A Taguchi Parameter Design (PDE) Experiment consists of two orthogonal arrays. The inner array accommodates the controllable factors, while the noise (or uncontrollable) factors are imbedded into the outer orthogonal array. The objectives of a PDE (parameter design experiment) for a nominal dimension is three-fold, the last two of which are optimization steps: (1) To classify the design factors into 3 categories of Control, Signal, and Weak factors. A Control factor is one that impacts process variability and may or may not impact the process mean response. A Signal factor significantly impacts the mean response but has no (or trivial) impact on the variability of the response. A Weak factor has no impact on the mean or variability of the response. (2) To use the levels of the Control factors to reduce process variability. (3) To use the levels of Signal factors to move the mean response toward the ideal target m.

When the response, y, is either STB or LTB, QI can be accomplished in one step by increasing the signal-to-noise ratio (which in turn lowers the signal for an STB and heightens it for an LTB QCH), and as a result the experimenter can accomplish objectives (2) and (3) in one step by setting the process conditions at those levels of influential factors that maximize Taguchi's S/N ratio, measured in decibels (dB), as defined below.

$$\eta_{db} = -10\log_{10}(\text{MSD}) = \begin{cases} -10\log_{10}\left(\sum_{i=1}^{n} y_i^2/n\right), & \text{if y is STB} \\ -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}(1/y_i^2)\right], & \text{if y LTB} \end{cases} \qquad (3)$$

It should be highlighted that classical design of experiments have until early 1980's emphasized methods that improve only the mean response and hence one OA would generally be sufficient, and in some cases more efficient, to carry out QI only when the response y is STB or LTB. However, when the response is of nominal type, the variability of response plays a very important role in QI, and hence an outer OA is needed to imbed the noise factors that cause process variability. In a PDE, the experimenter intentionally induces noise into the response y through the use of an outer array and then takes advantage of the interactions between the noise factors in the outer array with the controllable factors in the inner array to assess and then diminish the impact of noise on the response y (using appropriate S/N ratios). The impact of noise factors is diminished by selecting those levels of the controllable influential factors imbedded in the inner OA which are less sensitive to noise factors imbedded in the outer array, thereby producing a more robust product.

In Chapter 5 of the Manual we dealt with FFDs (or OAs) where the experimenter has taken only n = 1 observation at each FLC of the design matrix. Since at least 2 determinations are needed in order to assess variability (i.e., to compute the value of standard deviation S) at each FLC, the impact of different factors on the variability of the response variable (measured by σ_y) could not easily be evaluated unless one design factor's effect on the response is determined to be trivial. In other words, to determine if a factor affects σ_y, it is generally necessary (but not always) to run at least 2 experiments at some FLCs of the design matrix, and in order to maintain balance of the design matrix, it is best to observe y an equal number of times, n, at all FLCs. Thus, if the experimenter can afford only n = 1 observation per cell, then most likely only the impact of different effects (imbedded in the design matrix) on the mean of the output can be assessed. In such situations, the only hope the experimenter has is to identify one or more factors that have minimal impact on the mean response, E(Y), but do have significant impact on σ_y.

According to Taguchi, design factors should be classified into 2 types: (1) Controllable, (2) Noise. Factors whose levels can easily be controlled by the experimenter (such as Process Temperature and Pressure) are called controllable. Noise factors are those that are too difficult, time-consuming, or too expensive (or all three) to control (i.e., beyond the control of an experimenter), and as a result it is generally cheapest to take counter measures against noise factors at the secondary design stage. Further, noise factors are in turn categorized into Outer and Inner noise factors. Examples of outer noise factors are: Dust, Humidity, Air (or ambient) Temperature, Operator and/ or Customer behavior, etc. Examples of Inner Noise factors are: Oxidization, deterioration of parts and subcomponents, and unit to unit (within) variation.

Again, the objective of Taguchi's Parameter Design (PDE) is to identify the levels of controllable factors (or the FLC(s)) that are least sensitive to noise factors. Therefore, a Taguchi's PDE (or robust design) experiment must have 2 OAs: the Inner OA, and the Outer OA. The design factors that are easily controllable are imbedded in the inner OA, and noise factors (those that are difficult to control) are placed in the outer OA. As an example see page 83 of the manual, where there are 6 controllable factors (A = "Baffle", ..., F = "Gasket Thickness") that are imbedded in an L_8 inner OA, but it is not quite clear to this author whether there is only one noise factor (Oil level of compressor) or 2 noise factor – Oil Level and Compressor to Compressor variation. If I had to go out on the limb, my guess is that 2 compressors were made at each FLC of the inner array and their loudness were measured at the N_1 level of oil, and then the oil levels of the same 2 compressors were changed to N_2 and the resulting loudness's were measured. Thus, the outer array was an L_4 OA, not L_2.

