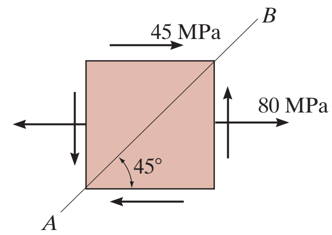
9–7. Determine the normal stress and shear stress acting on the inclined plane *AB*. Solve the problem using the stress transformation equations. Show the result on the sectioned element.



Stress Transformation Equations:

$\theta = +135^\circ$ (Fig. *a*) $\quad \sigma_x = 80$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 45$ MPa

we obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{80 + 0}{2} + \frac{80 - 0}{2}\cos 270 + 45\sin 270^\circ$$

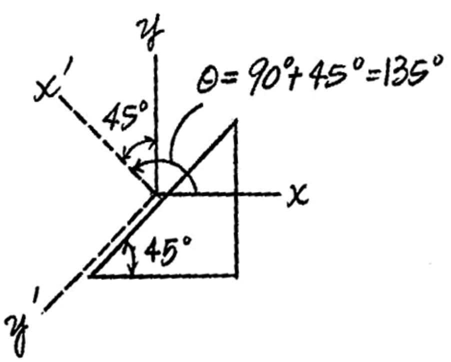
$$= -5 \text{ MPa}$$ **Ans.**

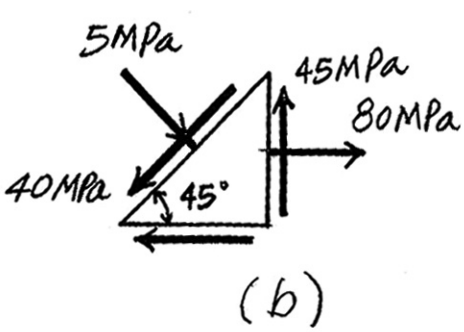
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{80 - 0}{2}\sin 270^\circ + 45\cos 270^\circ$$

$$= 40 \text{ MPa}$$ **Ans.**

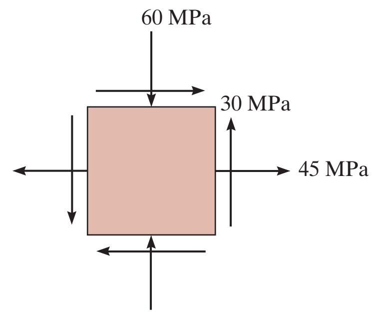
The negative sign indicates that $\sigma_{x'}$ is a compressive stress. These results are indicated on the triangular element shown in Fig. *b*.





Ans:
$\sigma_{x'} = -5$ MPa, $\tau_{x'y'} = 40$ MPa

9–15. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



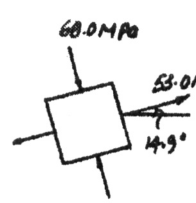
$\sigma_x = 45 \text{ MPa} \qquad \sigma_y = -60 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$

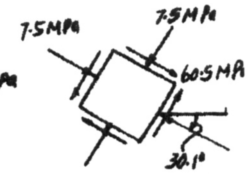
a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 53.0 \text{ MPa}$ **Ans.**

$\sigma_2 = -68.0 \text{ MPa}$ **Ans.**





Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87°, \qquad -75.13°$

Use Eq. 9–1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

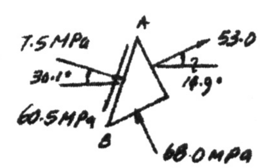
$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30 \sin 29.74° = 53.0 \text{ MPa}$$

Therefore $\theta_{p1} = 14.9°$ **Ans.** and $\theta_{p2} = -75.1°$ **Ans.**

b) $$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa}$$ **Ans.**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ **Ans.**

Orientation of maximum in-plane shear stress:



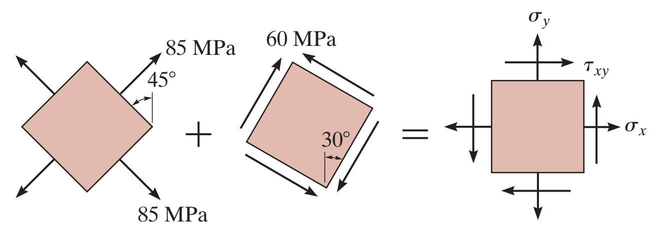
$$\tan 2\theta_y = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$\theta_y = -30.1°$ **Ans.** and $\theta_y = 59.9°$ **Ans.**

