^{E_a(\frac{1}{T_o} - \frac{1}{T_s})/k}$ = 4000 $e^{0.191(\frac{1}{323.15} - \frac{1}{473.15})/8.6171\times 10^{-5}}$ = 35191.33024 hours. The value of acceleration factor is A_f = 35191.33024 /4000 = 8.79783256.

<u>Example 16.</u> The TTF of n = 10 samples under an accelerated temperature of T_s = 100 Centigrade are $t_{(i)}$: 130, 140, 160, 180, 185, 195, 205, 205, 240, and 260 hours. The measurement of interest is the thermo-compression bond between two dissimilar metals, the strength of which reduces in time by the formation of voids by solid-state diffusion which has an activation energy of 0.90 eV. The normal operating temperature is T_o = 25 Celsius. The sample statistics are $\bar{x}_s$ =190, S_s = 40.8248290, and cv_s = 21.487% showing that the accelerated data is obviously not exponentially (i.e., IFR) and if it is Weibull, then the slope $\beta \cong 5.0$ (see my Table 1 on p. 10). Most probably, the accelerated data is lognormally distributed. The use of equation (104a) yields the normal operating sample mean to failure $mttf_o$ = 190 × $e^{0.90(\frac{1}{298.15} - \frac{1}{373.15})/(8.6171\times 10^{-5})}$ = 190×1142.3450161 = 217045.5531 hours → A_f = 1142.3450161. If we wish to be more conservative about our estimate of MTTF in normal operating use, we could estimate it as $mttf_o = 130\, e^{0.90(\frac{1}{298.15} - \frac{1}{373.15})10^5/8.6171}$ = 148504.8521 hours, giving an acceleration factor of A_f = 148504.8521/190 =781.6044847, where 130 is the value of the 1st order statistic, $x_{(1)} = t_1$, under stressed condition. Note that we are using *mttf* as the sample MTT failure.

It is reported in the literature that the value of E_a ranges in the interval 0.30 – 0.60 for semiconductor failures, for intermetallic diffusion (like in Example 16) it ranges in the interval 0.90 −1.1 eV, and for silicon junction defects E_a = 0.80 eV. The question arises how high the stressed temperature should be for a unit under accelerated test so that the resulting stressed life can be extrapolated to the expected life under normal operating conditions. Almost all metals change properties when the testing temperature exceeds 50% of their melting temperature T_m. Therefore, the accelerated testing temperature, T_s, must not exceed $0.50\times T_m$.

Example 17. The lifetimes of n = 50 PC components under an accelerated temperature of $T_s = 100^oC$ yielded the sample mean $\bar{x}_s$ = 232.2 hours and a standard deviation of S_s = 82.8 hours, with E_a = 0.85 eV and $T_o = 27^oC$. The use of equation (104b) gives an acceleration factor of $A_f = e^{E_a\left(\frac{1}{300.15} - \frac{1}{373.15}\right)/k}$ = 619.695651 giving an estimated $mttf_o = A_f \times \bar{x}_s$ = 619.695651×232.2 = 143893.3301 hours ≅ 16.42618 years. Since the sample size n > 20, then we may use the normal approximation to the SMD of $\bar{x}_s$ to obtain an approximate lower 95% CI for the $MTTF_s$, given by $L_s = 232.2 - 1.645\times 82.8/\sqrt{n}$ = 212.937563 hours $\rightarrow L_o$ = 619.695651×212.938 = 131956.75253264 hours → 15.0635562252 years ≤ $MTTF_o < \infty$ at the 95% confidence level. Note that this normal approximation would not be permissible unless n > 20. Methods of analysis for the exponential, Weibull, and lognormal underlying distribution of TTF_s, for any n, are given by Wayne Nelson, (1990), " Accelerated Testing, Wiley, New York, ISBN: 0-471-52277-5.

