INSY 7330 Summer Term 2013 Maghsoodloo

Prerequisite STAT/INSY 7300

e-mail: maghsood@eng.auburn.edu

Web Site: www.eng.auburn.edu/~maghsood/homepage.html

Tel: (334) 844-1405 Fax: 1381

Taguchi's Major Contributions to QE (Quality Engineering): (1) Redefined Quality, (2) Introduced OAs (orthogonal arrays) as an aid for engineers to design experiments, (3) Introduced Robust (i.e., Parameter and Tolerance) Designs and recommended the use of his S/N (Signal-to-Noise) ratios in analyzing data from a Robust design.

There are 3 types of Static Quality Characteristics (QCHs): STB (Smaller-The-Better), LTB, NTB (Nominal-The-Best)

Examples of STB type QCH: Error Rate, Tire Imbalance, Noise level of an engine, Tool Wear, Braking Distance, Warpage, etc. All STB QCHs have 2 common features: Their ideal target is m = 0, and they have only a single consumers' USL (Upper Specification Limit) denoted by $y_u$.

Examples of LTB QCH: % Yield, TTF (Time-to-Failure), Weld Strength, (Fuel) Efficiency, Net Profit, Tape adhesiveness, etc. All LTB QCHs have 2 common features: Their ideal target m = $\infty$, and they all have only a single LSL = $y_L$

Examples of NTB QCHs: pH level in a chemical compound, Output voltage (of a TV set), Tape Edge Weave, Clearance, etc. All NTB QCHs have 2 common features: They have an ideal target denoted by m $\neq \infty$, and they all have 2 consumers' specifications $y_L$ = LSL = m$-\Delta$ and $y_u$ = USL = m + $\Delta$, where $\Delta$ is called the manufacturing allowance by Taguchi. Majority of nominal dimensions are symmetrical, i.e., m = (LSL + USL)/2, but asymmetric tolerances occur often in manufacturing processes, in which case $y_L$ = LSL = m $-$ $\Delta_1$ and $y_u$ = USL = m + $\Delta_2$, where $\Delta_1 \neq \Delta_2$.

Traditional (or Conventional) View of Quality

If a unit's dimension is within specification limits (LSL, USL), then the amount of quality losses imparted to society is 0. Further, if a unit is out of specs, no matter how far out of specs, then the amount of quality losses imparted to society is only $$A_c$.

Thus, $QL_{Trad}(y, STB) = \begin{cases} \mathbf{0}, & \mathbf{0 < y \le y_u} \\ \mathbf{A_c}, & \mathbf{y > y_u} \end{cases}$,

$QL_{Trad}(y, LTB) = \begin{cases} \mathbf{0}, & \mathbf{y_L \le y < \infty} \\ \mathbf{A_c}, & \mathbf{y < y_L} \end{cases}$, and $QL_{Trad}(y, NTB) = \begin{cases} \mathbf{0}, & \mathbf{y_L \le y \le y_u} \\ \mathbf{A_c}, & \textbf{Otherwise} \end{cases}$.

Taguchi's View of Quality

The farther a unit's dimension is from the ideal target m, the more quality losses is imparted to society by that unit, i.e., Quality is the amount of losses an item imparts to society from the instant that is shipped $\rightarrow$ Lack of Quality for one item = y – m $\rightarrow$ L(y) = $k(y - m)^2$, where L(y) is called Taguchi's QLF (Quality Loss Function). For the case of asymmetric tolerances, this last QLF generalizes to

$L(y) = \begin{cases} \mathbf{k_1(y-m)^2}, & \mathbf{y \le m} \\ \mathbf{k_2(y-m)^2}, & \mathbf{y > m} \end{cases}$, where $k_1 = \dfrac{\mathbf{A_1}}{\mathbf{\Delta_1^2}}$, $k_2 = \dfrac{\mathbf{A_2}}{\mathbf{\Delta_2^2}}$, $A_1$ and $A_2$ are the amount of QLs at the LSL and USL, respectively. Note that Taguchi's QLF is the same as Karl Gauss's quadratic loss. When $\Delta_1 = \Delta_2 = \Delta$, then $k = A_c/\Delta^2$.

<u>Example 1.</u> y = The Output Voltage of a TV Set, the ideal target m = 115 volts, $\Delta$ = 20 volts $\rightarrow$ $y_L$ = LSL = 115 - 20 = 95, $y_u$ = USL = 135 volts, $A_c$ = $150.00

Data from Manufacturer 1 ($M_1$): 121, 131, 117, 94, 110, 112, 118, 109, 114, 93 $\rightarrow$ $n_1$ = 10

Data from $M_2$: 97, 110, 116, 129, 133, 101, 96, 98, 134, 96, 137, 99 $\rightarrow$ $n_2$ = 12. Since the amount of QL at 95 and 135 volts is $150.00, then 150 = k(115 $\pm$ 20–

$m)^2 \rightarrow k = 150/400 = 0.375 \rightarrow M_1$: $L_1 = 0.375\,(121-115)^2 = \$13.50$, $L_2 = 0.375\times(131-115)^2 = \$96.00$, ..., $L_{10} = \$181.50 \rightarrow \bar{L}_1 = \frac{1}{n_1}\sum_{j=1}^{10} L_{1j} = 487.875/10.$

$\longrightarrow \bar{L}_1 = \$48.7875$; Similarly, $\bar{L}_2 = \frac{1}{n_2}\sum_{j=1}^{12} L_{2j} = 99.3125 \rightarrow$

QDPU (quality difference per unit) = \$50.525 for $M_1$ over $M_2$.

