MULTISCALE TERRAIN CHARACTERIZATION USING FOURIER AND WAVELET TRANSFORMS FOR UNMANNED GROUND VEHICLES

Jeremy J. Dawkins, David M. Bevly, Robert L. Jackson
Department of Mechanical Engineering
Auburn University
Auburn, AL 36849
jjd0006@auburn.edu, bevlydm@auburn.edu, jacksr7@auburn.edu

ABSTRACT

This paper investigates the use of the Fourier transform and Wavelet transform as methods to supplement the more common root mean squared elevation and power spectral density methods of terrain characterization. Two dimensional terrain profiles were generated using the Weierstrass-Mandelbrot fractal equation. The Fourier and Wavelet transforms were used to decompose these terrains into a parameter set. A two degree of freedom quarter car model was used to evaluate the vehicle response before and after the terrain characterization. It was determined that the Fourier transform can be used to reduce the profile into the key frequency components. The Wavelet transform can effectively detect discontinuities of the profile and changes in the roughness of the profile. These two techniques can be added to current methods to yield a more robust terrain characterization.

NOMENCLATURE

D Fractal dimension
G Fractal roughness parameter
L Length of profile
R Roughness amplitude constant
c_d Damping coefficient
h Profile height
k Wave number spectrum slope
k_s Spring coefficient
k_t Spring coefficient of tire
m_s Sprung mass
m_u Unsprung mass
n Frequency iteration integer
z_u Position of unsprung mass
z_s Position of sprung mass
Φ Power spectral density
γ Phase difference parameter
ϕ Random phase shift
ω Wave number

INTRODUCTION

One of the key aspects to be considered in the control of unmanned ground vehicles (UGVs) is the effect of the terrain on the vehicle's dynamics and handling. The terrain plays a significant role in the mobility and dynamics of an off-road vehicle. There are several factors that can inhibit a vehicle from passing a given section of terrain. These factors occur on a large variety of length scales. At a large length scale this may include steep hills. Moving down in length scale, large rocks or boulders can make a path very difficult for a vehicle to pass. At an even smaller length scale the roughness of the surface, such as small rocks can have a significant influence on traction, which affects the mobility of the vehicle over larger length scales.

For the control of a UGV it is important that ways to characterize the roughness of the terrain in numerical terms are developed. The vehicle must be able to gather information about the terrain on which it is traveling and make decisions on the optimal speed, steer angle, or braking pressures required to perform various maneuvers. In a manned vehicle, the driver is using input from the vehicle as well as visual cues to make decisions on how to control the vehicle. For example, if the driver feels the road is too bumpy, they will slow down to minimize the loads on the vehicle. The ultimate task of this work is to simulate the driver's decision making ability in the control of the UGV. Improperly interpreting the terrain could result in a loss of efficiency, the vehicle rolling over, the vehicle getting stuck, or the vehicle having a premature component failure. This requires relating terrain information to critical vehicle parameters to determine possible roll over or spin out.

Two methods which are commonly used to characterize a terrain are the root mean squared elevation (RMSE) and power spectral density (PSD) [1, 2]. The RMSE has been popular because it is a simple calculation which returns only one value. However, using RMSE requires some strong statistical assumptions to hold for the terrain profile being characterized. The data set must be stationary and Gaussian in order for the RMSE to be an accurate statistical representation of the profile. A limitation of this method for analyzing off road terrain profiles is that they are generally non-stationary and non-Gaussian [3-5]. Another key limitation of the RMSE method is that it gives no indication of the frequency contents of the terrain.

To overcome this limitation, the RMSE is often supplemented with the PSD which can give an estimate of the energy associated with the various frequency levels of the terrain [1, 2, 6, 7]. This adds a level of sophistication to the terrain characterization beyond the RMSE alone. Yet, this too comes with several drawbacks. The PSD will also only be accurate for the case where the terrain profile is stationary and Gaussian [7]. Additionally, taking the PSD of the profile does not track the phase shift information at each scale which can drastically change the vehicle response when driving over that terrain. Terrain profiles are also likely to contain transient events and large irregularities which must be accounted for, due to their significant affect on the vehicle response. Taking the PSD alone cannot capture these important features. It can however give a good understanding of the frequency content of a specific window of data.