By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

Ans:
$\sigma_1 = 53.0 \text{ MPa}, \sigma_2 = -68.0 \text{ MPa}$,
$\theta_{p1} = 14.9°$ and $\theta_{p2} = -75.1°$,
$\sigma_{avg} = -7.50 \text{ MPa}, \tau_{\substack{\max \\ \text{in-plane}}} = 60.5 \text{ MPa}$,
$\theta_s = -30.1°$ and $59.9°$

9–18. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



For element *a*:

$\sigma_x = \sigma_y = 85$ MPa $\qquad \tau_{xy} = 0 \qquad \theta = -45°$

$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

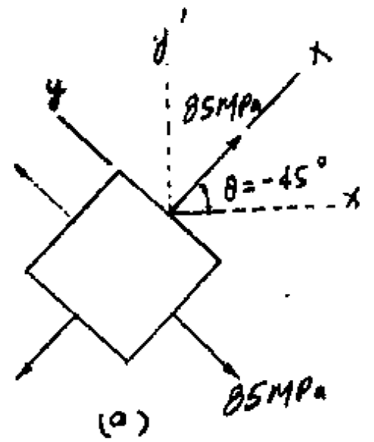
$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85 \text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85 \text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$



+

For element *b*:

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 60$ MPa $\qquad \theta = -60°$

$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

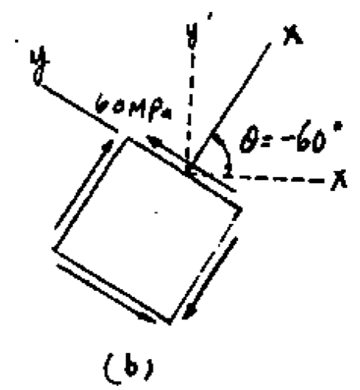
$$= 0 + 0 + 60\sin(-120°) = -51.96 \text{ MPa}$$

$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 0 - 0 - 60\sin(-120°) = 51.96 \text{ MPa}$$

$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30 \text{ MPa}$$



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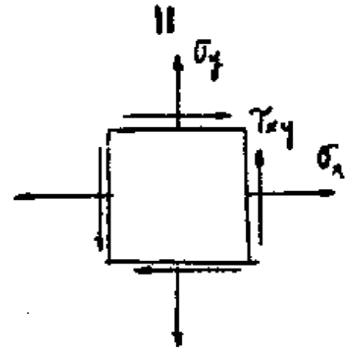


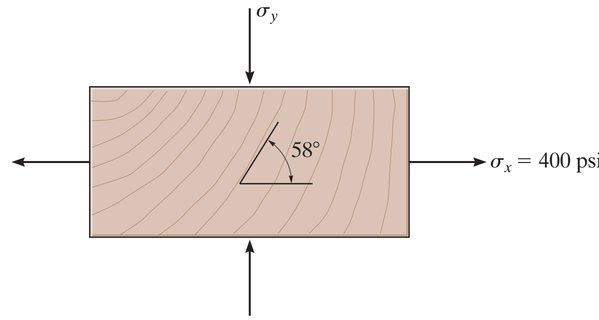
$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0$ MPa **Ans.**

$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137$ MPa **Ans.**

$\tau_{xy} = (\tau_{x'y'})_a + (\tau_{x'y'})_b = 0 + (-30) = -30$ MPa **Ans.**

Ans:
$\sigma_x = 33.0$ MPa, $\sigma_y = 137$ MPa, $\tau_{xy} = -30$ MPa

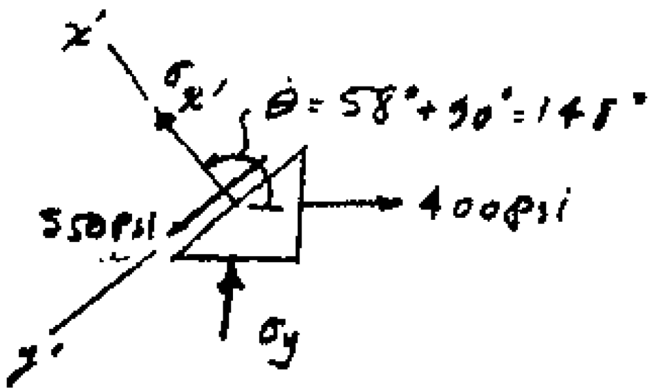
9–25. The wooden block will fail if the shear stress acting along the grain is 550 psi. If the normal stress $\sigma_x = 400$ psi, determine the necessary compressive stress σ_y that will cause failure.



