INTRODUCTION TO TAGUCHI-BASED QUALITY DESIGN AND IMPROVEMENT

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Introduction

TAGUCHI'S CONTRIBUTIONS

1. Redefined quality using the quadratic loss function.
2. Introduced parameter design and tolerance design.
3. Introduced the Signal-to-Noise concept as an analytical tool.

PARAMETER DESIGN OBJECTIVES

1. Identify process parameters (or factors)

$$x_1, x_2, \ldots, x_k$$

that significantly affect a product's performance characteristic y :

$$y = f\,(x_1, x_2, \ldots, x_k; \xi_1, \xi_2, \ldots, \xi_\ell)$$

The inputs $x_1, x_2, \ldots, x_k$ are controllable factors and $\xi_1, \xi_2, \ldots, \xi_\ell$ are noise (or uncontrollable factors).

2. Optimize the quality characteristic y by selecting the factor level combination

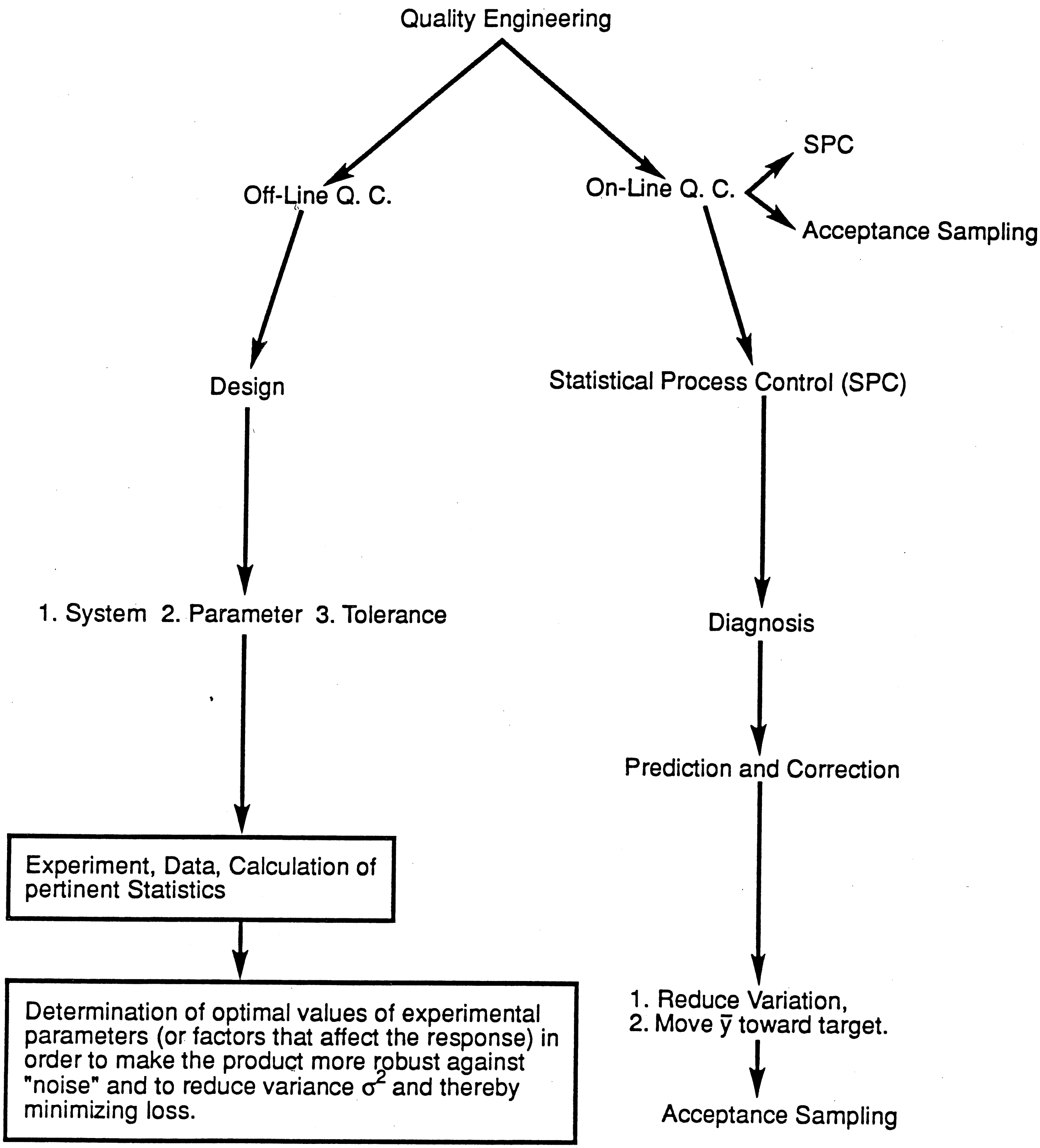
$$x_o = [x_1^*, x_2^*, \ldots, x_k^*]$$

that makes the output y insensitive (or robust) to noise factors $\xi_1, \xi_2, \ldots, \xi_\ell$. That is, the optimal process conditions are selected such that

$$y^* = f(x_1^*, x_2^*, \ldots, x_k^*, \xi_1, \xi_2, \ldots, \xi_\ell)$$

is least sensitive to noise.

Parameter design is the key step in Taguchi method to achieve high quality without much increase in cost.


Quality Engineering
Off-Line Q. C.
On-Line Q. C.
SPC
Acceptance Sampling
Design
Statistical Process Control (SPC)
1. System 2. Parameter 3. Tolerance
Diagnosis
Prediction and Correction
Experiment, Data, Calculation of pertinent Statistics
Determination of optimal values of experimental parameters (or factors that affect the response) in order to make the product more robust against "noise" and to reduce variance σ^2 and thereby minimizing loss.
1. Reduce Variation,
2. Move $\bar{y}$ toward target.
Acceptance Sampling

CHAPTER I

1. TYPES OF STATIC QUALITY CHARACTERISTICS

<u>Definition.</u> Quality is the loss(es) a product imparts to society from the time the product is shipped, according to Taguchi. Types of losses may be (a) monetary loss to the consumer, (b) dissatisfaction due to the fact that the product is imperfect, (c) time loss, and (d) hazards to the environment.

A static quality characteristic (QCH) can be measured either in the form of attributes (good/bad, defective/effective, success/failure, grade A/B/C/, etc), or by variables. Quality Characteristics by variables are continuous and further divided into three types: (1) Smaller the Better (STB), (2) Larger The Better (LTB) or Bigger the Better (BTB), and (3) Nominal The Best (NTB). These three QCH types are also referred to as type S, type B, and type N, respectively.

Any measurable dimension y with only an upper specification limit (USL), denoted by y_u, is an STB QCH. Examples of the STB QCH are rate of wear, shrinkage or warpage, deterioration, noise level of an engine, tire imbalance, and waiting time in a queue.

Any dimension y with only a lower specification limit (LSL), denoted by y_ℓ, is an LTB QCH. Examples of the LTB QCH are tensile or welding strength, life length, fuel efficiency, tape adhesiveness, and percent yield.

Any dimension, y, with both LSL and USL is an NTB QCH. Examples of the NTB type QCH are clearance, output voltage of a TV set, tire tread length, tire sidewall width, magnetic tape edge weave, and chemical content level.

2. QUALITY LOSS FUNCTION FOR ONE UNIT OF A PRODUCT

Consider a dimension, y, which is an NTB type QCH. The nominal (or target) value is denoted by m where generally m=(LSL+USL)/2. QCH's where the LSL and USL are not symmetrically specified relative to the target value m can occur, but here discussion is limited to the symmetrical case. The Taylor expansion of the loss function, L(y), about m is

$$L(y) = k_0 + k_1(y-m) + k_2(y-m)^2 + k_3(y-m)^3 + \ldots = \sum_{i=0}^{\infty} k_i(y-m)^i \qquad (1)$$

Clearly, $k_0 = L(m)$, and assuming the target is set such that the loss is minimum when y = m, it follows that

$$\left.\frac{dL(y)}{dy}\right|_{y=m} = L'(m) = 0 \quad . \qquad (2)$$

Equations (1) and (2) result in $k_1 = 0$. To determine k_2, take the 2nd derivative of L(y) in (1) with respect to y:

$$\frac{d^2L}{dy^2} = 2k_2 + 6\, k_3(y-m) + 12k_4(y-m)^2 + \ldots$$

$$k_2 = \frac{1}{2!}\left.\frac{d^2L}{dy^2}\right|_{y=m} = L''(m)/2! \ .$$

It is simple to show that in general

$$k_j = \frac{1}{j!}\left.\frac{d^jL}{dy^j}\right|_{y=m} \ , \ j = 2,3,4,\ldots \qquad (3)$$

From the above discussion, the loss function in (1) reduces to:

$$L(y) = L(m) + k_2(y-m)^2 + k_3(y-m)^3 + k_4(y-m)^4 + \ldots \qquad (4)$$

$$\text{where } k_j = \frac{1}{j!}\left[d^jL/dy^j\right]_m \ , j = 2,3,4,\ldots \ .$$

Therefore, the possible approximations for the loss, beyond what is incurred at m, are

$$L(y) \doteq k_2 \ (y-m)^2 \quad , \qquad (5)$$

$$L(y) \doteq k_2 \ (y-m)^2 + k_3 \ (y-m)^3 \quad , \qquad (6)$$

or

$$L(y) = \sum_{j=2}^{4} k_j \ (y-m)^j \qquad \text{per unit.} \qquad (7)$$

Note that equation (7) incorporates the effects of the variance, skewness and kurtosis into the quality loss function (QLF). Better approximations may be obtained by including the 5th, 6th, etc, central moments, but this is unnecessary since moments beyond the 4th have little practical statistical meaning. The quadratic portion of (4), $L(y) = k\ (y-m)^2$, given in (5), is selected to represent the QLF, i.e., the society's loss function is

$$L(y) = k(y-m)^2 \quad . \qquad (8)$$

The constant k in eq. (8) is generally determined using the average consumer's tolerance points LSL = m - Δ and USL = m + Δ, where Δ is called the allowance. The reason for the use of these points is the fact that there is generally more information available about the loss incurred at these points than at other points on the y scale. Given that $L(y) = \$A_c$ per unit when $y = m \pm \Delta$, we get by substitution into (8):

$$\$\ A_c = k(m \pm \Delta - m)^2 = k\ \Delta^2$$

or

$$k = A_c/\Delta^2 \qquad (9)$$

Example 1. A power supply circuit for a color T.V. set is designed to attain an output of 115 DC volts. The typical consumer's tolerance is 115 ± 20 DC volts. The repair cost to the consumer or the warranty cost to the producer is \$100.00; the time loss to the consumer is \$35.00; and the producer's loss of market share is \$15.00. Therefore, the value of L(y) at LSL = 95 or USL = 135 is A_c = \$(100.00 + 35.00 + 15.00) = \$150.00. The QLF is determined by considering the loss incurred when y = 95 or 135 volts.

Let y represent the output in DC volts; then m = 115 volts, Δ = 20, LSL = 95, USL = 135 volts, A_c = \$150.00, and inserting into (8) results in

$$150 = k(y-m)^2 = k(95-115)^2.$$

The QLF constant is therefore k = 0.375, or from formula (9):

$$k = \frac{A_c}{\Delta^2} = \frac{150}{400} = .375 \text{ \$/volt}^2 \text{ per set as before.}$$ Thus

$$L(y) = 0.375\ (y-115)^2.$$

The QLF can then be used to determine the loss incurred when, say, y = 105 volts as follows:

$$L(105) = .375\ (105 - 115)^2 = \$37.5/\text{set}.$$

3. **THE USE OF L(y) IN SETTING PRODUCTION TOLERANCE**

Suppose in the above example the output voltage can be adjusted toward the target by changing a resistor at the end of the production line at a cost of \$3.00. For what values of y is it financially justified to spend the \$3.00?

<u>Solution</u> $3.00 = L(y) = 0.375\ (y-115)^2$. Solving for y gives

$$y = 115 \pm 2.83 \text{ volts.}$$

Therefore, the voltage should be adjusted only if a TV set's output voltage is outside the range (112.17,117.83). The quantity 2.83 is called the production or factory allowance and is denoted by $\Delta_f = 2.83$.

<u>Exercise 1</u>. Prove that in general

$$\Delta_f = \Delta_c \sqrt{A_f/A_c} = \sqrt{A_f/k}, \qquad (10)$$

where $\Delta_c = \Delta$ = consumer's tolerance.

Note that for the Example 1, $\Delta_c = 20$, $A_f = 3.00$ and $A_c = \$150$ and thus

$$\Delta_f = 20\sqrt{3/150} = 2.83, \text{ or } \Delta_f = \sqrt{3/.375} = \sqrt{8} = 2.83 \text{ as before.}$$

Exercise 2. (Borrowed from reference [9]) A company manufacturers vinyl sheets for building agricultural houses. The specifications for the thickness, y, is 1.00 ± 0.20 (mm) with $A_c = \$100.00$ per sheet. (a) Determine the loss function L(y). (b) For what values of y will the consumer's loss be less than \$49 per sheet? (c) If it costs \$5.00 to calibrate the thickness toward the target, for what values of y should we go ahead with the expenditure?

Answers: (b) (.86, 1.14), (c) $\Delta_f = .04472$.

4. THE LOSS FUNCTION FOR AN STB QUALITY CHARACTERISTIC

Since the target value is generally zero, we can insert m = 0 in (8) resulting in

$$L(y) = ky^2 \quad \text{(for one unit).} \tag{11}$$

Given that at the USL = y_u, the amount of loss is $\$A_c$, we get (See Figure 1)

$$k = A_c/y_u^2 \quad . \tag{12}$$

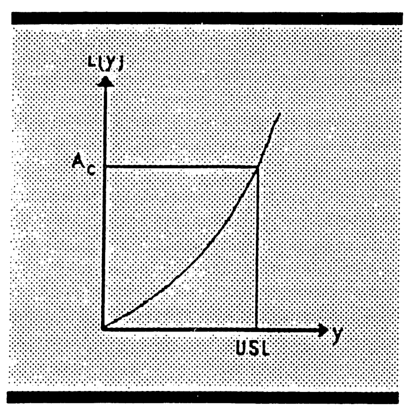


Figure 1

Exercise 3. The USL for radial force harmonic (RFH) of certain tire brands is $y_u = 26$ lbs. Given $A_c = \$10.00$, (a) determine the QLF per unit. (b) If it costs \$2.00 to reduce the radial force, for what values of y should the \$2.00

be spent? ANS: k = .0148, (b) y > 11.63.

5. THE LOSS FUNCTION FOR AN LTB QUALITY CHARACTERISTIC

Since the nominal (or target) value is $m = \infty$, then larger values of y must yield smaller quality losses, i.e., L(y) must be inversely proportional to y^2. Further, 1/y is an STB type QCH with zero target value. Hence

$$L(y) = k\left(\frac{1}{y} - 0\right)^2 = k/y^2, \text{ per unit} \tag{13}$$

Given that $L(y) = \$A_c$ at $y_\ell = LSL$, then substitution into (13) results in

$$k = A_c y_\ell^2. \tag{14}$$

Exercise 4. The lower specification limit for tensile strength of a product is 17000 psi. Given that $A_c = \$25.00$ at the LSL, (a) determine the loss function and graph the loss versus y. (b) If the strength of one unit can be increased at an additional cost of \$4.00 during production, below what value of y should such an expenditure be undertaken?

ANS: (a) $k = 7225 \times 10^6$, (b) y < 42500 psi.

6. SUMMARY

Since its inception the quality control (QC) literature has been developed on the basis of the following traditional QLF (based on nonconformance to specifications)

$$(15) \qquad L_s(y) = \begin{cases} 0, & LSL \le y \le USL \\ A_c, & \text{otherwise} \end{cases}$$

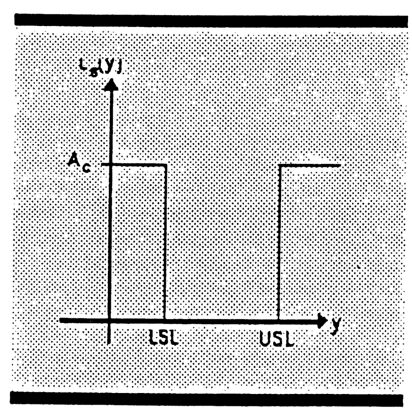


Figure 2.

which is depicted by Figure 2. Taguchi's concept of quality is revolutionarily different from the above step function and is expressed by the quadratic loss function given in equation (8). Unlike equation (15), the quadratic loss function implies that the quality loss is zero only when the value of the QCH is exactly equal to the target (or the ideal) value. Further, the loss due to poorer quality increases exponentially (in quadratic fashion) as shown in Figure 3 when the lack of quality, $|y-m|$, becomes larger. The quadratic loss function was developed well over 100 years ago, but Taguchi was the first to recommend its use to evaluate the quality (or lack of it) of a continuous and measurable unit of a product. Therefore, producing parts that just meet specifications is no longer good enough and most likely will cause the manufacturer to lose its competitive edge in the long run. In a later section, we will compare the quality evaluation of a product from both the traditional and Taguchi's points of view.

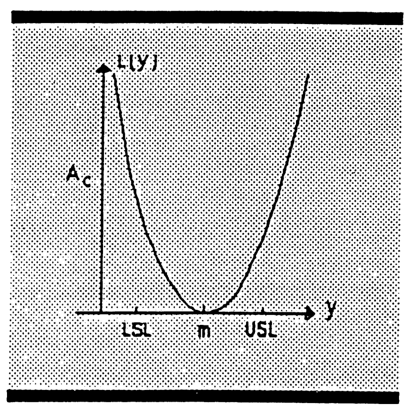


Figure 3.

CHAPTER II

1. THE QLF FOR n > 1 UNITS OF A PRODUCT

So far we have dealt with the QLF only for one unit of a product. Equations (8, 11, 13) give the definitions for L(y) for a random sample of size $n = 1$. If we wish to evaluate the quality of n randomly selected units of a product, then we use the average of the individual losses over the n items, i.e.,

$$\bar{L} = \frac{1}{n}\sum_{i=1}^{n} L_i = \begin{cases} k(\sum_{i=1}^{n} y_i^2/n) & \text{for STB} \quad (16a) \\ k \sum_{i=1}^{n} (y_i - m)^2/n & \text{for NTB} \quad (16b) \\ k \sum_{i=1}^{n} y_i^{-2}/n & \text{for LTB QCH}, \quad (16c) \end{cases}$$

where $k = A_c/y_u^2$ (for STB), $k = A_c/\Delta^2$ (for NTB), and $k = A_c y_\ell^2$ (for LTB) quality characteristics. First, consider L for a NTB performance characteristic, i.e.,

$$\bar{L} = \frac{k}{n} \sum_{i=1}^{n} (y_i - m)^2 \quad .$$

The quantity $\Sigma(y_i-m)2/n$ represents the average of squared deviations from the target for n sample units and is called the mean squared deviation (MSD), i.e.,

$$\begin{aligned} MSD &= \frac{1}{n}\Sigma(y_i-m)^2 = \frac{1}{n}\Sigma[(y_i-\bar{y}) + (\bar{y}-m)]^2 \\ &= \frac{1}{n}\Sigma[(y_i-\bar{y})^2 + 2(\bar{y}-m)(y_i-\bar{y}) + (\bar{y}-m)^2] \\ &= \frac{1}{n}\Sigma(y_i - \bar{y})^2 + (\bar{y}-m)^2 \quad , \end{aligned}$$

where the middle term on the RHS vanishes because $\Sigma(y_i-\bar{y}) = 0$. Therefore, for the NTB case

$$MSD = s_n^2 + (\bar{y}-m)^2 \quad , \tag{17}$$

where $s_n^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i-\bar{y})^2$ is the sample variance and $\bar{y} = \frac{1}{n} \Sigma y_i$. Note that the estimator

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i-\bar{y})^2$$

is generally and loosely referred to as the sample variance, but as Kendall M.G. and Stuart (1962, vol.2, p. 4) indicate, the sample variance is indeed

$$\hat{\sigma}_n^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i-\bar{y})^2 \quad ,$$

where the symbol "^" universally stands for estimate. To be consistent with Taguchi's notation, we will use s_n^2 for $\hat{\sigma}_n^2$. Since there may be a controversy as to which of the two, S^2 or s_n^2, is the better estimate of the process variance σ^2, we will make a complete comparison between the two estimators in Section 4 of this chapter. Substituting eq. (17) into (16) yields

$$\bar{L} = k(MSD) = k[s_n^2 + (\bar{y}-m)^2]. \tag{18}$$

Equation (18) shows that for the NTB case, to reduce quality losses, we must reduce either s_n^2 or $(\bar{y}-m)^2$ (or both). This has to be accomplished as follows:

(1) To reduce variance, we must use off-line QC methods (i.e., secondary or parameter design) to reduce variation about the mean. In a later chapter we will discuss that at least 50% of a manufacturer's gains in quality (per dollar invested) must be attained through statistical experimental design (SED) or Taguchi's parameter design (PDE). If

sufficient reduction in variance cannot be achieved through secondary experimentation, then resort must be made to the use of higher cost tolerance design.

(2) To move the mean toward the target, or reduce $(\bar{y}-m)^2$, we must make use of the signal* factors identified after PDE. This is done by setting the levels of the signal factors in such a manner that the mean is just below the target m [See Box (1988), p.3]. The above two statements imply that SED or PED (whichever is deemed appropriate) is the key component of quality engineering, and failure to seize the opportunity to apply it will certainly result in a manufacturer's loss of competitive edge. In a later chapter, we will illustrate how to use the QLF to measure the amount of quality improvement (QI) after parameter design.

Once sufficient reduction in MSD is attained by the use of experimental design (SED or PDE), then on-line QC methods (specifically SPC) should be applied to monitor process variation and to maintain the mean on target.

Equation (18) implies that if a process is centered (i.e., $\mu = m$), then for an NTB response the expected value of QL is

$$E(L) = k\sigma^2 \qquad (19)$$

so that $E(L) \geq k\sigma^2$, where $k\sigma^2$ represents the quality loss due to process variation. Generally, in a manufacturing process the standard deviation of a QCH is assumed to be no more than $\Delta/3$ or $\frac{1}{6}$ of the tolerance (2Δ), and the distribution of the QCH is assumed normal with mean μ and variance $\sigma^2 \leq \Delta^2/9$. Symbolically, this is

*A signal or adjustment factor is one that affects the mean of the output y but does not affect σ_y^2.

expressed as $y \sim N(\mu, \Delta^2/9)$. Before we describe the relation between quality loss and natural tolerances of a process, we first give a brief review of statistical tolerance limits.

Henceforth $Z \sim N(0,1)$ and $\Phi(Z)$ will represent the cumulative of the standard normal distribution. The values of $\Phi(Z)$ are tablulated on pages F-E and F-F.

2. STATISTICAL TOLERANCE LIMITS

The Case of Known Population Mean and Standard Deviation

For the sake of illustration, suppose the length, y, of certain steel pipes is N(12.00", 0.0064 in^2) with specifications 12 ± 0.20". The distribution of y is depicted in Figure 4.

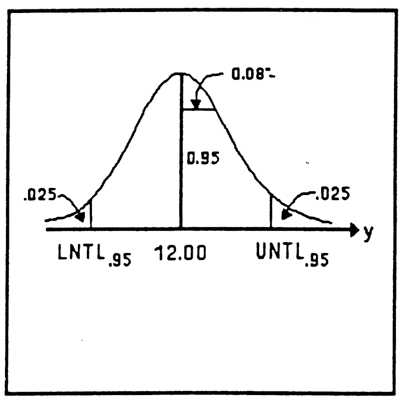


Figure 4

As illustrated by Figure 4, the 95% (i.e., $\alpha = .05$) natural tolerance limits (NTL) are defined such that with certainty 95% of the steel pipes have dimensions within the interval ($LNTL_{.95}$, $UNTL_{.95}$). Therefore, $LNTL_{.95} = 12.00 - Z_{.025}(\sigma) = 12 - 1.96(.08) = 11.8432$, and $UNTL_{.95} = 12.1568$. Therefore, we are 100% sure that 95% of the produced units will have their y values within the tolerance interval (11.8432, 12.1568"). Note that $Z_{.025} = 1.96$ is the 2½ percentage point (or the 97.5 percentile) of a standard normal distribution.

<u>Exercise 2.1</u> For the steel pipe example above, obtain the 99% NTL's. ANS: (11.794, 12.206").

In general, natural tolerances of a normal process are defined such that the corresponding interval contains exactly the proportion $1-\alpha = 0.9973$ of the population. Since $Z_{\alpha/2} = Z_{.00135} = 3$, Figure 5 shows that NTL's of a normal process correspond to $\mu \pm 3\sigma$. Throughout this book, it is implied that LNTL = $LNTL_{.9973}$ and UNTL = $UNTL_{.9973}$

For the steel pipe example, the natural tolerance limits are LNTL = 11.76" and UNTL = 12.24, i.e., we are 100% sure that 99.73% of the pipes have length within the natural tolerance interval (11.76, 12.24""). Since specifications are given by consumers (or determined by a designer) and in this case LSL = 11.80 and USL = 12.20, the process fraction nonconforming to specifications is $p = 2\ \Phi\ (-2.5) = 0.01242$. Thus, in this case, if the amount of tolerable fraction nonconforming is set at $\alpha = .0027$, then the process is unacceptable because p exceeds α. This implies that in general NTL's must lie within specification limits in order for the fraction nonconforming to be tolerable (i.e., $p \leq \alpha$); otherwise, resort must be made to Taguchi's tolerance design to reduce variance, or variation reduction must be achieved thru new technology. Exercise 2.2 below further illustrates the concepts discussed in this section.

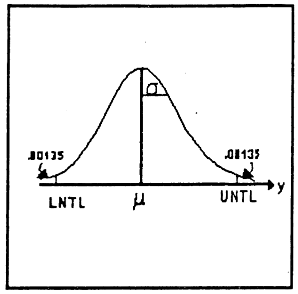


Figure 5

<u>Exercise 2.2</u>. The specifications for tire tread length are 75.00 ± 0.25", and it is assumed that the process is N(75", 0.0081). (a) Given that the tolerable fraction nonconforming is $\alpha = .0027$, determine if the process is capable ~~acceptable~~. (b) If not, how much reduction in σ must be attained in order that the fraction nonconforming is barely acceptable. (c) If the standard deviation is improved to $\sigma = .06$ but the process is not centered with $\mu = 74.93"$, would this yield an capable ~~acceptable~~ process ($\alpha = .0027$)? ANS.: (a) p = .005473, No., (b) $\sigma \leq .08333$, (c) p = .00135, Yes.

It should be clear by now that when NTL's of a process exactly coincide with specifications, then $p = \alpha = .0027$ and USL-LSL=6σ. Assuming that the process is normal and centered, μ=m=(LSL+USL)/2, and tolerable fraction nonconforming is α=.0027, then the process is acceptable only if USL-LSL $\geq 6\sigma$. This leads to the definition of the Process Capability Ratio (usually referred to as the C_p index)

$$PCR = \frac{USL-LSL}{2\sigma Z_{\alpha/2}} \qquad (NTB) \qquad , \qquad (20)$$

which measures the ability of a centered process to conform to specifications. Note that the most prevalent value of $Z_{\alpha/2}$ is equal to 3, which occurs when α is set at .0027. Therefore, a normal process that is centered (i.e., μ=m), is capable of conforming to specifications only if PCR ≥ 1. Most practitioners, to be on the safe side, recommend PCR $\geq 4/3$ due to the possibility that, a process may be nonnormal and/or that its parameters may change "slightly" without making the process unacceptable. If it is known that $\mu \neq m$, then for the NTB case, conformance to specifications is measured by the capability index

$$C_{pk} = \frac{1}{3} Z_{min} = \frac{1}{3} \min \left\{ \frac{\mu - LSL}{\sigma} , \frac{USL - \mu}{\sigma} \right\} \qquad (21)$$

If tolerable fraction nonconforming is $\alpha \neq .0027$, then replace 3 with $Z_{\alpha/2}$ in the denominator of equation (21).

In actual practice, the parameters μ and σ are generally unknown and their sample estimates $\bar{y}$ and S (or s_n) are used in equations (21) and (22). Again values of $C_{pk} \leq 1$ are undesirable, and for STB and LTB performance characteristics, C_{pk} should be used only relative to USL and LSL, respectively.

For magnitude (or size) quality characteristics, conformance to specifications is measured by comparing

$$PCR = \begin{cases} \dfrac{USL - \mu}{3\sigma} , & (STB) , \qquad (22a) \\[2ex] \dfrac{\mu - LSL}{3\sigma} , & (LTB) , \qquad (22b) \end{cases}$$

against 1. As before, values of PCR in (22) not exceeding 1 are undesirable. Note that for equations (22) the implied value of α is .00135 not .0027. If the tolerable fraction nonconforming is set at $\alpha = .0027$, then the denominator of (22) can be as small as 2.7825σ.

