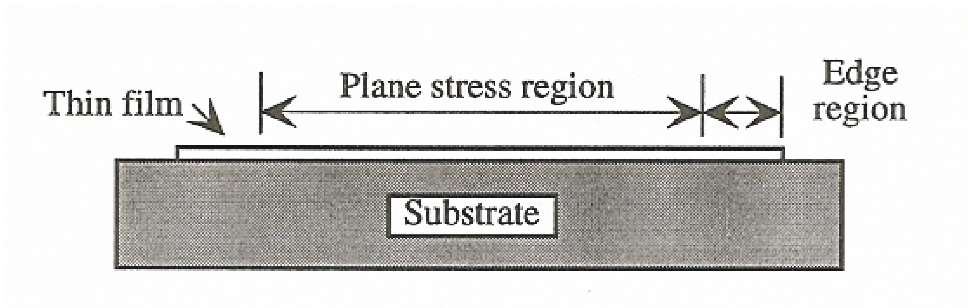
Mechanics of Microstructures

Topics

- Plane Stress in MEMS
- Thin film Residual Stress
- Effects of Residual Stress

Reference: Stephen D. Senturia, “Microsystem Design,” Kluwer Academic Publishers, January 2001.

Plane Stress in MEMS



- In MEMS devices, a thin film deposited or formed on a substrate has some in-plane stress, referred to as *Residual Stress.*
- Plane stress arises typically due to mismatches in thermal expansion between the film and the substrate.
- The two components of in-plane normal stress along the principle axes are

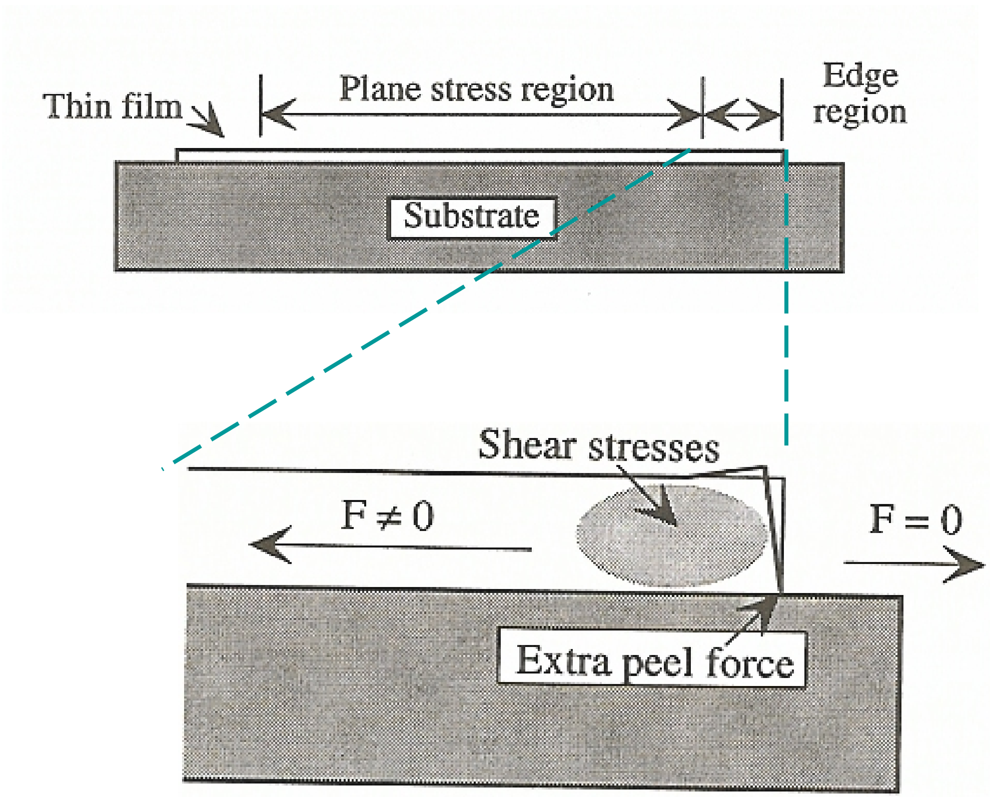
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \quad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \quad \varepsilon_z = \frac{1}{E}\left(-\nu\sigma_y - \nu\sigma_x\right)$$