Determination of the Acceleration Factor A_f Using Linear Regression

In order to use regression to estimate A_f, the Arrhenius model must first be linearized as shown below. From equation (103), $TTF = C e^{E_a/(kT)}$, which can be linearized by taking the natural logarithm of both sides only once. This leads to y = ln(TTF) = ln (C) + $E_a/(kT)$ = ln (C) + $E_a x$, where $x = 10^5/(8.6171T)$, and T must be in units of Kelvin. I used the data provided by Boris Gnedenko *et al* (Statistical Reliability Engineering, Wiley, Example 5.2 on pp. 171-172, ISBN: 0-471-12356-0) and W. Nelson (1990), which are listed for your convenience below, to estimate C and E_a using regression analysis. The experiment from the above two authors involved a new Class-H motor insulation with a design temperature of T_o = 180 °C = 453.15 Kelvin, where n = 40 units were equally and randomly divided and tested to failure at the accelerated temperatures 190, 220, 240, and 260 Celsius. The accelerated times TF_S in hours are provided in Table 5.2 of B. Gnedenko and duplicated herein. I used Minitab to regress y on x, where x = $10^5/(8.6171T)$, and its output is provided below.

Table 5.2 of Boris Gnedenko *et al* (page 171)

190 °C	220 °C	240 C°	260 °C
7228 hours	1764 hours	1175 hours	600 hours
7228	2436	1175	744
7228	2436	1521	744
8448	2436	1569	744
9167	2436	1617	912
9167	2436	1665	1128
9167	3108	1665	1320
9167	3108	1713	1464
10511	3108	1761	1608
10511	3108	1953	1896
mttf **= 8782.20**	**2637.6000**	**1581.4000**	**1116.0000**
S = 1244.0117	**453.5654**	**244.2745**	**439.2357**
CV **= 14.165%**	*CV* **= 17.196%**	*CV* **= 15.447%**	*CV* **= 39.358%**

Regression Analysis: y versus x

```
The regression equation is
y = - 7.28341 + 0.64936 x, x = 10^5/(8.6171 T)

Predictor        Coef      SE Coef          T        P
Constant      -7.2837       0.7719      -9.44    0.000
x             0.64938      0.03317      19.58    0.000

S = 0.2557      R-Sq = 91.0%     R-Sq(adj) = 90.7%

Analysis of Variance
Source            DF        SS          MS         F        P
Regression         1    25.073      25.073    383.36    0.000
Residual Error    38     2.485       0.065
  Lack of Fit      2     0.368       0.184      3.12    0.056
  Pure Error      36     2.118       0.059
Total             39    27.558
```

```
Unusual Observations
Obs        x        y          Fit      SE Fit    Residual     St Resid
 11       23.5   7.4753      7.9973     0.0416    -0.5220       -2.07R
 39       21.8   7.3827      6.8509     0.0635     0.5319        2.15R
 40       21.8   7.5475      6.8509     0.0635     0.6966        2.81R

R denotes an observation with a large standardized residual
```

The above regression output clearly shows that $\hat{\mathbf{y}} = -7.28341 + 0.64936\,x$ is an excellent model because $\mathbf{R^2_{Model}} = 91\%$ so that $\ln(C) = -7.28341 \rightarrow C = 0.00068684$ and $E_a = 0.649358$eV. To extrapolate the expected life to the operating temperature of 180°C = 453.15 K, we insert $x_O = 100000/(8.6171\times453.15) = 25.609251$ into our regression model $\rightarrow \hat{y}_O = -7.28341 + 0.649358x_O = 9.34617 \rightarrow mttf_o = e^{9.34617} = 11459.8130$ hours. I will next convert the above regression model $\hat{\mathbf{y}} = -7.28341 + 0.64936x$, $\ln(TTF) = \ln(C) + E_a/(k\,T)$ to the Arrhenius format:

$$mttf_o = 0.00068684\,e^{0.64936\times10^5/(8.6171T)}, \tag{105}$$

where $0.00068684 = e^C = e^{-7.28341}$ and the temperature T must be measured in Kelvin. Inserting $T_o = 180 + 273.15 = 453.15$ into equation (105) again yields $mttf_O$ (180°C) = 11459.8130 hours. The acceleration factors from 180 to 190°C is $A_f = 11459.8130/8782.20 = 1.3043$. In practice, I would use only the stressed-Temperature that is closest to T_O to compute the AF. Further, I attempted to improve the above model by adding the regressors x^2 and x^3 to the model, unfortunately the $\mathbf{R^2_{Model}}$ improved by a minute amount to 92.3% but all the coefficients in the model became highly insignificant (i.e., a worthless model).