$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

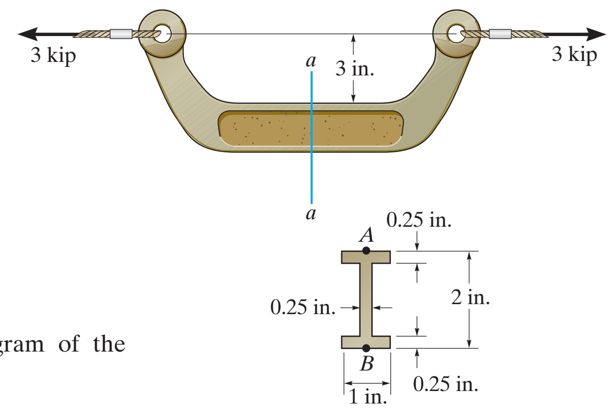
$$550 = -\left(\frac{400 - \sigma_y}{2}\right)\sin 296° + 0$$

$$\sigma_y = -824 \text{ psi}$$ **Ans.**



Ans:
$\sigma_y = -824$ psi

9–27. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point B on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.



Section $a-a$

Internal Loadings: Consider the equilibrium of the free-body diagram of the bracket's left cut segment, Fig. a.

$\xrightarrow{+} \Sigma F_x = 0;\quad N - 3 = 0 \qquad N = 3\,\text{kip}$

$\Sigma M_O = 0;\ 3(4) - M = 0 \qquad M = 12\,\text{kip}\cdot\text{in}$

Normal and Shear Stresses: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

The cross - sectional area and the moment of inertia about the z axis of the bracket's cross section is

$$A = 1(2) - 0.75(1.5) = 0.875\ \text{in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) - \frac{1}{12}(0.75)\left(1.5^3\right) = 0.45573\ \text{in}^4$$

For point B, $y = -1$ in. Then

$$\sigma_B = \frac{3}{0.875} - \frac{(-12)(-1)}{0.45573} = -22.90\ \text{ksi}$$

Since no shear force is acting on the section,

$$\tau_B = 0$$

The state of stress at point A can be represented on the element shown in Fig. b.

In - Plane Principal Stress: $\sigma_x = -22.90$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0$. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -22.90\ \text{ksi} \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the elements shown in Fig. b.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\text{max}\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-22.90 - 0}{2}\right)^2 + 0^2} = 11.5\ \text{ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(-22.9 - 0)/2}{0} = -\infty$$

$$\theta_s = 45^\circ \text{ and } 135^\circ \qquad \textbf{Ans.}$$

9–27. Continued

Substituting $\theta = 45°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-22.9 - 0}{2}\sin 90° + 0$$

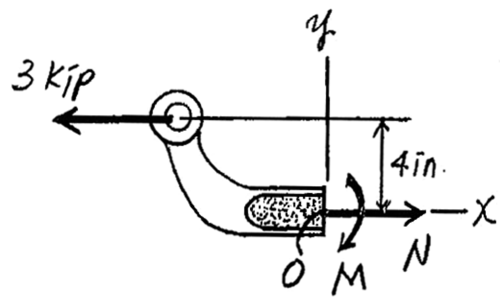
$$= 11.5 \text{ ksi} = \tau_{\substack{\max \\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ is directed in the positive sense of the y' axes on the element defined by $\theta_s = 45°$.

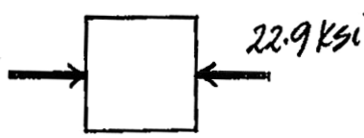
Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-22.9 + 0}{2} = -11.5 \text{ ksi}$$

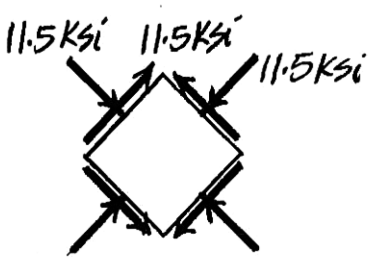
The state of maximum in - plane shear stress is represented by the element shown in Fig. *c*.