Biaxial Plane Stress: x and y strain components are equal to each other

$$\varepsilon_x = \varepsilon_y = \varepsilon \quad \Longrightarrow \quad \sigma_x = \sigma_y = \sigma \qquad \sigma = \left(\frac{E}{1-\nu}\right)\varepsilon$$

Plane Stress in MEMS

Peel Forces



- High stress concentration at the attachment point of the film leads to extra forces called peel forces that tend to detach the film from the substrate
- Debonding of tensile films tends to occur at the edges of patterned features

Residual Stress due to Thermal Expansion

- The linear thermal expansion coefficient (CTE) of a material is defined as the rate of change of uniaxial strain with temperature

$$\alpha_T = \frac{d\varepsilon_x}{dT} \text{ (1/K)}$$

Typical range: 10^{-6} – 10^{-7}

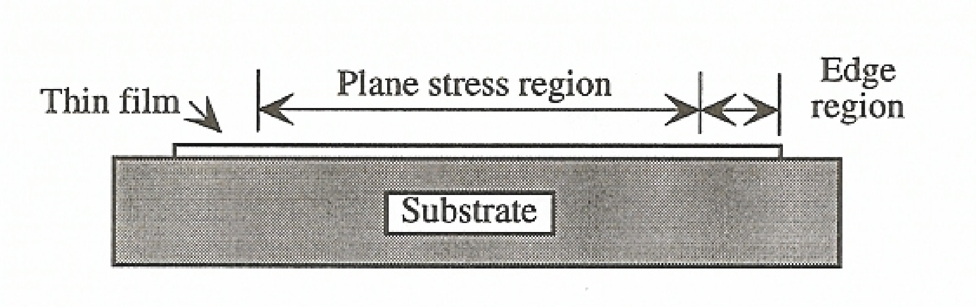
- Assuming that the CTE remains constant for moderate temperature excursions, the increase (or decrease) in strain for a finite temperature change can be written as

$$\varepsilon_x(T) = \varepsilon_x(T_o) + \alpha_T \Delta T$$

$\varepsilon_x(T_o)$: Strain at the original temperature T_o

$\Delta T = T - T_o$ Incremental change in temperature

Residual Stress in Thin Films due to Thermal Expansion

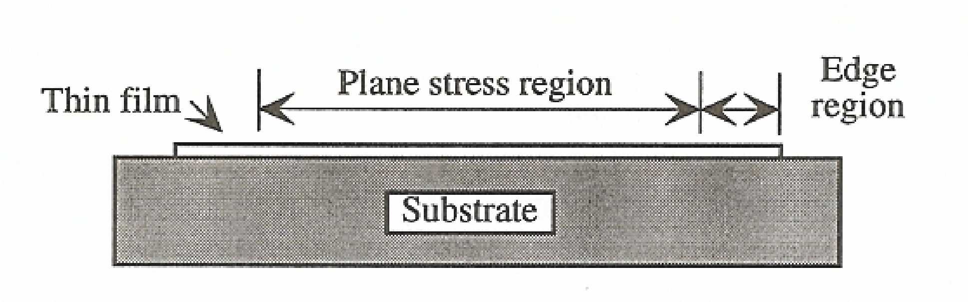


- Let us assume that the film is much thinner than the substrate and is deposited onto the substrate in a stress-free state at temperature T_d
- The sample is then cooled to room temperature T_r
- The thermal strain of the substrate is given by

$$\varepsilon_s = -\alpha_{Ts} \Delta T$$

$$\Delta T = T_d - T_r$$

Residual Stress in Thin Films due to Thermal Expansion



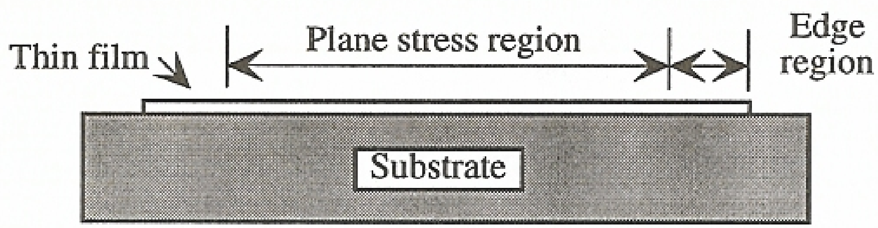
- Assume that the film is much thinner than the substrate.
- In this case, as a good approximation we can assume that the film must contract by the same amount as the substrate.