3. TOLERANCE LIMITS WHEN μ AND σ ARE UNKNOWN

In practice, the mean μ and standard deviation σ of a manufacturing process are unknown constants and have to be estimated from a random sample of size n by

$$\hat{\mu} = \bar{y} = \frac{1}{n} \Sigma y_i$$

and

$$\hat{\sigma}^2 = S^2 = \frac{1}{n-1} \Sigma(y_i - \bar{y})^2 = \frac{ns_n^2}{n-1} \tag{23}$$
$$= \frac{1}{n-1} \left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right] = \frac{1}{n-1} \left[\Sigma y_i^2 - n(\bar{y})^2\right] .$$

However, the tolerance internal $\bar{y} \pm (1.96\ S)$ no longer has 100% probability (pr.) of containing 95% of the manufactured items since $\bar{y}$ and S are both random variables and change from sample to sample. Fortunately, it is possible to determine a constant K, for a normal universe, such that the interval $\bar{y} \pm K\ S$ contains at least (1-α) proportion of the population with a confidence pr. equal to γ. Table 1, borrowed from Bowker and Lieberman [2], gives the values of K for n = 1(1)50, γ = .95, .99 and 1 - α = .75, .90, .95, .99, .999. The notation 1 (1) 50 means for n=1 to 50 in increments of 1. Note that $K \geq Z_{\alpha/2}$ and K slowly approaches $Z_{\alpha/2}$ as $n \rightarrow \infty$. Therefore, for a random sample of size n from a $N(\mu,\sigma^2)$, we are γx100% certain such that at least 1-α proportion of the population lies within the (γ,1-α) tolerance interval $\bar{y} \pm KS$. The original article by Wald and Wolfowitz (1946), contains the procedure for obtaining values in the table.

Example 2.1. A random sample of n = 25 tires had an average tread length $\bar{y}$ = 75.10" with S = .070. (a) Determine a tolerance interval that contains 99% of the total output with a confidence pr. of 0.95. (b) Given that specifications are 75 ± .25" and the tolerable fraction nonconforming is α = .01, determine the capability of the process to conform to specifications.

Solution. (a) Since α = .01, γ = .95 and n=25, Table 1 gives K = 3.457. Thus the (.95,.99) tolerance interval is 75.10 ± 3.457 x .07 = (74.858, 75.342").

Table I   Tolerance Factors for Normal Distributions

n \ α	γ = 0.95					γ = 0.99 = conf. prob.				
	0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001
2	22.858	32.019	37.674	48.430	60.573	114.363	160.193	188.491	242.300	303.054
3	5.922	8.380	9.916	12.961	16.208	13.378	18.930	22.401	29.055	36.616
4	3.779	5.369	6.370	8.299	10.502	6.614	9.398	11.150	14.527	18.383
5	3.002	4.275	5.079	6.634	8.415	4.643	6.612	7.855	10.260	13.015
6	2.604	3.712	4.414	5.775	7.337	3.743	5.337	6.345	8.301	10.548
7	2.361	3.369	4.007	5.248	6.676	3.233	4.613	5.488	7.187	9.142
8	2.197	3.136	3.732	4.891	6.226	2.905	4.147	4.936	6.468	8.234
9	2.078	2.967	3.532	4.631	5.899	2.677	3.822	4.550	5.966	7.600
10	1.987	2.839	3.379	4.433	5.649	2.508	3.582	4.265	5.594	7.129
11	1.916	2.737	3.259	4.277	5.542	2.378	3.397	4.045	5.308	6.766
12	1.858	2.655	3.162	4.150	5.291	2.274	3.250	3.870	5.079	6.477
13	1.810	2.587	3.081	4.044	5.158	2.190	3.130	3.727	4.893	6.240
14	1.770	2.529	3.012	3.955	5.045	2.120	3.029	3.608	4.737	6.043
15	1.735	2.480	2.954	3.878	4.949	2.060	2.945	3.507	4.605	5.876
16	1.705	2.437	2.903	3.812	4.865	2.009	2.872	3.421	4.492	5.732
17	1.679	2.400	2.858	3.754	4.791	1.965	2.808	3.345	4.393	5.607
18	1.655	2.366	2.819	3.702	4.725	1.926	2.753	3.279	4.307	5.497
19	1.635	2.337	2.784	3.656	4.667	1.891	2.703	3.221	4.230	5.399
20	1.616	2.310	2.752	3.615	4.614	1.860	2.659	3.168	4.161	5.312
21	1.599	2.286	2.723	3.577	4.567	1.833	2.620	3.121	4.100	5.234
22	1.584	2.264	2.697	3.543	4.523	1.808	2.584	3.078	4.044	5.163
23	1.570	2.244	2.673	3.512	4.484	1.785	2.551	3.040	3.993	5.098
24	1.557	2.225	2.651	3.483	4.447	1.764	2.522	3.004	3.947	5.039
25	1.545	2.208	2.631	3.457	4.413	1.745	2.494	2.972	3.904	4.985
26	1.534	2.193	2.612	3.432	4.382	1.727	2.469	2.941	3.865	4.935
27	1.523	2.178	2.595	3.409	4.353	1.711	2.446	2.914	3.828	4.888
28	1.514	2.164	2.579	3.388	4.326	1.695	2.424	2.888	3.794	4.845
29	1.505	2.152	2.554	3.368	4.301	1.681	2.404	2.864	3.763	4.805
30	1.497	2.140	2.549	3.350	4.278	1.668	2.385	2.841	3.733	4.768
31	1.489	2.129	2.536	3.332	4.256	1.656	2.367	2.820	3.706	4.732
32	1.481	2.118	2.524	3.316	4.235	1.644	2.351	2.801	3.680	4.699
33	1.475	2.108	2.512	3.300	4.215	1.633	2.335	2.782	3.655	4.668
34	1.468	2.099	2.501	3.286	4.197	1.623	2.320	2.764	3.632	4.639
35	1.462	2.090	2.490	3.272	4.179	1.613	2.306	2.748	3.611	4.611
36	1.455	2.081	2.479	3.258	4.161	1.604	2.293	2.732	3.590	4.585
37	1.450	2.073	2.470	3.246	4.146	1.595	2.281	2.717	3.571	4.560
38	1.446	2.068	2.464	3.237	4.134	1.587	2.269	2.703	3.552	4.537
39	1.441	2.060	2.455	3.226	4.120	1.579	2.257	2.690	3.534	4.514
40	1.435	2.052	2.445	3.213	4.104	1.571	2.247	2.677	3.518	4.493
41	1.430	2.045	2.437	3.202	4.090	1.564	2.236	2.665	3.502	4.472
42	1.426	2.039	2.429	3.192	4.077	1.557	2.227	2.653	3.486	4.453
43	1.422	2.033	2.422	3.183	4.065	1.511	2.217	2.642	3.472	4.434
44	1.418	2.027	2.415	3.173	4.053	1.545	2.208	2.631	3.458	4.416
45	1.414	2.021	4.408	3.165	4.042	1.539	2.200	2.621	3.444	4.399
46	1.410	2.016	2.402	3.156	4.031	1.533	2.192	2.611	3.431	4.383
47	1.406	2.011	2.396	3.148	4.021	1.527	2.184	2.602	3.419	4.367
48	1.403	2.006	2.390	3.140	4.011	1.522	2.176	2.593	3.407	4.352
49	1.399	2.001	2.384	3.133	4.002	1.517	2.169	2.584	3.396	4.337
50	1.396	1.969	2.379	3.126	3.993	1.512	2.162	2.576	3.385	4.323

"Bowker and Lieberman **[2]"**

Thus, we are 95% sure that 99% of the product output have dimensions within (74.858, 75.342). (b) Since the sample indicates the process may not be centered (i.e., $\mu \neq 75.00$), it is not appropriate to measure process capability at $\alpha = .01$ with

$$PCR = \frac{USL - LSL}{2\sigma Z_{\alpha/2}}$$

if 6-sigma tolerances

$$\doteq \frac{0.50}{2(0.07)(2.5762)} = 1.3863$$

and compare against 1. We observe that on the lower side, the $LTL_{.99} = 74.858$ conforms well to LSL = 74.75, but $UTL_{.99} = 75.342$ does not conform to USL = 75.25. Therefore, we must use the capability index

$$C_{pk} = \frac{1}{Z_{\alpha/2}} Z_{min} = \frac{1}{Z_{.005}} \left(\frac{75.25-75.10}{.07}\right) = 0.8318$$

and compare it against 1. Since $C_{pk} < 1$, then altogether the TL's do not conform to specifications and thus $p > .01$. An estimate of p is obtained from $\hat{p} = 1 - \Phi\left(\frac{75.25-75.10}{.07}\right) = .0161$ which exceeds $\alpha = .01$.

We next illustrate how the ability of process tolerances to conform to specifications relate to Taguchi's quality loss given in equations (16b) and (17). For the sake of illustration, consider the situation of Exercise 2.2, and assume the consumer's loss at specification limits is $A_c = \$50.00$. Then equation (9) gives $k = 50/(0.25)^2 = 800$, and $L(y) = 800\ (y-75.00)^2$. In parts (a) through (d) listed below, we consider different possibilities and provide the corresponding solution.

(a) If process variance $\sigma^2 = 0.0081$ and $\mu = m = 75.00$, then the average (or expected) loss per unit will be

$$E(L) = k\sigma^2 = 800\ (0.0081) = \$6.48/\text{tire},$$

where E denotes the expected value operator. On the other hand, the expected traditional loss per unit (i.e., from the standpoint of nonconformance to specifications) will be

$$E(L_s) = A_c p = 50(.005473) = \$0.274 << E(L).$$

The reason for the above large discrepancy between E(L) and $E(L_s)$ is the fact that the traditional loss totally ignores process variability and grades tread lengths such as 75.24 and 75.00 just as good as each other, while Taguchi's loss at y = 75 is zero and at y = 75.24 is \$46.08/tire.

(b) If process variance is unspecified, $p = \alpha = .0027$ and $\mu = m$, then NTL's and specifications coincide and thus σ is 1/6th of the tolerance, i.e., $\sigma = \frac{1}{6}(2\Delta) = \frac{2x.25}{6} = 0.25/3$. Hence $E(L) = k\sigma^2 = 800\ (0.25/3)^2 = \$5.56$ as compared to the traditional expected loss of $E(L_s) = 50 \times .0027 = \$0.135$/tire.

(c) In part (b), if thru new technology the standard deviation is improved to 1/8th of the tolerance, then $\sigma = \Delta/4$ and $E(L) = 800(0.25/4)^2 = \$3.125$/tire so that the amount of quality improvement (QI) per tire will be \$2.435.

(d) If process variance $\sigma^2 = .0081$ and $\mu = 75.13 \neq m$, then to obtain the expected loss per unit, we note from equation (16b) that

$$E(L) = \frac{k}{n} E\left\{\Sigma\ (y_i - m)^2\right\} = \frac{k}{n}\ E\ \Sigma\left[(y_i - \mu) + (\mu - m)\right]^2$$

$$= \frac{k}{n} E\ [\Sigma\ (y_i - \mu)^2 + n(\mu - m)^2 + 2(\mu - m)\ \Sigma(y_i - \mu)] \qquad (24)$$

Since $E(y_i - \mu) = 0$ for all i, the last term on the RHS of (24) vanishes and as result

$$E(L) = k\sigma^2 + k\ (\mu - m)^2 = k\ [\sigma^2 + (\mu - m)^2]. \qquad (25)$$

Inserting the given values into equation (25) yields

$$E(L) = 800\ [.0081 + (75.13 - 75.00)^2] = \$20$$

as compared to E(L) = \$6.48 of case (a).

To compute the expected traditional loss per tire, we use the distribution of y (assumed normal) in Figure 6 that shows $p = p_\ell + p_u = .0000121 + 0.09121 = 0.09122$, resulting in $E(L_s) = A_c p = \$4.56$ as compared to \$0.274 of case (a). Therefore, depending on the variance, the quality loss per unit can become prohibitive when the mean is far away from the target.

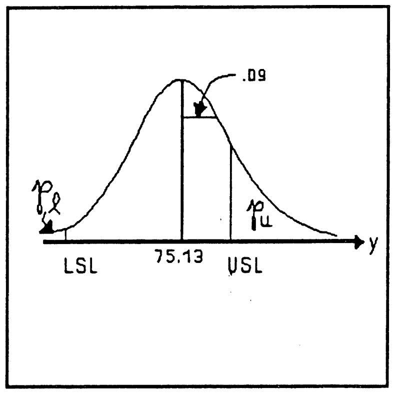


Figure 6

<u>Exercise 2.3</u>. The data 8, 4, 5, 6, 7, 9, 4, 5, 6 are collected for the QCH y = windshield sag in mm, where the target value is m = 7. (a) Compute the MSD in 2 different ways, thereby verifying eq. (17). (b) Assuming $\Delta = \pm 2$, compute the average loss per unit from both the traditional and Taguchi points of view, given $A_c = \$10.00$. (c) Compute the proper process capability index if $\alpha = .05$. ANS: (b) $2.22\overline{2}$, 9.167. (c) $C_{pk} = 0.2946$ (using S as the estimator of σ).

<u>Exercise 2.4</u>. The specifications for tread length of a certain brand of passenger tire are 75 ± .25". The consumer's cost for a tire with tread at the specs is \$50.00. (a) Determine the QLF. (b) If a nonconforming tread can be calibrated toward nominal (=75.00") at a cost of 40¢ during production, for what

values of the length should the 40¢ be spent? (c) If thru improved technology, the standard deviation can be reduced from $\Delta_c/3$ to 7/24 of Δ_c and 5000 such tires are produced per day, what is the QI/day if $\mu = m$. (d) If $\mu = 74.91$, compute E(L) and $E(L_s)$. ANS: (b) $\Delta_f = .022361$, (c) \$6510.42. (d) \$12.04, \$1.373 when $\sigma = \Delta_c/3$.

Exercise 2.5. In the previous exercise, there are 2 machines for making tire treads. Data from machine 1 gave the coded (from 75.00) results: -.22(→74.78), 0.27, 0.30, -.15, 0.26, -.18, -.18, -.23, 0.15, 0.11, -0.31. Machine 2's data are -.07, 0.12, .04, 0.10, .16, -.08, .09, .10, .06, .05. Determine the QD/day of machine 2 over 1 if PR = 5000 tires per day. [Hint: Compute the MSD's 1st]. ANS: \$165,087.273; traditional QD/day = \$90,909.091.

Exercise 2.6. An automobile component has specs m ± 5 microns (μm). The loss caused by a defective unit is \$6.00. (a) Determine the expected loss/unit if σ=1/6 of the tolerance and the process in centered. (b) The company is trying to improve their process capability to 1/8th of the tolerance. Determine the average QL/unit and the amount of QI per month if 100,000 units are built in one month, i.e., PR = 100,000/month. Answers: (a) 67¢, (b) 37.5¢, \$29166.67.

4. A STATISTICAL COMPARISON OF S^2 AND s_n^2

1. S^2 is unbiased, i.e., its long run average over all possible random samples or its expected value, $E(S^2)$, is equal to the process variance σ^2. Therefore, for an infinite universe the bias in S^2

$$B(S^2) = E(S^2) - \sigma^2$$

is zero. It can be shown that the bias in s_n^2 is $B(s_n^2) = -\sigma^2/n$, implying that

s_n^2 is asymptotically (i.e., as $n \to \infty$) unbiased. Further, for a finite population of size N the amount of bias in S^2 is $\sigma^2/(N-1)$.

2. The quantity $\sum_{i=1}^{n} (y_i - \bar{y})^2$ has n-1 degrees of freedom (df) since the constraint $\sum_{i=1}^{n}(y_i - \bar{y}) = 0$ implies that only (n-1) of the $(y_i - \bar{y})$'s are independent. Therefore, S^2 has the advantage that the sum of squares, SS = $\Sigma(y_i - \bar{y})^2$, is divided by its df to yield the value of S^2.

3. Unfortunately, S^2 is not a 100% efficient estimator (or simply efficient) in the sense that it does not use all the statistical information in the sample. It can be shown (see Hogg & Craig, p. 241) that for a normal universe its efficiency is (n-1)/n. Furthermore, it can be proved that for a normal universe, the mean square errors of the two estimators are

$$MSE(S^2) = V(S^2) = 2\sigma^4/(n-1)$$

and

$$MSE(s_n^2) = E[(s_n^2 - \sigma^2)^2] = V(s_n^2) + B^2(s_n^2) = (2n-1)\,\sigma^4/n^2.$$

These last two equations show that MSE (s_n^2) < MSE (S^2). Therefore, for a normal universe s_n^2 is a more accurate estimator of σ^2 than S^2. If a population is not normal, then s_n^2 has a smaller MSE than S^2 only if

$$\mu_4 > \frac{3n^2 - 8n + 3}{(n-1)(2n-1)}\,\sigma^4 \quad , \qquad (26)$$

where $\mu_4 = E[(y-\mu)^4]$ is the 4th central movement of y. Inequality (26) shows that, at n=2, s_n^2 is a more accurate estimator of σ^2 than S^2 for any population.

5. THE LOSS PER UNIT FOR AN STB QCH WHEN $n \geq 2$

In general, since the target (or the ideal value of y) is m = 0, then equation (17) reduces to

$$MSD = s_n^2 + (\bar{y})^2 \quad (27a)$$

and as before

$$L = k(MSD) = k(s_n^2 + \bar{y}^2) \quad (27b)$$

where $k = A_c/y_u^2$.

Exercise 2.7. The ten observations 16.3, 21.9, 10.2, 27.5, 14.2, 29.8, 8.2, 14.8, 22.1, 21.4 represent the RFH in lbs of certain passenger tires. (a) Given that the USL = 26.00 lbs and A_c = \$10.00, compute the MSD using its definition given by (16a, removing k) and then using equation (27a). (b) Compare the Taguchi loss with the traditional loss (based on nonconformance to specs) and comment on their discrepancy.

ANS: (a) MSD = 392.792 lbs^2, (b) $\bar{L}$ = 5.8105, L_s = \$2.00/tire.

Exercise 2.8. Data from a different brand of tires gave the following RFH values in lbs: 23.5, 18.6, 10.7, 25.9, 24.6, 19.9, 22.3, 26.8, 24.3, 17.2, 20.6, 25.4. Evaluate the quality of this brand from both the Taguchi and traditional standpoints and compare with those of Exercise 2.7. Assume A_c = \$10.00 at y_u = 26.00 lbs. ANS: L = 7.2176, L_s = \$0.83$\overline{3}$.

6. THE LOSS PER UNIT FOR AN LTB QUALITY CHARACTERISTIC WHEN $n \geq 2$

Since the target is $m = \infty$ (or zero for $1/y$) then from (16c)

$$L = k(MSD) = \frac{k}{n} \sum_{i=1}^{n} (1/y_i^2)$$

where $k = A_c y_\ell^2$ and y_ℓ = LSL. Taguchi and Wu [9] give the following approximation for the sample mean squared deviation

$$MSD = \frac{1}{n} \Sigma(1/y_i^2) \doteq \frac{1}{(\bar{y})^2} \left(1 + \frac{3s_n^2}{(\bar{y})^2}\right) , \quad (28)$$

which is adequate for some data and not for others. In order to show how approximation (28) was arrived at, consider the following algebraic manipulation:

$$\text{MSD} = \frac{1}{n}\sum_{i=1}^{n} [(y_i - \bar{y}) + \bar{y}]^{-2} = \frac{1}{n(\bar{y})^2}\sum_{i=1}^{n} (1 + \frac{y_i - \bar{y}}{\bar{y}})^{-2} \quad . \tag{29}$$

Since the Taylor expansion of $f(x) = (1 + x)^{-2}$ about $x = 0$ is $f(x) = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$, equation (29) has the expansion

$$\text{MSD} = \frac{1}{n(\bar{y})^2}\sum_{i=1}^{n} [1 - 2 (\frac{y_i - \bar{y}}{\bar{y}}) + 3 (\frac{y_i - \bar{y}}{\bar{y}})^2 - 4 (\frac{y_i - \bar{y}}{\bar{y}})^3 + \ldots]. \tag{30}$$

Equation (30) now confirms the approximation given in (28), which includes the first three terms inside the Σ in (30) and incorporates only the effects through second central moment. Further, it shows that a better approximation should be

$$\text{MSD} \doteq \frac{1}{(\bar{y})^2} (1 + \frac{3s_n^2}{(\bar{y})^2} - \frac{4\hat{\mu}_3}{(\bar{y})^3}) \tag{31}$$

and even a better approximation may be

$$\text{MSD} \doteq \frac{1}{(\bar{y})^2} (1 + \frac{3s_n^2}{(\bar{y})^2} - \frac{4\hat{\mu}_3}{(\bar{y})^3} + \frac{5\hat{\mu}_4}{(\bar{y})^4}) \quad , \tag{32}$$

where $\hat{\mu}_3 = \frac{1}{n}\Sigma (y_i - \bar{y})^3$ and $\hat{\mu}_4 = \frac{1}{n}\Sigma (y_i - \bar{y})^4$ are biased estimators of the third and fourth central moments of y, respectively. From the above developments, we conclude that approximation (28) is adequate only if the distribution of the QCH y is not too skewed (i.e., $\mu_3 \doteq 0$) and/or the population norm, $|\mu|$, far exceeds $|\mu_3|$ and μ_4.

<u>Exercise 2.9</u>. The production lower specification for welding strength is 1.20 ksi, and the average loss per unit due to not meeting specs is \$20.00. (a) Set up the Taguchi loss function for one unit. (b) Given the data 4.0, 2.0, 1.0, 1.3, 1.1, 6.0, 4.8, 1.5, 0.9 ksi, compute the exact value of the MSD and the

resulting average loss per unit. (c) Estimate the MSD using equations (28), (31) and (32). (d) Compare the Taguchi loss with the traditional loss. ANS: (a) $L(y) = 28.80/y^2$, (b) .4979, \$14.34, (c) .4038, .2063, .6488, (d) $L_t = \$6.67$.

Note that none of the approximations to the MSD are adequate for the data of Exercise 2.9.

Exercise 2.10. Data from a different welding method gave the following ksi's: 2.3, 3.6, 1.60, 1.20, 1.0, 1.80, 1.70, 2.8, 1.10, 1.50, 1.4, 1.30. Compare with the method of Exercise 2.9 from both the Taguchi concept of quality and conforming to specification concept of quality given that $A_c = \$20.00$ at the LSL $= y_\ell = 1.20$ ksi. ANS: $L = \$13.22$, $L_t = \$3.3\overline{3}$ (This is better than the welding method used in Exercise 2.9).

MISCELLANEOUS FORMULAS

PAGES F-A THROUGH F-H

Formulas Applicable to All Static Measurable Quality Characteristics

1. The sample mean $= \overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

2. The sample variance $= s_n^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \overline{y})^2 = \hat{\sigma}_n^2$

   or $s_n^2 = \frac{1}{n} \Sigma y_i^2 - (\overline{y})^2 = \frac{1}{n} [\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}]$,

   $S^2 = ns_n^2/(n-1)$ is an unbiased estimator of process variance σ^2

3. The sample coefficient of variation $= cv_y = S/\overline{y}$

4. The average \$ quality loss per unit = $\overline{L}$

   $$\overline{L} = k\,(MSD),$$

   where MSD = Mean Squared Deviation and k is a constant. The formulas for MSD and k are different for the three QCH types.

5. A_c = Consumers' QL at a specification limit.

6. $\hat{\eta}_o$ = S/N at the optimal FLC.

Formulas for a STB Performance Characteristic

1. The target $= m = 0$

2. The USL $= y_u$

3. $L(y)$ = Taguchi's QLF $= ky^2$, $k = A_c / y_u^2$, \$ A_c = Consumer's loss at y_u .

4. $\text{MSD} = \frac{1}{n}\Sigma y_i^2 = s_n^2 + (\bar{y})^2$

   $\bar{L} = k\,(\text{MSD}) = k[s_n^2 + (\bar{y})^2]$

5. Signal/Noise $= \text{S/N} = \eta_{db} = -10 \log_{10} \ (\text{MSD}) = -10 \log_{10} \ (\frac{1}{n}\Sigma y_i^2)$

   $= 10 \log_{10} \ (k/\bar{L})$

   $= \log_{10} \ [(k/\bar{L})^{10}]$

6. $\bar{L} = k \ (10^{-\eta_{db}/10}) = k\,(\text{MSD})$

Formulas for a LTB Performance Characteristic

1. The target ∞ and $1/y$ is an STB QCH

   Hence the target for $1/y$ is zero.

2. The LSL $= y_l$

3. $L(y) = k/y^2$, $k = A_c y_l^2$

4. $MSD = \frac{1}{n} \sum_{i=1}^{n} 1/y_i^2 \doteq \frac{1}{(\bar{y})^2} [1 + \frac{3s_n^2}{(\bar{y})^2}]$,

   $\bar{L} = k(MSD) = \frac{k}{n} \sum_{i=1}^{n} (\frac{1}{y_i^2})$

5. $S/N = \eta_{db} = -10 \log_{10} (MSD) = -10 \log_{10} [\frac{1}{n} \sum_{i=1}^{n} (\frac{1}{y_i^2})]$, decibels

   $= 10 \log_{10} (k/\bar{L})$

   $= \log_{10} [(k/\bar{L})^{10}]$.

6. $\bar{L} = k(10^{-\eta_{db}/10}) = k(MSD)$

Formulas for a NTB Performance Characteristic

1. The target = $m \neq 0$ (generally)

2. The LSL = $m - \Delta$, and USL = $m + \Delta$

3. $L(y) = k(y - m)^2$, $k = A_c / \Delta^2$

   \$ A_c = Consumer's loss at $m \pm \Delta$.

4. $MSD = \frac{1}{n} \Sigma (y_i - m)^2 = s_n^2 + (\bar{y} - m)^2$

   $\bar{L} = k\,(MSD) = k\,[\sigma_n^2 + (\bar{y} - m)^2]$

5. $S/N = \eta_{dB} = 10 \log_{10} \left(\frac{1}{cv^2} - \frac{1}{n} \right)$, $cv = S/\bar{y}$

   $\doteq 10 \log_{10} \left(\frac{1}{cv^2} \right) = 10 \log_{10} \left(\frac{\bar{y}}{S} \right)^2$

   $\eta_{dB} \doteq 20 \log_{10} (\bar{y}/S)$.