(a)



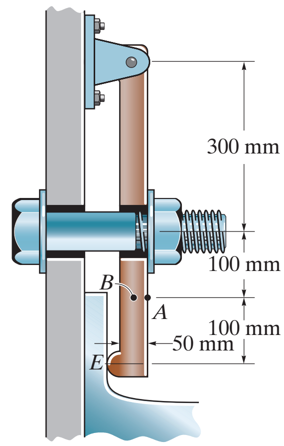
(b)

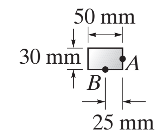


(c)

Ans:
$\sigma_1 = 0, \sigma_2 = -22.90$ ksi, $\tau_{\substack{\max \\ \text{in-plane}}} = 11.5$ ksi,
$\theta_s = 45°$ and $135°$

9–33. The clamp bears down on the smooth surface at E by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stress at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.





Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right)\text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\dfrac{My}{I}$.

$$\sigma_A = -\frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192\text{ MPa}$$

$$\sigma_B = -\frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$

$$\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

$$\tau_B = \frac{24.0(10^3)\left[9.375(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 24.0\text{ MPa}$$

In-Plane Principal Stresses: $\sigma_x = 0, \sigma_y = -192$ MPa, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = -192\text{ MPa} \qquad \textbf{Ans.}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -24.0$ MPa for point B. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (-24.0)^2}$$

$$= 0 \pm 24.0$$

$$\sigma_1 = 24.0\text{ MPa} \qquad \sigma_2 = -24.0\text{ MPa} \qquad \textbf{Ans.}$$

9–33. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty$$

$$\theta_p = -45.0° \qquad \text{and} \qquad 45.0°$$

Substituting the results into Eq. 9-1 with $\theta = -45.0°$ yields

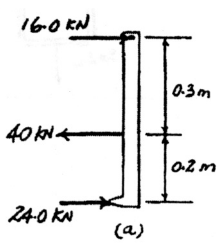
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

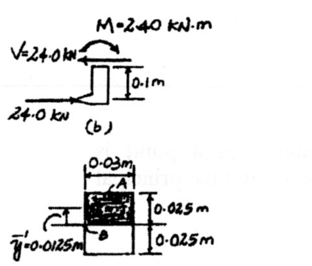
$$= 0 + 0 + [-24.0 \sin(-90.0°)]$$

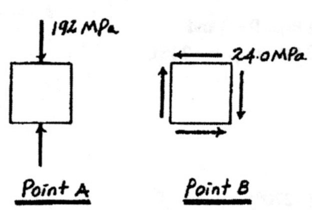
$$= 24.0 \text{ MPa} = \sigma_1$$

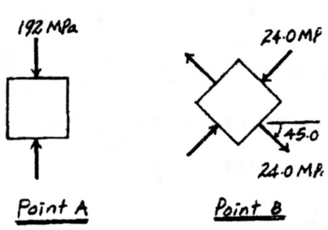
Hence,

$$\theta_{p1} = -45.0° \qquad \theta_{p2} = 45.0°$$ **Ans.**



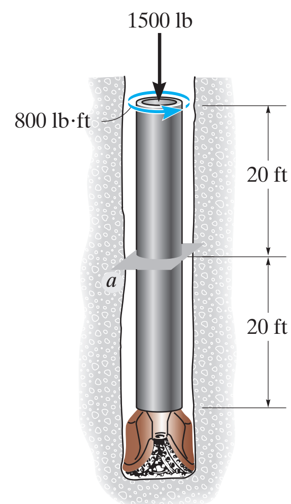




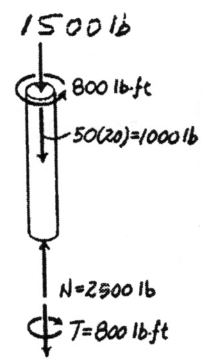


Ans:
Point A: $\sigma_1 = 0$, $\sigma_2 = -192$ MPa,
$\theta_{p1} = 0$, $\theta_{p2} = 90°$,
Point B: $\sigma_1 = 24.0$ MPa, $\sigma_2 = -24.0$ MPa,
$\theta_{p1} = -45.0°$, $\theta_{p2} = 45.0°$

9–42. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stresses and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Internal Forces and Torque: As shown on FBD(a).