$$\varepsilon_{f,attached} = \varepsilon_s \Longrightarrow \varepsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

- If the film were not attached to the substrate, it would experience a thermal strain according to its own thermal expansion coefficient

$$\varepsilon_{f,free} = -\alpha_{Tf}\Delta T$$

Residual Stress in Thin Films due to Thermal Expansion



- The extra strain, i.e., the difference between the actual strain (attached case) and the unattached strain in the film (free case) is called the *thermal mismatch strain*

$$\varepsilon_{f,mismatch} = \left(\alpha_{Tf} - \alpha_{Ts}\right)\Delta T$$

- The biaxial strain develops an in-plane biaxial stress give by

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\varepsilon_{f,mismatch}$$

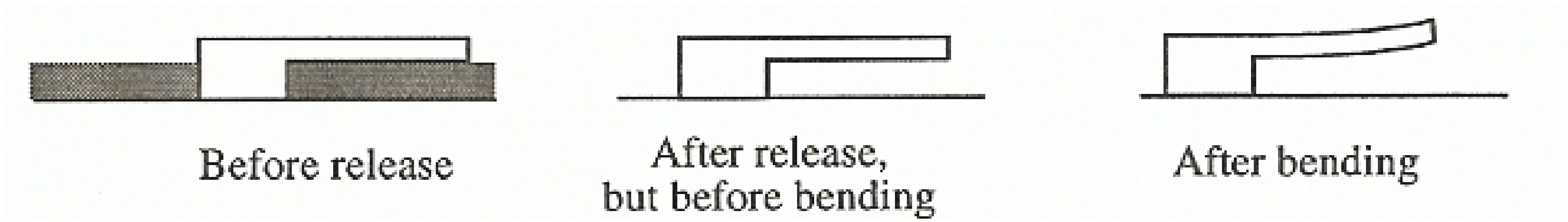
Effects of Residual Stresses

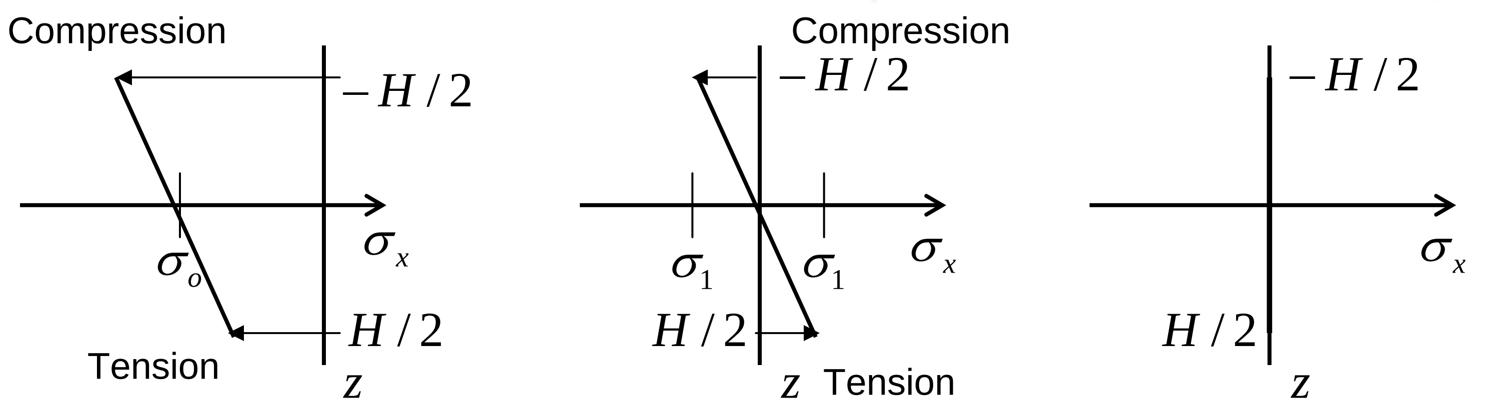
What are the effects of Residual stresses?