Table II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = \int_{-\infty}^{z} \phi(t)\, dt$$

z	.00	.01	.02	.03	.04	z
.0	.500 00	.503 99	.507 98	.511 97	.515 95	.0
.1	.539 83	.543 79	.547 76	.551 72	.555 67	.1
.2	.579 26	.583 17	.587 06	.590 95	.594 83	.2
.3	.617 91	.621 72	.625 51	.629 30	.633 07	.3
.4	.655 42	.659 10	.662 76	.666 40	.670 03	.4
.5	.691 46	.694 97	.698 47	.701 94	.705 40	.5
.6	.725 75	.729 07	.732 37	.735 65	.738 91	.6
.7	.758 03	.761 15	.764 24	.767 30	.770 35	.7
.8	.788 14	.791 03	.793 89	.796 73	.799 54	.8
.9	.815 94	.818 59	.821 21	.823 81	.826 39	.9
1.0	.841 34	.843 75	.846 13	.848 49	.850 83	1.0
1.1	.864 33	.866 50	.868 64	.870 76	.872 85	1.1
1.2	.884 93	.886 86	.888 77	.890 65	.892 51	1.2
1.3	.903 20	.904 90	.906 58	.908 24	.909 88	1.3
1.4	.919 24	.920 73	.922 19	.923 64	.925 06	1.4
1.5	.933 19	.934 48	.935 74	.936 99	.938 22	1.5
1.6	.945 20	.946 30	.947 38	.948 45	.949 50	1.6
1.7	.955 43	.956 37	.957 28	.958 18	.959 07	1.7
1.8	.964 07	.964 85	.965 62	.966 37	.967 11	1.8
1.9	.971 28	.971 93	.972 57	.973 20	.973 81	1.9
2.0	.977 25	.977 78	.978 31	.978 82	.979 32	2.0
2.1	.982 14	.982 57	.983 00	.983 41	.983 82	2.1
2.2	.986 10	.986 45	.986 79	.987 13	.987 45	2.2
2.3	.989 28	.989 56	.989 83	.990 10	.990 36	2.3
2.4	.991 80	.992 02	.992 24	.992 45	.992 66	2.4
2.5	.993 79	.993 96	.994 13	.994 30	.994 46	2.5
2.6	.995 34	.995 47	.995 60	.995 73	.995 85	2.6
2.7	.996 53	.996 64	.996 74	.996 83	.996 93	2.7
2.8	.997 44	.997 52	.997 60	.997 67	.997 74	2.8
2.9	.998 13	.998 19	.998 25	.998 31	.998 36	2.9
3.0	.998 65	.998 69	.998 74	.998 78	.998 82	3.0
3.1	.999 03	.999 06	.999 10	.999 13	.999 16	3.1
3.2	.999 31	.999 34	.999 35	.999 38	.999 40	3.2
3.3	.999 52	.999 53	.999 55	.999 57	.999 58	3.3
3.4	.999 66	.999 68	.999 69	.999 70	.999 71	3.4
3.5	.999 77	.999 78	.999 78	.999 79	.999 80	3.5
3.6	.999 84	.999 85	.999 85	.999 86	.999 86	3.6
3.7	.999 89	.999 90	.999 90	.999 90	.999 91	3.7
3.8	.999 93	.999 93	.999 93	.999 94	.999 94	3.8
3.9	.999 95	.999 95	.999 96	.999 96	.999 96	3.9

TABLE II Cumulative Standard Normal Distribution (continued)

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = \int_{-\infty}^{z} \phi(u)\, du$$

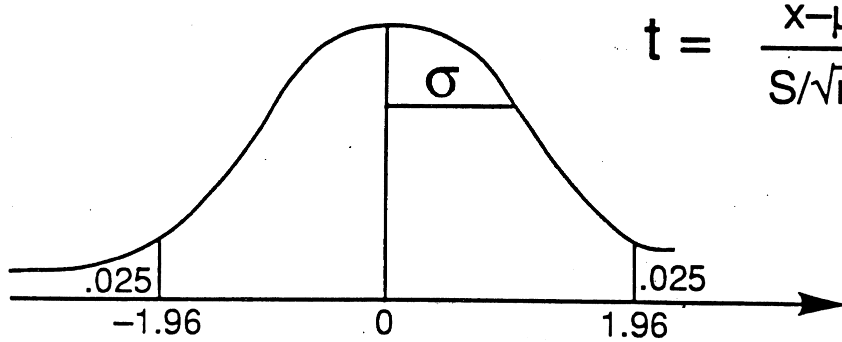
z	.05	.06	.07	.08	.09	z
.0	.519 94	.523 92	.527 90	.531 88	.535 86	.0
.1	.559 62	.563 56	.567 49	.571 42	.575 34	.1
.2	.598 71	.602 57	.606 42	.610 26	.614 09	.2
.3	.636 83	.640 58	.644 31	.648 03	.651 73	.3
.4	.673 64	.677 24	.680 82	.684 38	.687 93	.4
.5	.708 84	.712 26	.715 66	.719 04	.722 40	.5
.6	.742 15	.745 37	.748 57	.751 75	.754 90	.6
.7	.773 37	.776 37	.779 35	.782 30	.785 23	.7
.8	.802 34	.805 10	.807 85	.810 57	.813 27	.8
.9	.828 94	.831 47	.833 97	.836 46	.838 91	.9
1.0	.853 14	.855 43	.857 69	.859 93	.862 14	1.0
1.1	.874 93	.876 97	.879 00	.881 00	.882 97	1.1
1.2	.894 35	.896 16	.897 96	.899 73	.901 47	1.2
1.3	.911 49	.913 08	.914 65	.916 21	.917 73	1.3
1.4	.926 47	.927 85	.929 22	.930 56	.931 89	1.4
1.5	.939 43	.940 62	.941 79	.942 95	.944 08	1.5
1.6	.950 53	.951 54	.952 54	.953 52	.954 48	1.6
1.7	.959 94	.960 80	.961 64	.962 46	.963 27	1.7
1.8	.967 84	.968 56	.969 26	.969 95	.970 62	1.8
1.9	.974 41	.975 00	.975 58	.976 15	.976 70	1.9
2.0	.979 82	.980 30	.980 77	.981 24	.981 69	2.0
2.1	.984 22	.984 61	.985 00	.985 37	.985 74	2.1
2.2	.987 78	.988 09	.988 40	.988 70	.988 99	2.2
2.3	.990 61	.990 86	.991 11	.991 34	.991 58	2.3
2.4	.992 86	.993 05	.993 24	.993 43	.993 61	2.4
2.5	.994 61	.994 77	.994 92	.995 06	.995 20	2.5
2.6	.995 98	.996 09	.996 21	.996 32	.996 43	2.6
2.7	.997 02	.997 11	.997 20	.997 28	.997 36	2.7
2.8	.997 81	.997 88	.997 95	.998 01	.998 07	2.8
2.9	.998 41	.998 46	.998 51	.998 56	.998 61	2.9
3.0	.998 86	.998 89	.998 93	.998 97	.999 00	3.0
3.1	.999 18	.999 21	.999 24	.999 26	.999 29	3.1
3.2	.999 42	.999 44	.999 46	.999 48	.999 50	3.2
3.3	.999 60	.999 61	.999 62	.999 64	.999 65	3.3
3.4	.999 72	.999 73	.999 74	.999 75	.999 76	3.4
3.5	.999 81	.999 81	.999 82	.999 83	.999 83	3.5
3.6	.999 87	.999 87	.999 88	.999 88	.999 89	3.6
3.7	.999 91	.999 92	.999 92	.999 92	.999 92	3.7
3.8	.999 94	.999 94	.999 95	.999 95	.999 95	3.8
3.9	.999 96	.999 96	.999 96	.999 97	.999 97	3.9

TABLE IV Percentage Points of the *t* Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.320	318.310	636.620
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.842	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Source: This table is adapted from *Biometrika Tables for Statisticians,* Vol. 1, 3rd edition, 1966, by permission of the Biometrika Trustees.

$$t = \frac{\bar{x}-\mu}{S/\sqrt{n}} = \frac{(\bar{x}-\mu)\sqrt{n}}{S}$$



$$z = \frac{(\bar{x}-\mu)}{\sigma/\sqrt{n}}$$

Quality Engineering: Exercises

LOSS FUNCTION (Nominal the best)

= "Shift pressure of transmission" Nominal for y = 75 lbs. (m=75)

Consumers' tolerance (LD-50) = 75 ± 40 lbs.
Repair cost after shipped out = $500.00

Q.1) Set up the loss function for y, i.e. find constant k. Draw the loss function L(y).

$$L = \begin{cases} k\,(y-m)^2 = L(y) \text{..............for one piece} \\ k\,(MSD) = \bar{L} \text{.................for n pieces} \end{cases}$$

Q. 2) Rework cost at plant before shipping out is $40.00. Set up appropriate production tolerance for y and state for what values of y the $40.00 should be spent.

Q. 3) Following data are "Shift pressure" from two different transmission designs. Compare their quality level ($Q\ell$).

TYPE-1	76	82	84	75	68	66	72	63	72	92
TYPE-2	70	62	82	65	68	74	70	67	75	77

	$\bar{y}$	$(\bar{y}-m)^2$	$\hat{\sigma}_n^2$	MSD	Average QL/Unit $= \bar{L}$
TYPE-1					
TYPE-2					

What is the difference in monthly $Q\ell$'s if production is 40,000 units/month?

CHAPTER III

1. NOISE AND ROBUSTNESS AGAINST NOISE

Taguchi defines noise as those variables that are either too difficult or too expensive to control, i.e., a noise factor is an uncontrollable factor. There are 3 types of noise: (1) Outer noise, such as environmental conditions, dust, humidity, operators, etc., (2) Inner noise, such as oxidization and/or deterioration of parts, material, subcomponents, etc., (3) Between product noise, i.e., piece to piece variation.

From a design standpoint, the main difference between classical (fractional) factorial designs and Taguchi's parameter design (PDE) is that in the former case, the extraneous factors (or experimental noise factors) are averaged out over all treatment combinations (TCs) through randomization. In Taguchi's method (i.e., PDE), noise is imbedded in the design of the experiment so that randomization is generally unnecessary. This implies that Taguchi recommends data be taken across all possible noise levels so that optimal factor levels can be selected in order to make the product more robust against noise (as depicted in Figure 7).

Taguchi's PDE generally consists of two orthogonal arrays (OAs): the inner OA and the outer OA as depicted in Figure 8.

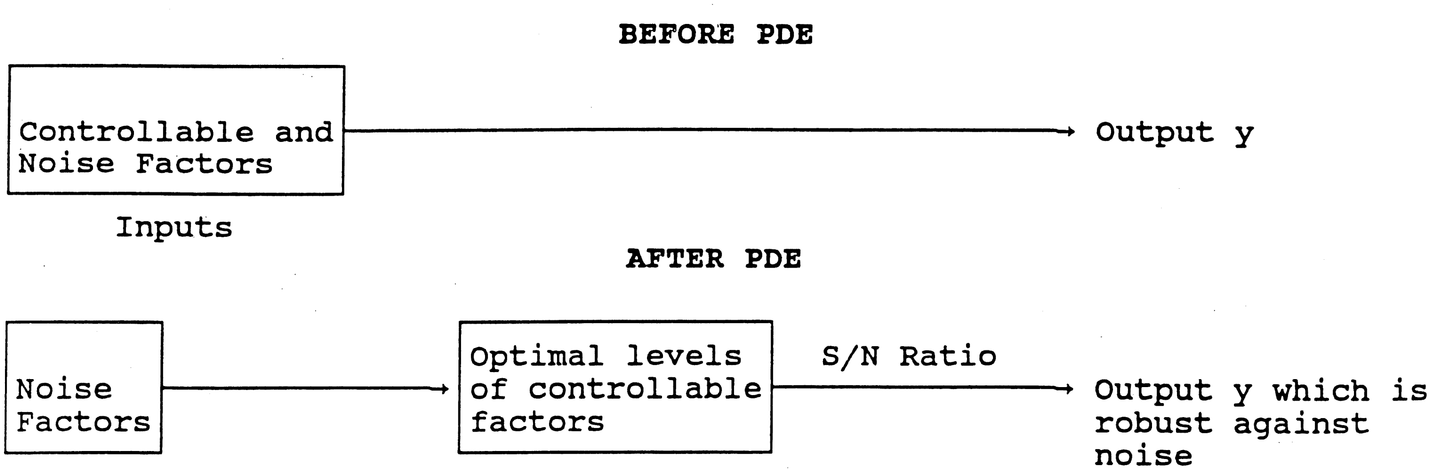


Figure 7

Experiment	Controllable Factors	Noise Factors
1 2 3 . . .	Inner OA	Outer OA

Figure 8

In Taguchi's experimental design method, factors that may affect a QCH (or response variable), y, are of 2 types: (a) experimentally controllable factors, (2) noise factors. The controllable factors will be imbedded in the inner array and the noise factors in the outer OA as shown in Figure 8.

<u>Example</u> Suppose the QCH is y = welding strength measured in psi. Controllable factors considered are: A = shell thickness, B = speed, C = pressure, D = surface finish, E = voltage, F = current, G = angle, H = plating thickness, where all these factors are at 2 levels - low and high. Factor I = weld material has 2 levels also, which are type 1 and type 2.

Noise factors considered are S = shift (at levels 1 and 2), M = Machine (A and B) and 0 = operator (1 and 2).

<u>Exercise 3.1</u> Construct another experiment citing the QCH, y, the controllable and noise factors.

In summary, according to Taguchi's philosophy, noise is a factor that affects product function and is generally too expensive or too difficult (or nearly impossible) to control, and therefore, countermeasures against it must be taken at the (secondary) design stage. That is, the controllable factors should be designed such that the product achieves maximum robustness against noise. This leads to another definition of quality: high quality implies robustness against noise.

2. THE THREE STAGES OF DESIGN

In order to minimize noise effects, countermeasures must be taken at the design stage, of which there are three: (1) System (or primary) design, (2) Parameter (or secondary) design, (3) tolerance design.

(i) System Design

This stage describes the design of a system which functions under normal (or ideal) conditions. An example would be a prototype. In this step, knowledge from a particular field is necessary so that this stage is not statistical in its development.

(ii) Parameter (or Secondary) Design

Once system design is completed, the next step is to determine the optimal levels of all the controllable factors affecting the product quality, which, according to Taguchi, should be done through the use of a parameter design. This stage is similar to a (fractional) factorial experiment, but noise factors are imbedded in the outer OA so that optimal controllable factor levels can be determined in the presence of noise thereby minimizing its effect. In formal notation, the objective of Taguchi's parameter design is to optimize

$$y = f(F_1, F_2, \ldots, F_k, N_1, N_2, \ldots, N_L),$$

where y = a quality measure (or characteristic), F_i = the i^{th} controllable factor, and N_j = the j^{th} noise factor. After data is taken, the determination of optimal parameter levels, denoted by F_1^*, F_2^*, ..., F_k^*, is carried out through computing the signal to noise ratio for each treatment combination, containing n observations, and then maximizing the signal to noise (S/N) ratio. Generally the S/N ratio is denoted by η_{dB} (or η_{db}), measured in decibels, and is defined as

$$\eta_{dB} = \frac{f(\text{Data mean})}{g(\text{Data variance})} \quad . \tag{33}$$

Therefore, Taguchi's parameter design objective is to optimize y across different noise levels N_1, N_2, ..., N_L, and this is carried out by always maximizing η_{dB} in (33) where the optimum value of y is

$$y^* = f(F_1^*, F_2^*, \ldots, F_k^*, N_1, N_2, \ldots, N_L) \ .$$

Note that in the case of an STB QCH, y* will be the minimum value of y, while for an LTB QCH, y* will be the maximum value of y.

(iii) Tolerance (or Allowance) Design

Parameter design helps identify the best (or optimum) combination of controllable factor levels, reducing the influence of noise rather than removing the causes of variation. After the mid- (or optimum) values of the controllable factors are determined through parameter design, the influence of inner, outer or piece to piece noise may not have been sufficiently reduced. Our final option, which is the last topic of this book, is to reduce the influence of noise (probably through higher cost) by the use of an allowance design.

To start the tolerance design (TDE), high quality subcomponents (such as super quality resistors) are tried to reduce variation. Then 3 levels of each factor are used in an inner OA with the mid-level at the optimum values obtained in the parameter design. There is one outer OA for each TC of the inner OA. The noise factors imbedded in the outer array will consist of the tolerances of the controllable factors considered in the experiment. Significant factors are identified from the ANOVA table. The contribution of the significant factors to the error variance is diminished through the use of higher quality subcomponents, thereby reducing the allowance Δ.

In this chapter we have pointed out the differences between the classical statistical experimental design (CSED) and Taguchi's PDE (if necessary followed by TDE). In Chapter IV, we will discuss classical SED whose basic principles are replication, randomization, blocking and fractionalization. Furthemore, the principal objective of CSED is to identify the controllable factors that affect the mean of the QCH, while the objective of Taguchi's PDE is also to identify the factors that affect process variation. Taguchi refers to the two types of controllable factors as signal and control factors, respectively.

CHAPTER IV

1. REVIEW OF FACTORIAL EXPERIMENTS

An experimental arrangement is factorial if observations are taken at every treatment combination (TC) of all the factors that may affect the response variable, y. For example, suppose we wish to study the effects of (post-cure) inflation time, T, and pressure, P, on the QCH shoulder-drop (measured in inches) of certain passenger tires. We consider 2 levels of T and 3 levels of P as depicted below. Specifications are: drop = .20 ± .015.

T \ P	25 psi	30 psi	35 psi	$y_{i..}$
5 minutes	y_{111}, y_{112} y_{113}	y_{121}	y_{131}, y_{132}	
15 minutes	y_{211}	y_{221} y_{222}	y_{231} y_{232}	
$y_{.j.}$				$y_{...}$

The experiment would constitute a factorial arrangement only if observations are taken at all 6 TCs (5,25), (5,30), (5,35), (15,25), (15,30), (15,35). The design is balanced if the number of repetitions, n_{ij}, at each of the 6 cells is the same, i.e., $n_{ij} = n$ for all (i,j).

Traditionally experiments were done by changing the levels of one factor while keeping the levels of the remaining factors fixed. Experimentation would continue until the response, y, would change direction at which point, a second factor's levels were changed, keeping the remaining factor levels fixed. The procedure was continued until the presumed optimum was reached. Such a procedure is called one-factor-at-a-time experiment. Unfortunately, if the factors in the experiment interact (defined later), then the one-factor-at-a-time experimental method will generally not lead to the optimum conditions in the region of the factor space. Further, factorial arrangements may actually need fewer experiments to locate the (near) optimum conditions thereby reducing cost.

As examples of a one-factor-at-a-time experiment, see Figures 11 and 12 of the article by J. Stuart Hunter, p. 14, given in the next page.

Exercise 4.1 Suppose y is an LTB type QCH. Construct a one-factor-at-a-time experiment where the maximum value of y is not correctly determined. Assume that there are only 2 factors, A and B, that affect y, and $y_{max} = 125$.

2. INTERACTION

Two factors, A and B, interact in affecting the response y if the effect of A on y at B_1 (= level 1 of B) is not the same as the effect of A on y at Bj (j = 2,3,4,...) for at least one j. In short, A and B interact if their joint effect on y is not equal to the sum of their individual effects on y.

As an example consider the following 2x3 factorial experiment where factor A represents time and factor B represents pressure. The QCH is shoulder-drop measured in inches.

A \ B	1	2	3
1	0.193	0.212	0.183
2	0.217	0.202	0.186

Figure 9 clearly shows that the effect of B at A_1 is quadratic while the effect

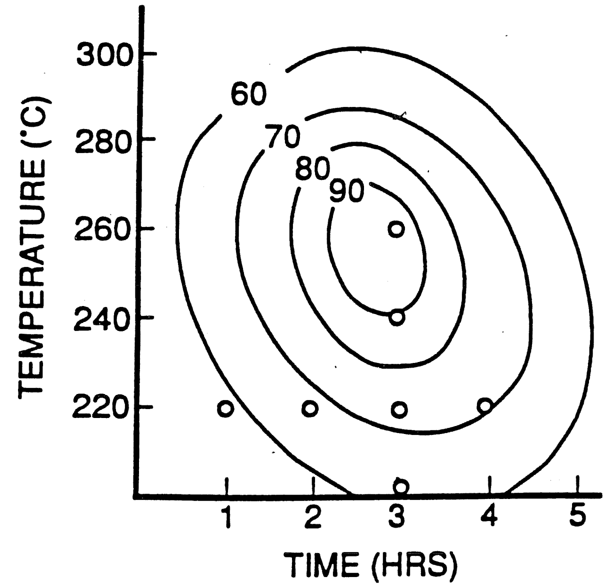


Figure 11 — Contours of Equal Response for y = Percent of Theoretical Yield (Illustration of Method of One Factor at a Time)

A favorite and classical attack is the method of one factor at a time. This method required that the experimenter hold all the controlled variables save one at some constant level and vary the remaining single variable until a maximum response is observed. Then holding this variable at its optimum value, a second variable is varied, and so on. The method is illustrated below for the ease of k = 2 controlled variables.

Suppose a response y (percent of theoretical yield) is a function of time (measured along the x_1 axis), and temperature (measured along the x_2 axis). Suppose further that the response, when viewed geometrically, has the appearance of a mound with a single maximum point. This response is illustrated by means of the contour diagram in Fig. 11.

Using the method of one factor at a time, the experimenter might hold the temperature constant at 220 degrees and vary the time in one hour increments. The result experimental trials are shown by the horizontal line of heavy dots in Fig. 11. Deciding that three hours was the best time; he would then hold the time constant at this value and vary the temperature in, say, increments of 20 degrees. This gives the series of experiments illustrated by the vertical line of dots. Thus the procedure leads the experimenter to the point of maximum response.

However, suppose that instead of a mound (which clearly leads to a single maximum point, the response were slightly more complicated and had the appearance of a rising ridge as shown in Fig. 12. The identical procedure illustrated by the dots in Fig. 12 leaves the experimenter with the conviction that the setting of three hours and 220 degrees is optimum, for steps higher or lower in either time or temperature from this point will produce a decrease in the response. Obviously, from the "map" higher yields by varying time and temperature simultaneously and in the proper amounts so that he proceeds in the proper direction.

A method which guarantees that the experimenter will tend toward the maximum point regardless of the form or the response surface save that it is continuous) is the method of steepest ascents as described in the paper "On the Experimental Attainment of Optimum Conditions" by G. E. P. Box and K. B. Wilson in the Journal of the Royal Statistical Society Series B, in 1951. This technique makes use of the first-order mathematical model, and a first-order experimental design, that is either the two-level factorial or the fractional factorial design. An important concept behind the idea of predicting a path of steepest ascent is simply that a plane does a good job of approximating a curved surface within a limited area. This is analogous to the old argument that straight lines do a good job of approximating curved lines over small intervals. **The plan then is to predict the best fitting plane in some small sub-space of the experimental region. Noting the tilt of the fitted plane, a path of steepest ascent can be predicted. Experiments are than performed along this path until a decline is reponse is noted. Additional observations taken in and around this point can confirm whether a maximum has been reached, whether a new path of steepest ascent can be predicted. Experiments are then performed along this path until a decline in response is noted. Additional observations taken in and around this point can confirm whether a maximum has been reached, whether a new path of steepest ascent can be predicted, or whether the response surface should be mapped in this important region.**

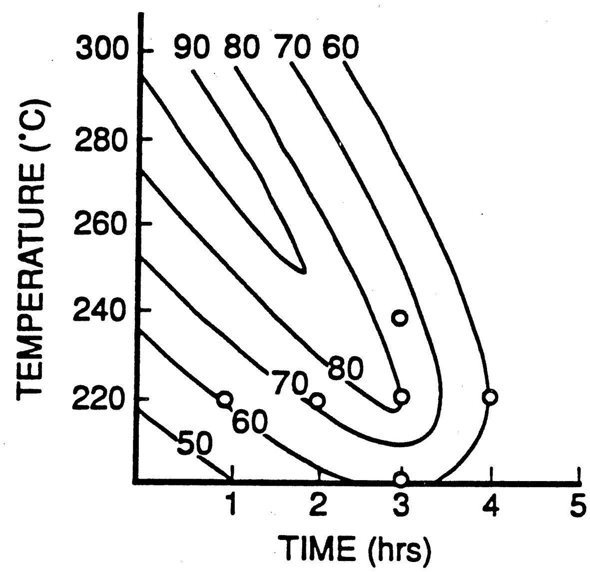


Figure 12 — Contours of Equal Response for y = Percent of Theoretocal Yield (Illustration of Method of One Factor at a Time)

For example, imagine that a two-level factorial design had been run in the region illustrated by the small circles in Fig. 13. The following table of results would be observed.

Controlled Var. Levels		Exper. Design Levels		Response
Time (hours)	*Temp (degrees)*	x_1	x_2	y
0.5	210	-1	-1	51%
1.0	210	1	-1	57%
0.5	220	-1	1	55%
1.0	220	1	1	61%

Fitting the first-order model one obtains for the best fitting plane in this region.

$$\hat{y} = 56 + 3x_1 + 2x_2$$

The path of steepest ascent is not determined by the coefficients of x_1 and x_2. In this example we are advised that for every three units x_1 is charged, x_2 should be simultaneously changed two units.

effect of B at A_2 is linear. Therefore, factors A and B have a strong interaction in affecting y.

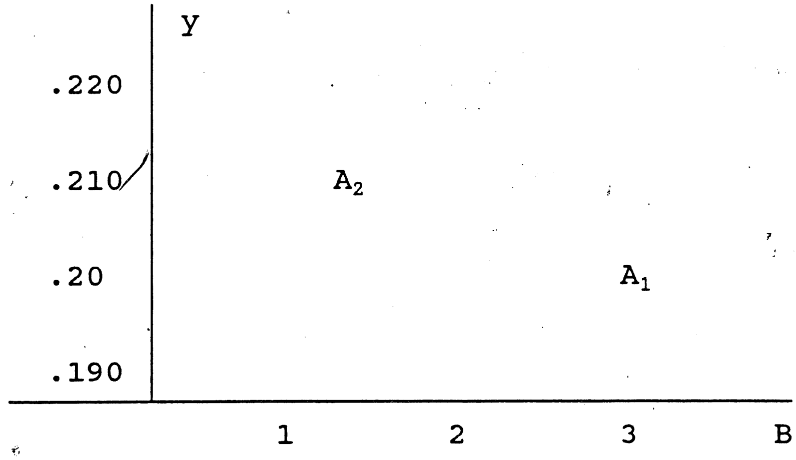


Figure 9. Interaction Between A and B

<u>Exercise 4.2.</u> Construct a 2x2 (or 2^2) design, with n=1 observation per cell, such that there is (a) no interaction between A and B, (b) a moderate amount of interaction between A and B, and (c) a strong interaction between the two factors.

For further discussions on Factorial designs, see chapter 7 of D. C. Montgomery, 3rd Ed., "Design and Analysis of Experiments." Furthermore, for a review of SS's computation in a <u>balanced</u> factorial design, obtain the ANOVA table for the problem below.

<u>Exercise 4.3.</u> The percentage of hardwood concentration in raw pulp, the freeness, and the cooking time of pulp are being investigated for their effects on the strength of paper; [The data was coded and the marginal totals were computed by subtracting 95]. Obtain the ANOVA table and identify the effects that influence the mean response at the 5% level.

Percentage of Hardwood Concentration i	Cooking Time 1.5 Hours k			Cooking Time 2.0 Hours k			$y_{i...}$
	Freeness (F)			Freeness			
	400	500	650 j	400	500	650 j	
10	1.6 1.0	2.7 1.0	4.4 4.8	3.4 3.6	4.6 5.4	5.6 5.9	44.0
15	3.5 2.2	1.0 1.9	3.4 2.6	2.5 3.1	3.7 3.0	4.6 4.0	35.5
20	2.5 1.6	0.6 1.2	2.4 3.1	2.6 3.4	2.0 2.8	3.5 4.8	30.5
Column Totals	12.4	8.4	20.7	18.6	21.5	28.4	$y_{....}$ = 110.0

Borrowed from Hines and Montgomery [4]. ANS: SS_{CxT} = 2.08165, SS_{Error} = 6.58 SS_{CxF} = 6.0911, SS_{CxFxT} = 1.9733.

Some factorial designs, such as 2^k and 3^k, have specific practical value. The 2^k design, where there are k factors each at 2 levels, is generally used in screening experiments to identify which of the k factors significantly affect the response y. The joint effects of the significant factors are subsequently studied in a more elaborate design, such as 3^k, where the nonlinearity effects of the significant factors can be determined.

Further, for the purposes of block confounding and fractional replications, it turns out that the design base, in a b^k factorial, has to be a prime. If b is not a prime number, then the SS_{AxBxC} does not break down into additive orthogonal components. Then use has to be made of pseudo factors for the purposes of block confounding and fractionalizing.

As an example, consider a 3^3 design, where b = 3 is indeed a prime number. Then it can be shown that SS_{AxBxC} decomposes into the following 4 orthogonal components: AB^2C^2, ABC^2, AB^2C and ABC each with 2 degrees of freedom. Then such an orthogonal breakdown makes confounding in 3 blocks of 9 TCs each possible. Similarly, if only 1/3 fraction of a 3^3 design can be run, then any of the above 2-df components of AxBxC interaction can be used as the generator of the design.

For further detail, see Chapters 10 and 11 of D.C. Montgomery, 3rd Ed., "Design and Analysis of Experiments." Since Taguchi's orthogonal designs are so closely related to fractional factorials, we review fractional factorial designs in the next section.

Exercise 4.4. Construct a 2^2 design (with n=1 observation per cell) for which the effects B and AxB are ~~significant~~ large while A is not.

Exercise 4.5. Construct a 2^3 design such that the effects B, C, AC, ABC, are significant while A, AB and BC are not. $n = 1$

Exercise 4.6. Consider the data of Example 12-1 pages 392-395 of D.C. Montgomery 3rd edition. (a) Compute the linear and quadratic effects of factors A and C. (b) Compute the effect $A_L \times C_Q$. (c) Use the contrast function

$$\xi = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$$

to compute SS's due to ABC, ABC^2, AB^2C, AB^2C^2 and check to see if these SS's sum to SS_{AxBxC}.

TABLE V Percentage of the F Distribution (*continued*)

$F_{.05, \upsilon_1, \upsilon_2}$

Degrees of freedom for the numerator (υ_1); rows: Degrees of freedom for the denominator (υ_2)

υ_2 \ υ_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.63
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.28	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	7.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.66	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.08	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.38	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.28	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.48	1.39	1.32	1.22	1.00

Note that this table provides threshold values for the statistic F_0, i.e., an effect will be declared significant at the 5% level only if its F_0 statistic exceeds $F_{.05}(\upsilon_1, \upsilon_2)$.

TABLE V Percentage Points of the F Distribution (*continued*)

$F_{.01,\, \upsilon_1,\, \upsilon_2}$

υ_2 \ υ_1 Degrees of freedom for the denominator (υ_2)	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.00	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.48
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.87	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.61	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.88	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.88	6.51	5.58	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.36	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.76	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.59
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.48	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.98	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

CHAPTER V

1. FRACTIONAL FACTORIAL DESIGNS

As the number of factors in a 2^k, 3^k, 5^k,..., factorials increases, the number of experimental runs (or observations) required for one complete replicate rapidly outgrows the resources of most experimenters. One complete replicate of a 2^7 factorial requires 128 experimental observations. The problem with such a 2^7 design is 2-fold:

(1) perhaps too many observations giving rise to prohibitive cost.

(2) Out of the 127 degrees of freedom (df), only 7 are absorbed by the main factors and only 21 are absorbed by the 1st-order interactions AB, AC, ..., FG. The remaining 99 df correspond to 3-way, 4-way, ..., 7-way interactions. These high order (3-way or higher) interactions, if found significant, are difficult to interpret and in most cases are assumed nonexistent. Therefore, their SS's are generally pooled together and used as the error term in the ANOVA table.

The above problem worsens more rapidly for the 3^k and 5^k series; e.g., one full replicate of a 3^5 factorial requires 243 experiments. But out of the 242 total df, only 10 are absorbed by the main factors and 40 df correspond to 2-way interactions and the other 192 correspond to HOI's (high order interactions).

If it can be reasonably assumed that HOI's are negligible, then information on the main factors and 1st-order interactions can be gleaned by running only a fraction of one replicate. Such designs are called fractional factorials. Fractional factorial designs are widely used in industrial research as screening experiments where many factors are considered with the purpose of identifying those factors (if any) that have large effects relative to others. The factors that are identified as having large effects on the response, y, are then investigated more completely in subsequent experiments.

$\frac{1}{2}$ FRACTION of a 2^k FACTORIAL

Consider a 2^4 factorial (4 factors each at 2 levels) and for the sake of discussion assume that the experimenter can afford to conduct only 8 experiments. This leads to a 1/2 replicate of a 2^4 factorial and clearly $1/2(2^4) = 2^{4-1} = 2^3 = 8$. Therefore, we will have 2 fractions each forming one block. Using I = AxBxCxD as the generator of the design, the contrast function is

$$\xi = x_1 + x_2 + x_3 + x_4.$$

The contrast function $\xi = x_1 + x_2 + x_3 + x_4$ and the use of mod 2 algebra leads to the following 2 fractions:

$\xi = 0$:	(1), ab, ac, ad, bc, bd, cd, abcd
$\xi = 1$:	a, b, c, d, abc, abd, acd, bcd

where acd represents the TC (1,0,1,1), etc.