Section Properties:

$$A = \frac{\pi}{4}\left(3^2 - 2.5^2\right) = 0.6875\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.25^4\right) = 4.1172 \text{ in}^4$$

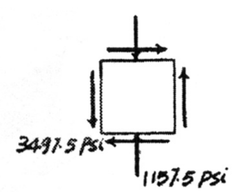
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-2500}{0.6875\pi} = -1157.5 \text{ psi}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}$$

a) **In-Plane Principal Stresses:** $\sigma_x = 0$, $\sigma_y = -1157.5$ psi and $\tau_{xy} = 3497.5$ psi for any point on the shaft's surface. Applying Eq. 9-5,



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= -578.75 \pm 3545.08$$

$\sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi}$ **Ans.**

$\sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi}$ **Ans.**

b) **Maximum In-Plane Shear Stress:** Applying Eq. 9–7,

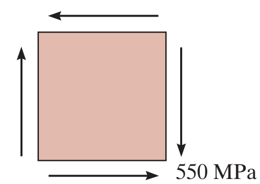
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$= 3545 \text{ psi} = 3.55 \text{ ksi}$ **Ans.**

Ans:
$\sigma_1 = 2.97$ ksi, $\sigma_2 = -4.12$ ksi, $\tau_{\substack{\max \\ \text{in-plane}}} = 3.55$ ksi

9–58. Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



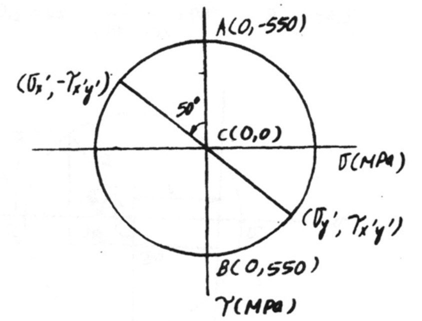
$A(0, -550)$ $B(0, 550)$ $C(0, 0)$

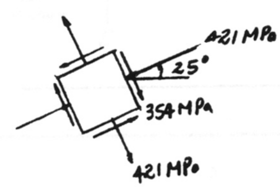
$R = CA = CB = 550$

$\sigma_{x'} = -550 \sin 50° = -421 \text{ MPa}$ **Ans.**

$\tau_{x'y'} = -550 \cos 50° = -354 \text{ MPa}$ **Ans.**

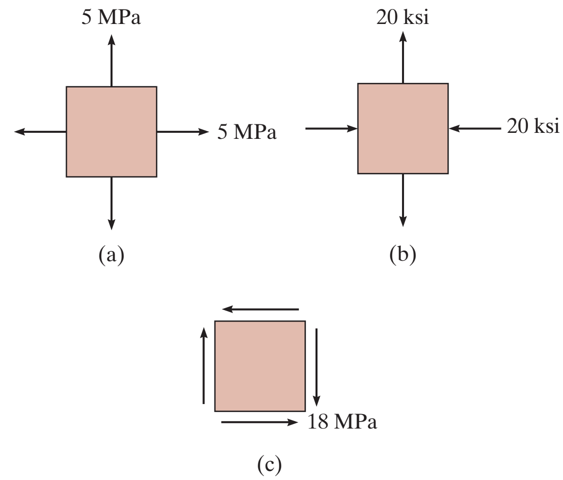
$\sigma_{y'} = 550 \sin 50° = 421 \text{ MPa}$ **Ans.**

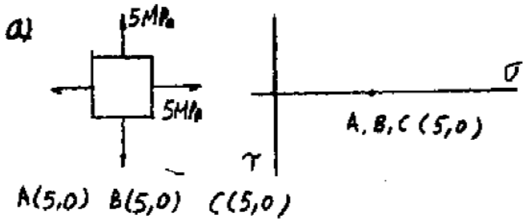


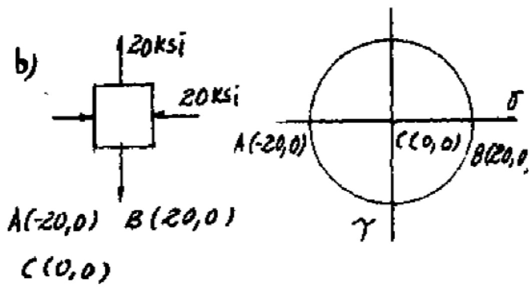


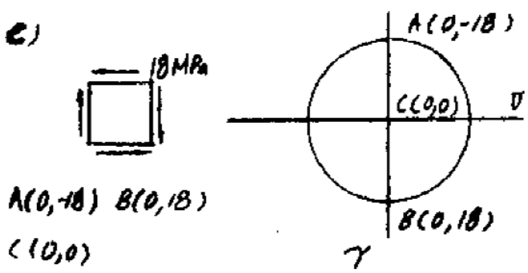
Ans:
$\sigma_{x'} = -421 \text{ MPa}$, $\tau_{x'y'} = -354 \text{ MPa}$,
$\sigma_{y'} = 421 \text{ MPa}$

9–61. Draw Mohr's circle that describes each of the following states of stress.