Recall that there are basically two types of static QCHs:
Magnitude (STB and LTB) type, and NTB. It must be emphasized that if y is an STB or LTB, then the magnitude of the signal (i.e., the mean of y = E(Y)) takes precedence over σ_y, while if y is an NTB type QCH, then it is also essential for the experimenter to 1st identify factors that impact σ_y, and then identify factors that merely affect E(y).

Taguchi defined a (controversial) measure called "Signal-to-Noise' ratio that simultaneously reduces variation and improves the signal, and this in turn reduces societal QLs. For a magnitude type QCH (STB and LTB), signal-to-noise (S/N) ratio defined previously and given below.

$$\eta_{db} = -10\log_{10}(\text{MSD}) = -10\log_{10}(\mathbf{S_n^2} + \text{mean}^2),$$

where

$$\text{MSD} = \begin{cases} \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} y_i^2, & \text{y is STB} \\ \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} (1/y_i^2), & \text{y is LTB} \end{cases},$$

and for the LTB case, mean = $\bar{\mathbf{x}}$, $\mathbf{S}_{\mathbf{n}}^{\mathbf{2}}$ = $\mathbf{S}_{\mathbf{n}}^{\mathbf{2}}$(x) , and x = 1/y. Since Taguchi's MQLs (mean quality losses) is $\bar{\mathbf{L}}$ = k(MSD), then our experimental objective must be to always minimize MSD in order to minimize societal QLs. Further, η_{db} = –10log$_{10}$ (MSD) = –10log$_{10}$ ($\bar{\mathbf{L}}$/ k) = 10log$_{10}$(k /$\bar{\mathbf{L}}$) shows that maximizing S/N ratio in turn minimizes average societal QLs. Therefore, in all PDE experiments the objective will be to maximize the S/N ratio of the system, which in turn will minimize societal quality losses.

Therefore, for both STB and LTB it is sufficient to identify the FLC(s) that maximize(s) the S/N = η_{db} = –10log$_{10}$($\bar{\mathbf{L}}$/ k), and this in turn minimizes log$_{10}$($\bar{\mathbf{L}}$/ k) = –η_{db}/10, which also minimizes $\bar{\mathbf{L}}$ = k($\mathbf{10}^{-\eta_{\mathbf{db}}/\mathbf{10}}$) = $\mathbf{k} / (\mathbf{10}^{\eta_{\mathbf{db}}/\mathbf{10}})$.

For a NTB QCH, however, the problem is two-fold because the ideal target, m, hardly ever is zero and is never equal to infinity. Therefore, a 2-step procedure is needed to arrive at the (nearly) optimal condition, X_0, which is outlined below.

1. Identify the design factors that significantly impact process variation; such factors are called "Controls". Set the levels of control factors in order to maximize the system S/N ratio given by

$$\eta_{dB} = 10\log_{10}[(\bar{\mathbf{y}}/\mathbf{S})^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{n}}] \cong \log_{10}(\bar{\mathbf{y}}/\mathbf{S})^{\mathbf{20}} \text{ , or } \eta_{dB} \cong 20\log_{10}(\bar{\mathbf{y}}/\mathbf{S})$$

2. Identify design factors that significantly impact the process mean, E(Y), but have no effect on σ_y. Such process parameters are called Signal (or adjustment) factors. Use the levels of signal factors to move μ_y toward the ideal target m.

The above 2-step procedure in turn will minimize societal average societal QLs given by $\bar{\mathbf{L}}$ = k[$\mathbf{S}_{\mathbf{n}}^{\mathbf{2}}$ + ($\bar{\mathbf{y}}$ – m)^2].

This brings us to the Example 7.1 on pp. 82-87 of your Manual, which will be discussed in details in class.

Example 8.1 (The contents borrowed from ASI, Inc.) on pages 95-105 of my Manual

The RT at the bottom of page 96 clearly shows that the factors that impact process variability in the order of their strength are H, B, D, G and F. These are called process Controls, i.e., they control process S/N ratio (or variability). The levels of these factors must be set in such a manner as to always maximize S/N ratio (or in turn to minimize variability). Therefore, the optimal settings of these factors are

$$B_3D_{3(1)}F_3G_3H_{3(1)}$$

which provide 4 choices with which to reduce process variation.

The RT for the mean pull force is given in the middle of page 96 of my Manual, which shows that the remaining 3 factors (A, C, and E) have a significant impact on the process mean μ. The levels of these 3 signal factors must be selected in such a manner as to move the mean as close as possible to the ideal target of m = 40 lbs.