To understand what information will be lost in one of the above two 1/2 replicates, consider the table below where a + sign indicates the factor is at its high level (i.e., level 1) and a minus sign indicates the factor is at its low level (i.e., level 0).

Treatment Combinations	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
(1)*	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
a	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
b	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
ab*	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
c	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
ac*	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
bc*	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
abc	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
d	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
ad*	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
bd*	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
abd	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
cd*	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
acd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
bcd	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
abcd*	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Note that (AB)(BC) = AB^2C = AC so that exponents are obtained using modulus 2 algebra.

$$A^2 = 1 = A^0$$

In the above table we have put an * above every TC that pertains to the block with $\xi = 0$. Furthermore, in those TCs with an asterisk, all the signs of I = ABCD are positive. This means that ABCD is the identity element for this fractional factorial design. To further understand this concept, the table below gives the signs for the 1/2 replicate with $\xi = 0$.

TC	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	I = ABCD	Observed values
(1)	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	8
ab	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	19
ac	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	28
bc	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	14
ad	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	18
bd	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	10
cd	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	7
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	21

Consider now the effect of A from the above table.

$Contrast_A = ab + ac + ad + abcd - (1) - bc - bd - cd = 47$. But we also observe that the effect of BCD interaction is

$$BCD = -(1) + ab + ac - bc + ad - bd - cd + abcd = 47$$

Therefore, if we run the 8 experiments indicated by the block $\xi = 0$, then in the ANOVA table we cannot distinguish between the effects A and BCD, i.e., A and BCD will have exactly the same SS's. Two or more effects that have the same numerical value (and thus the same SS's) are called aliases.

Similarly, aliases of B, C, D, AB, AC, BC effects are ACD, ABD, ABC, CD, BD, and AD, respectively. A simple way to obtain the alias structure of a fractional factorial design is to multiply an effect by its identity elements mod (2, 3, 5,...), depending on the base b of the design.

In the above example the identity element is I = ABCD. Therefore the alias of A is

$A = A \times I = A(ABCD) = A^2BCD = BCD.$

$AB = AB \times I = AB(ABCD) = CD$, etc.

If we choose to run the TCs in the block with $\xi = 1$, the table of signs on page 41 shows that A = - BCD, B = -ACD, ..., BC = -AD, i.e., the contrast for A is the negative of that of BCD, etc. However, still $SS_A = SS_{BCD}$, $SS_B = SS_{ACD}$, ... so that the same alias structure will remain in tact.

Now consider the responses given in the table on page 42. Then

Contrast of A = (19 + 28 + 18 + 21) - (8 + 14 + 10 + 7) = 47

B = 19 + 14 + 10 + 21 - (8 + 28 + 18 + 7) = 3

C = 28 + 14 + 7 + 21 - (8 + 19 + 18 + 10) = 15

D = 18 + 10 + 7 + 21 - (8 + 19 + 28 + 14) = 13

Contrast of A x B (or CD) = (8 + 19 + 7 + 21) - (28 + 14 + 18 + 10) = -15

AC (or BD) = + 9, and AD (or BC) = 3.

Relatively influential effects are: A, C, D and C x D.

2. GENERAL RULES FOR CONSTRUCTING A 2^{k-p} FRACTIONAL FACTORIAL DESIGN.

1. Select the identity elements such that no main factor is aliased with any other main factor. As a matter of fact, the higher the interaction order of the aliases of the main effects (i.e., main factors and their 1st order interactions) are, the higher will be the resolution of the design.

2. For a $1/2^p$ fraction of a 2^k factorial, exactly p independent generators must be selected for the defining relation (or the identity elements) I. The total number of elements in I is

$$p + {}_pC_2 + {}_pC_3 + \ldots + {}_pC_p = 2^p - 1,$$

where

$${}_pC_i = \binom{p}{i} = \frac{p!}{i!(p-i)!}$$

To obtain the other ${}_pC_2 + {}_pC_3 + \ldots + {}_pC_p = 2^p - p - 1$ interactions, called the generalized interactions, multiply the p chosen independent generators 2 at a time, 3 at a time, ..., p at a time (mod 2) until the entire set of $2^p - p - 1$ generalized interactions is identified.

3. Then use the $2^p - 1$ words in I to obtain the aliases of each effect. This is done by multiplying the effect by each word in I (mod 2).

Example 5.1 Construct a 1/4th fraction of a 2^6 factorial.

Solution $1/4\ (2^6) = 2^{6-2} \rightarrow p = 2$.

Thus we must select 2 independent HOI's as the generators of the design. So, let I = ABCD and CDEF.

Number of generalized interactions $= 2^p - 1 - p = 1$.

$$(ABCD)(CDEF) = ABC^2D^2EF = ABEF \pmod 2.$$

The total number of words in I is $2^p - 1 = 3$, which are: I = ABCD, CDEF and ABEF. This further implies that each effect will have exactly 3 aliases.

The alias of A is BCD, ACDEF, BEF, i.e.,

$$A = BCD = ACDEF = BEF.$$

Similarly

$$B = BI = B(ABCD) = ACD$$

$$B = BI = B(CDEF) = BCDEF$$

$$B = BI = B(ABEF) = AEF,$$

and so on.

$$AB = CD = ABCDEF = EF$$

$$BD = AC = BCEF = ADEF$$

$$BC = AD = BDEF = ACEF, \text{ etc.}$$

Note that the resolution for the design is IV since the minimum length of the words in I is 4 letters.

Finally to actually set up a $\frac{1}{2^p}\ 2^k$ fraction, simply write the plus-minus signs for a complete replicate of a 2^{k-p} and determine the signs for the remaining p factors using the words in the identity I. This is illustrated for the above example.

I = ABCD, CDEF, ABEF

a 2^{6-2}_{IV} design

A	B	C	E	D=ABC	F=CDE,	TC	The response y
-	-	-	-	-	-	(1)	4
+	-	-	-	+	+	adf	7
-	+	-	-	+	+	bdf	15
+	+	-	-	-	-	ab	8
-	-	+	-	+	-	cd	9
+	-	+	-	-	+	acf	14
-	+	+	-	-	+	bcf	20
+	+	+	-	+	-	abcd	6
-	-	-	+	-	+	ef	9
+	-	-	+	+	-	ade	2
-	+	-	+	+	-	bde	5
+	+	-	+	-	+	abef	13
-	-	+	+	+	+	cdef	17
+	-	+	+	-	-	ace	11
-	+	+	+	-	-	bce	10
+	+	+	+	+	+	abcdef	16
							Total = 166

2 3

g_1 =

Assuming interactions of order 3 and higher are nonexistent, the average effects of the main factors and 1st-order interactions are as follows:

Ave. Eff. of (A + BCD + BEF) = [(7 + 8 + 14 + 6 + 2 + 13 + 11 + 16) -

$$- (4 + 15 + 9 + 20 + 9 + 5 + 17 + 10)]/8 = \frac{-12}{8} = -1.5.$$

Similarly

Ave. Effect of (B + ACD + AEF) = 2.50

Ave. Effect of (C + ABD + DEF) = 5.0

Ave. Effect of (D + ABC + CEF) = -1.5

Ave. Effect of (E + CDF + ABF) = 0.0

Ave. Effect of (F + CDE + ABE) = 7.0

Ave. Effect of (AB + CD + EF) = [(4 + 8 + 9 + 6 + 9 + 13 + 17 + 16) -

- (7 + 15 + 14 + 20 + 2 + 5 + 11 + 10) = .25

Ave. Effect of (AC + BD) = -6/8 = -.75

Ave. Effect of (AD + BC) = -18/8 = -2.25

Ave. Effect of (AE + BF) = 14/8 = 1.75

Ave. Effect of (AF + BE) = -10/8 = -1.25

Ave. Effect of (CE + DF) = 10/8 = 1.25

Ave. Effect of (CF + DE) = 0.75.

Possible important effects are: B, C, F, BxC and BxF

The above effects are summarized in the following response table (RT).

Effect	A	B	C	D	E	F	AB=CD=EF	AC=BD	AD=BC	AE=BF	AF=BE	CE=DF	CF=DE	ACE=BDE=ADF=BCF	ACF=BDF=ADE=BCE
ℓ_o	89	73	63	89	83	55	84	86	92	76	88	78	80	81	83
ℓ_1	77	93	103	77	83	111	82	80	74	90	78	88	86	85	83
$\lvert\ell_1-\ell_o\rvert$	12	20	40	12	0	56	2	6	18	14	10	10	6	4	0
R_i	6½	3	2	6½	14½	1	13	10½	4	5	8½	8½	10½	12	14½

Note that in the above RT the strongest effect (F) has received a rank of 1 and the 2nd most influential effect was assigned a rank of 2, and so on.

If the response, y, were an STB type performance characteristic (PCH), then the optimal conditions would be $A_1B_0C_0D_1E_7F_0$. However, if y were an LTB type PCH, then $X_o = A_0B_1C_1D_0E_7F_1$, where both BxC and BxF were examined in determination of X_o.

Exercise 5.1 Construct a 2_{IV}^{7-3} fractional factorial design where 2 of the generators are ABCD, CDEF and the remaining generators are determined such that the design has a resolution R = IV. Note that in a resolution IV design all main factors A, B, C, ... are free from 2-way interactions, while in an R = III design main factors are aliased with 2-way interactions. The experimental layout is give below.

(1) A	(2) B	(3) C	(4) E	D=	F=	G=	TC	Response y (STB)
-	-	-	-					3
+	-	-	-					-5
-	+	-	-					0
+	+	-	-					7
-	-	+	-					12
+	-	+	-					15
-	+	+	-					9
+	+	+	-					-1
-	-	-	+					14
+	-	-	+					6
-	+	-	+					3
+	+	-	+					-3
-	-	+	+					7
+	-	+	+					8
-	+	+	+					12
+	+	+	+					13

Total = 100.00

(a) Obtain the aliases of B and BxE.

(b) Find the average effect of B, E and B x E.

(c) Determine the optimal conditions assuming that y is an STB type QCH.

Exercise 5.2 In determining which of four factors A, B, C, or D might affect the underpuffing of cereal in a plant, the following one-half replicate was run. Assume y is an LTB type QCH.

		A_0		A_1	
		B_0	B_1	B_0	B_1
C_0	D_0			96.6 (a)	125.7 ab
	D_1	43.5 d	22.4 bd		
C_1	D_0	14.1 c	9.5 bc		
	D_1			28.8 acd	52.5 abcd

a. Determine which interaction was the generator of this design.

b. Analyze the data, i.e., develop a RT and determine the optimal FLC, and comment on the inadequacy of the above design.

3. FRACTIONAL FACTORIALS FOR BASE-3 DESIGNS

For the sake of illustration, suppose we are to study the joint effects of 3 controllable factors: post-inflation time (A), pressure (B) and rim-spacing (C) on the shoulder-drop (y) of a tire. The 3 fixed levels of A are 7, 14 and 21 minutes corresponding to levels 0, 1 and 2, respectively. The 3 levels of B are 25, 30 and 35 psi, which we refer to as levels 0, 1, and 2, respectively. The 3 levels of C are 5, 5½ and 6 inches → (0, 1, 2). Thus we have a 3^3 factorial and one full replicate will require 27 experimental runs performed in a completely randomized order.

If our resources allow only 9 experiments, then we can afford to conduct a 1/3 fraction of this 3^3 factorial. However, we must design in such a manner that the main effects, A, B and C are not aliased with one another. In order to design this $1/3\ (3^3)=3^{3-1}$ fractional factorial, we must first observe that there are 3 fractions each with 9 TCs, and therefore we must confound 2 df with blocks. This implies that our generator, I, must have exactly 2 df. The candidates for confounding purposes are: AB, AB^2, AC, AC^2, BC, BC^2, ABC, AB^2C, ABC^2, AB^2C^2. If we select AC^2 as the identity I, then the main factors A and C become aliased. To give our design the highest resolution, we select one of the effects ABC, AB^2C, ABC^2 or AB^2C^2. Selecting $I = AB^2C$ as our defining relation, the contrast function (CF) is $\xi = X_1 + 2X_2 + X_3$ and the 3 fractions are obtained according to $\xi = 0, 1, 2 \pmod 3$.

<u>Example 5.2</u> Construct a 3^{3-1}_{III} design using $I = AB^2C$ as its generator. As stated earlier, the contrast function is $\xi = X_1 + 2X_2 + X_3$, and since the elements of base 3 are 0, 1 and 2, the possible values of ξ are 0,1,2 leading to the three fractions. To determine which TC (000, 001, 010, 011, ... , 212,

222) goes into which blocks, we compute the values of ξ for each TC using mod 3 algebra. For the TC, 000, the value of $\xi = 0 + 2x0 + 0 = 0$. For 011, $\xi = 0 + 2 + 1 = 3 = 0$ (mod 3). Similarly $\xi = 0$ mod (3) for TCs 022, 110, 102, 121, 201, 212, 220. Hence the principle fraction consists of the 9 TCs given below.

$\xi = 0$	000, 011, 022, 110, 102, 121, 201, 212, 220

To generate the fraction with $\xi = 1$, it is sufficient only to obtain one of its elements and use the principle block (i.e., the one with $\xi = 0$) to generate the rest. Clearly the TC (100) belongs to the fraction with $\xi = 1$. Adding 100 to the nine elements of the principle fraction yields: 100 + 000 = 100, 100 + 011 = 111, 100 + 022 = 122, 210, 202, 221, 001, 012, 020. The remaining nine TCs belong to the fraction with $\xi = 2$.

<u>Exercise 5.3</u> The experimenter randomly decides to run the fraction with $\xi = 2$ and obtains the coded data below.

B	0			1			2		
A \ C	0	1	2	0	1	2	0	1	2
0			13.4	13.9				12.6	
1		14.8				12.8	13.1		
2	15.3				15.6				16.4

where A $\triangleq$ time, B $\triangleq$ Pressure, C = rim-spacing, and y in inches/10. Then the linear effect of A = A_ℓ = -1(39.9) + 0(40.7) + 1(47.3) = 7.4, and its quadratic effect is $A_q = 1(39.9) - 2(40.7) + 1(47.3) = 5.8$. Similarly, the student should compute B_ℓ, B_q, C_ℓ and C_q and obtain the aliases of A, B, and C.

4. $1/3^P$ FRACTION OF A 3^K FACTORIAL

Suppose in the example of page 49 there is another factor D, such as Breaker Building Drum Diameter, affecting the response variable y = shoulder drop, measured in 10th of an inch. Then the experimenter is faced with a 3^4 factorial and one complete replicate will require 81 observations taken in a completely random order. If our resources allow only 9 experiments, then we have a 1/9th fraction of a 3^4 factorial or simply a 3^{4-2}_{II} fractional factorial design, where p = 2. The steps for designing a 3^{k-p} design are as follows:

1. Select p HOI's as generators of the design such that the main factors A, B, C, ..., K are not aliased with each other. Call these g_1, g_2, ..., g_p.

2. Obtain the remaining generators in I by multiplying the p generators in step 1 (mod 3) in the manner below:

$$g_1g_2,\ g_1g_2^2,\ g_1g_3,\ g_1g_3^2,\ \ldots,\ g_2g_3,\ g_2g_3^2,\ \ldots$$

The defining relation, I, will have the p independent initially chosen effects plus their $(3^p - 2p - 1)/2$ generalized interactions (each one called a word). That is, the defining relation will have $(3^p - 1)/2$ HOI's, and the resolution of the design is equal to the length of the shortest word in I.

3. Use the p independent generators to partition the 3^k TCs into 3^p blocks. Then select one of these blocks at random to run as a 3^{k-p} fractional factorial design.

4. Obtain the aliases of each effect by multiplying the effect by the

terms in I and I^2 (mod 3).

Example 5.3 Construct a 3^{4-2} fractional factorial design.

Step 1. $p = 2$, $g_1 = AB^2C$, $g_2 = BCD^2$

Step 2. $g_3 = g_1g_2 = AC^2D^2$, $g_4 = g_1g_2^2 = ABD$

Step 3. $(3^p - 1)/2 = 4$, $I = AB^2C, BCD^2, AC^2D^2, ABD$.

Note that this is a resolution III design because the shortest word in I has length 3.

We arbitrarily use $g_1 = AB^2C$ and $g_2 = BCD^2$ to partition the 81 TCs into 9 blocks of 9 TCs each. For $g_1 = AB^2C$, $\xi_1 = x_1 + 2x_2 + x_3$, and for $g_2 = BCD^2$, $\xi_2 = x_2 + x_3 + 2x_4$.

The principle fraction is the one for which $\xi_1 = \xi_2 = 0$. The TCs in the principle fraction are

0000, 1101, 1022, 2011, 0112, 0221, 2202, 1210, 2120.

Step 4. Using Step 3 results, the aliases of A are

I: $ABC^2, ABCD^2, ACD, AB^2D^2$

I^2: BC^2, AB^2C^2D, CD, BD .

Exercise 5.4 List the ξ values for the other 8 fractions of the above 3^{4-2} design. List the TCs in the fractions whose CF values are $\xi_1 = 2$ and $\xi_2 = 1$.

Exercise 5.5 For the Example 5.3, obtain the aliases of C and AD^2.

Exercise 5.6 A company manufacturing engines was concerned about the emission characteristics of these engines. To try to minimize various undesirable emission variables, the company proposed to build and operate experimental engines varying five controllable factors: throat diameter to be set at three levels, ignition system at three levels, temperature at three levels, velocity of the jet stream at three levels, and timing system at three levels. As this would require 243 experimental runs, it was decided that a one-9^{th} ~~replicate~~ fraction would be conducted. (a) Set up a fractionalizing scheme for this problem and indicate a few terms in the principal block. Also show an ANOVA outline when running one of the 9 fractions, indicating some aliases. Comment on the design. (b) Comment on the deficiency of the design with generators I = ABC^2DE and BDE. (c) Taking $g_1 = ACDE^2$ and $g_2 = BC^2DE^2$, obtain the remaining generators of the $1/9^{th}$ fraction and list 9 of the TCs for which $\xi_1 = 1$ and $\xi_2 = 0$. Give the aliases of A and BC^2.

Note that Exercises 5.2 and 5.6 were borrowed from Fundamental Concepts in the Design of Experiments, 3rd Ed., by Charles R. Hicks.

CHAPTER VI

1. TAGUCHI'S ORTHOGONAL ARRAYS

It has been pointed out earlier that as the number of factors in an industrial experiment increases, the required number of experimental runs for one complete replicate rapidly increases. As Taguchi notes "there are often too many factors that affect the output in most industrial settings", and therefore running even a fraction of a b^k ($b = 2, 3, 5, 7$) factorial can easily become too time-consuming and cost prohibitive! For example, 1/9th fraction of a 3^7 factorial requires 243 runs. It is clear that if k (= no. of factors) exceeds 4, then we must frequently settle for a fraction of a full replicate. As noted in Chapter V, such designs are called fractional factorials. As in the case of all classical experimental designs, fractional factorials require complete randomization over all observations to average out the effect of noise. However, Taguchi has developed fractional designs that generally imbed noise factors in the outer OA, and are ready/easy to use, so that randomization is generally unnecessary. Dr Taguchi's inner and outer orthogonal arrays, depicted in Figure 8, p. 27, form the basis for a PDE. It turns out that almost all Taguchi's OAs are either fractional factorials or closely linked to them.

Definition According to Taguchi, an array is orthogonal (or balanced) if the number of times that an (i,j) pair appears in any 2 columns is the same for all $i,j = 1,2,\ldots,b$ where b is the number of levels of a factor. As an example, see the L_8 OA below.

L_8 OA

H

TC \ Columns	(1) A	(2) B	(3) C	(4) D	(5) E	(6) F	(7) G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2
	Group 1	Group 2			Group 3		

Use the definition above to prove to yourself that the L_8 array is indeed orthogonal.

$L_8(2^7)$

The most common Taguchi's OAs for the 2^k series are provided on pages 58 thru 64. The notation L_8 (2^7) means that up to 7 effects can be studied with a total of 8 experimental runs, where all factors are at 2 levels. Similarly, L_{16} (2^{15}) means that with the aid of 16 runs we can study up to 15 effects where all factors are at 2 levels. Since each effect in a 2^k series carries 1 df, then each column in an $L_N(2^k)$ will have exactly 1 df.

Taguchi's linear graphs, obtained from interactions between two columns (IBTCs) tables, are used to assign interaction effects to different columns of an OA. In fact, the assignments (of interactions) can be made directly from the IBTC tables so that linear graphs are unnecessary for a Taguchi design. Generally, the main factors A, B, C,... may be assigned arbitrarily, but once A and B are assigned to two different columns, say (2) and (4), of an L_8 array, then the interaction AB must be assigned to the column provided by the

IBTCs table, which is given at the top of page 58 on the RHS. That IBTCs table shows that AB must be assigned to column 6. Similarly if we further assign C to column 3, then AxC must be assigned to column 1.

<u>Exercise 6.1</u> For the example in the last paragraph, determine the column assignment for AB if A→(3) and B is assigned to column (7).

<u>Example 6.1</u> Suppose we have four 2-level controllable factors A, B, C, D and also wish to study the interaction effects AC, AB and AxD. Our objective is to make proper assignments to a Taguchi OA.

<u>Solution</u>

<u>Step 1</u> Design Considerations.

The seven effects A, B, C, D, AB, AC and AD carry 7 degrees of freedom and hence the minimum number of required runs is 8. Note that each experimental observation carries one df and 1 df is always lost to the constraint

$\sum_{i=1}^{N} (y_i - \bar{y}) = 0$ so that the 8 runs carry 7 df. This suggests that an L_8 OA is adequate.

<u>Step 2</u> Column Assignments of Effects.

Since the main factor A is more involved in the 3 interactions, we 1st assign it to Column (1). Next, arbitrarily assign B to Column (2). Now the IBTCs table at the top of page 58 dictates that we must assign AxB to column 3. If we assign C to column 4, then the same IBTCs table shows that we must assign AC to column 5. Therefore, one possible complete set of assignments is as follows.

Effects	A	B	AB	C	AC	AD	D
Columns	(1)	(2)	(3)	(4)	(5)	(6)	(7)

<u>Exercise 6.2</u> Obtain another Taguchi design for the 7 effects of Example 6.1. That is, make different column assignments using the Table at the NE corner of page 58. For example, assign A to (5) and B to (3).

<u>Exercise 6.3</u> Make column assignments for the following experiments.

(a) 2-level factors: A,B,C,D,E; interactions BC,BE

(b) 2-level factors A,B,C,D; interactions AC, BD, AD.

(c) 2-level factors: A,B,C,D,E,F,G,H,I

Interactions: DE, EF, FG, EH, HI and BE.

(d) 2-level factors: A,B,C,D,E,F,G

Interactions: BD, DE, DF, EG, BC, BE, BG, FG.

$L_8(2^7)$

No. \ Column	1	2	3	4	5	6	7
	A	C	AC	D	AD	B	AB
I = BCD 1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2 → bc
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1 → abd
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2
	Group 1*	Group 2		Group 3			

* A factor that is placed in Group 1 is generally the one whose levels are most difficult to change in actual practice.

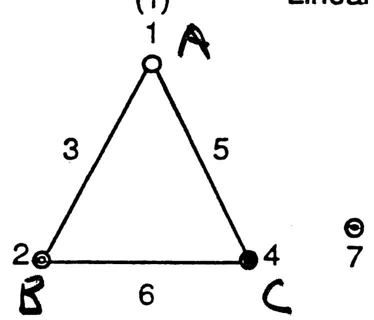
Interactions Between Two Columns

Column \ Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
A →	(1)	3	2	5	4	7	6
		(2)	1	6	7	4	5
			(3)	7	6	5	4
				(4)	1	2	3
					(5)	3	2
						(6)	1
							(7)

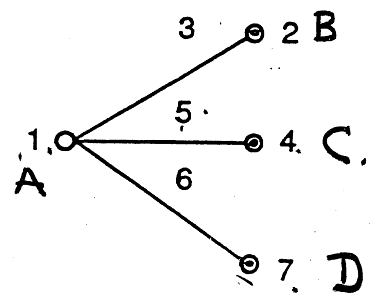
* This table is only for the L_8

Linear Graphs of L_8 Table

(1)



(2)



No. \ Column	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1
	Group 1		Group 2								

Note: The interaction between two columns confounds with the remaining 9 columns. It is necessary to use sequential analysis to find interactions. Therefore, do not use this table for the experiment which requires interactions.

Up to 11 factors may be studied with this array. No interaction can be studied with this table since 12 cannot be written as 2^{k-p} for integer values of k and p, i.e., L_{12} is not a complete fractional factorial.

Note that each column in a base-two design carries exactly 1 df.

$L_{16}(2^{15})$

No. \ Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
Groups	1	2		3				4							

This Interaction Between Two Columns is Only for L_{16}

Column \ Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
		(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
			(3)	7	6	5	4	11	10	9	8	15	14	13	12
				(4)	1	2	3	12	13	14	15	8	9	10	11
					(5)	3	2	13	12	15	14	9	8	11	10
						(6)	1	14	15	12	13	10	11	8	9
							(7)	15	14	13	12	11	10	9	8
								(8)	1	2	3	4	5	6	7
									(9)	3	2	5	4	7	6
										(10)	1	6	7	4	5
											(11)	7	6	5	4
												(12)	1	2	3
													(13)	3	2
														(14)	1

Interactions between Two Columns

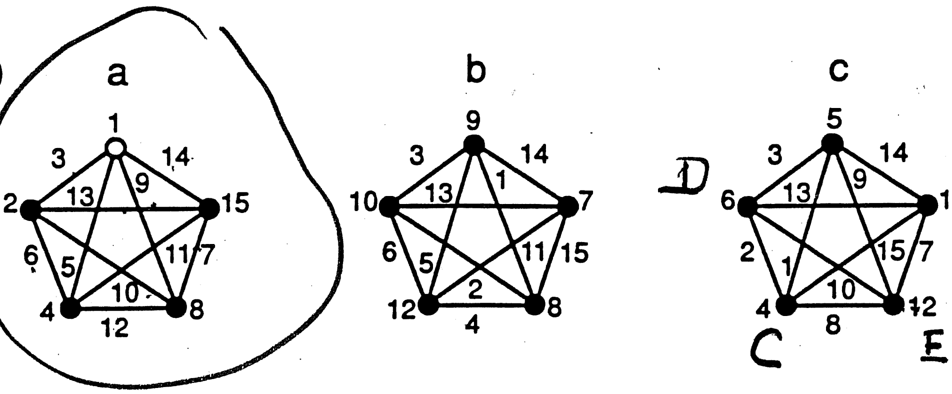
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
	(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
		(3)	7	6	5	4	11	10	9	8	15	14	13	12
			(4)	1	2	3	12	13	14	15	8	9	10	11
				(5)	3	2	13	12	15	14	9	8	11	10
					(6)	1	14	15	12	13	10	11	8	9
						(7)	15	14	13	12	11	10	9	8
							(8)	1	2	3	4	5	6	7
								(9)	3	2	5	4	7	6
									(10)	1	6	7	4	5
										(11)	7	6	5	4
											(12)	1	2	3
												(13)	3	2
													(14)	1

Note that the interaction of columns (7) and (12) is column (11), i.e., (7) x (12) = (11).

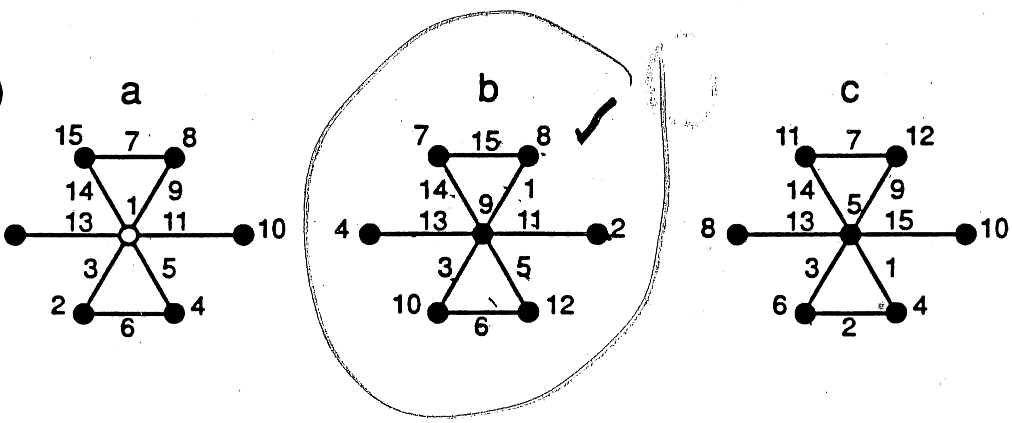
IBTC's Table for an $L_{16}(2^{15})$ O.A..

