Computing the Field Standard Errors such that the Corresponding $MA_t(W_{Field})$ Will Have the Same Control Limits **By S. Maghsoodloo**

For a Sieve, suppose the target process mean is μ_0 = 40% passing and the Lab measuring device has a standard deviation denoted by $\sigma^{\mathbf{D}}_{\mathbf{Lab}}$. Then, the control limits for lab moving average of span W_L, based only on measuring device variability, is given by

$$LCL_{Lab} = \mu_0 - 3\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}}\text{ , and }\quad UCL_{Lab} = \mu_0 + 3\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \tag{1}$$

The total length (or width) of the above two Lab control limits is given by

$$\text{Lab-Width} = 6\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \tag{2}$$

Similarly, given that the field measuring device standard deviation is denoted by $\sigma^{\mathbf{D}}_{\mathbf{FLD}}$, the corresponding Field 3-σ control limits are given by

$$LCL_{FLD} = \mu_0 - 3\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}}\text{ , and }\quad UCL_{FLD} = \mu_0 + 3\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} \tag{3}$$

The Field-Width, based only on device measurement variability, is given by

$$\text{Field-Width} = 6\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} \tag{4}$$

Our objective is to obtain the value of $\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}$ such that the two Widths in equations (2) and (4) are the same given that $\sigma^{\mathbf{D}}_{\mathbf{FLD}} = k_1\sigma^{\mathbf{D}}_{\mathbf{Lab}}$, where $k_1 > 1$ is a constant of proportionality. To this end, we equate (4) to (2) :

$$6\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} = 6\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \;\rightarrow\; k_1\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} = \sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \;\rightarrow$$

$$\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} = k_1\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \quad \rightarrow \quad \mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}} = \mathbf{k}_1^2\mathbf{W}^{\mathbf{D}}_{\mathbf{L}} \tag{5}$$

Equation (5) implies that if $k_1 = 2$ (i.e., $\sigma^{\mathbf{D}}_{\mathbf{FLD}} = 2\,\sigma^{\mathbf{D}}_{\mathbf{Lab}}$), and the Lab control procedure is carried out with MAs of span 4, then the Field MAs, based only on device variability, must be set at the span (or width) $\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}} = 2^2\times 4 = 16$.

2. In order to increase the power of the Field control-chart, we consider the case of 2-σ control chart for the Field. The objective now is to ascertain how this will impact the relationship in Eq. (5). The 2-σ Field control limits for the measurement device are given by

$$LCL_{FLD} = \mu_0 - 2\,\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}}\text{ , and }\quad UCL_{FLD} = \mu_0 + 2\,\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} \tag{6}$$

The control limits in Eq. (6) raise the false-alarm rate from 27 in 10000 to roughly 455 in 10000 samples, but substantially reduce Type II error probability. Equating the width of control limits in (6) to that of Eq. (2), we obtain

$$4\,\sigma^{\mathbf{D}}_{\mathbf{FLD}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} = 6\,\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \rightarrow 4k_1\,\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}}} = 6\,\sigma^{\mathbf{D}}_{\mathbf{Lab}}/\sqrt{\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}} \rightarrow$$

$$\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}} = (4\,\mathbf{k}_1^2/9)\times \mathbf{W}^{\mathbf{D}}_{\mathbf{L}} \tag{7}$$

As an example, if $\sigma^{\mathbf{D}}_{\mathbf{FLD}} = 2\,\sigma^{\mathbf{D}}_{\mathbf{Lab}}$, (i.e., $k_1= 2$), then Eq. (7) shows that $\mathbf{W}^{\mathbf{D}}_{\mathbf{FLD}} = (4\times 4/9)\times \mathbf{W}^{\mathbf{D}}_{\mathbf{L}} = 16\,\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}/9 \cong 1.778\,\mathbf{W}^{\mathbf{D}}_{\mathbf{L}}$

Inclusion of all sources of Variation

The variability in measurements on %Passing (or any other performance characteristic of an asphalt mix) denoted y, generally originates from 3 sources:

(1) due to material (or the Mix M), (2) due to the process (P), and (3) due to the measuring device (D). That is to say,

$$V(y) = V(M) + V(P) + V(D) \qquad (8)$$

Eq. (8) assumes that M(Material) variability, P(process), and measuring device (D) variability are independent. In sections 1 and 2 above, we have excluded the 1st two sources (Material and Process) of variation in the measurement y. It is a reasonable assumption that Material variability in the Lab and Field are approximately the same; however, process variability in the field may exceed that of the lab, say $\sigma_{\mathbf{FLD}}^{\mathbf{P}} = k_2 \sigma_{\mathbf{Lab}}^{\mathbf{P}}$, $k_2 \geq 1$. Eq. (8) shows that the overall Lab variance is given by

$$V(y_L) = V(M_L) + V(P_L) + V(D_L), \qquad (9a)$$

while the corresponding field variance is given by

$$V(y_F) = V(M_F) + V(P_F) + V(D_F) \qquad (9b)$$

Assuming 2-σ MA charts for the field, we equate $4\sigma_{\mathbf{FLD}}/\sqrt{\mathbf{W_{FLD}}}$ to

$6\sigma_{\mathbf{Lab}}/\sqrt{\mathbf{W_L}}$, i.e., $4\sigma_{\mathbf{FLD}}/\sqrt{\mathbf{W_{FLD}}} = 6\sigma_{\mathbf{Lab}}/\sqrt{\mathbf{W_L}} \rightarrow \sqrt{\mathbf{W_{FLD}}} = (2\sigma_{\mathbf{FLD}}\sqrt{\mathbf{W_L}}/(3\sigma_{\mathbf{Lab}})$

$$\rightarrow \qquad \mathbf{W_{FLD}} = \frac{4\sigma_{\mathbf{FLD}}^2}{9\sigma_{\mathbf{Lab}}^2}\mathbf{W_L}, \qquad (10)$$

where $\sigma_{\mathbf{FLD}}^2 = V(M_F) + V(P_F) + V(D_F) \cong V(M_L) + \mathbf{k}_2^2 V(P_L) + \mathbf{k}_1^2 V(D_L)$. I have provided an Excel file that obtains the value of $\mathbf{W_{FLD}}$ for specified values of k_1, k_2, $V(M_L)$ $V(P_L)$ and $V(D_L)$.