- Nonuniform residual stresses in cantilevers, due either to a gradient in the material properties through the cantilever thickness or to the deposition of a different material onto a structure, can cause the cantilevers to curl
- In doubly-supported beams, residual stress modify the bending stiffness, and lead to important nonlinear spring effects when the deflections become comparable to the beam thickness
- Compressive residual stresses can cause buckling by out of plan bending

Residual Stress in Cantilevers

Before Release





Axial Stress: Prior to release the beam material has an average compressive stress σ_o, and also a stress gradient from the deposition

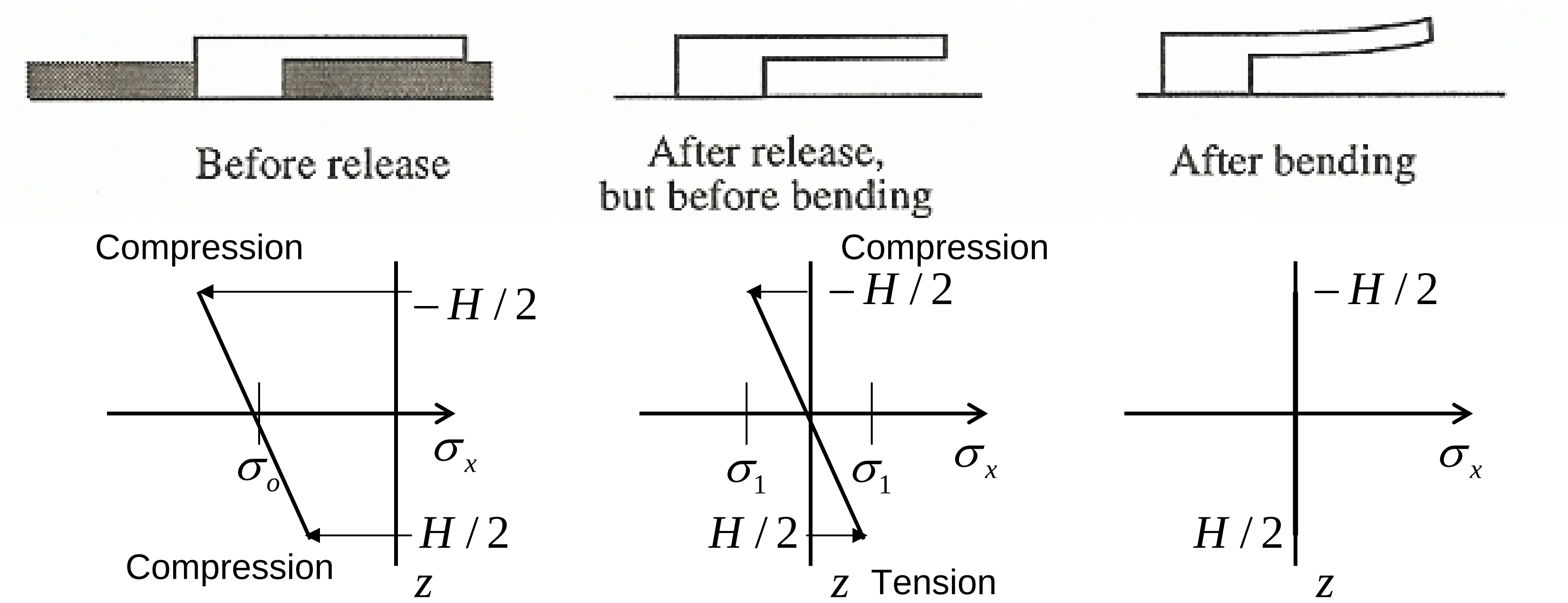
$$\sigma = \sigma_o - \frac{\sigma_1}{(H/2)}$$

Internal bending moment:

$$M_x = \int_{-H/2}^{H/2} Wz\sigma dz \quad \Longrightarrow \quad M_x = -\frac{1}{6} W H^2 \sigma_1$$