Linear Graphs
for L_{16} Only

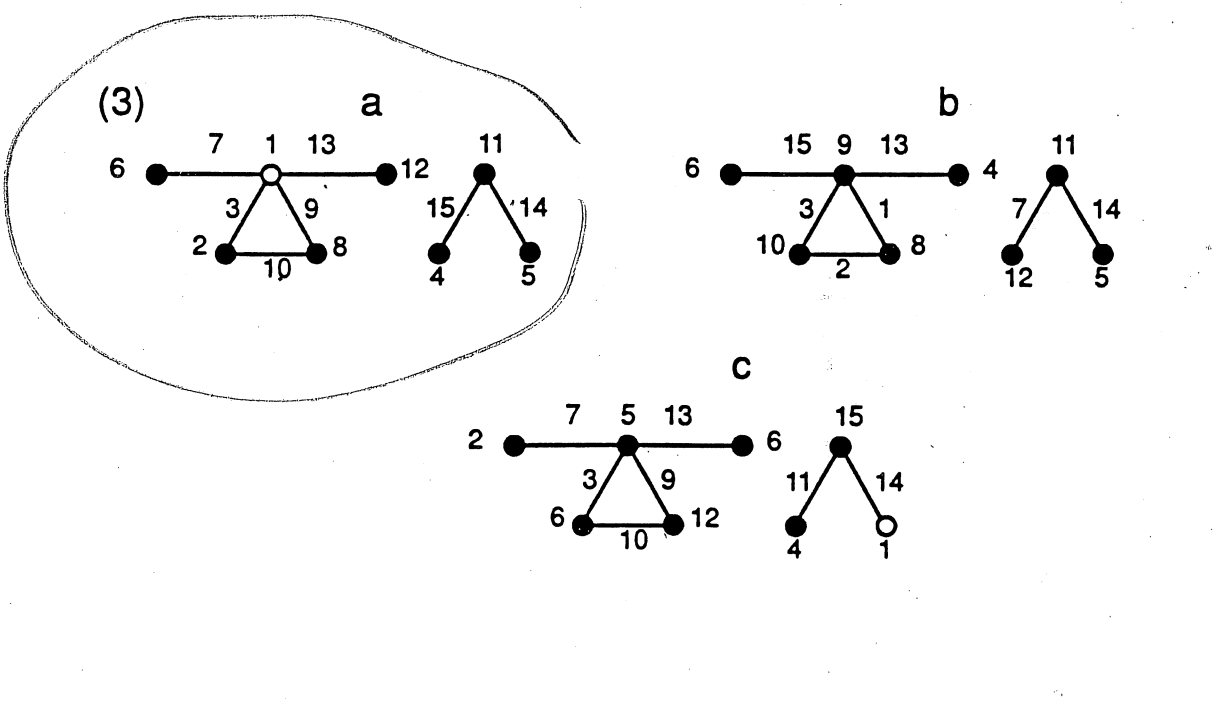
(1)



(2)



(3)

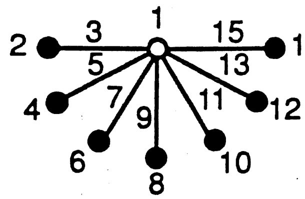


9-5
2 III
Interactions: AB, CD, EF, AG, CF, DG

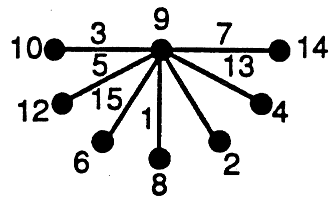
Linear Graphs for L_{16} Only

(4)

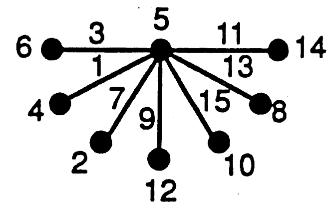
a



b

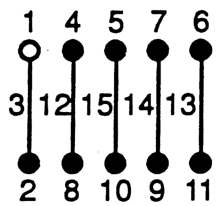


c

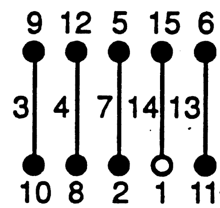


(5)

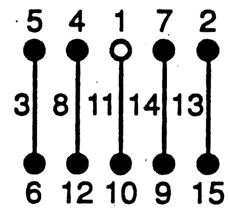
a



b

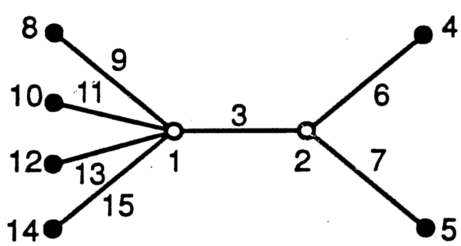


c

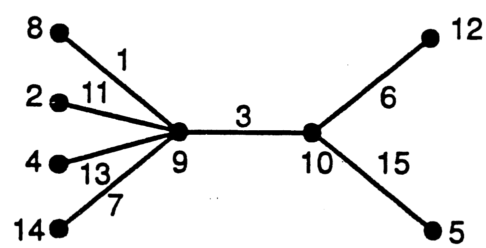


(6)

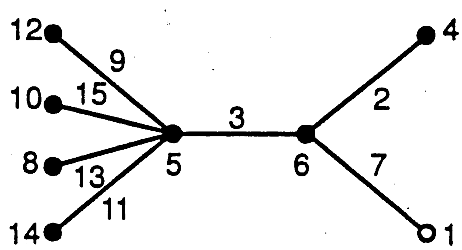
a



b



c



$L_{32}(2^{31})$

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
5	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
6	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
7	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
8	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
9	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
10	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
11	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
12	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
13	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
14	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
15	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
16	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
17	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
18	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
19	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
20	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
21	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
22	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
23	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
24	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
25	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
26	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
27	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
28	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
29	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
30	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
31	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
32	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2
	a	b	a b	c	a c	b c	a b c	d	a d	b d	a b d	c d	a c d	b c d	a b c d	e	a e	b c	a b c	c c	a c c	b c c	a b c c	d c	a d c	b d c	a b d c	c d c	a c d c	b c d c	a b c d e
	1	2		3				4								5															

For L_{32} only!

IBTC'S

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22	25	24	27	26	29	28	31	30
		(2)	1	6	7	4	5	10	11	8	9	14	15	12	13	18	19	16	17	22	23	20	21	26	27	24	25	30	31	28	29
			(3)	7	6	5	4	11	10	9	8	15	14	13	12	19	18	17	16	23	22	21	20	27	26	25	24	31	30	29	28
				(4)	1	2	3	12	13	14	15	8	9	10	11	20	21	22	23	16	17	18	19	28	29	30	31	24	25	26	27
					(5)	3	2	13	12	15	14	9	8	11	10	21	20	23	22	17	16	19	18	29	28	31	30	25	24	27	26
						(6)	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30	31	28	29	26	27	24	25
							(7)	15	14	13	12	11	10	9	8	23	22	21	20	19	18	17	16	31	30	29	28	27	26	25	24
								(8)	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16	17	18	19	20	21	22	23
									(9)	3	2	5	4	7	6	25	24	27	26	29	28	31	30	17	16	19	18	21	20	23	22
										(10)	1	6	7	4	5	26	27	24	25	30	31	28	29	18	19	16	17	22	23	20	21
											(11)	7	6	5	4	27	26	25	24	31	30	29	28	19	18	17	16	23	22	21	20
												(12)	1	2	3	28	29	30	31	24	25	26	27	20	21	22	23	16	17	18	19
													(13)	3	2	29	28	31	30	25	24	27	26	21	20	23	22	17	16	19	18
														(14)	1	30	31	28	29	26	27	24	25	22	23	20	21	18	19	16	17
															(15)	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
																(16)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
																	(17)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
																		(18)	1	6	7	4	5	10	11	8	9	14	15	12	13
																			(19)	7	6	5	4	11	10	9	8	15	14	13	12
																				(20)	1	2	3	12	13	14	15	8	9	10	11
																					(21)	3	2	13	12	15	14	9	8	11	10
																						(22)	1	14	15	12	13	10	11	8	9
																							(23)	15	14	13	12	11	10	9	8
																								(24)	1	2	3	4	5	6	7
																									(25)	3	2	5	4	7	6
																										(26)	1	6	7	4	5
																											(27)	7	6	5	4
																												(28)	1	2	3
																													(29)	3	2
																														(30)	1

Note that when O.A. columns are occupied only by controllable factors, then the design is said to be saturated. For example, an L_8 (2^7) with 7 controlling factors A,B,C,D,E,F and G is a saturated design.

2. Taguchi's Linear Graphs

Examples of linear graphs (LGs) are given on page 58 for L_8, and pages 61-62 for an L_{16} OA. The LG's are obtained directly from the IBTCs tables and they can be used to construct a Taguchi design. In an LG, a dot (•) represents a main factor (such as A,B,C, etc) and the line connecting 2 dots represents the corresponding 2-way interation. For example, suppose in an experiment, we assign the controllable factor, A, to column 2 of an L_8 array and factor B to column 5 of the same L_8 array. These assignments are depicted below:

```
                A          B
                ⊙          ⊙
     Column    (2)        (5)
```

First of all, the line connecting the above 2 dots will represent AxB, i.e.,

```
            A     AxB     B
            ⊙-------------⊙
           (2)           (5)
```

and further, the IBTCs table at the NE corner of page 58 shows that interaction of (2) with (5) is (7) so that our complete LG looks as follows.

```
            A             B
            •-------------•
           (2)     7     (5)
```

We emphasize again that in a Taguchi LG, a dot represent a main factor and a line always represents a 1st-order interaction. The student should now examine the LGs on pages 58, 61 and 62 to see if they are consistent with the corresponding IBTCs tables.

Taguchi's OAs and LGs for the 3^k series are given on pages 67-71. Note that some of these arrays are not the same as the classical fractional factorials. Attempt to see which of the OAs are not fractional factorials. Furthermore, there are more extensive and complicated OAs that are available in the literature so that the OAs presented on pages 58-71 are the basic and most widely used for the 3^{k-p} fractional factorial designs.

<u>Exercise 6.4</u> Design the following experiments using the proper Taguchi OA.

(a) 3-level factors: A, B, C, D, E, F, G, H, I
Interactions: AxD, AxE

(b) 2-level factors: A, B, C, D, E, F, G, H
Interactions: AB, AC, AD, AE, BC, CD, EF

(c) 6-level factor: A
3-level factors: B,C,D,E,F,G

(d) 2-level factors: A,B, C, D, E, F
4-level factors: G, H
Interactions: AB, BD, DE

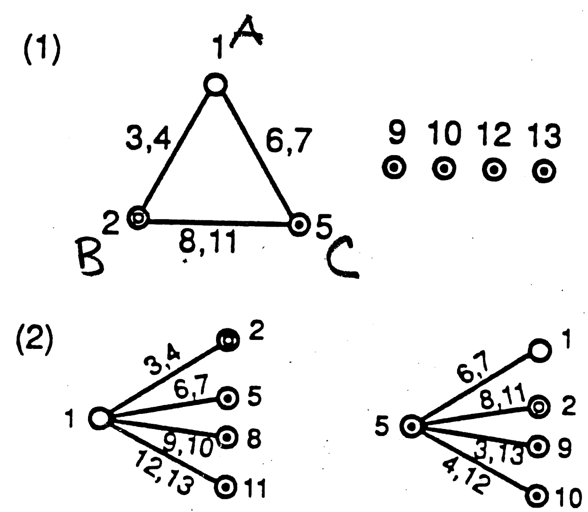
Draw the Taguchi LG only for part (b) above.

$L_{27}(3)^{13}$

Experiment #	1 (B/A)	2	3	4	5	6	7	8	9 (ABC)	10 (A^2BC)	11	12 (AB^2C)	13 (ABC^2)
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

Group 1: column 1; Group 2: columns 2–4; Group 3: columns 5–13

Linear Graphs



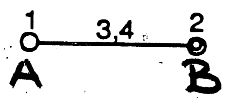
Interactions between Two Columns for L_{27}

1	2	3	4	5	6	7	8	9	10	11	12	13
(1)	3	2	2	6	5	5	9	8	8	12	11	11
	4	4	3	7	7	6	10	10	9	13	13	12
	(2)	1	1	8	9	10	5	6	7	5	6	7
		4	3	11	12	13	11	12	13	8	9	10
		(3)	1	9	10	8	7	5	6	6	7	5
			2	13	11	12	12	13	11	10	8	9
			(4)	10	8	9	6	7	5	7	5	6
				12	13	11	13	11	12	9	10	8
				(5)	1	1	2	3	4	2	4	3
					7	6	11	13	12	8	10	9
					(6)	1	4	2	8	3	2	4
						5	13	12	11	10	9	8
						(7)	3	4	2	4	3	2
							12	11	13	9	8	10
							(8)	1	1	2	3	4
								10	9	5	7	6
								(9)	1	4	2	3
									8	7	6	5
									(10)	3	4	2
										6	5	7
										(11)	1	1
											13	12
											(12)	1
												11

$L_9(3)^4$

Experiment #	1 (A)	2 (B)	3 ($C = AB$)	4 ($D = A^2B$)
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1
	Group 1	Group 2		

Linear Graph



mixed-level design

$L_{18}(2^1 x 3^7)$

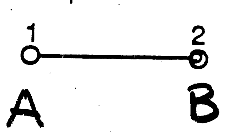
Experiment #	A (1)	B (2)	C (3)	D (4)	E (5)	F (6)	G (7)	H (8)
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1
Groups	1	2	3					

A x B

A \ B	1	2	3
1			
2			

Note that the main factors carry 15 df and A x B has 2 df.

Linear Graph



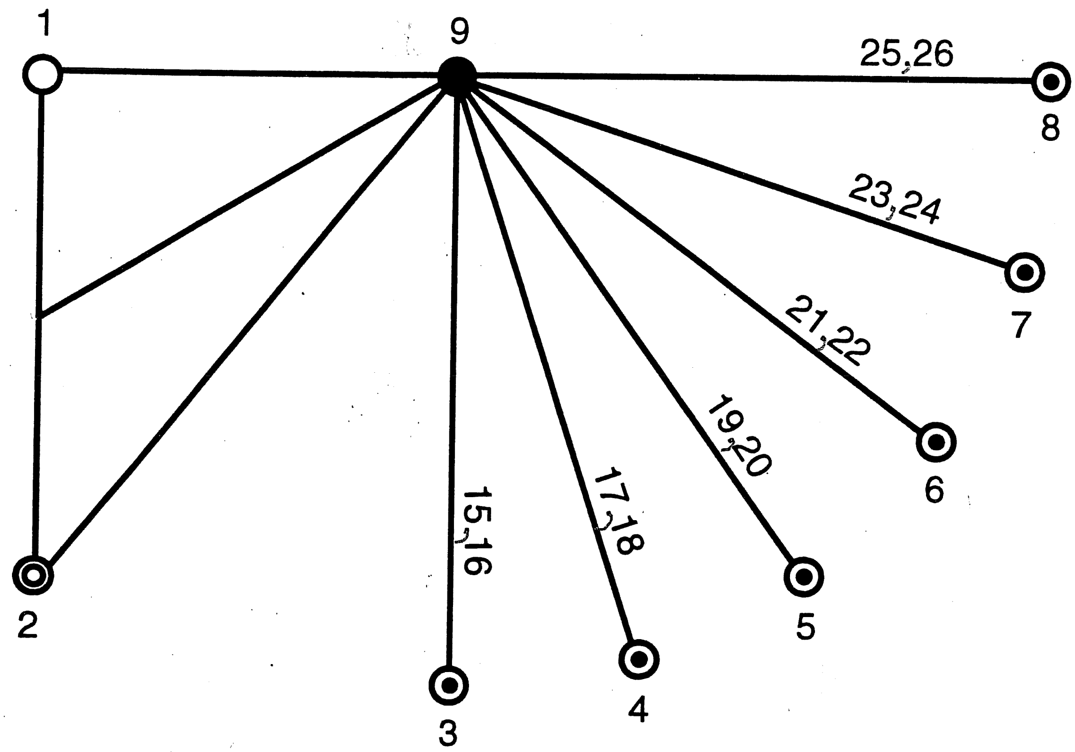
Note that no interaction can be imbedded in the L_{18} OA, but (1) x (2) can be studied after data has been collected.

$A \times B = 2$ df

$L_{54}(2^1 \times 3^{25})$

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	1	1	2	2	2	2	2	2	1	1	1	1	1	1	2	3	2	3	2	3	2	3	2	3	2	3
5	1	1	2	2	2	2	2	2	2	2	2	2	2	2	3	1	3	1	3	1	3	1	3	1	3	1
6	1	1	2	2	2	2	2	2	3	3	3	3	3	3	1	2	1	2	1	2	1	2	1	2	1	2
7	1	1	3	3	3	3	3	3	1	1	1	1	1	1	3	2	3	2	3	2	3	2	3	2	3	2
8	1	1	3	3	3	3	3	3	2	2	2	2	2	2	1	3	1	3	1	3	1	3	1	3	1	3
9	1	1	3	3	3	3	3	3	3	3	3	3	3	3	2	1	2	1	2	1	2	1	2	1	2	1
10	1	2	1	1	2	2	3	3	1	1	2	2	3	3	1	1	1	1	2	3	2	3	3	2	3	2
11	1	2	1	1	2	2	3	3	2	2	3	3	1	1	2	2	2	2	3	1	3	1	1	3	1	3
12	1	2	1	1	2	2	3	3	3	3	1	1	2	2	3	3	3	3	1	2	1	2	2	1	2	1
13	1	2	2	2	3	3	1	1	1	1	2	2	3	3	2	3	2	3	3	2	3	2	1	1	1	1
14	1	2	2	2	3	3	1	1	2	2	3	3	1	1	3	1	3	1	1	3	1	3	2	2	2	2
15	1	2	2	2	3	3	1	1	3	3	1	1	2	2	1	2	1	2	2	1	2	1	3	3	3	3
16	1	2	3	3	1	1	2	2	1	1	2	2	3	3	3	2	3	2	1	1	1	1	2	3	2	3
17	1	2	3	3	1	1	2	2	2	2	3	3	1	1	1	3	1	3	2	2	2	2	3	1	3	1
18	1	2	3	3	1	1	2	2	3	3	1	1	2	2	2	1	2	1	3	3	3	3	1	2	1	2
19	1	3	1	2	1	3	2	3	1	2	1	3	2	3	1	1	2	3	1	1	3	2	2	3	3	2
20	1	3	1	2	1	3	2	3	2	3	2	1	3	1	2	2	3	1	2	2	1	3	3	1	1	3
21	1	3	1	2	1	3	2	3	3	1	3	2	1	2	3	3	1	2	3	3	2	1	1	2	2	1
22	1	3	2	3	2	1	3	1	1	2	1	3	2	3	2	3	3	2	2	3	1	1	3	2	1	1
23	1	3	2	3	2	1	3	1	2	3	2	1	3	1	3	1	1	3	3	1	2	2	1	3	2	2
24	1	3	2	3	2	1	3	1	3	1	3	2	1	2	1	2	2	1	1	2	3	3	2	1	3	3
25	1	3	3	1	3	2	1	2	1	2	1	3	2	3	3	2	1	1	3	2	2	3	1	1	2	3
26	1	3	3	1	3	2	1	2	2	3	2	1	3	1	1	3	2	2	1	3	3	1	2	2	3	1
27	1	3	3	1	3	2	1	2	3	1	3	2	1	2	2	1	3	3	2	1	1	2	3	3	1	2
28	2	1	1	3	3	2	2	1	1	3	3	2	2	1	1	1	3	2	3	2	2	3	2	3	1	1
29	2	1	1	3	3	2	2	1	2	1	1	3	3	2	2	2	1	3	1	3	3	1	3	1	2	2
30	2	1	1	3	3	2	2	1	3	2	2	1	1	3	3	3	2	1	2	1	1	2	1	2	3	3
31	2	1	2	1	1	3	3	2	1	3	3	2	2	1	2	3	1	1	1	1	3	2	3	2	2	3
32	2	1	2	1	1	3	3	2	2	1	1	3	3	2	3	1	2	2	2	2	1	3	1	3	3	1
33	2	1	2	1	1	3	3	2	3	2	2	1	1	3	1	2	3	3	3	3	2	1	2	1	1	2
34	2	1	3	2	2	1	1	3	1	3	3	2	2	1	3	2	2	3	2	3	1	1	1	1	3	2
35	2	1	3	2	2	1	1	3	2	1	1	3	3	2	1	3	3	1	3	1	2	2	2	2	1	3
36	2	1	3	2	2	1	1	3	3	2	2	1	1	3	2	1	1	2	1	2	3	3	3	3	2	1
37	2	2	1	2	3	1	3	2	1	2	3	1	3	2	1	1	2	3	3	2	1	1	3	2	2	3
38	2	2	1	2	3	1	3	2	2	3	1	2	1	3	2	2	3	1	1	3	2	2	1	3	3	1
39	2	2	1	2	3	1	3	2	3	1	2	3	2	1	3	3	1	2	2	1	3	3	2	1	1	2
40	2	2	2	3	1	2	1	3	1	2	3	1	3	2	2	3	3	2	1	1	2	3	1	1	3	2
41	2	2	2	3	1	2	1	3	2	3	1	2	1	3	3	1	1	3	2	2	3	1	2	2	1	3
42	2	2	2	3	1	2	1	3	3	1	2	3	2	1	1	2	2	1	3	3	1	2	3	3	2	1
43	2	2	3	1	2	3	2	1	1	2	3	1	3	2	3	2	1	1	2	3	3	2	2	3	1	1
44	2	2	3	1	2	3	2	1	2	3	1	2	1	3	1	3	2	2	3	1	1	3	3	1	2	2
45	2	2	3	1	2	3	2	1	3	1	2	3	2	1	2	1	3	3	1	2	2	1	1	2	3	3
46	2	3	1	3	2	3	1	2	1	3	2	3	1	2	1	1	3	2	2	3	3	2	1	1	2	3
47	2	3	1	3	2	3	1	2	2	1	3	1	2	3	2	2	1	3	3	1	1	3	2	2	3	1
48	2	3	1	3	2	3	1	2	3	2	1	2	3	1	3	3	2	1	1	2	2	1	3	3	1	2
49	2	3	2	1	3	1	2	3	1	3	2	3	1	2	2	3	1	1	3	2	1	1	2	3	3	2
50	2	3	2	1	3	1	2	3	2	1	3	1	2	3	3	1	2	2	1	3	2	2	3	1	1	3
51	2	3	2	1	3	1	2	3	3	2	1	2	3	1	1	2	3	3	2	1	3	3	1	2	2	1
52	2	3	3	2	1	2	3	1	1	3	2	3	1	2	3	2	2	3	1	1	2	3	3	2	1	1
53	2	3	3	2	1	2	3	1	2	1	3	1	2	3	1	3	3	1	2	2	3	1	1	3	2	2
54	2	3	3	2	1	2	3	1	3	2	1	2	3	1	2	1	1	2	3	3	1	2	2	1	3	3
	1	2	3						4																	

$L_{54}(2^1 \times 3^{25})$



3. RESPONSE TABLE ANALYSIS (RTA) FOR ATTRIBUTE DATA

The following is an actual example of a tile manufacturing experiment, performed by Ina-Tile Company in 1953. The flow of the manufacturing process is:

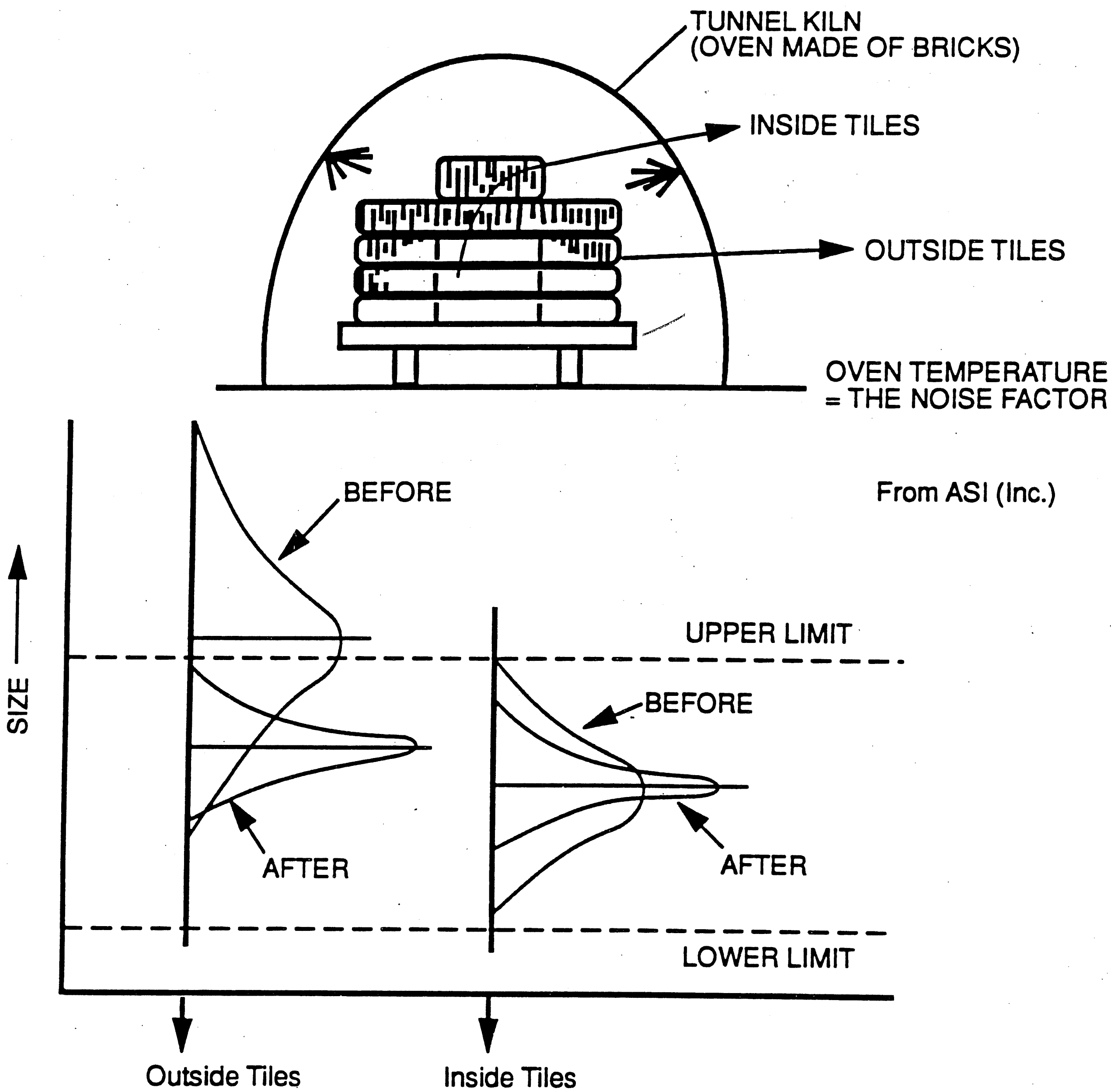
Raw material combination crushing and mixing → Moulding → Calcining → Glazing → Calcining

The purpose of the manufacturing scale experiment was to confirm the optimum raw material combination that had been obtained from the small-scale experiments in the laboratory. In this experiment, a one-ton actual manufacturing scale was used. As soon as the confirmation was made, they planned to adopt the combination for actual manufacturing.

All seven factors in the experiment concern the raw material combination. In factors A, B, C, D and E, one of the two levels denote the existing manufacturing conditions, and the other levels were expected to be better than the existing levels either by quality or cost.

Factor E does not aim for better quality but for better productivity. It compares the existing charging quantity, 1200 kg, with a larger quantity, 1300 kg. If there is no bad effect with 1300 kg, a productivity increase of about 10% will result. For productivity increase or cost-down purposes, we should cite those factors that may not affect quality very much by different levels but result in a large productivity difference. Through production field experiments, i.e., those experiments using actual production, equipment and production output, many benefits such as gains of millions of dollars having been reported. It is not an exaggeration to say that most of these benefits came from the discovery of factors that do not affect quality very much but

Tile Manufacturing Process



ASI ≜ The American Supply Institute.

Table 6.2 Layout by Orthogonal Array L8

	Orthogonal table $L_8(2^7)$	A	B	C	D	E	F	G	
Factor	A B C D E F G	Content of a certain lime(%)	Fine-ness	Content of agal-matolite (%)	Kind of agal-matolite	Charg-ing quantity (kg)	Content of waste return (%)	Content of feldaspar (%)	Number of defectives in 100 tiles=y
Column No.	1 2 3 4 5 6 7	1	2	3	4	5	6	7	
1	1 1 1 1 1 1 1	5	Coarser	43	Existing	1300	0=F_1	0	y_1=16
2	1 1 1 2 2 2 2	A_1 5	Coarser	43	New	1200	4=F_2	5	17=y_2
3	1 2 2 1 1 2 2	5	Finer	53	Existing	1300	4	5	12
4	1 2 2 2 2 1 1	5	Finer	53	New	1200	0	0	6 / 51
5	2 1 2 1 2 1 2	1	Coarser	53	Existing	1200	0	5	6*
6	2 1 2 2 1 2 1	A_2 1	Coarser	53	New	1300	4	0	68
7	2 2 1 1 2 2 1	1	Finer	43	Existing	1200	4	0	42
8	2 2 1 2 1 1 2	1	Finer	43	New	1300	0	5	26 / 142
								Total	193

Table 6.3 Effect of Each Factor (Response Table)

Factor and level	Total number of defectives	Fraction defective (%)
A_1 A_2	51 142	12.75 35.50
B_1 B_2	107 86	26.75 21.50
C_1 C_2	101 92	25.25 23.00
D_1 D_2	76 117	19.00 29.25
E_1 E_2	122 71	30.50 =122/400 17.75
F_1 F_2	54 139	13.50 34.75
G_1 G_2	132 61	33.00 15.25
overall percent defective = 24.125 =193/800		

Weak Factors:
B and C

Moderately Strong Factors:
E and D

Strong Factors:
A, F and G

Borrowed from "Introduction to Off-Line Quality Control" by G. Taguchi and Y. Wu make a big difference in cost when levels are changed.

Note that this example was borrowed from the text by Taguchi and Wu with title: "Introduction To Off-Line QC." We now list the actual factor levels.