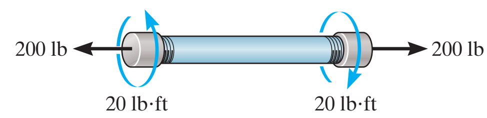








9–65. The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.



Section Properties:

$$A = \pi\left(0.275^2 - 0.25^2\right) = 0.013125\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(0.275^4 - 0.25^4\right) = 2.84768\left(10^{-3}\right) \text{ in}^4$$

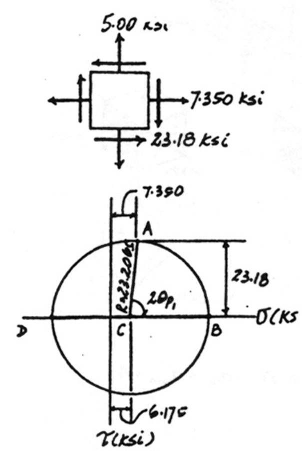
Normal Stress: Since $\frac{r}{t} = \frac{0.25}{0.025} = 10$, thin wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{200}{0.013125\pi} + \frac{500(0.25)}{2(0.025)} = 7.350 \text{ ksi}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{500(0.25)}{0.025} = 5.00 \text{ ksi}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{20(12)(0.275)}{2.84768(10^{-3})} = 23.18 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention $\sigma_x = 7.350$ ksi, $\sigma_y = 5.00$ ksi, and $\tau_{xy} = -23.18$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5.00}{2} = 6.175 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(7.350, -23.18) \qquad C(6.175, 0)$$

The radius of the circle is

$$R = \sqrt{(7.350 - 6.175)^2 + 23.18^2} = 23.2065 \text{ ksi}$$

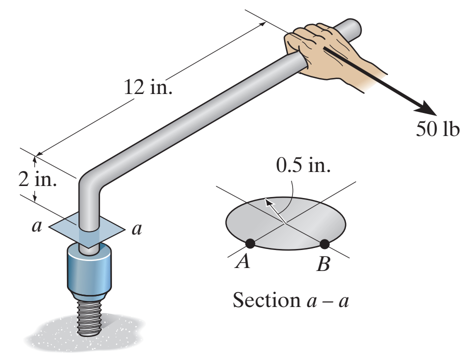
In-Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 6.175 + 23.2065 = 29.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.175 - 23.2065 = -17.0 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 29.4$ ksi, $\sigma_2 = -17.0$ ksi

9–75. If the box wrench is subjected to the 50 lb force, determine the principal stress and maximum in-plane shear stress at point B on the cross section of the wrench at section a–a. Specify the orientation of these states of stress and indicate the results on elements at the point.

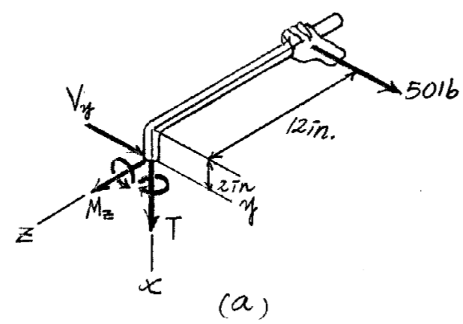


Internal Loadings: Considering the equilibrium of the free-body diagram of the wrench's cut segment, Fig. a,

$\Sigma F_y = 0;\quad V_y + 50 = 0 \quad V_y = -50 \text{ lb}$

$\Sigma M_x = 0;\quad T + 50(12) = 0 \quad T = -600 \text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\quad M_z - 50(2) = 0 \quad M_z = 100 \text{ lb}\cdot\text{in}$



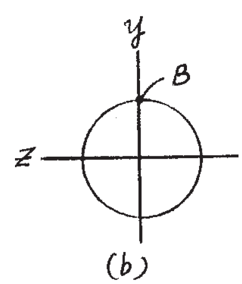
Section Properties: The moment of inertia about the z axis and the polar moment of inertia of the wrench's cross section are

$$I_z = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$$

$$J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$$

Referring to Fig. b,

$$(Q_y)_B = 0$$



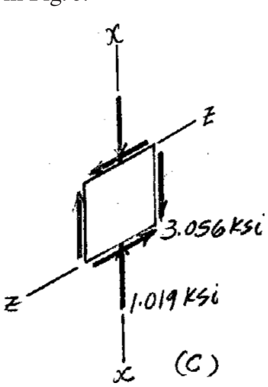
Normal and Shear Stress: The normal stress is caused by the bending stress due to $\mathbf{M}_z$.