Clearly, Taguchi's average QL per unit must be $\bar{L} = \frac{1}{n}\sum_{j=1}^{n} L_j = \frac{1}{n}\sum_{j=1}^{n} k(y_j - m)^2 =$

$k[\frac{1}{n}\sum_{j=1}^{n}(y_j - m)^2] = k(MSD)$. It can be shown (study pp. 9-10 of my manual) that for a nominal dimension $MSD = \frac{1}{n}\sum_{j=1}^{n}(y_j - m)^2 = S_n^2 + (\bar{y} - m)^2$, where $S_n^2 = \frac{1}{n}\sum_{j=1}^{n}(y_j - \bar{y})^2$ is the sample variance, as illustrated in the following table.

k = 0.375

| | $\bar{y}$ | $(\bar{y} - 115)^2$ | $S_n^2$ | MSD | $\bar{L} = k(MSD)$ |
|---|---|---|---|---|---|
| $M_1$ | 111.90 | 9.61 | 120.49 | 130.10 | \$48.7875 |
| $M_2$ | 112.1666 | 8.027778 | 256.8055 | 264.8333 | \$99.3125 |

From the traditional view of quality, $AQL_{Trad1} = (2\times150)/10 = \$30.00$/ TV set, $AQL_{Trad2} = (1\times150)/12 = \$12.50$/set $\rightarrow$ $QDPU_{Trad}$ of $M_2$ over $M_1$ is \$17.50 per set. The amount of contradiction between the two views of quality evaluations is \$68.025 per unit.

## The Use of Taguchi's QLF, L(y), to Set Up Production Tolerances

For the sake of illustration, consider the QLF for the Example 1, where $L(y) = 0.375(y - 115)^2$ for both manufacturers. Suppose that during (or at the end) of production the output voltage of a TV set, y, can be adjusted (or calibrated) toward the ideal target of $m = 115$ volts with the use of a resistor. Further, the total cost of such an adjustment before shipment is \$6.00. The question arises "for what values of y should the \$6.00 be spent" ? Answer : Spend the \$6.00 before shipment only if the unit's quality loss $L(y) >$ \$6.00. That is, spend the \$6.00 only if $0.375\,(y - 115)^2 >$ \$6.00 $\rightarrow (y - 115)^2 > 16 \rightarrow |y - 115| > 4 \rightarrow$ Either $y - 115 < -4$, or $y - 115 > 4 \rightarrow$ Spend the \$6.00 only if $y < 111$ or $y > 119$.

Therefore, the production (or on the drawing) tolerances for the output voltage are 115 $\pm$ 4 volts, as opposed to $115 \pm 20$ volts for consumers' tolerances.

It can be shown that for an NTB type QCH, in general, in-factory allowance $\Delta_f = \Delta_c\sqrt{A_f / A_c}$, where for the above example $\Delta_c = 20$ volts, $A_f =$ \$6.00 and $A_c =$ \$150.00.

Exercise 1. Work the QE Exercise on page F-H of my manual, carrying at least 4 decimals.

## The QLF for an STB QCH

Substituting the value of $m = 0$ into the Taguchi's QLF $L(y) = k\,(y - m)^2$ results in $L(y) = k(y - 0)^2 = ky^2$. Since it is assumed that the amount of societal QLs at the USL = $y_u$ is $A_c$, then $A_c = k\,y_u^2 \rightarrow k = A_c/ y_u^2$ .

Exercise 2. Show that for an STB type QCH, $\bar{L} = k(MSD) = k[S_n^2 + (\bar{y})^2]$ , where the $MSD = \frac{1}{n}\sum_{i=1}^{n} y_i^2$ .

Example 2. As an example, consider Exercises 2.7 and 2.8 on page 23 of my manual. Clearly, the Radial Force Harmonic (RFH) of a passenger tire is of an STB type QCH. (a) Since $k = A_c/ y_u^2 = 10.00 / 26^2 = 0.014793$, then for one tire $L(y) = 0.014793y^2$ . (b) $AQL_{Trad1} = (2\times10)/10 =$ \$2.00/ tire, and $AQL_{Trad2} = (1\times10) /12 =$ \$0.8333/ tire $\rightarrow$ $QDPU_{Trad}$ of brand 2 over brand 1 = \$1.166667.
(c) $k = 0.014793$.

| Brands | $\bar{y}$ | $(\bar{y})^2$ | $S_n^2$ | MSD | $\bar{L}$ = k(MSD) |
|---|---|---|---|---|---|
| $B_1$ | 18.64 | 347.4496 | 45.3424 | 392.792 | \$5.8106 |
| $B_2$ | 21.65 | 468.7225 | 19.2825 | 488.005 | 7.2191 |

From the above Table, $QDPU_{Tag}$ of Brand 1 over $B_2$ = \$1.4085 $\rightarrow$ Quality contradiction between the traditional and Taguchi's view = \$2.5752 per tire.