Factors and levels

A: Content of a certain lime
A_1 = 5% (new suggestion) A_2 = 1% (existing condition)

B: Fineness of the additive
B_1 = Coarser (existing condition) B_2 = Finer (new suggestion)

C: Content of agalmatolite
C_1 = 43% (new suggestion) C_2 = 53% (existing condition)

D: Kind of agalmatolite
D_1 = Existing combination D_2 = New combination with lower cost

E: Raw material charging quantity
E_1 = 1300 kg (new suggestion) E_2 = 1200 kg (existing condition)

F: Content of waste return
F_1 = 0% F_2 = 4% (existing condition)

G: Content of feldspar
G_1 = 0% G_2 = 5% (existing condition)

The above seven factors were signed to an orthogonal array L_8 as shown on page 58; the experiments were carried out according to that layout. Table 6.2 (of Taguchi and Wu) shows the assignment and eight raw material combinations. The prepared raw materials were moulded and then calcined in a tunnel kiln.

In experimental run No. 1, all of the figures in the columns A, B, ..., G are "1"; this means that all the factors in this experiment are at their first (or low) level, expressed as $A_1B_1C_1D_1E_1F_1G_1$. Actually, the first level of A, or A_1, means "the content of a certain lime" is 5%, and B_1 means "the additive is prepared in the courser-ground condition." Similarly, C_1 means "to use 43% agalmatolite", D_1 means "to use the existing agalmatolite combination". F_1 and G_1 mean "to contain neither waste return nor feldspar," and E_1 shows that the "raw material charging quantity is 1300 kg.

Orthogonal array L_8 is a method of conducting eight particular experimental runs to impartially compare the effects between A_1 and A_2, B_1 and B_2 ..., G_1 and G_2 in a completely balanced (or orthogonal) manner.

One hundred pieces of tile were sampled from each one of the eight different treatment combinations (or eight runs). The figures in the right column of Table 6.2 show the number of defectives observed in each 100 tiles.

The object of this experiment was to select the combination of parameter (or factor) levels that minimized the number of defective tiles. Table 6.2 gives the experimental layout for an L_8 OA and the last column gives the number of defectives, y, in a sample of n = 100 tiles. Therefore, a total of 800 tiles were sampled and examined in the 8 runs. Table 6.2 at first glance might indicate that either the TC number 4(=$A_1B_2C_2D_2E_2F_1G_1$) or the TC 5(=$A_2B_1C_2D_1E_2F_1G_2$) will provide optimal conditions for the 7 parameters A, B, ..., G. However, since a full replicate was not run (i.e., there were 120 other TCs that were not examined), further analysis is necessary to estimate optimum conditions. Accordingly, the response table for the number of defectives, y, is provided in Table 6.3, page 74.

In Table 6.3, the total number of defectives under condition E_1 was computed as follows:

$$y_1 + y_3 + y_6 + y_8 = 16 + 12 + 68 + 26 = 122$$

which results in % defective of 122/400 = 30.50%.

Similarly, the other 13 total number of defectives and percent defectives were computed and are given in columns 2 and 3 of Table 6.3. The response table of page 74 indicates that the optimal parameter conditions are estimated as $A_1B_2C_2D_1E_2F_1G_2$. Note that only factors C and B seem to be relatively weak so that it will probably not make a difference whether their levels are set at 1 or 2 from the standpoint of percent defective. If indeed it costs less to set C at its level 1 (henceforth ℓ_1), then optimal conditions are $A_1B_?C_1D_1E_2F_1G_2$.

Further, the above optimal conditions were estimated and may not indeed provide the true optima. To make fairly sure that optimal conditions have been obtained, the experimenter must conduct at least n = 100 "confirmation runs", i.e., at least 100 more items must be inspected at the TC:

$$A_1B_?C_?D_1E_2F_1G_2 \qquad (B_1 \text{ or } B_2,\ C_1 \text{ or } C_2?)$$

and number of defectives in $n \geq 100$ counted. If the percent defectives observed is fairly close to the expected improved percent defective, $\hat{p}$, then the optimal conditions have been closely identified. The value of $\hat{p}$ is obtained under optimal conditions $A_1D_1E_2F_1G_2$ as follows.

$$\hat{p}_o = \bar{p} + (\hat{p}_{A1} - \bar{p}) + (\hat{p}_{D1} - \bar{p}) + (\hat{p}_{E2} - \bar{p}) + (\hat{p}_{F1} - \bar{p}) + (\hat{p}_{G2} - \bar{p})$$

$$= \hat{p}_{A1} + \hat{p}_{D1} + \hat{p}_{E2} + \hat{p}_{F1} + \hat{p}_{G2} - 4\bar{p} \qquad (6.1)$$

where $\bar{p}$ = 193/800 = 0.24125, $\hat{p}_{A1}$ = 51/400 = 0.1275, $\hat{p}_{D1}$ = 76/400 = 0.19, $\hat{p}_{E2}$ 0.1775, $\hat{p}_{F1}$ = 54/400 = 0.135, and $\hat{p}_{G2}$ = 0.1525. Substituting these values into equation (6.1) yields

$$\hat{p}_o = .1275 + 0.19 + 0.1775 + .135 + .1525 - 4(.24125)$$

$$= -0.1825 = -18.25\%,$$

which is impossible and it clearly requires that we resort to another form of analysis.

Accordingly, we now define the general form of Signal To Noise (S/N) ratio, where "signal" implies the mean $\bar{y}$ and the "noise" implies the standard deviation S_y so that the general form of S/N ratio is some function, $f(\bar{y}/S)$, of $\bar{y}$ divided by S. Taguchi's S/N ratio for fraction nonconforming is measured in decibels and is defined as

$$\eta_{dB} = 20 \log_{10} \left[\left(\frac{p}{\sqrt{p(1-p)}} \right) \right] \quad \text{decibels} \tag{6.2}$$

where clearly for fraction nonconforming $\bar{y} = p$ and $S_y = \sqrt{p(1-p)}$. Equation (6.2) can be rearranged as

$$\eta_{dB} = 20 \log_{10} \left[\left(\frac{p}{\sqrt{p(1-p)}} \right) \right] = 10 \log_{10} \left(\frac{p}{1-p} \right). \tag{6.3a}$$

$= 10 \log_{10} (p/q)$

Note that the S/N ratio given by Taguchi in (6.3a) is troublesome because larger values of p (i.e., worse quality) leads to larger value of η_{dB}, which is inconsistent with the premise of always maximizing S/N ratio. Recall that in order to improve quality, S/N must be maximized. Therefore, we take the liberty of redefining S/N for fraction nonconforming p as

$$\eta_{dB} = 20 \log_{10} \left[\left(\frac{1-p}{\sqrt{p(1-p)}} \right) \right] = 10 \log_{10} \left[\frac{(1-p)^2}{p(1-p)} \right] \tag{6.3b}$$

$$= 10 \log_{10} \left(\frac{1-p}{p} \right)$$

The expected S/N ratio under optimal conditions is

$$\hat{\eta}_{dB} = \bar{\eta} + (\hat{\eta}_{A1} - \bar{\eta}) + (\hat{\eta}_{D1} - \bar{\eta}) + (\hat{\eta}_{E2} - \bar{\eta}) + (\hat{\eta}_{F1} - \bar{\eta}) + (\hat{\eta}_{G2} - \bar{\eta})$$

$$= \hat{\eta}_{A1} + \hat{\eta}_{D1} + \hat{\eta}_{E2} + \hat{\eta}_{F1} + \hat{\eta}_{G2} - 4\bar{\eta} \tag{6.4}$$

where $\bar{\eta} = 10 \log_{10}(\frac{.75875}{.24125}) = 4.9763$, $\hat{\eta}_{A1} = 10 \log_{10}(\frac{.8725}{0.1275}) = +8.3526$, $\hat{\eta}_{D1} = +6.2973$, $\hat{\eta}_{E2} = +6.6594$, $\hat{\eta}_{F1} = +8.0668$ and $\hat{\eta}_{G2} = +7.4486$. Substitution into (6.4) yields $\hat{\eta}_{dB} = +16.9195$. The use of equation (6.3b) now gives

$$+16.9195 = 10 \log_{10} \left[\frac{(1-\hat{p}_o)}{\hat{p}_o} \right]$$

which results in $\hat{p}_o \doteq 0.019921 \doteq 2\%$. Therefore, under optimal parameter settings, $A_1B_7C_7D_1E_2F_1G_2$, we expect approximately 2% defectives for titles. To determine how much improvement this is over the existing process capability, we assume that the present fraction nonconforming is 24.125% so that the reduction in p is 22.133%. Note that the present existing condition $A_2B_1C_2D_1E_2F_2G_2$ was not tried in the experiment so that we had no other way to approximate p but from all the data.

To determine the reduction in \$ losses, suppose each defective tile results in A_c = \$25.00 societal loss. Then from a traditional standpoint, the average loss per unit before process optimization is $L_s = \frac{193 \times 25}{800} = A_c p =$ \$6.031/tile and under optimal conditions is 25.00 x .019921 = \$0.498, which represents a 91.74% reduction in average loss per unit.

Note that the Taguchi loss per unit, L, cannot be estimated because the inspections were purely by attributes, i.e., no continuous measurements, y, were made in the experiment. However, in later chapters we will provide an approximate answer to this question by using the decibel improvement δ_{db} = 16.9195 - 4.9763 = 11.9432.

<u>Exercise 6.5</u> The following data are on the QCH: y = appearance of painted surface, which takes on classes: Good, Fair and Bad.

$L_8(2^7)$	A B C D E F G			
Experiment Number Col.	(1) (2) (3) (4) (5) (6) (7)	Good	Fair	Bad
1		3	2	1
2		1	4	1
3		2	4	0
4		0	3	3
5	Inner OA for an L_8.	6	0	0
6		1	5	0
7		2	4	0
8		0	0	6

The factors are A - curing time, B - paint thickness, C - paint viscosity, D - pressure, E - drying temperature, F - surface pretreatment, G - solvent type, and n=6 observations per TC. (a) Construct the RT. (b) Identify weak, moderate and strong factors. (c) Determine the optimal conditions. (d) How reliable are the factor settings obtained in part (c).

Note that similar RT analysis as above may be performed on y for STB, LTB and NTB quality characteristics as long as there is only n=1 observation per TC. However, if there are $n \geq 2$ observations per TC, then for continuous measurements RTs should be constructed not for y values but for the S/N ratios of each TC. Again, we remind the reader that the n observations should be taken generally across n different noise levels embedded in the outer OA for each TC of the inner OA. For a balanced PDE this will give a total of nxN observations in the entire experiment.

WORKSHEETS OF ORTHOGONAL ARRAY TABLES

PAGES W-A THROUGH W-I

The L_8 (2^7) Orthogonal Array

Table of Interactions Between Two Columns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	(1)	3	2	5	4	7	6
(2)		(2)	1	6	7	4	5
(3)			(3)	7	6	5	4
(4)				(4)	1	2	3
(5)					(5)	3	2
(6)						(6)	1
(7)							(7)

Orthogonal Array

Row No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	y values
1	1	1	1	1	1	1	1	
2	1	1	1	2	2	2	2	
3	1	2	2	1	1	2	2	
4	1	2	2	2	2	1	1	
5	2	1	2	1	2	1	2	
6	2	1	2	2	1	2	1	
7	2	2	1	1	2	2	1	
8	2	2	1	2	1	1	2	

The L_{16} (2^{15}) Orthogonal Array

Table of Interactions Between Two Columns

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
	(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
		(3)	7	6	5	4	11	10	9	8	15	14	13	12
			(4)	1	2	3	12	13	14	15	8	9	10	11
				(5)	3	2	13	12	15	14	9	8	11	10
					(6)	1	14	15	12	13	10	11	8	9
						(7)	15	14	13	12	11	10	9	8
							(8)	1	2	3	4	5	6	7
								(9)	3	2	5	4	7	6
									(10)	1	6	7	4	5
										(11)	7	6	5	4
											(12)	1	2	3
												(13)	3	2
													(14)	1

Orthogonal Array

Row No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	y values
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	

Interactions Between Two Columns

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
		(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
			(3)	7	6	5	4	11	10	9	8	15	14	13	12
				(4)	1	2	3	12	13	14	15	8	9	10	11
					(5)	3	2	13	12	15	14	9	8	11	10
						(6)	1	14	15	12	13	10	11	8	9
							(7)	15	14	13	12	11	10	9	8
								(8)	1	2	3	4	5	6	7
									(9)	3	2	5	4	7	6
										(10)	1	6	7	4	5
											(11)	7	6	5	4
												(12)	1	2	3
													(13)	3	2
														(14)	1

Note that (6) x (11) = (13)

The $L_9 (3^4)$ Orthogonal Array

Table of Interactions Between Two Columns

	1	2	3	4
(1)	(1)	3,4	2,4	2,3
(2)		(2)	1,4	1,3
(3)			(3)	1,2
(4)				(4)

Orthogonal Array

Row No.	(1)	(2)	(3)	(4)	
1	1	1	1	1	
2	1	2	2	2	
3	1	3	3	3	
4	2	1	2	3	
5	2	2	3	1	
6	2	3	1	2	
7	3	1	3	2	
8	3	2	1	3	
9	3	3	2	1	

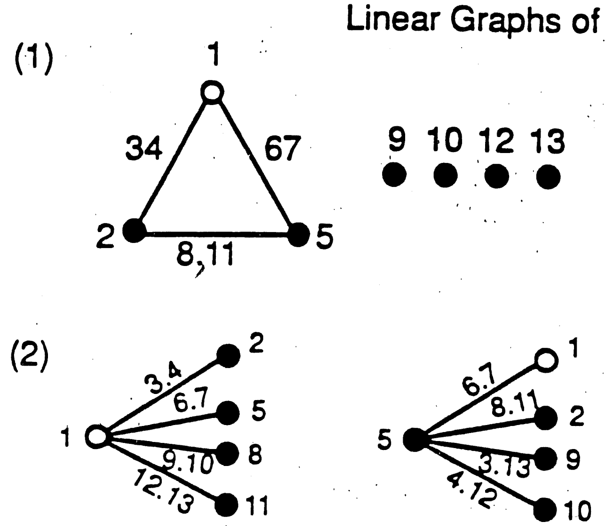
$L_{27}(3^{13})$

No. \ Column	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2
Groups	1		2						3				

Interactions between Two Columns

Column \ Column	1	2	3	4	5	6	7	8	9	10	11	12	13
	(1)	3	2	2	6	5	5	9	8	8	12	11	11
		4	4	3	7	7	6	10	10	9	13	13	12
		(2)	1	1	8	9	10	5	6	7	5	6	7
			4	3	11	12	13	11	12	13	8	9	10
			(3)	1	9	10	8	7	5	6	6	7	5
				2	13	11	12	12	13	11	10	8	9
				(4)	10	8	9	6	7	5	7	5	6
					12	13	11	13	11	12	9	10	8
					(5)	1	1	2	3	4	2	4	3
						7	6	11	13	12	8	10	9
						(6)	1	4	2	3	3	2	4
							5	13	12	11	10	9	8
							(7)	3	4	2	4	3	2
								12	11	13	9	8	10
								(8)	1	1	2	3	4
									10	9	5	7	6
									(9)	1	4	2	3
										8	7	6	5
										(10)	3	4	2
											6	7	7
											(11)	1	1
												13	12
												(12)	1
													11

Linear Graphs of L_{27} Table



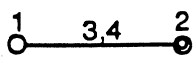
$L_9(3)^4$

Experiment #	1 (A)	2 (B)	3 (C = AB)	4 (D ∝ A²B)
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1
	Group 1	Group 2		

→ 9, 8

→ 25, 16

Linear Graph



$L_{18}(2^1 \times 3^7)$

Experiment #	1 (A)	2 (B)	3 (C)	4 (D)	5 (E)	6 (F)	7 (G)	8 (H)	y (STB)
1	1	1	1	1	1	1	1	1	4
2	1	1	2	2	2	2	2	2	17
3	1	1	3	3	3	3	3	3	5
4	1	2	1	1	2	2	3	3	
5	1	2	2	2	3	3	1	1	
6	1	2	3	3	1	1	2	2	
7	1	3	1	2	1	3	2	3	
8	1	3	2	3	2	1	3	1	
9	1	3	3	1	3	2	1	2	
10	2	1	1	3	3	2	2	1	
11	2	1	2	1	1	3	3	2	
12	2	1	3	2	2	1	1	3	
13	2	2	1	2	3	1	3	2	
14	2	2	2	3	1	2	1	3	
15	2	2	3	1	2	3	2	1	
16	2	3	1	3	2	3	1	2	
17	2	3	2	1	3	1	2	3	
18	2	3	3	2	1	2	3	1	39
Groups	1	2	3						

A x B

A \ B	1	2	3
1			
2			

Note that the main factors carry 15 df and A x B has 2 df.

Linear Graph

Note that no interaction can be imbedded in the L_{18} OA, but (1) x (2) can be studied after data has been collected.

$L_{36} (3^{13})$

Experiment #	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	1
3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	1	1	1	1	2	2	2	2	3	3	3	3	1
5	2	2	2	2	3	3	3	3	1	1	1	1	1
6	3	3	3	3	1	1	1	1	2	2	2	2	1
7	1	1	2	3	1	2	3	3	1	2	2	3	1
8	2	2	3	1	2	3	1	1	2	3	3	1	1
9	3	3	1	2	3	1	2	2	3	1	1	2	1
10	1	1	3	2	1	3	2	3	2	1	3	2	1
11	2	2	1	3	2	1	3	1	3	2	1	3	1
12	3	3	2	1	3	2	1	2	1	3	2	1	1
13	1	2	3	1	3	2	1	3	3	2	1	2	2
14	2	3	1	2	1	3	2	1	1	3	2	3	2
15	3	1	2	3	2	1	3	2	2	1	3	1	2
16	1	2	3	2	1	1	3	2	3	3	2	1	2
17	2	3	1	3	2	2	1	3	1	1	3	2	2
18	3	1	2	1	3	3	2	1	2	2	1	3	2
19	1	2	1	3	3	3	1	2	2	1	2	3	2
20	2	3	2	1	1	1	2	3	3	2	3	1	2
21	3	1	3	2	2	2	3	1	1	3	1	2	2
22	1	2	2	3	3	1	2	1	1	3	3	2	2
23	2	3	3	1	1	2	3	2	2	1	1	3	2
24	3	1	1	2	2	3	1	3	3	2	2	1	2
25	1	3	2	1	2	3	3	1	3	1	2	2	3
26	2	1	3	2	3	1	1	2	1	2	3	3	3
27	3	2	1	3	1	2	2	3	2	3	1	1	3
28	1	3	2	2	2	1	1	3	2	3	1	3	3
29	2	1	3	3	3	2	2	1	3	1	2	1	3
30	3	2	1	1	1	3	3	2	1	2	3	2	3
31	1	3	3	3	2	3	2	2	1	2	1	1	3
32	2	1	1	1	3	1	3	3	2	3	2	2	3
33	3	2	2	2	1	2	1	1	3	1	3	3	3
34	1	3	1	2	3	2	3	1	2	2	3	1	3
35	2	1	2	3	1	3	1	2	3	3	1	2	3
36	3	2	3	1	2	1	2	3	1	1	2	3	3

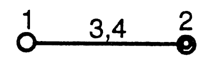
Don't consider interactions when using this array. Interaction effects are aliased almost equally between all columns

SS_A =

$L_9(3^4)$

Column / No.	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1
Groups	1	2		

(1)



$L_{18}(2^1 \times 3^7)$

n = 1

Column / No.	A 1	B 2	C 3	D 4	E 5	F (6)	G 7	H 8	y (STB)
1	1	1	1	1	1	1	1	1	4
2	1	1	2	2	2	2	2	2	17
3	1	1	3	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3	6
5	1	2	2	2	3	3	1	1	20
6	1	2	3	3	1	1	2	2	15
7	1	3	1	2	1	3	2	3	13
8	1	3	2	3	2	1	3	1	11
9	1	3	3	1	3	2	1	2	9 /98
10	2	1	1	3	3	2	2	1	8
11	2	1	2	1	1	3	3	2	3
12	2	1	3	2	2	1	1	3	28
13	2	2	1	2	3	1	3	2	40
14	2	2	2	3	1	2	1	3	31
15	2	2	3	1	2	3	2	1	14
16	2	3	1	3	2	3	1	2	7
17	2	3	2	1	3	1	2	3	17
18	2	3	3	2	1	2	3	1	39 /187
Groups	1	2	3					Total	285

(1)

Interaction is obtainable without sacrificing any column. It is obtained from the two-way table of columns 1 and 2.

Note: The interactions between 3-level columns are partially confounded with the remaining 3-level columns. The situation is similar to the note of L_{12} Table.

$L_{36}\,(3)^{13}$

Experiment #	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	1
3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	1	1	1	1	2	2	2	2	3	3	3	3	1
5	2	2	2	2	3	3	3	3	1	1	1	1	1
6	3	3	3	3	1	1	1	1	2	2	2	2	1
7	1	1	2	3	1	2	3	3	1	2	2	3	1
8	2	2	3	1	2	3	1	1	2	3	3	1	1
9	3	3	1	2	3	1	2	2	3	1	1	2	1
10	1	1	3	2	1	3	2	3	2	1	3	2	1
11	2	2	1	3	2	1	3	1	3	2	1	3	1
12	3	3	2	1	3	2	1	2	1	3	2	1	1
13	1	2	3	1	3	2	1	3	3	2	1	2	2
14	2	3	1	2	1	3	2	1	1	3	2	3	2
15	3	1	2	3	2	1	3	2	2	1	3	1	2
16	1	2	3	2	1	1	3	2	3	3	2	1	2
17	2	3	1	3	2	2	1	3	1	1	3	2	2
18	3	1	2	1	3	3	2	1	2	2	1	3	2
19	1	2	1	3	3	3	1	2	2	1	2	3	2
20	2	3	2	1	1	1	2	3	3	2	3	1	2
21	3	1	3	2	2	2	3	1	1	3	1	2	2
22	1	2	2	3	3	1	2	1	1	3	3	2	2
23	2	3	3	1	1	2	3	2	2	1	1	3	2
24	3	1	1	2	2	3	1	3	3	2	2	1	2
25	1	3	2	1	2	3	3	1	3	1	2	2	3
26	2	1	3	2	3	1	1	2	1	2	3	3	3
27	3	2	1	3	1	2	2	3	2	3	1	1	3
28	1	3	2	2	2	1	1	3	2	3	1	3	3
29	2	1	3	3	3	2	2	1	3	1	2	1	3
30	3	2	1	1	1	3	3	2	1	2	3	2	3
31	1	3	3	3	2	3	2	2	1	2	1	1	3
32	2	1	1	1	3	1	3	3	2	3	2	2	3
33	3	2	2	2	1	2	1	1	3	1	3	3	3
34	1	3	1	2	3	2	3	1	2	2	3	1	3
35	2	1	2	3	1	3	1	2	3	3	1	2	3
36	3	2	3	1	2	1	2	3	1	1	2	3	3

We cannot study interactions with this table since 36 cannot be written as 3^{k-p} for integer values of k and $p \leq k$.

CHAPTER VII

1. S/N RATIOS FOR TAGUCHI'S PARAMETER DESIGN

Generally a parameter design consists of two OAs: (1) the columns of inner array always represent the effects pertaining to controllable factors, and (2) the columns of the outer array always represent the levels of noise factors. For example, if the inner array is an L_{16} and the outer array is an L_4, then the parameter design calls for at least 64 observations with $n \geq 4$ experiments at each TC of the controllable factors. The objective of a parameter design is to utilize the interaction amongst controllable with noise factors in order to make the product more robust to noise. Put differently, the objective is to remove (or diminish) the effect of noise on product quality instead of removing the noise from the system.

Taguchi uses the symbol η_{dB} to represent S/N ratio; the units of η_{dB} is decibels, i.e., η_{dB} = S/N ratio is measured in decibels.

Before defining η_{dB}, it is important for the reader to know that the analysis of S/N ratios for a NTB QCH is treated differently from those of STB and LTB QCHs. This is due to the fact that for optimal decision making, the variance plays a more important role for a NTB response than for a magnitude (i.e., STB or LTB) type QCH. For this reason, the analysis of parameter design for an NTB QCH will be presented last.

Definition Suppose y is either an STB or LTB type QCH. Then

$$\eta_{db} = -10 \log_{10} (\text{MSD}), \text{ decibels} \tag{7.1}$$

where $\text{MSD} = \frac{1}{n} \sum_{i=1}^{n} y_i^2$ for STB, and $\text{MSD} = \frac{1}{n} \sum_{i=1}^{n} 1/y_i^2$ for a LTB performance characteristic. Equation (7.1) can be rearranged as

$$\eta_{db} = \log_{10} (\text{MSD})^{-10} = \log_{10} \left(\frac{1}{\text{MSD}}\right)^{10} = \log_{10} \left(\frac{k}{L}\right)^{10}, \tag{7.2}$$

where L is Taguchi's average loss per unit. Equation (7.2) shows that S/N ratio is a decreasing function of L, i.e., as L decreases, η increases and vice versa. Therefore, in order to determine optimal conditions from the response table (RT) of a parameter design, it is always necessary to <u>maximize</u> η_{db} for <u>all</u> types of QCHs.

<u>Example 7.1.</u> Consider the experiment involving a parameter design with 6 controllable factors and one noise factor, where the QCH is y = the noise level of a compressor. The experimental layout is given on page 83. The reader should compute the first 2 or 3 values of η_{db} given in the last column. The analysis of RT is performed below, where ℓ_1 = level 1 and ℓ_2 = level 2.

N_1 + N_2 Response Table for η_{db}

Effects	A	B	C	D	BxF	E	F
ℓ_1	-140.5	-145.29	-143.95	-147.50	-143.40	-145.39	-147.71
ℓ_2	-149.04	-144.25	-145.59	-142.04	-146.14	-144.15	-141.83
$\|\ell_1 - \ell_2\|$	8.54	1.04	1.64	5.46	2.74	1.24	5.88
R_i	1	7	5	3	4	6	2

The above RT shows that the weak Effects are B, C and E; BxF is moderately strong, and strong Effects are A, D and F.

Remember we must select parameter levels with the highest S/N ratios in order to optimize the response y.

optimal parameter levels are estimated as: $A_1, B_2, C_?, D_2, E_?$ and F_2

B x F Interaction Analysis

	F_1	F_2
B_1	-73.43	-71.86
B_2	-74.28	-69.97*

max. S/N

EXAMPLE OF PARAMETER DESIGN LAYOUT (From ASI, Inc).

y = "Sound power (loudness) of compressor (Smaller the better) Assume USL = 65

CONTROLLABLE FACTORS

Assume A_C = $80.275

		Level 1	Level 2
A :	Baffle	With	Without
B :	Piston Clearance	Low	High
C :	Bearing Clearance	Low	High
D :	Valve Design	Existing	New
E :	Shell Design	Existing	New
F :	Gasket Thickness	Low	High

INTERACTION B x F

NOISE FACTOR

		Level 1	Level 2
N :	Oil Level	Low	High

Layout & Data

$L_8(2^7)$ parameter design

	A 1	B 2	C 3	D 4	B x F 5	E 6	F 7	outer array N_1		N_2		$-10\log_{10}(\frac{1}{n}\Sigma y_i^2) = \eta_{db}$
1	1	1	1	1	1	1	1	65	63	72	60	-36.28
2	1	1	1	2	2	2	2	46	50	54	46	-33.82*
3	1	2	2	1	1	2	2	50	62	48	55	-34.65
4	1	2	2	2	2	1	1	60	61	65	59	-35.75
5	2	1	2	1	2	1	2	80	85	75	79	-38.04
6	2	1	2	2	1	2	1	81	83	60	61	-37.154
7	2	2	1	1	2	2	1	92	85	80	80	-38.53
8	2	2	1	2	1	1	2	63	57	59	54	-35.32

N_1 total: 1083 N_2 total: 1007 grand total = -289.54

$\Sigma y_{i1}^2 = 76457$ $\Sigma y_{i2}^2 = 65235$ → $\Sigma\Sigma y_{ij}^2 = 141692$

ASI, Inc.

If it is important to decide which noise level is optimum, then one ıld compute S/N ratio for N_1 and N_2 separately. Since η_{db} (at N_1) = -36.793 ıd η_{db} (at N_2) = -36.104*, then N_2 is optimal, i.e., N_2 provides maximum S/N ratio. However, it is generally best to 1st select the optimal conditions across all noise levels and then compute η_{db} for N_j at the optimal levels and make a decision accordingly.

Once optimal parameter levels have been estimated, the experimenter must check on reproducibility by conducting n confirmation runs at the designed optimal levels. If the value of S/N ratio from the confirmation runs, η_c, is close to

$$\hat{\eta}_{db} = \bar{\eta}_{db} + (\bar{\eta}_{A1} - \bar{\eta}_{db}) + (\bar{\eta}_{B2} - \bar{\eta}_{db}) + (\bar{\eta}_{D2} - \bar{\eta}_{db}) + (\bar{\eta}_{F2} - \bar{\eta}_{db}), \quad (7.3)$$

then the optimal conditions have been reproduced (or verified).