Since the Taguchi L_{18} OA allows studying only the interaction of column (1) with column (2), the RTs on page 99 of my manual show that factors A and B do not interact from the standpoint of S/N ratio but they do interact from the standpoint of the mean (see the interaction RT at the bottom of page 99 of the Manual), then we have to consider the A$\times$B interaction only when computing $\hat{\mu}$.

For the sake of illustration, suppose we set the process at the $FLC_1 = A_2B_3 C_1D_3E_2F_3G_3H_3$. Then, the estimated S/N ratio under this FLC is

$$\hat{\eta}_1 = 14.0770 + 18.239 + 13.721 + 16.812 + 16.011 + 16.819 + 17.881 + 17.050 - 7(14.7872) = 27.0992 \text{ dB}$$

and the corresponding mean (see the interaction table at the bottom of page 99 of the Manual) is estimated from

$$\hat{\mu}_1 = 51.333 + 45.542 + 48.125 + 43.5 + 46.208 + 48.75 + 62.292 - 6(52.4861) = 30.8333 \text{ lbs.}$$

Clearly, the above FLC_1 does improve the presumed (default) existing S/N of $\bar{\eta}$ = 14.7872 by roughly 12.30 dB but under adjusts the mean by roughly 9.17 lbs. The FLC_1 leads to a reduction to average societal QLs of 61.86%. Hence, there may well be another FLC that will produce much more reduction in predicted MQLs (mean quality losses).

Next we try the $FLC_2 = A_2B_3C_3D_3E_2F_3G_3H_1$. Similar computations as above lead to $\hat{\eta}_2$ = 29.1814 dB and $\hat{\mu}_2$ = 43.00 lbs, which are superior to those of FLC_1. We now compute the MQLs for both the default (Def) values $\bar{\eta}_{\mathbf{dB}}$ = 14.7872 dB, $\bar{\bar{\mathbf{y}}}$ = 52.4861 lbs and those at the FLC_2. Recall that for a nominal dimension, $\bar{\mathbf{L}} = k[\mathbf{S}_{\mathbf{n}}^{\mathbf{2}} + (\bar{\mathbf{y}} - m)^2]$.

Thus, we must first obtain the value of default $\mathbf{S}_{\mathbf{n}}^{\mathbf{2}}$ from $\bar{\eta}_{\mathbf{dB}}$ = 14.7872 dB as follows:

$$14.7872 = 10\log_{10}\left(\frac{\mathbf{52.4861^2}}{\mathbf{S}_{\mathbf{Def}}^{\mathbf{2}}} - \frac{\mathbf{1}}{\mathbf{4}}\right) \rightarrow \mathbf{S}_{\mathbf{Def}}^{\mathbf{2}} = 90.7356$$

$\rightarrow \mathbf{S}_{\mathbf{n}}^{\mathbf{2}}(\mathbf{Def})$ = 68.05172, where again Def stands for default. $\rightarrow \bar{\mathbf{L}}_{\mathbf{Def}} = 0.05\times[68.05172 + (52.4861 - 40)^2] = 11.19774 \cong \$ 11.20$. Similarly, $\hat{\eta}_{\mathbf{2}}$ = 29.1814 leads to $\mathbf{S}_{\mathbf{n}}^{\mathbf{2}}(\mathbf{2})$ = 1.67405 and $\bar{\mathbf{L}}_2 = 0.05[1.67405 + (43 - 40)^2] = \$0.5337$, which leads to a % reduction in societal QLs of 95. 234%.

This brings about the question " what is the best, or optimum, estimated FLC" ? Clearly, it is the one that minimizes $\bar{\mathbf{L}}$. For a nominal dimension, the only way to find the $FLC_o = X_0$ is thru a complete computer search. Recall that our controls provided 4 choices (D_1H_1, D_1H_3, D_3H_1, D_3H_3). The signal factors A, E, and C provide $2\times3\times3$ more choices, and thus a total of $4\times18 = 72$ possible different FLCs that we have to examine in order to arrive at near optimum condition. The computer program by H-H (Kevin) Hsu shows that $X_0 = A_1B_3C_2D_3E_2F_3G_3H_3$. This FLC has $\hat{\eta}_{\mathbf{o}}$ = 29.372 and $\hat{\mu}_{\mathbf{o}}$ = 40.4583 lbs, which in turn yield $\bar{\mathbf{L}}_{\mathbf{o}}$ = \$0.08142 and a percent reduction in societal QLs of 99.273%.