Exercise 26. Boris Gnedenko *et al* mention on their page 172 that the failure data at 260 Celsius in the above experiment looks very suspicious because it exhibits much higher sample *cv* than the other 3 accelerated temperatures. That is to say, the failure mechanism at 260 °C is different from failure modes at lower temperatures. Repeat my analysis of the above experiment but remove the data at 260 C. **ANS:** $mttf_o$ **(180 °C) > 12000 hours.**

(2) The Inverse Power Law (IPL)

This law is generally used when the TTF is inversely proportional to the applied (accelerated) stress, and the underlying lifetime distribution is almost always Weibull, or perhaps lognormal. As in the case of Arrhenius model, the IPL model is applicable only when there is a single type of stress, which in most cases is voltage accelerated stress, alternating temperature stress, or mechanical vibration in order to induce fatigue failure. The general form of the IPL is given by

$$\mathbf{TTF_s = C/ S^b} \qquad \textbf{(106a)}$$

where C > 0 and the exponent b > 0 are constants characteristics of the items under test, and S is the applied (accelerated) stress. The value of the exponent b = [2, 3] for metals and electronic solder joints, b = [4, 10] for microelectronic parts, and b = [4, 7] for intermetallic fatigue failures.

<u>Example 22 (borrowed from L. W. Condra, RE Improvement with DOE, pp 236-237, Marcel Dekker, ISBN: 0-8247-0527-0).</u> A sample of n electronic solder joints are placed on accelerated fatigue-testing at a displacement of S = 0.0008 inches with a $MTTf_s$ = 10 cycles. Assuming that under normal use the maximum displacement is S_o = 0.00005 inches and the exponent b = 2.50, our objective is to estimate $MTTF_o$. We need to compute the value of the A_f $= mttf_o/ MTTf_s = \dfrac{C/ S_o^b}{C/ S^b} = S^b/ S_o^b = (0.0008/0.00005)^{2.5} = 1024 \rightarrow mttf_o = 1024\times 10 =$ 10,240 cycles to failure.

Elsayed provides another form of IPL given in his equation (6.67) which is modified as

$$\mathbf{TTF_s = C'(V_o / V_s)^b} \qquad \textbf{(106b)}$$

where V_o is the standard specified (voltage) operating stress, V_S is the accelerated voltage stress, and the constant C′ is characteristic of the product under test , fabrication method, etc.

<u>The Example 6.13 on pages 410-411 of Elsayed</u>. In this experiment two samples of 20 CMOS integrated circuits each are put on accelerated life test, where V_S represents accelerated electric field stresses at 10 and 25 eV. The underlying distribution of TTF is assumed Weibull and there is only one stress factor, namely electric field, and hence the IPL is a plausible model for TTF_S. The normal operating stress is at $V_o = 5$ eV. For your convenience I have duplicated Table 6.7 of E. A. Elsayed on his page 410 below. I first used the data under the two accelerated stress levels, $V_1 = 10$ and $V_2 = 25$ eV, to obtain the MLEs of the Weibull parameters β and θ. Using my equations 111 (a &b) the MLEs are $\hat{\beta}_{1S} = 1.836028$, $\hat{\theta}_{1S} =$ 9343.5856011 hours, and at $V_2 = 25$ eV, $\hat{\beta}_{2S} = 1.981834234$, $\hat{\theta}_{2S} = 3916.9661061541$ hours. These MLEs under stressed conditions are consistent with those of Elsayed's. It seems that if the stressed data set is W(0, θ, β), then a rough value of the Weibull slope is close to $\beta \cong$ 1.910. However, it is not clear what the estimate of the characteristic life at normal operating stress 5 eV is, because $\hat{\theta}_{1S} = 9343.585601$ hours and $\hat{\theta}_{2S} = 3916.9661061541$ hours were obtained under accelerated testing conditions. I will now obtain the least-squares estimate of θ.