Residual Stress in Cantilevers

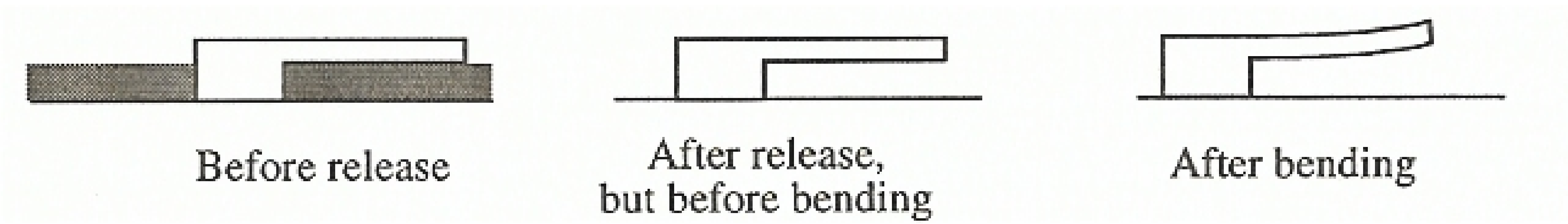
After Release (before bending)

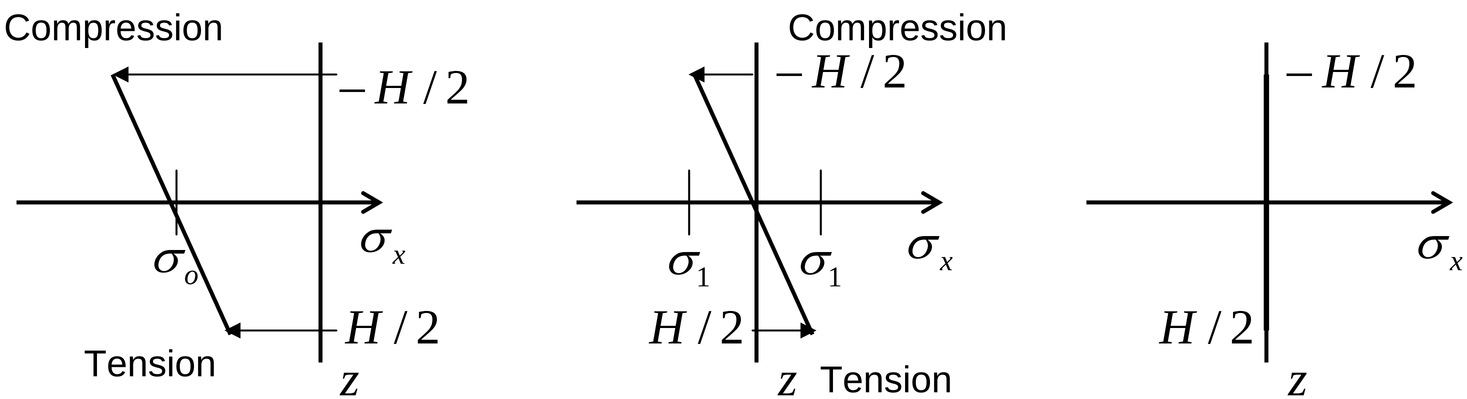


- Once the beam is released, the beam length increases slightly, relieving the compressive stress so that the average stress goes to zero.
- However the stress gradient is still present. This stress gradient creates a moment that bends the beam. In other words, the stress variation created by bending exactly cancels the initial stress gradient.
- This bending produces a decrease in the tensile stress at the bottom of the beam and simultaneously a decrease in the compressive stress at the top of the beam. Thus, after bending the stress is zero everywhere in the beam.

Residual Stress in Cantilevers

After Release





Radius of Curvature: The radius of curvature of the beam after bending is given by