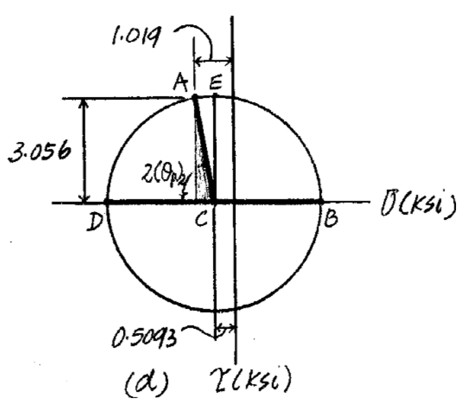
$$(\sigma_x)_B = -\frac{M_z y_B}{I_z} = -\frac{100(0.5)}{0.015625\pi} = -1.019 \text{ ksi}$$

The shear stress at point B along the y axis is $(\tau_{xy})_B = 0$ since $(Q_y)_B$. However, the shear stress along the z axis is caused by torsion.

$$(\tau_{xz})_B = \frac{Tc}{J} = \frac{600(0.5)}{0.03125\pi} = 3.056 \text{ ksi}$$

The state of stress at point B is represented by the two-dimensional element shown in Fig. c.





9–75. Continued

Construction of the Circle: $\sigma_x = -1.019$ ksi, $\sigma_z = 0$, and $\tau_{xz} = -3.056$ ksi. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ ksi}$$

The coordinates of reference point A and the center C of the circle are

$$A(-1.019, -3.056) \qquad C(-0.5093, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.056)^2} = 3.0979 \text{ ksi}$$

Using these results, the circle is shown in Fig. *d*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.5093 + 3.0979 = 2.59 \text{ ksi}$$ **Ans.**

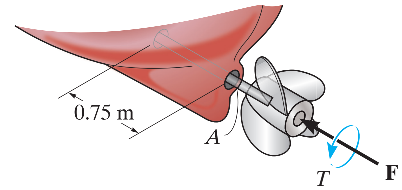
$$\sigma_2 = -0.5093 - 3.0979 = -3.61 \text{ ksi}$$ **Ans.**

Maximum In-Plane Shear Stress: The coordinates of point E represent the maximum in-plane stress, Fig. *a*.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 3.10 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_1 = 2.59$ ksi, $\sigma_2 = -3.61$ ksi, $\theta_{p1} = -40.3°$, $\theta_{p2} = 49.7$, $\tau_{\substack{max \\ in\text{-}plane}} = 3.10$ ksi, $\theta_s = 4.73°$

9–90. The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has a diameter of 250 mm, determine the maximum in-plane shear stress at any point located on the surface of the shaft.



Power Transmission: Using the formula developed in Chapter 5,

$$P = 900 \text{ kW} = 0.900\left(10^6\right) \text{ N} \cdot \text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0\left(10^3\right) \text{ N} \cdot \text{m}$$

Internal Torque and Force: As shown on FBD.

Section Properties:

$$A = \frac{\pi}{4}\left(0.25^2\right) = 0.015625\pi \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.3835\left(10^{-3}\right) \text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06 \text{ MPa}$$

Shear Stress: Applying the torsion formula.

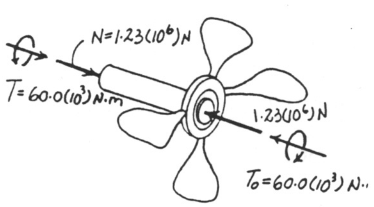
$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)\,(0.125)}{0.3835\,(10^{-3})} = 19.56 \text{ MPa}$$

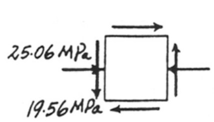
Maximum In-Plane Principal Shear Stress: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9-7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$$= 23.2 \text{ MPa}$$ **Ans.**

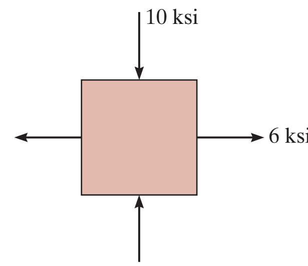




Ans:

$\tau_{\substack{\max \\ \text{in-plane}}} = 23.2$ MPa

9–93. Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr's circle.