Table 6.7 of Elsayed page 410 (TTF under accelerated testing condition)

10 eV	1037.39 hours, 3218.11, 3407.17, 3520.36, 3879.49, 3946.45, 6635.54, 6941.07, 7849.78, 8452.49
10 eV	9003.08, 9124.50, 9365.93, 9642.53, 10429.50, 10470.60, 11162.90, 12204.50, 12476.90, 23198.30
25 eV	809.10, 1135.93, 1151.03, 1156.17, 1796.53, 1961.23, 2366.54, 2916.91, 3013.68, 3038.61
25 eV	3802.88, 3944.15, 4095.62, 4144.03, 4305.32, 4630.58, 4720.63, 6265.99, 6916.16, 7113.82 hours

In order to obtain a LS estimate of θ, I first linearized the IPL model, $TTF_s = C/S^b$, by taking the natural logarithm of both sides. This leads to $y = \ln(TTF_s) = \ln(C) - b\ln(S)$, where S is at 2 levels, 10 and 25 eV. I used Minitab to regress $y = \ln(TTF_s)$ on $x = \ln(S)$, with the following output.

```
The regression equation is
y = 11.0 - 0.941 x,  x = ln(S)

Predictor       Coef      SE Coef        T        P
Constant     11.0065       0.6383     17.24    0.000
x            -0.9406       0.2281     -4.12    0.000

S = 0.6609      R-Sq = 30.9%     R-Sq(adj) = 29.1%

Analysis of Variance

Source             DF        SS          MS          F        P
Regression          1      7.4281      7.4281      17.01    0.000
Residual Error     38     16.5968      0.4368
Total              39     24.0248
```

```
Unusual Observations
Obs      x       y        Fit      SE Fit    Residual    St Resid
1       2.30    6.979    8.841    0.148      -1.862       -2.89R
```

I must caution the reader before using the above Minitab model for extrapolation! You must observe that the value of $\mathbf{R}^2_{\mathbf{Model}}$ = 30.9% is woefully too small to be an adequate model due to the fact that there is too much within (or experimental error) variability in the data. The data under level 1 of stress (10 eV) ranges from 1037.39 stressed hours to 23198.30 hours, which is very large, but still the regression is highly significant. The above model cannot be improved because there are only 2 levels of stress factor and hence regression can have only one df and any attempt to improve it by adding regressors such as x^2, x^3 and 1/x to the model will be futile because the design does not provide but one df for studying effects. Hence, we have to extrapolate with a model whose $\mathbf{R}^2_{\mathbf{Model}}$ = 30.9%. The estimate of the constant $\hat{C} = e^{11.0065} =$ 60264.591, and the estimate of the exponent $\hat{b} = 0.9406$ is very close to Elsayed's answer of 0.95318. To estimate the $MTTF_o$ at 5 eV, we insert x = ln(5) =1.609438 into the regression model. This yields $\hat{\mathbf{y}}(1.60944) = 11 - 0.9406\times1.60944 = 9.49266 \rightarrow mttf_{\mathbf{o}} = e^{9.49266} =$ 13262.06124 hours, which is fairly close to Elsayed's answer of 12140. Since the Weibull mean $E(T) = \theta\ \Gamma(1+\frac{1}{\beta})$, then $\hat{\theta}_o = 13262.06124 / \Gamma(1+\frac{1}{1.910}) = 14947.92193$ hours. The two acceleration factors are $A_{f1} = 13262.06124 / 8298.3295 = 1.59816036$, where 8298.3295 = *mttf(at 10 eV)* and $A_{f2} = 13262.06124 / 3464.2455 = 3.82827$.

(3) The Eyring Model

Both the Arrhenius and IPL models include only the effect of one accelerated stress. The Eyring model contains two stress factors, one of which is always temperature stress, and the other can be any stress type such as electric field, voltage, humidity, mechanical stress (load per area), even temperature cycling, or electrical current stress. The rate of reaction (or rate of failure) to the two stresses is given by

$$r = C_1\, \mathbf{e}^{-\mathbf{E_a}/(\mathbf{k\,T})} \times S^b \tag{107a}$$

where r is the rate of reaction to the two stresses; r may be thought of the parameter λ if the underlying distribution is exponential, but r $\cong 1/\theta$ if the TTF is W(0, θ, β), and if TTF is lognormal, then r = $1/T_{0.50}$ (the inverse of median life). Thus, from (107a) we deduce that

$$TTF_s = C\, e^{E_a/(kT_s)} / S^b = C\, e^{E_a/(kT_s)} \times S^{-b} \tag{107b}$$