## Taguchi's QLF For an LTB Type QCH

Since the ideal target is $\infty$, then $L(y) = k(y - \infty)^2 = \infty$ for all values of y, no matter how large y is or how far y exceeds the LSL = $y_L$. Therefore, in the case of LTB QCH, we have to make a transformation in order to evaluate the quality of an item by letting x = 1/y. Because y is LTB, then the random variable (rv) X is an STB type QCH, i.e., the ideal target for X is zero, leading to the Taguchi's QLF as $L(y) = k(x - 0)^2 = k(1/y)^2 = k/y^2$. The amount of societal QLs at the LSL = $y_L$ is $A_c$, and thus $k = A_c y_L^2$. The Taguchi's average QL for n randomly selected items is given by $\bar{L}$ = k (MSD), where $MSD = \frac{1}{n}\sum_{i=1}^{n}(1/y_i^2)$. The reader must not confuse the $\sum_{i=1}^{n}(1/y_i^2)$ with the $1/(\sum_{i=1}^{n} y_i^2)$.

In fact, if at least one $y_i \neq 0$ and n > 1, then $\sum_{i=1}^{n}(1/y_i^2) > 1/(\sum_{i=1}^{n} y_i^2)$. I have not been able to find a counter example to this last claim that I am making?! Can you?

Example 3. Consider the data of Exercise 2.9 on page 24 of your manual. Clearly welding strength is an LTB type QCH and hence $k = A_c y_L^2$ = (\$20.00) × $(1.20\text{ksi})^2 = 28.80\$\text{ksi}^2$ $\rightarrow$ $L(y) = 28.80/y^2$.

$$MSD_1 = \frac{1}{9}\left[\frac{1}{16} + \frac{1}{4} + 1 + \frac{1}{1.3^2} + \frac{1}{1.1^2} + \frac{1}{36} + \frac{1}{4.8^2} + \frac{1}{1.5^2} + \frac{1}{0.9^2}\right] = \frac{1}{9}[4.48085516]$$

= 0.4978728 $\rightarrow$ $\bar{L}_1$ = 28.8($MSD_1$) = \$14.33874 per unit.

Exercise 3. Repeat the above Example for the data of Exercise 2.10 on page 25 of the manual. (b) Compute the QDPU of the two welding methods from both the traditional and Taguchi's view of quality. Compute the amount of quality contradiction

(QCTDN) per unit if there is any. (c) Recompute the MSDs for both welding methods by transforming the data to the dimension x = 1/y, and then using the fact that MSD = $S_n^2$ + $(\bar{x})^2$. (d) For the welding method 1, compute the values of $L_1$, $L_2$, ..., $L_9$ and use these to re-compute $\bar{L}_1$ .

## Computation of Taguchi's Expected QL When Process Parameters Are Known and the QCH Y is an NTB Type

Suppose the process mean $\mu$ and process standard deviation $\sigma$ are known (or specified). Thus, sampling the process is totally unnecessary because the Taguchi's expected societal QLs, $E(QL_{Tag})$, for the entire population can easily be computed as follows:

$$E(QL_{Tag}) = E[k(y - m)^2] = kE\{[(y - \mu) + (\mu - m)]^2\} =$$

$$= k[E(y - \mu)^2 + E(\mu - m)^2] = k[\sigma^2 + (\mu - m)^2]. \qquad (1)$$

If the process is centered, i.e., if $\mu = m$, then $E(QL_{Tag}) = k\sigma^2$.

Example 4. Suppose the output voltage, y, ~ N(110, 121 volts$^2$). The consumers' Specs are 115 ± 20 volts and $A_c$ = \$150.00; then the societal QLs for one unit is $L(y) = 0.375\,(y - 115)^2$. However, over the life-cycle of this particular brand of TV sets, the average QLs over all sets produced and sold by the manufacturer from Eq. (1) is equal to $0.375[121 + (110 - 115)^2]$ = \$54.75 per set.

Note that sampling the manufacturer's product in this case is unnecessary because the process parameters are known and need not be estimated.