$$\rho_x = -\frac{1}{12}\frac{EWH^3}{M_x}$$

We know $\frac{1}{\rho} = -\frac{M}{EI}$ and $I = \frac{1}{12}WH^3$

Using

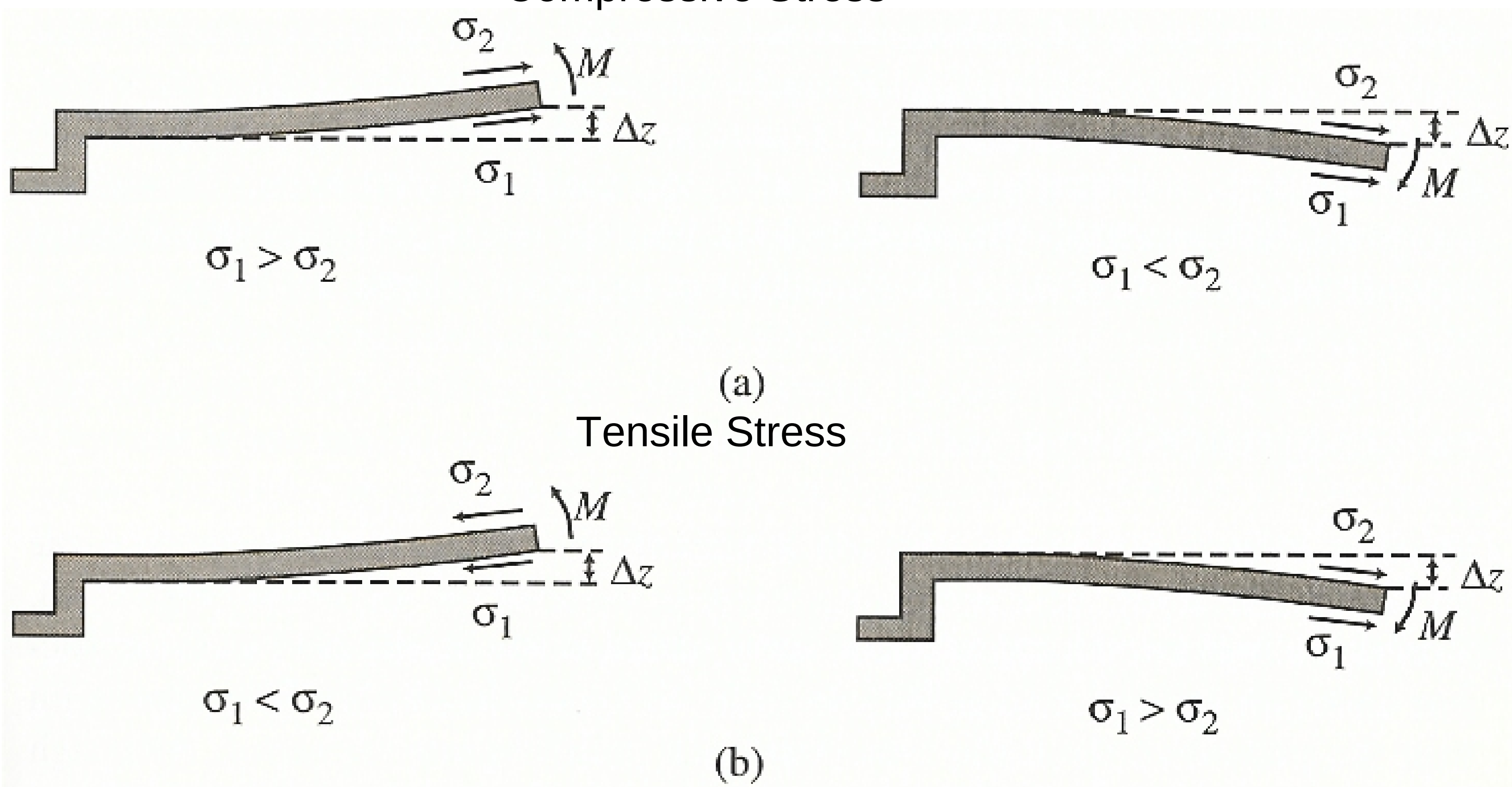
$$M_x = -\frac{1}{6}WH^2\sigma_1$$

$$\rho_x = \frac{1}{2}\frac{EH}{\sigma_1}$$

The radius of curvature is expressed as a function of stress gradient

Residual Stress Effects in Cantilevers

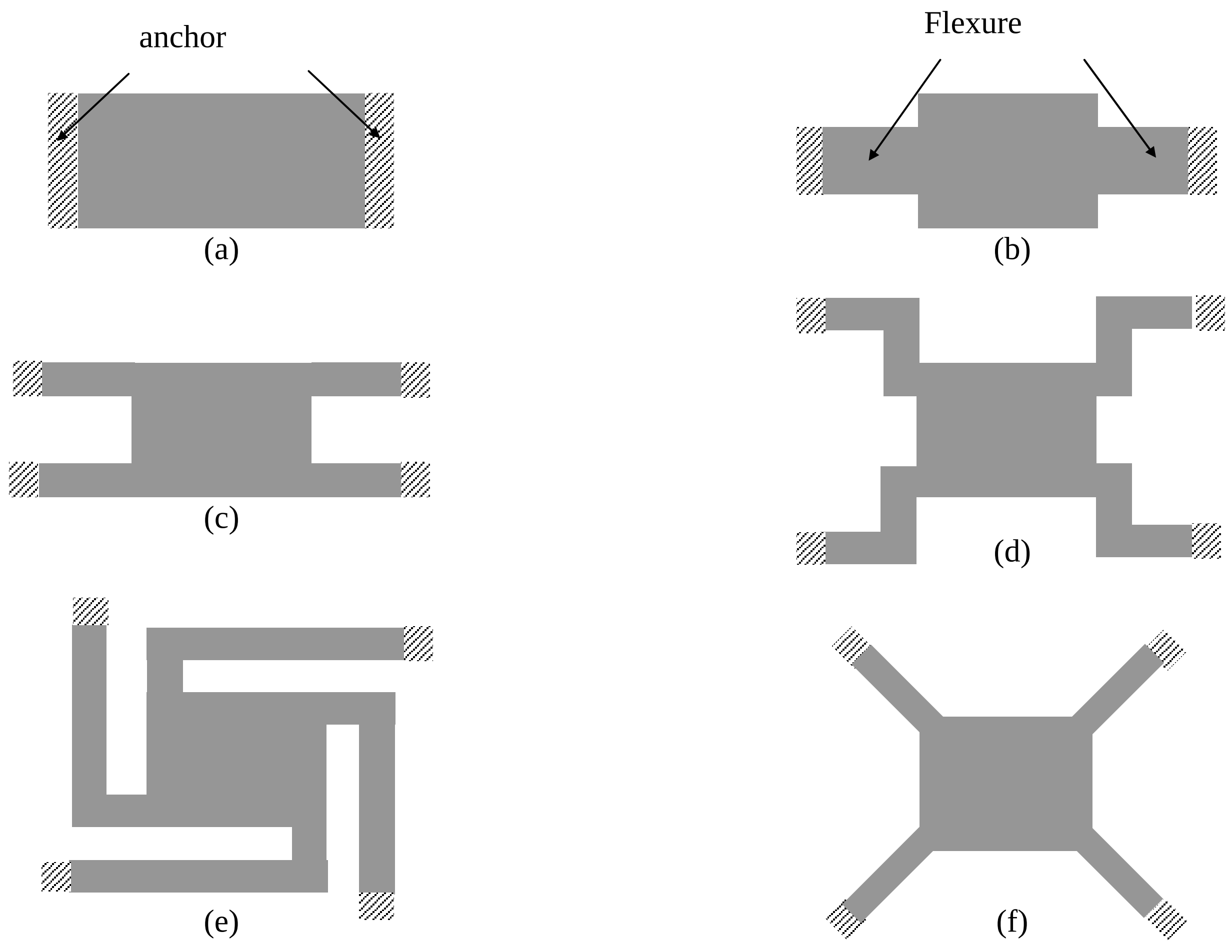
Compressive Stress



Is there something wrong in this figure!

Source: Gabriel M. Rebeiz, "RF MEMS: Theory, Design, and Technology," John Wiley & Sons, 1st edition (June 15, 2002).

Low spring constant Flexure Designs



Reference: W. Young and R. G. Budynas, “Roark’s Formulas for Stress and Strain,” McGraw Hill, 2002.

Residual Stress due to Thermal Expansion

- The *volume thermal expansion coefficient* that accounts for linear expansions in three dimensions is given by

$$\frac{\Delta V}{V} = (1+\varepsilon_x)(1+\varepsilon_x)(1+\varepsilon_y) - 1 \quad \Rightarrow \quad \frac{\Delta V}{V} = 3\alpha_T \Delta T$$