For the experiment on page 83, calculations yield $\bar{\eta}_{db}$ = -289.54/8 = -36.1925, $\bar{\eta}_{A1}$ = -35.125, $\bar{\eta}_{B2}$ = -36.0625, $\bar{\eta}_{D2}$ = -35.511 and $\bar{\eta}_{F2}$ = - 35.4575. Thus

$$\hat{\eta}_{db} = -35.125 - 36.0625 - 35.511 - 35.4575 - 3(-36.1925) \doteq -33.58.$$

In Equation (7.3) the moderate interaction effect BxF was excluded. If we include this interaction effect, our equation for expected S/N ratio under optimal conditions is

$$\hat{\eta}_{db} = \bar{\eta}_{A1} + \bar{\eta}_{B2} + \bar{\eta}_{D2} + \bar{\eta}_{F2} - 3\bar{\eta}_{db} + (\overline{BxF}_{22} - \bar{\eta}_{B2} - \bar{\eta}_{F2} + \bar{\eta}_{db}), \quad (7.4),$$

where $\overline{BXF}_{22}$ = -69.97/2 = -34.985. Equation (7.4) reduces to

$$\hat{\eta}_{db} = \bar{\eta}_{A2} + \bar{\eta}_{D2} + \overline{BxF}_{22} - 2\bar{\eta}_{db} = -33.236 \quad (7.5)$$

which is a bit better than the previous $\hat{\eta}_{db}$ = -33.58.

It is interesting to note that the above predicted value of $\hat{\eta}_{db}$ (= -33.236) is larger than all the 8 S/N ratios observed in the entire experiment (the largest observed S/N was -33.82). Had we used $\hat{\eta}_{db} = -10 \log_{10}$ $[\frac{1}{32}(141692] = -36.462$, we would expect $\hat{\eta}_{db} = -32.697$ from our confirmation runs for $\eta_{c.}$. This is even harder to believe (or to expect) than $\hat{\eta}_{db} = -33.236$. Further, we would recommend against using $\Sigma\Sigma y_{ij}^2 /32$ when estimating the present existing process S/N ratio.

2. LOSS FUNCTION ANALYSIS FOR AN STB RESPONSE

We now determine the amount of quality improvement (QI) per unit in dollars, where $A_c = \$80.275$ at the USL = y_u = 65 decibels. Then $k = A_c/y_u^2 = 0.019$. Exponentiating Equation (7.2) yields:

$$(k/L)^{10} = 10^{\eta_{db}} ,$$

$$k/L = 10^{\eta_{db}/10} ,$$

or

$$L/K = (10^{\eta_{db}/10})^{-1}$$

$$L = k(10^{-\eta_{db}/10}) \qquad (7.6)$$

In the absence of any information, we assume that the present existing S/N of the process is roughly $\bar{\eta}_{db} = -36.1925$ and from equation (7.6)

$$\bar{L} = .019\ (10^{3.61925}) = \$79.07.$$

Under optimal conditions our S/N ratio was expected to increase to -33.236, a decibel improvement of $\delta_{db} = -33.236 - (-36.1925) = 2.9565$. Therefore, our diminished loss per unit from (7.6) will be

$$L_0 = .019\ (10^{3.3236}) = \$40.03$$

which yields a reduction of 49.38% in average loss per unit.

<u>Exercise 7.1</u> The layout and data below are from the effect of 4

controllable factors A, B, C and D on the Quality Characteristic "rate of wear". There are 2 levels of the noise factor "machine".

	Inner L_8 OA							Outer L_4 OA					
								M_1		M_2			
TC	A (1)	B (2)	A x B (3)	C (4)	A x C (5)	B x C (6)	D (7)	O_1	O_2	O_1	O_2	Totals	
1	1	1	1	1	1	1	1	1.11,	1.30	0.90,	.81	4.12	$\Sigma y^2_{i,j} = 35.2637$
2	1	1	1	2	2	2	2	0.23,	.40	.30,	.38	1.31	
3	1	2	2	1	1	2	2	0.34,	.44	.26,	.34	1.38	
4	1	2	2	2	2	1	1	0.71,	.68	.5,	.55	2.44	
5	2	1	2	1	2	1	2	2.39.	1.80	1.20,	1.40	6.79	
6	2	1	2	2	1	2	1	0.19,	.25	.44,	.22	1.10	
7	2	2	1	1	2	2	1	2.08,	1.60	1.49,	1.06	6.23	
8	2	2	1	2	1	1	2	1.56,	0.90	1.10,	1.02	4.58	
							Totals	8.61	7.37	6.19	5.78	27.95	
									15.98		11.97		

(a) Given $SS_A = 2.7907$, $SS_B = 0.05363$, $SS_{AB} = .6413$, $SS_C = 2.5821$, $SS_{AC} = .9765$, compute SS's due to BC, D, noise and error. Obtain the ANOVA table that includes the 3 df for noise (machines, operators and M x O) and test the significance of factor A at $\alpha = .01$. (b) Compute the % contributions of A, C and BC. (c) Given the values $\eta_1 = -.4023$, $\eta_2 = 9.52$, $\eta_3 = 9.10$, $\eta_4 = 4.21$, $\eta_5 = -4.90$, $\eta_6 = 10.70$ and $\eta_7 = -4.08$, compute η_8 and obtain the response table for η_{db}. Determine the optimal conditions, examining only the most significant interaction. Predict the S/N ratio at the optimal setting and explain how you would check on reproducibility. (d) If y_u is .90 and $A_c = \$6.48$, compute the quality loss/piece at the optimal conditions and the % reduction in average loss per unit.

3. **Analysis of S/N Ratios for a Parameter Design Involving an LTB Response**

Example 7.2. Consider the experimental layout on page 88 where the QCH is y = welding strength measured in ksi. The LSL is y_ℓ = 1.20 ksi with A_c = \$20.00. The inner array is an L_{12} (2^{11}) and the outer array is an L_4 so that there are a total of 48 observations of welding strength (4 per TC). Clearly, there are 11 controllable factors (each at 2 levels) and 3 noise factors.

For the sake of illustration we compute the S/N ratio for the TC 1, i.e.,

$$\eta_1 = -10 \log_{10} [\frac{1}{4} (2.5^{-2} + 2.7^{-2} + 2.2^{-2} + 3.2^{-2})] =$$

$$-10 \log_{10} (\frac{.601442}{4}) = 8.228 \doteq 8.23.$$

The reader should take time to compute η_2 = 3.82 and η_3 = 0.710. The values of η_1, η_2, ..., η_{12} were used to obtain the RT below.

RT for S/N Ratios of the Welding Experiment

Factors	A	B	C	D	E	F	G	H	I	J	K
ℓ_1	21.06	31.94	26.89	28.67	34.13	24.76	33.57	22.03	30.09	30.29	32.30
ℓ_2	32.21	21.33	26.38	24.60	19.14	28.51	19.70	31.24	23.18	22.98	20.97
R_i	4	5	11	9	1	10	2	6	8	7	3

W W W

Note that R_i = The rank of the i[th] strongest factor, and E, which is the strongest factor, gets a rank of 1, etc.

The above RT indicates that the optimal parameter levels are estimated at

$$X_0 = A_2B_1C_?D_?E_1F_?G_1H_2I_1J_1K_1$$ = ak, ack

$X_o = A_2 B_1 C_? D_? E_1 F_? G_1 H_2 I_1 J_1 K_1$

EXAMPLE OF PARAMETER DESIGN LAYOUT -

= Welding Strength in ksi (Larger the better) = y, LsL = 1.20 KSi

		Level 1	Level 2
CONTROLLABLE FACTORS	A: SHELL THICKNESS	Low	High
	B: WELD MATERIAL	Type 1	Type 2
	C: SPEED	Low	High
	D: PRESSURE	Low	High
	E: WELDING THICKNESS	Low	High
A_C = \$20.00	F: SURFACE FINISH	Low	High
	G: ATMOSPHERE	Low	High
	H: VOLTAGE	Low	High
	I: CURRENT	Low	High
	J: ANGLE	Low	High
	K: PLATING THICKNESS	Low	High
NOISE FACTORS	S: SHIFT	1	2
	M: MACHINE	1	2
	O: OPERATOR	John	Gary

LAYOUT & DATA

L_{12} Inner OA; L_4 Outer Array

$\eta_{db} = -10 \log_{10} \left(\frac{1}{n} \Sigma \frac{1}{y_i^2}\right)$

	A 1	B 2	C 3	D 4	E 5	F 6	G 7	H 8	I 9	J 10	K 11	S 1, M 1, O 1	S 1, M 2, O 2	S 2, M 1, O 2	S 2, M 2, O 1	η_{db}
1	1	1	1	1	1	1	1	1	1	1	1	2.5	2.7	2.2	3.2	η_1 = 8.23
2	1	1	1	1	1	2	2	2	2	2	2	1.3	1.8	1.4	2.0	η_2 = 3.82
3	1	1	2	2	2	1	1	1	2	2	2	0.8	1.2	1.2	1.5	0.71
4	1	2	1	2	2	1	2	2	1	1	2	0.8	1.1	1.1	2.0	0.62
5	1	2	2	1	2	2	1	2	1	2	1	1.7	2.2	1.3	2.4	4.82
6	1	2	2	2	1	2	2	1	2	1	1	0.9	1.8	1.9	2.0	2.86
7	2	1	2	2	1	1	2	2	1	2	1	2.4	3.2	1.8	2.5	7.33
8	2	1	2	1	2	2	2	1	1	1	2	1.8	1.9	1.1	2.2	3.93
9	2	1	1	2	2	2	1	2	2	1	1	1.9	2.5	3.0	3.2	7.92
10	2	2	2	1	1	1	1	2	2	1	2	2.2	2.5	2.2	1.9	6.73
11	2	2	1	2	1	2	1	1	1	2	2	2.4	1.8	1.6	1.7	5.16
12	2	2	1	1	2	1	2	1	2	2	1	1.8	1.4	0.8	1.2	1.14
															Total	53.27

$\bar{\eta}$ = 4.4392 = 53.27/12.0

WEAK FACTORS: C, F, D

MODERATE FACTORS: I, J

STRONG FACTORS: E, G, K, A, B, H (in order of their strength)

$k = 20\,(1.2)^2 = 28.8$.

$\bar{\eta}_{db} = 53.27/12 \cong 4.44$.

from ASI, Inc.

To complete the analysis and verify the above optimal conditions, the experimenter must conduct n confirmation runs (i.e., obtain at least 4 observations at the TC $A_2B_1E_1G_1H_2I_1J_1K_1$) and compare the corresponding S/N ratio, η_c, with the predicted η_{db} from

$$\hat{\eta}_0 = \bar{\eta}_{A2} + \bar{\eta}_{B1} + \bar{\eta}_{E1} + \bar{\eta}_{G1} + \bar{\eta}_{H2} + \bar{\eta}_{I1} + \bar{\eta}_{J1} + \bar{\eta}_{K1} - 7\bar{\eta}_{db}$$

$$= 5.368 + 5.323 + \ldots -7(4.4392) = \underline{11.554}.$$

Again it is doubtful that η_c can be improved to as large as 11.554. Although $\hat{\eta}_{db} = 11.554$ exceeds the observed maximum of $(\eta_1, \eta_2, \ldots, \eta_{12}) = 8.23$, the experimenter must bear in mind that $\eta_1 = 8.23$ was not obtained under optimal conditions.

4. LOSS FUNCTION ANALYSIS FOR AN LTB RESPONSE

To illustrate the procedure, we compute the percent reduction in average loss per unit for the PDE of Example 7.2. In the absence of any information, we assume that the existing process decibel is roughly $\bar{\eta}_{db} = 53.27/12 = 4.4392$. Since $k = A_c y_\ell^2 = 20(1.2)^2 = 28.80$, then from Equation (7.6) the average existing loss per unit is $\bar{L} = 28.80(10^{-0.44392}) = \$10.363$.

Secondly, we make the assumption that our optimal conditions are reproducible so that the decibel improvement is $\delta_{db} = \eta_c - \bar{\eta}_{db} \doteq \hat{\eta}_{db} - \bar{\eta}_{db} = 11.554 - 4.4392 = 7.1148$. Again, from Equation (7.6) the diminished average loss per unit is $L_0 = 28.80(10^{-1.1554}) = \$2.014$. Therefore, the percent reduction in average loss per unit is $(1-r)100 = (\frac{\bar{L}-L_0}{\bar{L}}) \times 100\% \doteq 80.57\%$.

It is interesting to note that from the traditional standpoint, the average loss per unit is $L_s = 7 \times 20/48 = \$2.917$ as compared to the before PDE Taguchi loss of \$10.363/unit.

Exercise 7.2. An experiment was conducted to study the requisite pull-off force delivered by an elastometric connector. The LSL for y is 16.00 lbs at which a loss of A_c = \$10.00 is incurred. The controllable factors identified are A = Interference at low, medium and high levels; B = Connector wall thickness at 3 levels, C = Insertion Depth (at shallow, medium, deep) and D = Percent adhesive (at low, medium, high). The noise factors were E = conditioning time, F = conditioning temperature and G = conditioning humidity, each at 2 levels. The experimenter decided to use an L_9 inner array and an L_4 outer OA. (a) Give the loss function, and decide what fraction the L_9 OA is, and as a result each effect will have how many aliases? (b) The experimental layout is given on the next page. The values of S/N ratio for the 1st 8 TC's are 24.432, 24.852, 26.246, 26.546, 27.551, 25.299, 25.747, 24.178 decibels. Compute the S/N ratio for the 9th TC to three decimals. (c) Obtain the response table for S/N ratios and determine the optimal conditions. (d) Assuming reproducibility, compute to three decimals the % reduction in loss. (e) Compute the % contribution of the factor C, given that $\sum_{i,j} y_{ij}^2 = 14359.67$ and $y_{..} = 710.1$, where the index i refers to the TC number. Exclude the noise sum of the square from the SS_{Error}. ANS: $\hat{\eta}_0 = 27.708$, $\delta_{db} = 2.025$, $\rho_c = 29.94\%$.

Column					2	1	2	1	E	
					2	2	1	1	F	
TC	A	B	C	D	1	2	2	1	G	Totals
1	1	1	1	1	13	19.6	17.5	19.5		69.6
2	1	2	2	2	16.4	19.7	19.5	15.5		71.1
3	1	3	3	3	23	20.2	18	22		83.2
4	2	1	2	3	23.8	20	18.7	24		86.5
5	2	2	3	1	26.6	23.6	22.3	23.5		96.0
6	2	3	1	2	17.9	17.1	19.9	19.1		74.00
7	3	1	3	2	23.1	17.7	20.6	17.6		79.00
8	3	2	1	3	14.7	18.6	15.9	16.2		65.4
9	3	3	2	1	25.7	23	18.3	18.3		85.3
			Totals		184.2	179.5	170.7	175.70		$y_{..}$ = 710.10

$3^{4-2} \rightarrow \frac{1}{9}$th fraction → each effect will have 8 aliases.

Exercise 7.3 SIGNAL TO NOISE RATIO for "Larger the better characteristic"

y = Welding Strength in ksi

CONTROLLABLE FACTOR: A: Welding Method

NOISE FACTORS:
M: Machine
O: Operator
V: Voltage

Outer OA

L_4								
O 3	1	2	2	1				
M 2	1	2	1	2				
V 1	1	1	2	2	Ave.	Engineering Preference by Common Sense	LOSS / pc	η_{db}
A1	4.0	4.0	4.0	4.0				
A2	2.0	6.0	2.0	6.0				
A3	4.0	6.0	4.0	6.0				
A4	1.0	10.0	1.0	8.0				
A5	4.0	4.0	12.0	12.0				
A6	1.0	1.2	1.3	1.5				
A7	10.0	20.0	0.5	5.0				
A8	1.0	1.0	1.0	1.0				
	27.0	52.2	25.7	43.5				

$y_\ell = 1.20$

Q.1) Production Specification is y_o = 1.2 ksi and estimated average loss due to not meeting the spec. is $20.00.

Set up the loss function and calculate Loss / pc for each of A_i.

Q.2) Calculate Signal to Noise Ratio for each of A_i Discuss.

SIGNAL TO NOISE Ratio for "Larger the better"

$$= \eta_{db} = -10 \log_{10}(MSD)$$

$$MSD = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2}$$

From American Supplier Institute (ASI), Inc.

Then the diminished loss L' must be some fraction of the existing loss

$$L' = rL \quad (12)$$

where $0 < r < 1$. Combining (12), (11), and yields

$$\eta'_{db} = \eta_{db} + 10\log_{10}(1/r). \quad (13)$$

Equation (13) shows that if we manage, with the aid of parameter design, to increase the existing S/N ratio by a positive amount

$$\delta_{db} = 10\log_{10}(1/r) \quad (14a)$$

so that

$$r = 10^{-(\delta_{db}/10)} \quad (14b)$$

that our average loss per unit will be decreased to $L' = rL$, $0 < r < 1$. For example, if a 70% reduction in loss per unit is desired, then we must increase η'_{db} by $\log_{10}(1/0.30)^{10} = 5.229$ decibels. Table 1 gives the required amount of increases in η'_{db} for some desired values of percent reduction, $(1 - r)$ 100 %, in the expected loss per unit.

Conversely, if we wish to increase the known existing S/N ratio η_{db} to $\eta'_{db} = \eta_{db} + \delta_{db} > \eta_{db}$, what proportional reduction in the present MSD does this necessitate? To determine the required amount of reduction in MSD, we must note that for a STB response, η_{db} is generally negative unless in the unlikely event that $0 < \text{MSD} = (1/n)\Sigma\ 1/y_1^2 < 1$. While the exact opposite is true for a LRB performance characteristic unless unexpectedly $\text{MSD} = (1/n)\Sigma\ 1/y_1^2 > 1$. In short, we generally expect (not always) $\eta_{db} < 0$ for a STB and $\eta_{db} > 0$ for a LTB performance measure. Given $\delta_{db} > 0$, then the desired amount of proportional increase in S/N is

$$\text{PI} = \begin{cases} -\dfrac{\eta'_{db} - \eta_{db}}{\eta_{db}} = \dfrac{-\delta_{db}}{\eta_{db}} & \text{for STB} \quad (15a) \\[2ex] \dfrac{\eta'_{db} - \eta_{db}}{\eta_{db}} = \dfrac{\delta_{db}}{\eta_{db}} & \text{for LTB} \quad (15b) \end{cases}$$

Note that is (15), PI > 0 since $\eta_{db} < 0$ for a STB quality measure. Rearranging (15) yields

$$\eta'_{db} = -10\log_{10}(\text{MSD}')$$

$$= \begin{cases} (1 - \text{PI})\eta_{db} \text{ for STB} & (16a) \\ (1 + \text{PI})\eta_{db} \text{ for LTB} & (16b) \end{cases}$$

where MSD' is the new improved MSD after parameter design. Combining (9) and (16) results in

$$\text{MSD}' = \begin{cases} (\text{MSD})^{1-\text{PI}} \text{ for STB} & (17a) \\ (\text{MSD})^{1+\text{PI}} \text{ for LTB} & (17b) \end{cases}$$

Equations (17) indicate that if an amount of increase in δ_{db} is desired, then the necessary proportional decrease in the present known MSD is

$$d = \frac{\text{MSD} - \text{MSD}'}{\text{MSD}}$$

$$= \begin{cases} (\text{MSD})^{\text{PI}} \text{ for STB} & (17a) \\ (\text{MSD})^{\text{PI}} \text{ for LTB} & (17b) \end{cases}$$

where generally $0 < d < 1$ and $\text{PI} > 0$ is given is (15). Thus the required amount of reduction is MSD from (18) is

$$\text{MSD} - \text{MSD}' = d \times \text{MSD}$$

$$= \begin{cases} d\ \left[1 + \dfrac{(n-1)\text{CV}^2}{n}\right]\bar{y}^2 \text{ for STB} & (19a) \\[2ex] d\ \left[1 + \dfrac{3(n-1)\text{CV}^2}{n}\right]\Big/\bar{y}^2 \text{ for LTB} & (19b) \end{cases}$$

where we have made use of equations (7) and the fact that the REL-VAR of y is $\text{CV}^2 = S^2/\bar{y}^2$. Equations (19) indicate that if the coefficient of variation of y, CV, does not exceed 30% of STB and $0.30/\sqrt{3} = 17.32\%$ for LTB, than it is most economical (especially for STB) to spend nearly all available resources to adjust the mean toward the target m ($m = 0$ for STB and $m = \infty$ for LTB).

Table 1. Required Amount of Increases in S/N Ratio vs Desired Values of (1-r)

1 - r	0.05	0.10	0.15	0.20	0.25	0.30	0.333$\overline{3}$
δ_{db} decibels	0.2228	0.4576	0.7058	0.9691	1.2494	1.5490	1.7609
1 - r	0.40	0.45	0.50	0.55	0.60	0.65	0.666$\overline{6}$
δ_{db} decibels	2.2185	2.5964	3.0103	3.4679	3.9794	4.5593	4.7712
1 - r	0.70	0.75	0.80	0.85	0.90	0.95	0.99
δ_{db} decibels	5.2288	6.0206	6.9897	8.2391	10.000	13.0103	20.00

CHAPTER VIII

1. ANALYSIS OF DATA FROM A PARAMETER DESIGN WHEN Y IS AN NTB RESPONSE

For an NTB QCH, we need to be concerned not only about variation but also about how close the process mean is to the nominal (or target) value m. As a result, Taguchi uses the following measures to evaluate each parameter's effect on the response y from the standpoint of the variance and the mean.

(1) The effect on the mean relative to the standard deviation is measured from

$$\text{S/N Ratio} = \eta_{dB} = 10 \log_{10} \left[\frac{\frac{1}{n}(Sm - V_e)}{V_e}\right] \tag{8.1}$$

where $V_e = \frac{1}{n-1}\Sigma(y_{ij} - \bar{y})^2 = S^2 =$ Error variance $= \frac{1}{n-1}(\Sigma y_{ij}^2 - Sm)$.

(2) The effect on the mean is measured from

$$Sm_{dB} = \text{Sensitivity} = 10 \log_{10} (Sm) \tag{8.2}$$

where $Sm = T_i^2/n$ and $T_i = \sum_{j=1}^{n} y_{ij}$. Note that

$$\eta_{dB} = 10 \log_{10}\left(\frac{Sm/n}{V_e} - \frac{1}{n}\right) = 10\ \ell og_{10}\left[\left(\frac{(\bar{y})^2}{V_e} - \frac{1}{n}\right)\right] \doteq 10\ \ell og_{10}\left[\frac{(\bar{y})^2}{V_e}\right] = 10\ \ell og_{10}(1/CV^2).$$

where $CV = \frac{\sqrt{Ve}}{\bar{y}} = \frac{S}{\bar{y}}$ is the coefficient of variation and $S^2/(\bar{y})^2$ is called the relative variance. Thus we have established that for an NTB type QCH, the Taguchi S/N ratio is approximately

$$\eta_{dB} = 10 \log_{10} (CV)^{-2} = -20 \log_{10} (CV) \tag{8.3}$$

and since small values of CV are always desirable, it follows from (8.3) that larger values of η_{dB} are always desirable, i.e., in a Taguchi experiment one of the objectives is always to maximize η_{dB}.

Example 8.1 Consider the experimental layout on pages 96-97 (from ASI Inc) where y = force of cable in lbs., involving 8 controllable factors with factor A at 2 levels and factors B, C, ..., H each at 3 levels. The design and analysis are also given on pages 95-101. To obtain facility with the calculations, the student should verify the computations on pages 96-97 for at least 2 of the factors, say A and E.

Since factors B through H have 3 levels, we must actually assess both their linear and quadratic effects on S/N ratio and the mean, as illustrated below. (Recall that linear coefficients are -1,0,1 and quadratic coefficients are 1,-2,1.)

LINEAR AND QUADRATIC EFFECTS ON S/N RATIO

Effects	A*	B	C	D*	E	F	G	H*
Linear	-12.784	20.876	14.082	4.678	3.020	23.780	27.240	1.589
Ranks	5	3	4	6	7	2	1	8
Quadratic	N/A	61.63	3.85	58.85	-22.03	1.81	29.67	76.69
R_i	N/A	2	6	3	5	7	4	1

The above table shows that factors B, G, F and H strongly influence S/N ratio and D is moderately influential, while C, A and E are weak. Therefore, "control" factors are B, D, F, G and H, and any one of C, A and E may be an adjustment (or signal) factor. Thus, a "control" factor is one that affects the S/N (or the variance) and may or may not affect the mean. An adjustment (or signal) factor affects only the mean but not the S/N ratio.

Before we examine the linear and quadratic effects of the eight factors on sensitivity (or the mean), it is paramount for the reader to realize that factors A, D and H are qualitative and it is meaningless to compute their

* = Qualitative and hence not applicable.

Exercise X: DATA ANALYSIS for "Nominal the best characteristic" using SENSITIVITY & SIGNAL TO NOISE RATIO

$m \pm \Delta$

y = Core pull force of ignition cable (lbs), NTB. Specifications: 40 ± 15 (lbs), A_c = 11.25

CONTROLLABLE FACTORS

	Level 1	Level 2	Level 3
A: Extrusion Tooling	Type 1	Type 2	
B: Line Speed	Slow	Medium	Fast
C: Water Through Temp.	Low	Medium	High
D: Insulation Material	Type 1	Type 2	Type 3
E: CV Stream Pressure	Low	Medium	High
F: CV Speed	Slow	Medium	Fast
G: Braid Tension	Low	Medium	High
H: Release Coating	Type 1	Type 2	Type 3

NOISE FACTORS

S: Sample
P: Position within Sample

LAYOUT & DATA See next page.

~~Q.1) Complete response table for~~ "Averages" and "Signal to Noise Ratio"

AVERAGES in lbs	A	B	C	D	E	F	G	H
Level 1	46.583	52.958	45.542	61.458	52.833	42.125	59.458	55.917
Level 2	58.389	49.50	47.833	47.875	43.50	69.125	49.25	39.25
Level 3		55.00	64.083	48.125	61.125	46.208	48.75	62.292
Ranks						1		2

AVERAGE SIGNAL TO NOISE RATIOS in decibels	A	B	C	D	E	F	G	H
Level 1	15.4974	14.760	13.721	16.032	13.924	12.856	13.341	16.785
Level 2	14.0770	11.363	14.573	11.518	16.011	14.687	13.139	10.527
Level 3		18.239	16.068	16.812	14.427	16.819	17.881	17.050
R_i	6	2	7	3	8	5	4	1

Note that the divisors for the factor A are 9 while for the other factors the divisors are 6.

$X_0 = A_1 B_3 C_2 D_3 E_2 F_3 G_3 H_3$

Layout and Data for Example 8.1

L18

Sensitivity

L_{18}	A 1	B 2	C 3	D 4	E 5	F 6	G 7	H 8	S_1 P_1	S_1 P_2	S_2 P_1	S_2 P_2	T TOTAL	$\frac{\Sigma y_i^2 - S_m}{n-1}$ S^2	$\frac{T^2}{n}$ Sm	$10\log_{10}(S_m)$ S_{mdB}	$10\log_{10}(\frac{\bar{y}^2}{S^2} - \frac{1}{n})$ η_{dB}
1	1	1	1	1	1	1	1	1	30	40	38	49	157	60.9167	6162.25	37.8974	13.9863 = η_1
2	1	1	2	2	2	2	2	2	10	15	25	25	75	56.2500	1406.25	31.4806	7.7815
3	1	1	3	3	3	3	3	3	49	53	53	55	210	6.3333	11,025.00	40.4238	26.3844
4	1	2	1	1	2	2	3	3	62	58	52	68	240	45.3333	14,400.00	41.5836	18.9852
5	1	2	2	2	3	3	1	1	30	50	49	62	191	174.9167	9120.25	39.6001	11.0671
6	1	2	3	3	1	1	2	2	10	25	29	36	100	120.6667	2500.00	33.9794	6.9281
7	1	3	1	2	1	3	2	3	58	42	41	50	191	62.9167	9120.25	39.6001	15.5617
8	1	3	2	3	2	1	3	1	28	29	32	31	120	3.3333	3600.00	35.5630	24.3096
9	1	3	3	1	3	2	1	2	110	74	94	115	393	341.5833	33,612.25	45.8673	14.4731
10	2	1	1	3	3	2	2	1	76	88	66	103	333	254.2500	27,722.25	44.4283	14.3151
11	2	1	2	1	1	3	3	2	52	37	54	59	202	89.6667	10,201.00	40.0864	14.5012
12	2	1	3	2	2	1	1	3	55	79	62	98	294	368.3333	21,609.00	43.3463	11.5887
13	2	2	1	2	3	1	3	2	5	35	16	42	98	289.6667	2401.00	33.8039	2.6060
14	2	2	2	3	1	2	1	3	52	96	79	91	318	387.0000	25,281.00	44.0279	12.0632
15	2	2	3	1	2	3	2	1	50	70	56	65	241	80.2500	14,520.25	41.6197	16.5306
16	2	3	1	3	2	3	1	2	15	20	18	21	74	7.0000	1369.00	31.3640	16.8702
17	2	3	2	1	3	1	2	3	51	62	59	70	242	61.6667	14,641.00	41.6557	17.7163
18	2	3	3	2	1	2	3	1	77	83	66	74	300	50.0000	22,500.00	43.5218	20.5019 = η_{18}
Totals =									820	956	889	1114	3779				266.1702

Note: Because of L_{18}, A x B can be examined for free.

$\bar{S}^2 = 136.6713$

$\bar{\eta} = -10\log_{10}\frac{\bar{S}^2}{\bar{\bar{y}}^2} = 13.0441$

$\Sigma S_i^2 = 2460.083370$

$\bar{\bar{y}} = 52.48611$

$\bar{\eta} = \frac{1}{18}\Sigma\eta_i = 14.7872$

from ASI, Inc.

linear or quadratic effects. However, we have computed their linear and quadratic effects just to assess their influence on S/N ratio. The correct way to analyze the overall significance of a factor (specially a qualitative one) is thru an F test. However, for the data of Example 8.1, the only way we could conduct F tests for η_{dB} is to replicate the entire experiment in order to have degrees of freedom > 0 for the denominator of the F statistics.