The values of E_a and exponent b can be obtained empirically once accelerated data are available. For electronic applications, b $\cong$ 2 to 3 and E_a = 0.90 eV, and C is a constant characteristic of the product and testing conditions. Equation (107b) implies that under normal operating conditions the TTF is given by

$$TTF_o = C\, e^{E_a/(kT_o)} / S_o^b \tag{107c}$$

Combining equations (128 b&c) yields

$$A_f = \frac{TTF_o}{TTF_s} = \frac{e^{E_a/(kT_o)} / S_o^b}{e^{E_a/(kT_s)} / S^b} = (S/S_o)^b\, e^{(E_a/k)(1/T_o - 1/T_s)} \tag{108}$$

Note that A_f must be directly proportional to E_a because larger activation energy required to induce failure in the test unit generally implies longer $MTTF_o$.

<u>Example 18.</u> L. W. Condra (RE Improvement with DOE, 2nd edition, Marcel Dekker) reports (on his p. 239) the results of an accelerated life testing experiment of n (unspecified) microelectronic circuits conducted at the standard accelerated temperature stress of 85 °C and a standard accelerated relative humidity (RH) of S = 85%. (He refers to this type of accelerated testing as Temperature-Humidity Operating Bias test.) The sample $MTTf_s$ is reported to be 800 hours and normal operating conditions are T_o = 40 °C and RH_o = 60%. The Model (107b) when the 2nd stress represents S = RH (relative humidity) is referred to as Peck's relationship. Peck, D. S. (1986) *"Proc. International RE Physics Symposium, 24, pp. 44-45,* reports an exponent value of b $\cong$ 2.70 and an activation energy of E_a = 0.79 eV, but Condra in his example uses the rough values of b = 3 and E_a = 0.90 eV. I will use Peck's values in equation (129) to estimate the acceleration factor A_f.

$$A_f = (85/60)^{2.7} e^{(0.79\times10^5/8.6171)(1/313.16 - 1/358.16)} = 101.3547$$

which yields $mttf_O$ = 101.3547 ×800 hours = 81083.74783 hours = 9.2561356 years. The above estimated value of $mttf_O$ = 9.2561356 years does not conform well with that of Condra's 16.6 years. If we use Condra's values of b = 3 and E_a = 0.90 eV in equation (108), we obtain A_f = 187.780224 and an estimated $mttf_O$ = 187.780224 × 800 = 150224.179364 hours = 17.148879 years. I tried to obtain Condra's answer of A_f = 182 by using his values of T_o = 313 and T_s = 358 in equation (108) but I still got an answer of A_f = 188.5450005 which is not equal to Condra's answer of 182. The reader should be careful about interpreting the values of $mttf_O$ because if the underlying distribution is exponential, then $mttf_O$ is an estimate of MTTF; if the underlying distribution is Weibull, then $mttf_O$ is an estimate of the characteristic life $t_c = \theta$, and if the underlying distribution is lognormal, then $mttf_O$ is an estimate of the median life. Furthermore, the farther the operating conditions are from the stressed conditions, the less accurate the regression estimates of b and E_a become. This problem gets compounded when the baseline distribution is very highly skewed and /or there are outliers in the data.

Example 6.14 on page 412 of Elsayed. The data listed in Table 6.8 on page 412 of Elsayed presents the results of an ALT with 8 FLCs (factor level combinations) of Temperature and Voltage stresses. For your convenience, I am providing Elsayed's data below. The normal operating temperature T_o = 30 °C = 303.16 Kelvin and the operating voltage is V_o = 25 volts. Instead of using Elsayed's parametric approach to estimate $MTTF_o$, I linearized the Eyring Model (107b) as follows: $y = \ln(TTF_s) = \ln(C) - b\ln(V_s) + E_a x$, where $x = 100000/(8.6171\, T_s)$, and then I regressed y on $\ln(V_s)$ and x. The Minitab output is given below

```
The regression equation is
y = 2.25333 - 0.426737 LV + 0.199714 x,  LV = ln(Vs), x = 100000/(8.6171Ts),

Predictor        Coef      SE Coef         T        P
Constant        2.253        1.405      1.60    0.170
LV           -0.42674      0.03955    -10.79    0.000
x             0.19973      0.04057      4.92    0.004

S = 0.05824     R-Sq = 96.6%     R-Sq(adj) = 95.2%

Analysis of Variance

Source            DF         SS          MS          F        P
Regression         2    0.47708     0.23854      70.32    0.000
```

```
Residual Error     5     0.01696     0.00339
Total              7     0.49404

Source       DF      Seq SS
LV            1     0.39488
x             1     0.08220
```