## Computation of $E(QL_{Trad})$ When the Sampling Distribution of y is Known

For the sake of illustration, consider the scenario of the Example 4 above, as depicted in Figure 1 atop the next page. Figure 1 clearly shows that $Z_L = (95 - 110)/11 = -1.363\overline{36} \rightarrow p_L = \Phi(-1.363\overline{36}) = 0.086341$, and $Z_u = (135 - 110)/11$

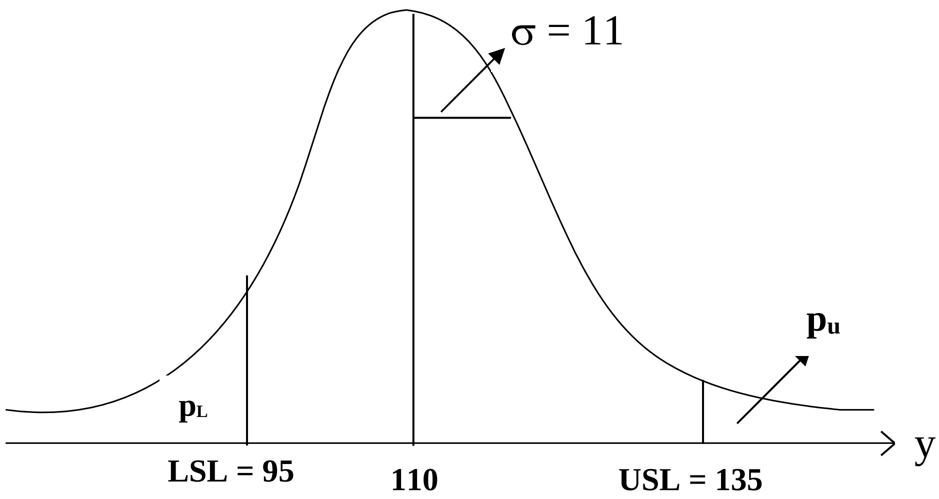


Figure 1

$= \mathbf{2.2727\overline{27}} \rightarrow p_u = 1 - \Phi(\mathbf{2.2727\overline{27}}) = \Phi(-\mathbf{2.2727\overline{27}}) = 1 - 0.98848 = 0.01152 \rightarrow p =$ $p_L + p_u = 0.097862 \rightarrow E(QL_{Trad}) = A_c p = 150 \times 0.097862 = \$14.68$. Note that, in general, $E(QL_{Tag}) \geq E(QL_{Trad})$. In this case \$54.75 vs \$14.68 → QCTDN/unit = quality contradiction per unit = \$40.07.

## Natural (Or Statistical) Tolerances (Known Parameters)

Natural tolerances of a machining process are determined by its standard deviation $\sigma$, while the consumers' (or design) tolerances are specified as $m \pm \Delta$. The reader must not confuse the two issues because $\sigma$ and $\mu$ practically determine the process capability while $y_L = LSL = m - \Delta$, and $y_u = USL =$
$m + \Delta$ are specified by the buyer or the designer (or some outside agency unrelated to the manufacturing process). Once $m \pm \Delta$ are specified, then a process is capable of meeting specs iff its natural tolerances are within, or inside, the interval (LSL, USL).

Definition. The $(1-\alpha)\times 100\%$ natural tolerances of a process are defined as the interval of y values that contain exactly $(1-\alpha)$ proportion of the entire population. For example, the 99% natural tolerance limits, $LNTL_{0.99}$ and $UNTL_{0.99}$, of a process contain the dimensions of 99% of all the units produced by the process. If the interval ($LNTL_{0.99}$, $UNTL_{0.99}$) lies within the interval (LSL, USL), then the process is said to be capable of meeting the tolerance level $\alpha = 0.01$. This in turn implies that the FNC (Fraction nonconforming) of the process, p, is less than $\alpha = 0.01$; if $p > \alpha$, then the process is not capable of meeting design specs, where $\alpha$ may be thought of the level of FNC below which the company will survive and prosper!

Example 5. The length of (steel) pipes, y, from a certain manufacturer is N(12", 0.0016 inches$^2$) with consumers' tolerances $12 \pm 0.10$.

Our objective is to determine if the 99% natural tolerances meet consumers' specs (11.90, 12.10). Since the process is normal and centered, then $LNTL_{0.99} = 12.00 - Z_{0.005}\times\sigma = 12 - 2.57583\times0.04 = 11.896967$, and $UNTL_{0.99} = 12.00 + 0.10303320 = 12.103033$. Thus, the tolerance interval (11.8970, 12.1030) contains 99% of the pipes' dimensions produced at a confidence probability of 100% (because $\mu$ and $\sigma$ are known). However, because the 99% tolerance interval (11.8970, 12.1030) contains the consumers' tolerance range of (11.90, 12.10), then the process is not capable of producing a maximum FNC of $\alpha = 0.01$. We may verify this fact by actually computing p for this Gaussian process from $Z_L = (11.90 - 12.00)/0.04 = -2.50 \rightarrow p/2 = \Phi(-2.50) = 0.00621 \rightarrow p = 0.01242 > \alpha = 0.01$. Thus,
the process is not capable of meeting specs because $p > \alpha$. In general, the $(1-\alpha)\times100\%$ natural tolerances for a Gaussian process is given by $\mu \pm Z_{\alpha/2}\times\sigma$. Because, $\mu$ and $\sigma$ are assumed known, the interval $(\mu - Z_{\alpha/2}\times\sigma,\ \mu + Z_{\alpha/2}\times\sigma)$ contains $(1-\alpha)$ proportion of the Gaussian process with certainty.