We next assess the linear and quadratic effects of each factor on the mean thru the following RT.

LINEAR AND QUADRATIC EFFECTS OF Sm_{dB}

Effects	A*	B	C	D*	E	F	G	H*
Linear	17.5887	-.091	20.081	-18.924	6.666	6.448	-7.121	8.007
Ranks	3	8	1	2	6	7	5	4
Quadratic	N/A	6.01	12.61	15.79	34.98	-42.88	11.56	60.11
Ranks	N/A	7	5	4	3	2	6	1

The above table shows that the strong and moderately strong factors are C, D, H, F, A and E, and the others do not relatively influence the mean. Since D, H and F were control factors, then A, C and E are the only candidates for adjustment factors.

Before optimal conditions can be estimated, the interaction (1)x(2) columns must always be examined for an L_{18} OA for significance. It is important to understand why the L_{18} OA allows the interaction (1)x(2) to be studied.

* - Qualitative and hence not applicable.

AxB INTERACTION FOR S/N RATIO

AxB for S/N

A(1) \ B(2)	1	2	3
1	48.1522	36.9804	54.3444
2	40.4050	31.1998	55.0884

The above table indicates no strong interaction (if any) between A and B from the standpoint of S/N ratio, and thus A will not play a role in maximizing η_{dB}.

We next examine AxB from the standpoint of sensitivity and the mean.

	B_1	B_2	B_3
A_1	109.81	115.163	121.03
A_2	127.86	119.45	116.54

Sm_{dB} Interaction RT

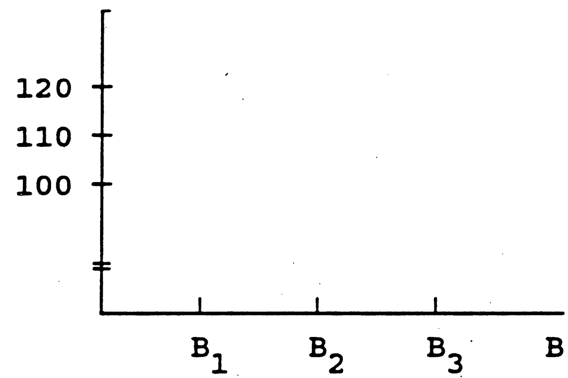


Figure 8.1

Since the two lines in Figure 8.1 cross, it seems that some interaction is present between A and B from the standpoint of sensitivity. Therefore, we also study AxB from the standpoint of averages.

INTERACTION RT FOR THE MEANS

	B_1	B_2	B_3
A_1	36.833	44.25	58.667
A_2	69.083	54.75	51.333

In nearly all quality engineering experiments, decision should be made first to minimize variation and second to force $\bar{y}$ toward the ideal target m. Hence, we first estimate the preliminary optimal conditions based solely on η_{dB} RT on page 96, and then will use the signal factors (A, C and E) to adjust $\bar{y}$ toward m. The RT for S/N on page 96 shows that the preliminary optimal conditions are estimated as

$$A_? \; B_3 \; C_? \; D_{3(or\ 1)} \; E_? \; F_3 \; G_3 \; H_{1\,(or\ 3)}.$$

The expected S/N under the above parameter conditions is

$$\hat{\eta}_{dB} = \bar{\eta}_{B3} + \bar{\eta}_{D3} + \bar{\eta}_{F3} + \bar{\eta}_{G3} + \bar{\eta}_{H3} - 4\bar{\eta}_{db}$$

$$= 18.239 + 16.812 + 16.819 + 17.881 + 17.050 - 4 \times 14.7872 = 27.652.$$

Again, note that the above predicted $\hat{\eta}_{db} = 27.652$ decibels is better than the largest observed S/N ratio in the entire experiment ($\eta_3 = 26.3844$). It is questionable whether $\eta_{dB} = 27.652$ can be achieved for the actual process, but then none of the 18 TCs run for the given experiment contained the optimal conditions $B_3 \; D_{3(or\ 1)} \; F_3 \; G_3 \; H_{3\,(or\ 1)}$.

We next check on the expected mean under the preliminary optimal condition

$$\hat{\mu}_1 = \bar{B}_3 + \bar{D}_3 + \bar{F}_3 + \bar{G}_3 + \bar{H}_3 - 4\bar{y}_{..}$$

$$= 55.00 + 48.125 + 46.208 + 48.75 + 62.292 - 4(52.4861)$$

$$= 50.4306$$

which is far away from the target of m = 40 and hence unacceptable. We 1st note that we can easily set factor H at H_1 instead of H_3 because S/N ratio will be reduced only by 0.265 decibels while the mean will be improved to

$$\hat{\mu}_2 = 50.4306 + (55.917 - 62.292) = 44.0556 \text{ lbs}.$$

In order to further force the mean toward m = 40, we no longer can change the levels of our control factors and have to resort to adjusting the levels of our signal factors (A, C and E).

There are a total of 2x3x3 possible ways that the levels of A, C and E can be selected. If we select the combination A_2 C_2 E_2, we will obtain $\hat{\eta}_{dB}$ = 27.686 and $\hat{\mu}_3$ = 26.75, where AxB interaction was considered in computing $\hat{\mu}_3$. After some search, we observe that the combination A_2 C_3 E_2 should improve the S/N ratio by (14.077 + 16.068 + 16.011 - 3 x 14.7872) = 1.7944 decibels and force the mean to $\hat{\mu}_4$ = 43.00 lbs. Therefore, our final optimal FLC is

$$A_1\ B_3\ C_2\ D_3\ E_2\ F_3\ G_3\ H_1$$

at which we expect $\hat{\eta}_0$ = (27.652 - .265 + 1.7994) = 29.182 decibels and $\hat{\mu}_0$ = 43.00 for our confirmation runs. Before performing a loss function analysis, the experimenter must conduct at least n ≥ 4 confirmation runs at this last optimal settings and compare the values of η_c and μ_c with the predicted values of 29.182 decibels and $\hat{\mu}$ = 43.00 lbs.

As a final note, recall that the A x B was influential from the standpoint of the mean and the interaction table on page 99 showed that A_2 B_3 is optimal. To show that is the case, obtain the solution of Exercise 8.1.

$$X_o = A_1 B_3 C_2 D_3 E_2 F_3 G_3 H_3.$$

Exercise 8.1 Compute $\hat{\eta}_{dB}$ and $\hat{\mu}$ for the factor settings A_1 B_3 C_3 D_3 E_2 F_3 G_3 H_1 and compare against those of the optimal FLC. (b) Verify the values of $\hat{\eta}_{dB}$ = 27.686 and $\hat{\mu}_3$ = 26.75 above.

Exercise 8.2 Consider the experimental layout below on a tire QCH y = shoulder drop.

Inner OA L_4

TC	C 1	E 2	B x D 3	B 4	A 5	B x E 6	D 7	M_1 O_1	M_1 O_2	M_2 O_1	M_2 O_2	Totals
1	1	1	1	1				137	142	161	158	598
2	1	1	1	2				152	140	137	137	566
3	1	2	2	1				171	160	170	168	669
4	Inner L_8 OA							151	153	156	149	609
5	2	1	2	1				164	157	151	156	628
6	2	1	2	2				162	155	168	160	645
7	2	2	1	1				153	154	150	143	600
8	2	2	1	2				135	139	136	134	544
							Totals	1225	1200	1229	1205	$y_{..} = 4859$

Specifications are 150 ± 15 in 100th of an inch and A_c = \$30.00. Controllable factors are as follows: A = Pressure, B = time, C = rim width, D = tread profile, E = shoulder set. Noise factors are: M = machines, O = operators.

(a) Identify weak, moderate and strong effects.

(b) Determine optimal conditions.

(c) What values of average response and S/N ratio will you expect from confirmation runs?

$SS_1 = 417$, $SS_2 = 153$, $SS_3 = 74.75$, $SS_4 = 26.75$; $SS_5 = 86.0$, $SS_6 = 86.75$, $SS_7 = 74.0$, $SS_8 = 14.0$. [Note that SS (Pure error + Noise) = $\sum_{i=1}^{8} SS_i = 932.25$. with 24 degrees of freedom].

2. LOSS FUNCTION ANALYSIS FOR AN NTB RESPONSE

Computation of average reduction in losses per unit for the NTB case is more difficult than the previous two cases because first the S/N ratio cannot perhaps be directly linked to L. Secondly, reduction in losses can originate from two sources - reducing variance and then adjusting $\bar{y}$ toward m. Unlike a magnitude type QCH, even if cv < 30%, for an NTB response it is profitable to reduce variance as much as possible by optimizing the levels of control factors and then adjusting the mean by the use of the signal factors. We illustrate the procedure by performing an LFA for Example 8.1.

Example 8.2 Our first step is to estimate the average loss per unit before PDE given by

$$E(L) = k\ [\sigma^2 + (\mu - m)^2] \tag{8.4}$$

where $k = A_c/\Delta^2 = 11.25/15^2 = 0.05$. In the absence of any other information, we estimate the present existing process average as $\hat{\mu} = \bar{\bar{y}} = \Sigma\Sigma\ y_{ij}/nN = 3779/72 = 52.48611$. As an estimate of before PDE process variance we use either the pooled biased estimator

$$\hat{\sigma}_1^2 = (n-1)\ \Sigma\ S_i^2/nN. \tag{8.5}$$

or the estimate $(n-1)\ S^2/n$ obtained from

$$\bar{\eta}_{dB} = 10\ \log_{10}\left[\frac{(\bar{\bar{y}})^2}{S^2} - \frac{1}{n}\right] \tag{8.6}$$

where $\bar{\eta}_{dB} = \sum_{i=1}^{N} \eta_i/N$. Equation (8.6) can be rearranged to yield

$$S^2 = \frac{n(\bar{\bar{y}})^2}{1+n\ (10^{\bar{\eta}_{dB}/10})} \tag{8.7}$$

and hence

$$\hat{\sigma}_2^2 = (n-1)\ S^2/n. \tag{8.8}$$

It is paramount to note that if the present existing process condition (PEPC) is one of the N treatment combinations actually conducted in the PDE experiment, then it is best to use the mean and S/N ratio under PEPC in preference to $\bar{\bar{y}}$ and equations (8.5) or (8.8).

If we use equation (8.5), then for the estimate of process variance we obtain

$$\hat{\sigma}_1^2 = (4-1)\ (2460.08337)/[(4)(18)] = 102.50347.$$

Substituting into equation (8.4) yields

$$\bar{L} = .05\ [102.50347 + (52.48611 - 40)^2] = \$12.92$$

i.e., one estimate of existing loss per unit is \$12.92.

<u>Exercise 8.3</u> Obtain another estimate of $\bar{L}$ by estimating σ^2 from equation (8.8). ANS: \$11.20.

Next, we need to estimate the mean and variance under optimal conditions $A_2\ B_3\ C_3\ D_3\ E_2\ F_3\ G_3\ H_1$. Assuming that the results of the n verification runs are consistent with $\hat{\mu}_o = 43.00$ <u>lbs</u> and $\hat{\eta}_0 = 29.182$ (See p. 101), then we use $\hat{\mu}_o = 43.00$ in equation (8.4) and estimate the variance under optimal conditions from

$$\sigma_o^2 = (n-1)\ S_o^2/n \quad , \tag{8.9a}$$

$$S_o^2 = \frac{n(\hat{\mu}_o)^2}{1+n\ (10^{\hat{\eta}_{dB}/10})} \tag{8.9b}$$

where $\hat{\eta}_{dB}$ is the estimated S/N under optimal conditions. Inserting $\hat{\mu}_o =$

43.00 and $\hat{\eta}_0$ = 29.182 into (8.9b) results in S_o^2 = 2.2315 so that $\hat{\sigma}_o^2$ = 1.674. Since the average QL per unit under optimal FLC is

$$L_0 = k\ [\sigma_o^2 + (\mu_o - m)^2] \tag{8.10}$$

we obtain $L_0 \doteq .05\ [1.674 + (43.00 - 40)^2] = \$.534$. Therefore, the percent reduction in average QL per unit is [(12.92 - .534)/12.92] x 100% = 95.87%. Note that the decibel improvement is δ_{dB} = 29.182 - 14.7872 = 14.395 and Table 1 of Maghsoodloo (1990), page 93, gives (l-r) x 100% $\doteq$ 96%, even though the values in Table 1 were specifically obtained for magnitude type (i.e., STB and LTB) QCHs.

Exercise 8.4 Recompute the percent reduction in average QL, $[(\bar{L} - L_0)/\bar{L}]$ x 100%, using the result of Exercise 8.3. ANS: 95.23%

Exercise 8.5 Perform an LFA for the PDE of Exercise 8.2.

CHAPTER IX

1. PERCENT CONTRIBUTION OF A FACTOR TO THE VARIATION IN THE RESPONSE

In order to compute percent contribution of a factor, we must first obtain the ANOVA table. Then the % contribution of the factor, say A, is defined as

$$\rho_A = \frac{SS_A - \nu_A\,(MS_{Error})}{SS_{Total}} \times 100\%, \qquad (9.1)$$

where ν_A = df of the factor A. To illustrate the procedure, we obtain the ANOVA table for the experiment whose data is given on page 97 (See Table 9.1).

$$SS_{total} = 30^2 + 40^2 + 38^2 + \ldots + 74^2 - \frac{3779^2}{72} =$$

$$= 243571 - 198345.0139 = 45225.98622,$$

lbs^2

$$SS_A = \frac{1667^2 + 2102^2}{36} - CF = 2508.6806,$$

$$SS_B = \frac{1271^2 + 1188^2 + 1320^2}{24} - CF = 371.0278,$$

$$SS_C = \frac{1093^2 + 1188^2 + 1538^2}{24} - 198345.0193 = 4904.8611.$$

Similarly, SS_D = 2898.7778, SS_E = 3732.0278, SS_F = 10166.7778, SS_G = 1753.0278, SS_H = 6794.6944.

The reader should recall that the OA L_{18} allows the interaction (1)x(2) to be studied. To compute SS_{AxB}, we should make use of a table that crosses A and B. Then

$$SS_{AxB} = \frac{442^2 + 531^2 + 704^2 + 829^2 + 657^2 + 616^2}{12} - CF - SS_A - SS_B = 4715.8611.$$

we next compute the SS's due to the noise factors.

$$SS(\text{due to Sample}) = \frac{1776^2 + 2003^2}{36} - CF = 715.6806$$

$$SS_{Position} = \frac{1709^2 + 2070^2}{36} - CF = 1810.0139$$

$$SS(Sample\ X\ Position) = \frac{820^2 + 956^2 + 889^2 + 1114^2}{18} - CF - SS_{sample} - SS_P =$$

$$= 110.0139$$

Note that generally it is best to pool all the noise SS's together and hence compute it from

$$SS_{noise} = \frac{820^2 + 956^2 + 889^2 + 1114^2}{18} - \frac{3779^2}{72}$$

$$= 2635.708\overline{3} \text{ (with df = 3)}$$

Therefore, the error SS's is computed from SS_{Error} = 45225.98611 - 2508.6806 - 371.0278- ... -110.0139 = 4744.5417, with df = 51. The percent contribution of factor A is

$$\rho_A = \frac{2508.6806 - 1\ (4744.5417/51)}{45225.98611} = 5.14\%$$

The percent contribution of factor H is

$$\rho_H = \frac{6794.6944 - 2\ (93.02023)}{45225.98611} = 14.61\%$$

The complete ANOVA table is given below where ρ_{Error} must be computed by subtraction. Note that the 51 degrees of freedom assigned to the experimental error is not due to pure error but rather due to the interaction between the controllable factors (with 17 df) and the noise factors (with 3 df).

Table 9.1 ANOVA For The Data Of Example 8.1

Source	df	SS's	MS	F_o	ρ in %
Total	71	45225.9862			
A	1	2508.6806	2508.6806	26.966**	5.34%
B	2	371.0278	185.5139	1.994	0.41%
AxB	2	4715.8611	2357.9306	25.346**	10.02%
C	2	4904.8611	2452.4306	26.36**	10.43
D	2	2898.7778	1449.3889	15.580**	6.00%
E	2	3732.0278	1866.0139	20.058**	7.84
F	2	10166.7778	5083.3889	54.642**	22.07%
G	2	1753.0278	876.5139	9.422**	3.46
H	2	6794.6944	3397.3472	36.519**	14.61
S	1	715.6806	** - Significant		ρ_{noise} = 5.21%
P	1	1810.0139	at the 1% level		
SxP	1	110.0139	$F_{.01,2,51}$ = 5.05		
Subtotal	3	2635.7083			
Experimental Error	51	4744.5417	93.0302		14.81%

The experimenter may pool the insignificant effects such as B and S x P with pure error to obtain a pooled MSE_{Error} = 96.7701 which leads to the pooled ANOVA table. But we recommend against such pooling because this will tend to make the effects of the remaining factors larger than they actually are.

The reader must bear in mind that the F statistics in the ANOVA Table determine the effects of each factor on only the mean and not the S/N ratio or the variance. This implies that the classical analysis of experimental design generally ignores the effects of factors and interactions on the variation of the response and concentrates solely on the mean of y.

2. THE UTILITY OF ANOVA FOR A TAGUCHI PDE

If the inner OA of a PDE has some 3-level factors that are qualitative, then the computation of linear and quadratic effects for those qualitative factors is meaningless. The proper method of assessing each factor's influence in such a design is first to obtain the ANOVA table for both the S/N ratio and sensitivity, and then assign a rank of 1 (i.e., the most influential) to the factor with the largest MS and a rank of 2 to the 2nd strongest factor, and so on. We first discuss the ANOVA table for S/N ratio that will identify the control factors, and then follow it up with the ANOVA table for Sm_{dB} to identify the signal factors.

The ANOVA Table For S/N Ratios

Consider the PDE of Example 8.1 where factors A, D and H are qualitative, and as stated before it is (actually) meaningless to compute their linear and quadratic effects. Therefore, the correct method of assessing the influence of each factor on the variance of y (or the S/N ratio) is through an ANOVA table for S/N ratios, which is given in Table 9.2.

To illustrate how the SS's in Table 9.2 were arrived at, we illustrate the computation of SS_{Total}, $SS(\eta_A)$, $SS(\eta_E)$ and $SS(\eta_A \times \eta_B)$.

$$SS_{total} = 13.9863^2 + 7.7815^2 + \ldots + 20.5010^2 - \frac{266.1702^2}{18} =$$

$$= 4519.69223 - 3935.92085 = 583.7714$$

$$SS(\eta_A) = \frac{139.4770^2 + 126.6932^2}{9} - 3935.92085 = 9.0792$$

$$SS(\eta_E) = \frac{83.5424^2 + 96.0658^2 + 86.562^2}{6} - CF = 14.2375$$

$$SS(\eta_A \times \eta_B) = \frac{48.1522^2 + 36.9804^2 + 54.3444^2 + 40.405^2 + 31.1998^2 + 55.0884^2}{3} -$$

$$- CF - SS(\eta_A) - 141.8217 = \underline{6.5855}$$

The ranks in the ANOVA Table 9.2 are nearly consistent with those indicated from the response table on page 96. As before, they indicate that factors H, B, D and G strongly influence S/N ratio while factor F moderately

Table 9.2. The ANOVA Table for S/N Ratio of Example 8.1

Source	df	SS	MS	Ranks
Total	17	583.7714		
A	1	9.0792	9.0792	6
B	2	141.8217	70.9109	2
AxB	2	6.5855	3.2928	9
C	2	16.9384	8.4692	7
D	2	98.0253	49.0127	3
E	2	14.2375	7.1188	8
F	2	47.2158	23.6079	5
G	2	86.2870	43.1435	4
H	2	163.5810	81.7905	1
Error	0	not retrievable		

influences process variance. Therefore, factors B, D, F, G and H are "control" factors. We next obtain the ANOVA table for sensitivity to identify the signal factors. The inquisitive reader may wonder why the ANOVA of Table 9.1 cannot be used to assess the influence of each factor on the mean? The answer is that it can; however, it may violate the two main assumptions (normality and equality of variances) required for the F tests. The variance-reduction transformation 10 $\log_{10}$ Sm is designed to diminish the impact of both requirements. The ANOVA table for sensitivity is provided in Table 9.3.

Table 9.3. The ANOVA Table for Sensitivity of Example 8.1

Source	df	SS	MS	Ranks
Total	17	342.4229		
A	1	17.7185	17.7185	7
B	2	1.0025	.5013	9
AxB	2	43.0608	21.5304	3
C	2	38.0196	19.0098	4
D	2	36.7687	18.3844	6
E	2	37.6875	18.8438	5
F	2	54.5381	27.2691	2
G	2	7.9358	3.9679	8
H	2	105.6914	52.8457	1
Error	0			

As discovered before, Tables 9.2 and 9.3 show that only C, E and A qualify as adjustment factors. Note that the ANOVA tables do not help identify the optimal conditions. A response table is needed to determine the optimum level of each factor.

3. CONFIDENCE INTERVALS FOR THE PREDICTED S/N RATIO AND THE MEAN RESPONSE

It has been mentioned numerously in the past that once optimal conditions are estimated for a Taguchi experiment, the predicted S/N ratio ($\hat{\eta}_o$) and the predicted mean ($\hat{\mu}_o$) have to be verified by repeated confirmation runs. Further, if the QCH is of a magnitude type (i.e., m = o or ∞) and the CV < 0.30, it is then necessary to be more concerned about the S/N ratio than the mean. The question arises: how close should the results of the r (each with n observations) confirmation runs be to the predicted values before the experimenter is fairly sure that the optimal conditions have been verified? To answer this question, we must first compute the standard errors for the predicted S/N ratio and the predicted mean. We will complete the first step before describing the second step.

Consider the layout of the PDE experiment on page 97, where it is inherently assumed that our objective function can be modeled as follows:

$$\eta_{db} = \bar{\eta} + a_i + b_j + c_k + d_\ell + e_m + f_n + g_r + h_t + \epsilon \ , \qquad (9.2)$$

$$i = 1,2;\ j, \ldots,\ t = 1,2,3 \quad ,$$

where $\bar{\eta}$ = the expected value of S/N = E (η_{dB}), a_i is the effect of the ith level of factor A on S/N ratio [i.e., $a_i = \eta(A_i) - \bar{\eta}$], . . . , and $h_t = \eta\ (H_t) - \bar{\eta}$. Equation (9.2) implies that the model for the system's S/N ratio is additive and of first order. Put differently, we are tacitly assuming that the behavior of the system's S/N ratio can be approximated by a 1st-order additive model. Therefore, if the result of the r confirmation runs, $\bar{\eta}_c$, is far from $\hat{\eta}_o$, one reason can be due to the inadequacy of an additive model, i.e., interactive terms such as a x $b_{ij} = \eta\ (AB_{ij}) - \eta\ (A_i) - \eta\ (B_j) + \bar{\eta}$ have to be added to the model (9.2) to make it more adequate. Therefore, one source of discrepancy between $\bar{\eta}_c$ and η_o is from the inadequacy of the model. The model error in estimating a_i, b_j, ... h_t is measured by σ_ϵ^2/n_o, where σ_ϵ^2 is the error variance for S/N ratio, and for the optimal conditions of Example 8.1 (A_1 B_3 C_3 D_3 E_2 F_3 G_3 H_1) the value of n_o is estimated from

$$\frac{1}{n_0} = \frac{1}{N} + \left(\frac{1}{n_{B3}} - \frac{1}{N}\right) + \left(\frac{1}{n_{D3}} - \frac{1}{N}\right) + \frac{1}{n_{F3}} - \frac{1}{N}) + \left(\frac{1}{n_{G3}} - \frac{1}{N}\right) + \left(\frac{1}{n_{H1}} - \frac{1}{N}\right). \tag{9.3}$$

In Equation (9.3), $N = 18$, $n_{B3} = 6$, $n_{D3} = 6$, etc.

The second source of error in predicting η_0 is due to inability of the r confirmation experiments conducted at the optimal conditions to yield exactly the same values of $\bar{y}$ and η_{dB} (i.e., the pure experimental error or replication error). The replication error is measured by σ_ϵ^2/r. Since the two components of error, from model inadequacy and from replication, are completely independent, the variance of the prediction error is

$$\sigma^2_{Pred}\ (\bar{\eta}) = \frac{\sigma_\epsilon^2}{n_0} + \frac{\sigma^2{}_\epsilon}{r} \tag{9.4}$$

Since σ_ϵ^2 is generally unknown, it has to be estimated from the pertinent ANOVA Table. For example, if we have to estimate the error variance for the S/N ratio of Example 8.1, then Table 9.2 shows that our only choice is to pool the SS's of the weak effects (A, AxB, C, E), which yields

$$\hat{\sigma}^2\ (\eta_\epsilon) = \frac{9.0792 + 6.5855 + 16.9384 + 12.2375}{7} = 6.6915 \text{ (with df} = 7).$$

Note that the df for error variance should be at least 5 (preferably 6). The value of n_0 for the first component of (9.4) is obtained from Equation (9.3), which is $n_0 = 18/11$.

Suppose now the experimenter conducts $r = 6$ confirmation experiments at A_1 B_3 C_3 D_3 E_2 G_3 H_1 (using the same noise levels) and obtains $(\eta_c, \bar{y}_c)$ = (22.3 db, 43.8 lbs), (28.6,39.4), (25.4,46.3), (21.6,42.7), (29.6,38.2), (24.5,31.6). The average S/N ration for this data is $\bar{\eta}_c = 25.334$ decibels.

Now, is this a consistent confirmation run with the predicted $\hat{\eta}_0 = 29.182$?

To answer this last question, as a 2nd step we need to construct a 95% prediction interval within which the future process S/N ratios under optimal conditions are expected to lie. From Equation (9.4) the prediction variance is

$$\hat{\sigma}^2_{Pred.} = \frac{6.6915}{18/11} + \frac{6.6915}{6} = 5.2045$$

so that the s.e. $= \sqrt{\hat{\sigma}^2_{Pred}}$ of predicted average s/N ratio is 2.2814 decibels.

The one-sided 95% predicted interval is give by

$$\hat{\eta}_0 - t_{.05;\nu} \text{ xs.e.}(\hat{\eta}), \qquad (9.5)$$

where ν is the df of σ^2_ϵ ($\nu = 7$ for this case). For the data of Example 8.1, we obtain the prediction interval $29.182 - 1.895 \times 2.2814 = (24.859, \infty)$. Since $\overline{\eta}_c = 25.334$ does lie in this interval, then our optimal conditions have been barely verified. In case of inconsistent confirmation runs, the experimenter has 2 choices:

(1) It is possible that the additive model is not appropriate, and an interactive model with some 2nd-order terms are needed.

(2) The estimated error variance, $\hat{\sigma}^2(\eta_\epsilon)$, may be too small in which case the experimenter must compare $S^2_\eta = 10.5427$ with $\hat{\sigma}^2(\eta_\epsilon) = 6.6915$ and examine their closeness. As a rule of thumb, if $S^2_\eta/\hat{\sigma}^2(\eta_\epsilon)$ exceeds 4, then our estimate of $\sigma^2(\eta_\epsilon)$ may be inaccurate.

Next, we examine the closeness of the mean of confirmation runs $\overline{y}_c$ with $\hat{\mu}_0$. Again, we need a prediction interval for $\overline{y}_c$ about $\hat{\mu}_0$, which is given by

$$\hat{\mu}_0 \pm t_{.025;\nu}\hat{\sigma}_{Pred} (\overline{y}), \qquad (9.6)$$

where $\hat{\sigma}_{Pred.} (\overline{y})$ is the s.e. for the predicted average.

To illustrate the use of (9.6), consider Example 8.1 where $\hat{\mu}_0 = 43.00$ lbs. From Table 9.1, a pooled estimate of σ^2_ϵ (y) is

$$\hat{\sigma}^2_\epsilon \ (y) = \frac{2635.7083 + 4744.5417}{3 + 51} = 136.6713$$

and

$$\hat{\sigma}^2 \ (\bar{y}) = \frac{136.6713}{4} = 34.1678.$$

The use of Equation (9.4) now yields

$$\hat{\sigma}^2_{Pred} \ (\bar{y}) = 34.1678 \ (\frac{1}{18/11} + \frac{1}{r}).$$

For r = 6 replications, we obtain $\hat{\sigma}_{Pred} \ (\bar{y}) = 5.1551$. Since $t_{.025;54} = 2.005$, then Equation (9.6) yields the 95% prediction interval 43.00 $\pm$ 10.3360 = (32.664,53.336). The mean of the confirmation runs is $\bar{y}_c = \frac{1}{6} \sum_{i=1}^{6} \bar{y}_i = \frac{1}{6}$ (43.8 + 39.4 + 46.3 + 42.7 + 38.2 + 31.6) = 40.333 which is well within the predicted interval. Again, if $\bar{y}_c$ falls outside the predicted interval, then either the additive model is inadequate or our estimate $\hat{\sigma}^2 \ (\bar{y}) = 34.1678$ is too small. For the given six confirmation runs $S^2_{\bar{y}} = 26.9827$ which is quite consistent with $\hat{\sigma}^2(\bar{y})$.

Exercise 9.1. Consider the PDE experiment of Exercise 8.2. The following 5 sets of confirmation runs have been obtained under optimal conditions:

Run No.	M_1		M_2	
	O_1	O_2	O_1	O_2
1	161	152	153	154
2	139	142	134	140
3	153	138	142	155
4	149	153	156	153
5	151	158	152	146

(a) Does the above data confirm the predicted S/N ratio $\hat{\eta}_o$ at $1-\alpha = .95$?

(b) Repeat part (a) for the predicted mean $\hat{\mu}_o$.

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ABOUT THE COURSE INSTRUCTOR

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