Another characterization method which has been used for analyzing road surfaces for quite a long time is the international roughness index (IRI) [8, 9]. This method uses the integral of the relative motion between the velocities of the sprung and unsprung masses to determine a value which represents the roughness of the profile. Several researchers have used this method as part of the terrain characterization [1, 2, 7]. It has also been combined with the PSD to yield a more complete characterization methodology [10]. Since this method relies solely on the vehicle motions and not directly on the terrain geometry, it is dependent on the vehicle parameters.

There has also been some work which has developed terrain characterization methods based on the Auto-Regressive Integrated Moving Average (ARIMA)[4, 5, 11]. By using this method the number of parameters used to characterize the terrain can be greatly reduced. This is important since typically it is impractical to keep the deterministic data for every type of road profile.

It is important to note that there are advantages and disadvantages to each of these characterization methods. To fully understand the terrain no single calculation will be able to capture all of the pertinent information, especially for an off road profile. Thus, the purpose of this work is to investigate methods which can be used to overcome the shortfalls of specifically the RMSE and PSD, and can ultimately be coupled with them to yield a more robust terrain characterization. Two methods which will be investigated are the Fourier transform and the Wavelet transform. It is proposed that analyzing the frequency content of the terrain profile using the Fourier transform can be used to determine the significance of various frequencies on the vehicle response. The Wavelet transform is another potential method for decomposing the terrain profile into a parameter set which can be analyzed more easily. Quarter car vehicle simulations will be run to validate the effectiveness of these methods.

METHODOLOGY

2-D Fractal Terrain Profile Generation

For the purposes of developing and testing terrain characterization methods, it is beneficial to be able to generate random profiles that match terrains with varying degrees of roughness. This allows vehicle simulations to be run on various terrains with less empirical data necessary. A terrain profile can be represented as self-similar fractal surface if the PSD of that profile exhibits a power law behavior with increasing frequency. There are several works which have modeled terrain profiles as self affine fractals for the purposes of terrain characterization [1, 2, 7, 12]. An ideal fractal surface will have a PSD of the following form

$$\Phi(\omega) = R(\omega)^{-k} \tag{1}$$

where R is the roughness amplitude constant, ω is the wave number (analogous to frequency in time domain) in cycles/m, and k is the slope of the wave number spectrum. This form can be fit to the PSD of a terrain profile to determine the constants R and k. As with any fit, this will average out the effects of certain features which may be important from a vehicle dynamics standpoint. However, this does allow a profile which will capture the general trend of the terrain roughness to be generated.

These parameters must be related to a function which can generate a fractal profile. A two-dimensional fractal surface profile $h(x)$, can be represented by a Weierstrass–Mandelbrot (W-M) function that satisfies the properties of continuity, nondifferentiabilty and self affinity. The W-M function classifies a rough surface based on two fractal parameters. For this study the following version has been used for dimensional consistency[13].

$$h(x) = L\left(\frac{G}{L}\right)^{(D-1)} \sum_{n=1}^{n_{max}} \frac{\cos\left(\frac{2\pi\gamma^n x}{L}+\phi_n\right)}{\gamma^{(2-D)n}} \qquad (2)$$

G and D are the fractal roughness parameter and fractal dimension of the surface profile respectively. n is an integer that represents a frequency level of the surface, γ is a parameter that determines the relative phase difference between fractal modes, and ϕ is a random phase shift for each frequency level of the surface. In this paper, γ = 1.5 and L is the length of the profile generated. ϕ is a random phase shift value for each frequency level such that $0 \le \phi \le \pi$.

Additionally, the continuous power spectrum for a fractal surface is given by [14],

$$\bar{\Phi}(\omega) = \frac{G^{2(D-1)}}{2\ln\gamma}\frac{1}{\omega^{(5-2D)}} \qquad (3)$$

setting Eq. (1) and Eq. (3) equal to each other the fractal parameters D and G can be determined for a given R and k. After determining the fractal parameters a surface can be generated using Eq. (2). By randomly selecting the phase shift, the profiles generated will be significantly different even though they were generated from the same fractal parameters. Figure 1 shows two profiles which were generated using the W-M function with different fractal parameters, resulting in two surfaces with different roughness.