Exercise 4. (a) For the Example 5 above, determine the maximum value of $\sigma$ at which the process is barely capable of meeting specs. (b) For the same example, suppose y ~ N(12.03, 0.0016), the specs as in part (a) are $12 \pm 0.10$, compute the process FNC, p, and determine if it meets the company-wide tolerance level of $\alpha = 0.01$. (c) For the situation of part (b) determine the maximum value of $\sigma$ for which $p \le \alpha = 0.01$.

It should be emphasized that in most manufacturing processes in the USA, the amount of tolerable company-wide FNC is set roughly at $\alpha = 0.0027$. This leads to 6-sigma natural tolerances if $y \sim N(\mu, \sigma^2)$ as depicted in Figure 2; please note that $\alpha/2 = 0.00135$. Further, note that y represents a single (n = 1) observation

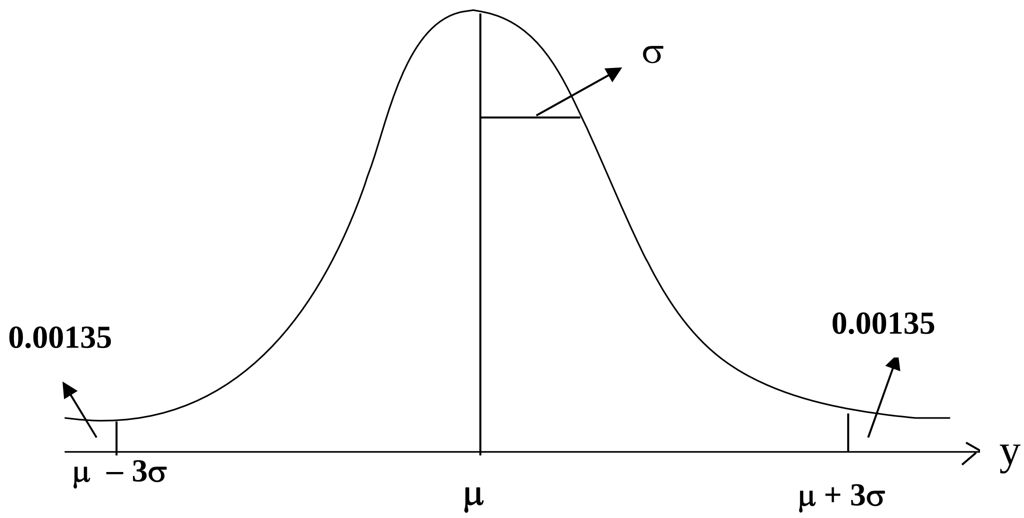


Figure 2.

from a N($\mu$, $\sigma^2$). Figure 2 illustrates that if LSL $\leq \mu - 3\sigma$, then $p_L \leq 0.00135$, and if USL $\geq \mu + 3\sigma$, then $p_u \leq 0.00135$. That is, if the tolerance range (LSL, USL) lies outside the 6-sigma natural tolerances ($\mu - 3\sigma$, $\mu + 3\sigma$), then $p \leq 0.00270$ no matter how off-centered the machining process is! Only then the process is said to be capable of meeting the Quality Level (QLEV) of 0.0027. As the result, it follows that in this case USL – LSL $\geq (\mu + 3\sigma) - (\mu - 3\sigma)$, i.e., USL – LSL $\geq 6\sigma$, which in turn will imply that the process capability ratio PCR = $C_p$ = [(USL – LSL)/(6$\sigma$ )] $\geq$ 1. It is generally best and preferable that we require the value of PCR to exceed 1 because $p < \alpha$ does not guarantee that the corresponding (1– $\alpha$)100% natural tolerances lie inside the design specifications for an off-centered process. For example, given design specs 12 $\pm$ 0.10, $\alpha$ = 0.01, and y ~ N(12.01, 0.001369), then $Z_L$= –0.11/0.037 = –2.972973, $p_L$ = 0.0014746521, $Z_u$= –0.09/0.037 = $-2.432\overline{432}$, $p_u$ = 0.00749889474, p = 0.00897355 < $\alpha$ = 0.01. However, the $UNTL_{0.99}$ = 12.01 + 2.5758$\times$0.037 =12.1053046 lies outside the USL = 12.10. It seems that if natural tolerances lie inside specs, then for certain $p \leq \alpha$. However, the converse of this statement is not necessarily true.

<u>Exercise 5.</u> It is known that the output dimension of a machining process is N($\mu$, $\sigma^2$) and the machine is capable of meeting a QLEV of $\alpha$ = 0.00046530. Given that the

process is centered, i.e., $\mu$ = m, compute the improved value of PCR = (USL – LSL)/(6$\sigma$). [Hint: Since the machine is capable of the QLEV 0.00046530, then $p \le \alpha$].