Table 6.8 of E. A. Elsayed (p. 412)

Voltage / Temperature	50	100	150	200 volts
60 °C	**1800 hours**	**1500**	**1200**	**1000**
70 °C	**1500**	**1200**	**1000**	**800 hours**

In the above Minitab output, LV = ln(V_s) and y = ln(TTF_s). Note that the value of E_a from the above regression output is E_a = 0.199714 which is in agreement with the reported value by Elsayed on his page 412. In order to estimate the $MTTF_o$, I used extrapolation (which is generally not a good idea in regression analysis) in the above regression model, which has an excellent $\mathbf{R^2_{Model}}$, as follows: $\hat{y}(30\,^{\text{o}}\text{C}, 25\,\text{volts}) = 2.25333 - 0.426737\ln(25) + 0.199714 \times 100000/(8.6171\times303.15) = 8.52494 \rightarrow mttf_O = e^{8.52494} = 5038.8636$ hours, which is a bit larger than L_o = 4484.11 hours reported by Elsayed. There will be 8 different values of A_f because there are 8 FLCs of the two stresses. The value of A_f from normal operating conditions (30°C, 25 volts) to stress FLC (60 °C, 50 volts) is $\hat{A}_f = 5038.8636/1800 = 2.79937$. To verify the adequacy of the Eyring model to the data, we also need to estimate this last acceleration factor A_f from equation (108) as follows: $\hat{\mathbf{A}}_{\mathbf{f}}\,(\text{Model}) = (\frac{50}{25})^{0.426737} \times e^{(19971.4/8.6171)(1/303.15-1/333.15)}$ = 2.6758, which is fairly consistent with the regression-value of 2.7994.

<u>Example 6.7 of Elsayed on pages 388-389.</u> This experiment makes no assumptions about the underlying distributions of Times TF (i.e., the nonparametric) and uses regression to estimate the MTTF by extrapolation. I used the data in Table 6.1 of Elsayed on his page 388 to regress the TTF on stress factor Temperature in Kelvin, and stress factor electric field measured in units of eV. For your convenience, I am duplicating Elsayed's Table 6.1 on the next page.

The resulting Minitab output is given below:

```
Regression Equation

TTFs  =  6061.97 - 17.8487 T + 160.159 eV

Coefficients
Term         Coef  SE Coef        T      P
Constant  6061.97  3.55841  1703.56  0.000
T          -17.85  0.01343 -1329.01  0.000
eV         160.16  0.22473   712.68  0.000
```

Table 6.1 (on page 388 of Elsayed)

Temperature C°	**100**	**100**	**100**	**100**	**100**	**100**
Electric Field (eV)	**10**	**10**	**10**	**10**	**10**	**10**
Stressed TTF (TTF_s)	**1000 hours**	**1002**	**1003**	**1004**	**1005**	**1006 hours**

Temperature C°	**150**	**150**	**150**	**150**	**150**	**150**
Electric Field (eV)	**10**	**10**	**10**	**10**	**10**	**10**
Stressed TTF (TTF_s)	**110**	**110.5**	**110.7**	**111**	**111.4**	**111.8 hours**