In practice, it is extremely doubtful (perhaps impossible) that we can attain 99.273% QI in a manufacturing process after one set of experiments. Even if we attain one 3rd of what we predicted, i.e. 33%, that is quite a bit of reduction in societal QLs. To check on the validity of our predicted 99.273% QI, we must set the process at $X_o = A_1B_3C_2D_3E_2F_3G_3H_3$ and make at least 12 cables (i.e., r = 6 confirmation experiments) and measure the pull force at two positions P_1 and P_2 on each cable, just like in the design matrix L_{18} on page 97 of the Manual. Note that it is necessary to make 2 cables per run. Suppose the results of the 6 confirmation runs are given below:

$(\eta_c, \bar{\mathbf{y}}_{\mathbf{c}})$ = (22.30 dB, 43.8 lbs), (28.6, 39.4), (25.4, 46.3), (21.6, 42.7), (29.6, 38.2), (24.5, 31.6). These 6 confirmation runs lead to the statistics $\bar{\eta}_{\mathbf{c}}$ = 25.3333, S_η =

3.2469, $\bar{\bar{y}}_c$ = 40.3333 and $S_{\bar{y}}$ = 5.1945. To determine whether these confirmation runs are consistent with the predicted values of $\hat{\eta}_o$ = 29.372 and $\hat{\mu}_o$ = 40.4583 lbs, we need to obtain the 95% CIs for E(η) and μ = E(Y). Since S/N ratio is a measure that must be maximized in all systems, it is judicious to first compare $\bar{\eta}_c$ against $\hat{\eta}_o$ (= 29.372 dB) before proceeding with any statistical inference. If the value of $\bar{\eta}_c \geq \hat{\eta}_o$, then immediately conclude that the predicted S/N ratio $\hat{\eta}_o$ has already been confirmed. Otherwise, if $\bar{\eta}_c << \hat{\eta}_o$, say by more than 1 dB, then a test of hypothesis becomes necessary. In this case, the null hypothesis is H_0: $\eta_{system} = \hat{\eta}_o$ = 29.372 versus the alternative H_1 : η_{system} < 29.372 dB. The statistic for this left-tailed test is given by

$$t_0 = \frac{(\bar{\eta}_c - \hat{\eta}_o)\sqrt{r}}{S_\eta},$$

where for our example r = 6 and the degrees of freedom of the above test statistic is ν = r – 1 = 5. The rejection interval for the above test statistic consists of values of t_5 in the range ($-\infty$, $t_{0.95,5}$) = ($-\infty$, – 2.015). For the observed 6 confirmation runs, the value of t_0 = – 3.047, which clearly lies in the rejection region implying that our confirmation S/N ratio, $\bar{\eta}_c$, is not consistent with the predicted optimal value of $\hat{\eta}_o$ = 29.372. However, the experimenter can rejoice the fact that the system S/N ratio has been improved from the presumed (default) value of $\bar{\eta}_{dB}$ = 14.7872 dB to $\bar{\eta}_c$ = 25.3333.

To determine if the observed value of $\bar{\bar{y}}_c$ = 40.3333 verifies the predicted mean value of $\hat{\mu}_o$ = 40.4583 lbs, it is generally best to obtain a 2-sided 95% CI for E(Y) = μ_y, although it is obvious in this case that the predicted mean has been well confirmed because $\bar{\bar{y}}_c$ = 40.3333 is closer to the ideal target of m = 40 lbs than is $\hat{\mu}_o$ = 40.4583 lbs. We, however, proceed with obtaining a 95% CI for μ_y just to illustrate the procedure. The requisite CI is given by

$$\bar{\bar{y}}_c - t_{0.025,5} \times se(\bar{\bar{y}}_c) \leq \mu_y \leq \bar{\bar{y}}_c + t_{0.025,5} \times se(\bar{\bar{y}}_c),$$

where the $se(\bar{\bar{y}}_c) = S_{\bar{y}} / \sqrt{r}$ = 5.1945/(6)$^{1/2}$ = 2.121. Substitution in the last interval yields

$$34.8812 \leq \mu_y \leq 45.7855 ,$$

which easily contains the predicted mean value $\hat{\mu}_o$ = 40.4583.

Before closing, we may wish to answer the question " has our societal QLs been diminished from process optimization"?

Recall that our $\bar{L}_{Def}$ = $ 11.20 before PDE, and assuming that now the values of our S/N ratio and the mean have been improved to $\bar{\eta}_c$ = 25.3333, and $\bar{\bar{y}}_c$ = 40.3333, then what is the expected QLs based on our confirmation experiments?

AS before, we need to compute the value of $S_n^2(c)$ from $\bar{\eta}_c$ = 25.333.

$$25.\,3333 = 10 \times \log_{10} \left[\frac{40.3333^2}{S_c^2} - \frac{1}{4}\right] \rightarrow \quad S_c^2 = 4.7608 \rightarrow \; S_n^2(c) = 3.5706.$$

Hence, $\bar{L}_c$ = 0.05[3.5706 +(40.3333 –40)^2] = $0.1841 → % reduction in societal QLs associated with confirmation runs = (11.20 –0.1841)/11.20 = 98.36%.