## Natural Tolerances When Process Parameters Are Unknown

When the population parameters $\mu$ and $\sigma$ are unknown, then the process has to be sampled in order to estimate $\mu$ and $\sigma$ with sample statistics $\bar{\mathbf{y}}$ and S, respectively. For a N($\mu$, $\sigma^2$) process, it was shown on pages 7-9 of these notes that the (1 – $\alpha$)$\times$100% natural tolerance limits are given by $\mu \pm Z_{\alpha/2}\times\sigma$ with 100% confidence probability (Pr). However, when $\mu$ and $\sigma$ of a N($\mu$, $\sigma^2$) process are unknown, then no longer the interval $\bar{\mathbf{y}} \pm Z_{\alpha/2}\times S$ contains at least (1– $\alpha$) proportion of the population at 100% probability. Further, the value of $Z_{\alpha/2}$ has to be adjusted upward to the level of tolerance factor K (see Table 1 on p. 17 of my manual ) in order to account for the error in estimating $\mu$ with $\bar{\mathbf{y}}$ and estimating $\sigma$ with S. The ($\gamma$, 1– $\alpha$) tolerance interval, $\bar{\mathbf{y}} \pm$ K$\times$S, has a confidence Pr (before drawing of the sample) of $\gamma$ to contain at least the (1– $\alpha$) proportion

of all y values produced by a manufacturing process. For n ≥ 30, the 2-sided tolerance factors can be approximated, to 3 decimals, from

$$K = \begin{cases} [(1+1/(2n+1.5)]\times Z_{\alpha/2}\sqrt{(n-1)/\chi^2_{\gamma,n-1}}, & \gamma = 0.95 \\ [(1+1/(2n+2.0)]\times Z_{\alpha/2}\sqrt{(n-1)/\chi^2_{\gamma,n-1}}, & \gamma = 0.99 \end{cases}$$

I do not presently know a good approximation for K of a one-sided tolerance factor, because the exact value of the tolerance factor in this case is given by

$$K = \text{noncentraltinv}(\gamma,\ n-1,\ Z_\alpha\sqrt{n})/\sqrt{n}$$

where the noncentrality parameter of the t distribution is given by $\delta = Z_\alpha\sqrt{n}$, and the noncentral t rv is defined as $T_{n-1}(\delta) = (Z+\delta)\sqrt{n-1}/\sqrt{\chi^2_{n-1}}$. The good news is the fact that Minitab will invert the noncentral t distribution. In Minitab, go to Calc → Probability distributions → t → in the dialogue box, select inverse cdf → insert the values of $\delta$, *df* (degrees of freedom), $\gamma$, and ok. The Matlab syntax for the one-sided tolerance factor is K = nctinv($\gamma$, n –1, $Z_\alpha\sqrt{n}$)/$\sqrt{n}$ .

Example 6. A random sample of size n = 30 steel pipes has a mean of $\bar{y}$ = 11.98 and S = 0.0525, where LSL = 11.90 and USL = 12.10. Obtain the (0.95, 0.99) natural tolerance limits of the process. Table 1 of my Manual shows that K = 3.35, and hence $LNTL_{0.99}$ = 11.98 – 3.35×0.0525 = 11.8041; similarly, $UNTL_{0.99}$ = 12.1559. This implies that the interval (11.8041, 12.1559) contains at least 99% of y values of all the pipes produced by this process at a confidence level of 0.95. Note that the Pr that at least 99% of all pipe dimensions to lie in the interval (11.8041, 12.1559) is either 0 or 1. Our next objective is to determine if this process is capable of meeting the QLEV of $\alpha$ = 0.01?

In the case of unknown $\mu$ and $\sigma$, a measure of process capability (PCP) is given by the process capability index

$$\hat{C}_{pk} = \frac{1}{Z_{\alpha/2}} Z_{min} = \frac{1}{Z_{\alpha/2}} \text{Min}\left(\left|\hat{Z}_L\right|,\ \hat{Z}_u\right), \qquad (2)$$

where $\hat{Z}_L$ = (LSL – $\bar{y}$)/S and $\hat{Z}_u$ = (USL – $\bar{y}$)/S. For our Example 6 above, $\hat{Z}_L$ = (11.90 – 11.98 )/0.0525 = –1.5238095 and $\hat{Z}_u$ = 2.2857143 so that $\hat{C}_{pk} = \frac{1}{Z_{\alpha/2}} \times Z_{min} = \frac{1}{2.57583}$ Min(1.52381, 2.28571) = 0.591580 << 1.00 $\rightarrow$ The process is not capable of meeting the QLEV of $\alpha$ = 0.01, i.e., $\hat{p}$ > 0.01. Note that this is consistent with the fact that the estimated tolerances (11.8041, 12.1559), at the confidence level of $\gamma$ = 0.95, do fall outside the consumers' specs of 12 ± 0.10.

Exercise 6. Compute the value of $\hat{p}$ for the above Example 6, assuming that y is N($\mu$, $\sigma^2$).

Exercise 7. The design specs for diameter of holes on a VCR board is 5 ± 0.003 cm. The drilling machine is capable of producing dimension y that is N (5.00, 0.000001 $cm^2$). (a) Assuming that 6 holes must be drilled on each board, and the resulting board is conforming only if all 6 holes meet design specs, compute the FNC of all boards, $p_b$, produced by the drilling machine. (b) If the production rate is PR = 50,000 boards/month, compute the expected number of boards that have to be scrapped annually. (c) Determine the machine's natural tolerances (i.e., its value of $\sigma$) that reduces the scrap rate to 228/year. ANS: (b) 9653.9014/year.