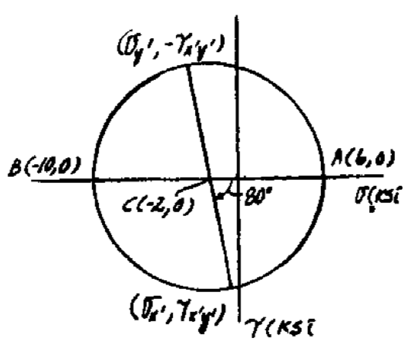
$A(6, 0) \qquad B(-10, 0) \qquad C(-2, 0)$

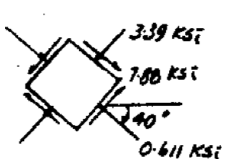
$R = CA = CB = 8$

$\sigma_{x'} = -2 + 8\cos 80° = -0.611\text{ ksi}$ **Ans.**

$\tau_{x'y'} = 8\sin 80° = 7.88\text{ ksi}$ **Ans.**

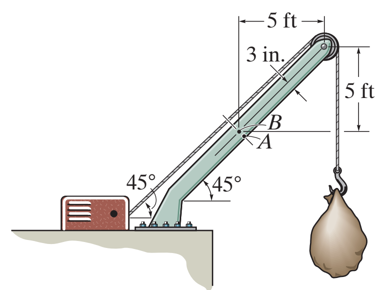
$\sigma_{y'} = -2 - 8\cos 80° = -3.39\text{ ksi}$ **Ans.**



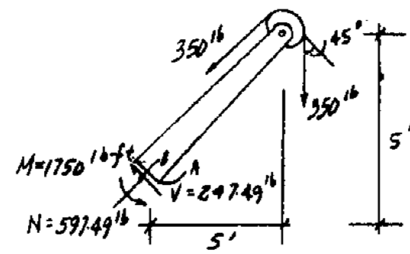


Ans:
$\sigma_{x'} = -0.611\text{ ksi}, \tau_{x'y'} = 7.88\text{ ksi}, \sigma_{y'} = -3.39\text{ ksi}$

9–94. The crane is used to support the 350-lb load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 6 in. and a thickness of 3 in. Use Mohr's circle.



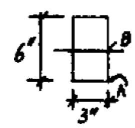
$A = 6(3) = 18\text{ in}^2 \qquad I = \frac{1}{12}(3)(6^3) = 54\text{ in}^4$



$Q_B = (1.5)(3)(3) = 13.5\text{ in}^3$

$Q_A = 0$

For point A:

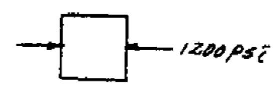


$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = \frac{597.49}{18} - \frac{1750(12)(3)}{54} = -1200\text{ psi}$$

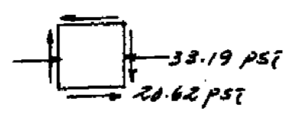
$\tau_A = 0$

$\sigma_1 = 0$ **Ans.**

$\sigma_2 = -1200\text{ psi} = -1.20\text{ ksi}$ **Ans.**



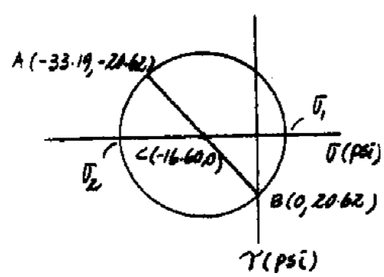
For point B:



$$\sigma_B = -\frac{P}{A} = -\frac{597.49}{18} = -33.19\text{ psi}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{247.49(13.5)}{54(3)} = 20.62\text{ psi}$$

$A(-33.19, -20.62) \qquad B(0, 20.62) \qquad C(-16.60, 0)$



$R = \sqrt{16.60^2 + 20.62^2} = 26.47$

$\sigma_1 = -16.60 + 26.47 = 9.88\text{ psi}$ **Ans.**

$\sigma_2 = -16.60 - 26.47 = -43.1\text{ psi}$ **Ans.**

Ans:
Point A: $\sigma_1 = 0$, $\sigma_2 = -1.20$ ksi,
Point B: $\sigma_1 = 9.88$ psi, $\sigma_2 = -43.1$ psi