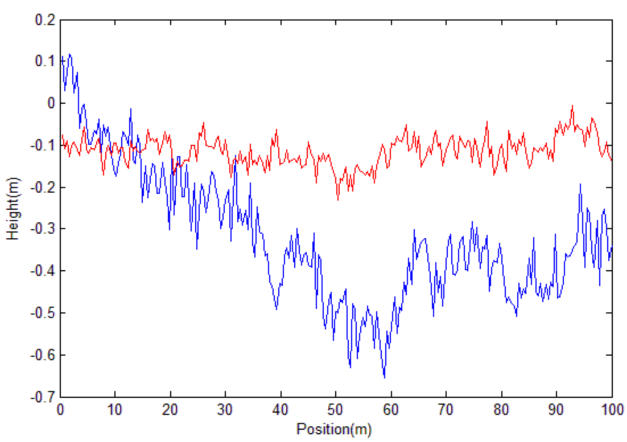


FIGURE 1. RANDOMLY GENERATED 2D TERRAIN PROFILES FOR MILD PROFILE (RED) AND ROUGH PROFILE (BLUE)

To validate the generated fractal profile, the PSD extracted from the generated surface is plotted against the ideal PSD line used to fit the fractal parameters. It can be seen from Fig. 2 that the PSD of the generated profile matches the trend of the ideal line.

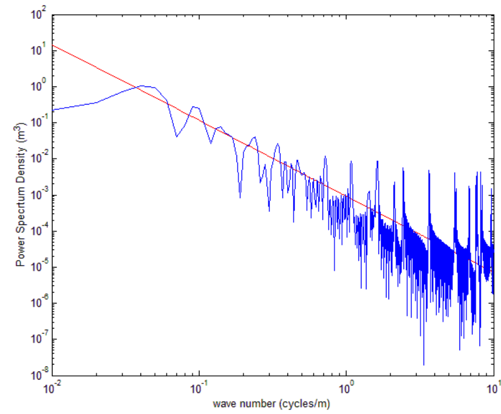


FIGURE 2. PSD OF GENERATED FRACTAL SURFACE COMPARED TO IDEAL PSD FRACTAL LINE.

Vehicle Simulation Model

It is important to understand the effects of the terrain on the vehicle response. The terrain can cause the vehicle to pitch and roll. In order to capture both of these effects a full 7 degree of freedom model is needed. However, since this study is focused on methods of characterizing the terrain, a 2 degree of freedom quarter car model was used to simplify the vehicle response. This makes the effect of changes in the terrain on the vehicle easier to detect and interpret. The derived equations for motion for the quarter car model shown in Fig. 3 are as follows:

$$m_s\ddot{z}_s = k_s(z_u - z_s) + c_d\,(\dot{z}_u - \dot{z}_s) \qquad (4)$$

$$m_u\ddot{z}_u = -k_s(z_u - z_s) - c_d\,(\dot{z}_u - \dot{z}_s) + k_t(h - z_u) \qquad (5)$$

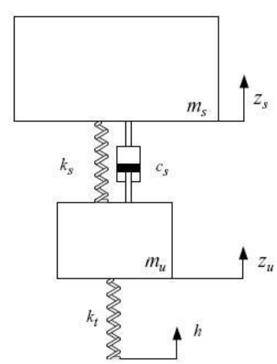


FIGURE 3. 2 DOF QUARTER CAR MODEL USED FOR VALIDATION OF TERRAIN CHARACTERIZATION.

In Eq. (4) and Eq. (5) m_s and m_u are the sprung and unsprung masses respectively. The spring constants used in the model are k_s for the suspension spring and k_t for the tire stiffness. The damping coefficient of the damper is denoted by c_d. The positions of the sprung and unsprung masses and their derivatives are z_s and z_u respectively. The values for these constants were derived from the international roughness index (IRI) golden car parameters [9]. This method allows approximate values of all the model constants to be calculated based on the sprung mass of the vehicle. The sprung mass represents one quarter of the total vehicle mass. The values used in the vehicle model are shown in Table 1.