Temperature C°	**200**	**200**	**200**	**200**	**200**	**200**	**200**	**200**	**200**	**200**
Electric Field (eV)	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Stressed TTF (TTF_s)	**19**	**19**	**19.1**	**19.2**	**19.3**	**19.32**	**19.38**	**19.4**	**19.44**	**19.49**

```
Summary of Model

S = 1.16308      R-Sq = 100.00%        R-Sq(adj) = 100.00%
PRESS = 36.9518  R-Sq(pred) = 100.00%

Analysis of Variance

Source      DF   Seq SS   Adj SS   Adj MS        F  P
Regression   2  3967238  3967238  1983619  1466367  0
  T          1  3280158  2389312  2389312  1766270  0
  eV         1   687080   687080   687080   507916  0
Error       19       26       26        1
Total       21  3967264

Fits and Diagnostics for Unusual Observations

Obs  TTFs      Fit    SE Fit  Residual  St Resid
 17  1000  1003.33  0.474824  -3.33333  -3.13951  R
```

```
 22  1006  1003.33  0.474824   2.66667   2.51161  R

R denotes an observation with a large standardized residual.


Predicted Values for New Observations

New Obs      Fit    SE Fit        95% CI              95% PI
      1   1541.19  0.836013  (1539.44, 1542.94)  (1538.19, 1544.18)

Values of Predictors for New Observations

New Obs       T  eV
 1  298.15    5  X

X denotes a point that is an outlier in the predictors.
```

General Regression Analysis: TTFs versus TempC, eV ; using the centigrade data

```
Regression Equation

TTFs  =  1186.61 - 17.8487 TempC + 160.159 eV

Coefficients

Term         Coef  SE Coef         T      P
Constant  1186.61  1.29166    918.67  0.000
TempC      -17.85  0.01343  -1329.01  0.000
eV         160.16  0.22473    712.68  0.000

Summary of Model

S = 1.16308      R-Sq = 100.00%        R-Sq(adj) = 100.00%
PRESS = 36.9518  R-Sq(pred) = 100.00%


Analysis of Variance

Source      DF   Seq SS   Adj SS   Adj MS        F  P
Regression   2  3967238  3967238  1983619  1466367  0
  TempC      1  3280158  2389312  2389312  1766270  0
  eV         1   687080   687080   687080   507916  0
Error       19       26       26        1
Total       21  3967264

Fits and Diagnostics for Unusual Observations

Obs  TTFs      Fit    SE Fit  Residual  St Resid
 17  1000  1003.33  0.474824  -3.33333  -3.13951  R
 22  1006  1003.33  0.474824   2.66667   2.51161  R

R denotes an observation with a large standardized residual.


Predicted Values for New Observations

New Obs      Fit    SE Fit        95% CI              95% PI
      1  1541.19  0.836013  (1539.44, 1542.94)  (1538.19, 1544.18)


Values of Predictors for New Observations

New Obs  TempC  eV
```

```
    1     25    5  X

X denotes a point that is an outlier in the predictors.
```

General Regression Analysis: LnTTF versus x, LV

```
Regression Equation

LnTTF  =  -11.6518 + 0.599347 x - 0.0332739 LV


Coefficients

Term          Coef    SE Coef         T      P
Constant  -11.6518  0.0659043  -176.798  0.000
x           0.5993  0.0011245   532.968  0.000
LV         -0.0333  0.0151534    -2.196  0.041


Summary of Model

S = 0.00715762      R-Sq = 100.00%        R-Sq(adj) = 100.00%
PRESS = 0.00124123  R-Sq(pred) = 100.00%


Analysis of Variance

Source      DF   Seq SS   Adj SS   Adj MS       F          P
Regression   2  58.9100  58.9100  29.4550  574939  0.0000000
  x          1  58.9098  14.5526  14.5526  284055  0.0000000
  LV         1   0.0002   0.0002   0.0002       5  0.0407246
Error       19   0.0010   0.0010   0.0001
Total       21  58.9110


Fits and Diagnostics for Unusual Observations

Obs    LnTTF      Fit     SE Fit    Residual  St Resid
  1  2.94444  2.95815  0.0022634  -0.0137082  -2.01878  R
  2  2.94444  2.95815  0.0022634  -0.0137082  -2.01878  R

R denotes an observation with a large standardized residual.
```

In order to estimate the $MTTF_o$ from the above regression models at the normal operating temperature T_o = 25°C = 298.15 Kelvin, and 5 eV, we insert these values into the first model as follows: $mttf_O$ = 17.8487×298.15 + 160.159×5 = 1541.19 hours which is consistent with that of Elsayed's answer. The value of the acceleration factor from normal operating conditions (25°C, 5 eV) to stress levels (100°C, 10 eV) is equal to $\hat{A}_{f1}$ = 1541.19/ $mttf(100^o C, 10\,eV)$ = 1541.19/1003.33333 = 1.5331. Equation (108) gives $\hat{A}_{f1}$ (Model) =

$(\frac{10}{5})^{0.0332739} e^{(59937.7/8.6171)(1/298.15 - 1/373.15)}$ = 2.367, which is not consistent with $\hat{A}_{f1}$ = 1.533, which may warrant the rejection of the Eyring model.