In the context of the above Exercise 7, if 9654.6 scraped boards /year is intolerable, i.e., $p_b$ = 0.016091 is too large and has to be reduced to an acceptable level, then 2 options exist. (1) Reduce $\sigma_{machine}$ as in the part (c) above to a level such that $p_b < \alpha$, if at all possible. (2) Widen the design tolerances on a hole from 5 $\pm$ 0.003 cm to, say, 5 $\pm$ 0.004 cm and manufacture the VCRs in such a manner that they still will function properly from consumers' standpoint. Such a design that tolerates larger variation (or larger tolerances) in manufacturing, and still works satisfactorily from consumers' standpoint, is called a robust design. Robust designs can withstand relatively more manufacturing noise. Note that in general option (1) is far more expensive than option (2).

## The Relationship Between Natural Tolerances and Taguchi's E(QL) For a Nominal Dimension

We will consider the 4 most common possibilities (out of infinite) as outlined below.

(i) $\mu$ = m and 6-sigma PCP $\rightarrow$ USL – LSL = $6\sigma \rightarrow$ $E(QL_{Tag}) = k\sigma^2 = \frac{A_c}{\Delta^2}[(USL - LSL)/6]^2 = \frac{A_c}{\Delta^2}(2\Delta/6)^2 = A_c/9$; note that in this case $\sigma = \Delta/3$.

(ii) $\mu$ = m and 8-sigma PCP, QI cost = $A_f$, $\rightarrow$ USL – LSL = $8\sigma \rightarrow$

$$E(QL_{Tag}) = \frac{A_c}{\Delta^2}[(USL - LSL)/8]^2 + A_f = \$A_c/16 + A_f$$

(iii) $\mu = m + 0.50\sigma$ and PCR = 1 $\rightarrow$ USL – LSL = $6\sigma$

$$\longrightarrow E(QL_{Tag}) = k[\sigma^2 + (\mu - m)^2] = \frac{A_c}{\Delta^2}[(\Delta/3)^2 + 0.25(\Delta/3)^2]$$

$$= A_c(1/9 + 0.25/9) = \$1.25A_c/9.$$

(iv) $\mu = m + 0.50\sigma$ and 8-sigma PCP, QI cost = $A_f$, $\rightarrow$ USL – LSL = $8\sigma$

$$\sigma = \Delta/4 \rightarrow E(QL_{Tag}) = \frac{A_c}{\Delta^2}[(\Delta/4)^2 + 0.25(\Delta/4)^2] + A_f = \$1.25A_c/16 + A_f$$

Exercise 8. Suppose that QI (quality improvement) at a cost of $A_f$ on a machine has improved the process mean from the off-target value of $m + 0.75\sigma$ to $\mu = m$ and the existing 6-sigma process capability (PCP) to 7-sigma PCP. Compute the % reduction in Taguchi's expected societal QLs.

## Review of Factorial Designs and the Associated ANOVA

We will review this topic quickly through an example, but those of you who do not have the background or need a more careful refresher, please refer to my website at

www.eng.auburn.edu/~maghssa/homepage.html

then scroll down to the section STAT 3610 (the prerequisite to this course), and clicking on Chapter 11 (DOE With Several Factors). Further, you will benefit from studying the chapter 4 of my manual.

Example 7. The shear strength of an adhesive in a chemical process is thought to be affected by the application of process variables A = Pressure, and B = Temperature. A factorial experiment was conducted to asses the effects of the inputs A and B on the process output y = shear strength. The coded data is displayed in the table below. Such a design is said to be a 2×3 complete factorial

| A \ B | 250 °F | 260 | 270 | $y_{i..}$ |
|---|---|---|---|---|
| 120 (lb/in²) | 7, 9 | 10, 13 | 14, 16 | 69 |
| 150 | 15, 19 | 9, 8 | 16, 17 | 84 |
| $y_{.j.}$ | 50 | 40 | 63 | $y_{...}$ = 153 |

experiment (implying that there are at least one response at each possible factor level combination), and since there are n = 2 observations per every cell (or FLC = factor level combination), the design is said to be balanced. Further, N = 2×3×2 = 12 total responses. The USS (Uncorrected Sum of Squares) = $\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} y_{ijk}^2$ = 2127.00 (with 12 df) , the CF = Correction Factor = $y_{...}^2/N = 153^2/12 = 1950.75$ (with 1 degree of freedom, df or DOF) → $SS_T$ = SS(Total) = USS – CF = 176.25 (with 12 – 1 = 11 degrees of freedom = df). $SS_A = (69^2 + 84^2)/6 - CF = 18.75$ (with 1 df); similarly you may verify that SS(B) = 66.50 (with 2 df). The experimental (pure) error SS has to be computed only from the within each cell variation as follows: $SS_{11} = 7^2 + 9^2 - 16^2/2 = 2.0$ (with 1 df), $SS_{12} = 10^2 + 13^2 - 23^2/2 = 4.5$, $SS_{13} = 2$, $SS_{21} = 8$, $SS_{22} = 0.50$, $SS_{23} = 0.50$ (with 1 df) $\rightarrow$ SS(Experimental Error) = 17.50 (with 6 df) $\rightarrow$ SS(Model) = SS(Total) – $SS_{Error}$ = 176.250 – 17.50 = 158.75 (with 5 df); however, SS(A) + SS(B) = 85.25 (with 3 df) $\rightarrow$ SS(A×B) = SS(Model) – ($SS_A + SS_B$) = 158.75 – 85.25 = 73.50 (with 2 df). The ANOVA Table from Minitab is provided below.