Chapter Summary

1. **The acceleration factor For the Arrhenius Model is given by A_f =**

$e^{E_a(\frac{1}{T_O} - \frac{1}{T_S})/k}$ → $MTTF_o$ = A_f ×$MTTF_S$, where k = Boltzman's constant = 8.6171×10^{-5}. Two cases exit: (a) The required activation energy E_a to induce failure is known, (b) E_a is not known and has to be empirically estimated from accelerated data. For Semiconductor failure $0.30 \le E_a \le 0.60$; for intermetallic diffusion $0.90 \le E_a \le 1.10$; For silicon junction defects $E_a = 0.80$.

(a) Assume E_a = 0.50 and normal operating temperature is 25 °C and accelerated testing is done at 50 °C. Then T_o = 25 + 273.15 = 298.16K and T_S = 50 + 273.15 = 323.1600K →

$$A_f = e^{0.50(\frac{1}{298.15} - \frac{1}{323.15})10^5/8.6171} = 4.506862$$

Note that A_f is an increasing function of E_a because larger values of E_a imply that more energy is required to induce failure which in turn would lead to higher $MTTF_O$. Note that some sources use the conversion Kelvin = °C + 273.15 and others use Kelvin = °C + 273.16.

(b) E_a is unknown.

Identify at least two stressed temperature levels, such as 50°C and 75°C (< $0.50T_m$) and obtain stressed failure data. Linearize the Arrhenius model TTF_S = $c\,e^{E_a/(kT_s)}$ and regress $\ln(TTF_S)$ on $x = 10^5/(8.6171T)$; then the rough estimate of E_a is given by the slope of the regression line. However, one must be cognizant of the fact that extrapolation is classical regression violates regression assumptions and is generally frowned upon. But than when there are no information about E_a (physical or otherwise), then the regression approach would be the only way to obtain a statistically unsound manner of obtaining a rough estimate of the activation energy E_a.

2. The IPL : $TTF_s = C/S^b$ $\rightarrow$ **Larger values of b induce higher failure rate reaction and smaller TTF. The value of b = [2, 3] for metals and electronic solder joints, b = [4, 7] for intermetallic fatigue failure, and b = [4, 10] for microelectronic parts, and very rarely b lies outside the range [2, 20].**

(a) b is known $\rightarrow A_f = S^b/S_o^b$. **For example, suppose the normal operating voltage is** $S_o = 110V$, **stressed voltage is** $S = 220$ **and** $b = 2.8$. **Then** $A_f = (220/110)^{2.8} = 6.9644$.

(b) b is unknown. First linearize $TTF_s = C/S^b \rightarrow$

$y = \ln(TTF_s)$, $x = \ln(S)$, **y-intercept=** $\ln(C)$, **and** $\hat{b} = -$ **slope of the LS line.**

3. The Eyring Model : $TTF_s = C\, e^{E_a/(kT_s)} / S^b = C\, e^{E_a/(kT_s)} \times S^{-b}$

$$A_f = (S/S_o)^b\, e^{(E_a/k)(1/T_o - 1/T_s)}$$

$$=(S/S_o)^b \times e^{(10^5 E_a/8.6171)(1/T_o - 1/T_s)}$$

(a) Both E_a and b are known.

(b) At least one is unknown. Use stressed data to extrapolate to estimate b and E_a. Note that this extrapolation often does not provide adequate and /or reasonable estimates of E_a and b, which implies that the Eyring model does not fit the data, and/or regression assumptions are grossly violated. Further, extrapolation is always on poor statistical ground and is used in accelerated testing because there are no other options, i.e., the constants b and E_a are unknown and testing under normal operating conditions involves well over thousands of hours.