Analysis of Variance for y From Minitab

| Source | DF | SS | MS | $F_0$ | *P-value* |
|---|---|---|---|---|---|
| A | 1 | 18.750 | 18.750 | 6.43** | 0.0440 |
| B | 2 | 66.500 | 33.250 | 11.40*** | 0.0090 |
| A*B | 2 | 73.500 | 36.750 | 12.60*** | 0.0070 |
| Error | 6 | 17.500 | 2.917 | | |
| Total | 11 | 176.250 | | | |

The reader must be cognizant of the fact that in all factorial experiments (involving 2 or more factors), SS(Model) will account for the effects of all inputs in the experiment. In the context of the experiment of Example 7, the two obvious process inputs are factors A and B, but SS(A) + SS(B) < SS(Model), and therefore, there has be another hidden input in the process that affects the output y, which is called the interaction between the two inputs A and B, denoted by A×B. You must use the symbol A×B for all factorial designs to denote the interaction between the two inputs A and B, except when all factors are at 2 levels where AB may also be used to denote the interaction effect A×B.

<u>Definition.</u> Two factors, A and B, interact if the impact of B on y at $A_1$ (the low level of A) is different from the effect of B at $A_2$ (the high level of A). Figure 3

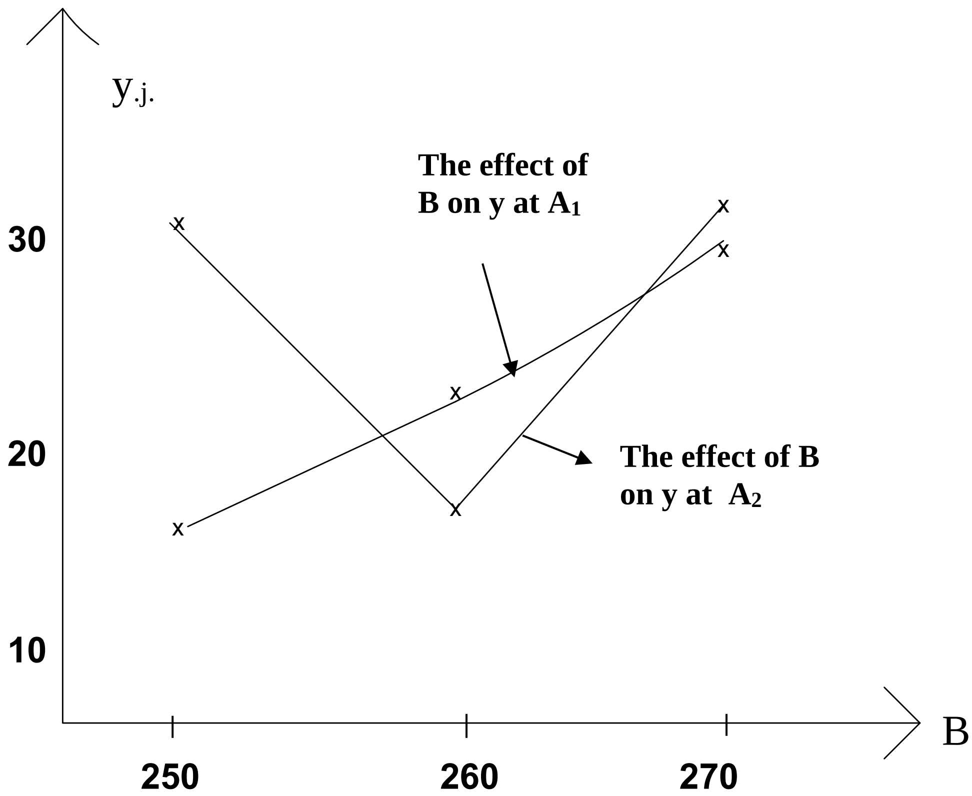


**Figure 3**

illustrates the interaction effect of A and B on y for the experiment of Example 7. Figure 3 shows that the effect of B at the low level of A (A = 120 lb/in$^2$ ) is almost completely linear, while the impact of B at $A_2$ (A = 150) is positively quadratic (or convex upward). Therefore, A and B interact in affecting the response variable y.

Exercise 9. Work Exercise 4.3 on page 34 of your manual. Use Minitab to verify your answers. In your ANOVA table put one * on effects that are significant at the 10% level, ** on effects that are significant at 5%, and place three *** on effects that are statistically significant at the 1% level.