TABLE 1. PARAMETERS USED IN QUARTER CAR MODEL

Model Parameter	*Value*
m_s	142 *kg*
m_u	21 *kg*
c_d	852 *N-s/m*
k_s	8.98×10^3 *N/m*
k_t	9.27×10^4 *N/m*

The vehicle simulations were conducted using Matlab/Simulink. The terrain profile height was used as the input to the system and the motion of the sprung mass was analyzed. The simulation was set up for a vehicle traveling 11.1 m/s (40kph) to cover the 100m terrain profile in approximately 9 seconds. The Runge-Kutta method with a variable time step was used to solve the differential equations. In Matlab this is implemented using the *ode45* command.

Fourier transform characterization

The Fourier transform has been widely used in many disciplines for a variety of applications. In this paper, the fast Fourier transform (FFT) is used to determine what information can be gained about the terrain from a vehicle dynamics standpoint. By decomposing the profile using the FFT into a parameter set and then reconstructing the surface based on a limited number of parameters, the effect that various frequencies have on the vehicle can be studied. Figure 4 shows how a given rough profile looks when being regenerated using different numbers of Fourier coefficients. Simulating the vehicle driving on these decomposed surfaces allows the effects on the dynamic response of the vehicle to be determined. For example when the vehicle is simulated driving over the original profile it will have a certain response. When the vehicle is simulated driving over one of the deconstructed profiles it will have a different response. However, as more terms are added to the regeneration the response will approach the response of the original surface.

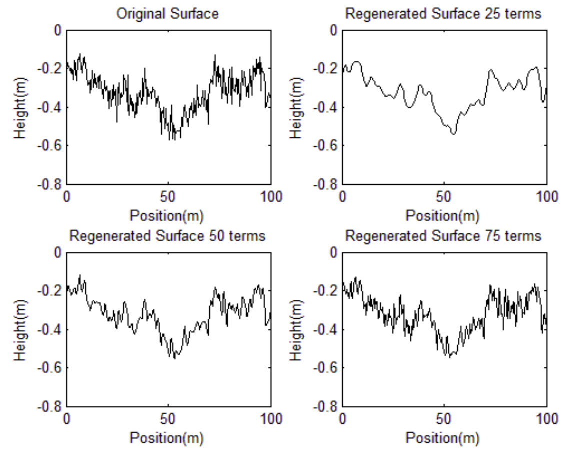


FIGURE 4. SURFACES DECOMPOSED USING FFT AND REGENERATED WITH DIFFERENT NUMBERS OF FOURIER COEFFICIENTS

Wavelet transform characterization

Another important aspect of terrain characterization is to develop a method for picking up discontinuities and changes in the surface. The previously introduced methods are not easily adapted to detecting these events. By effectively identifying these features we can supplement the traditional methods of terrain characterization resulting in a more robust solution.

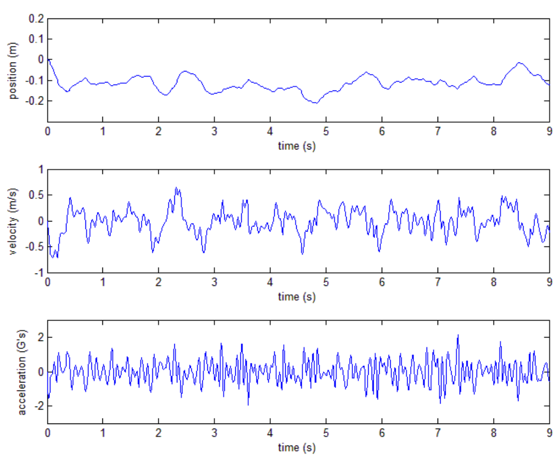
Wavelets are functions that decompose a signal into different frequency components and then analyze each frequency with a resolution matched to the scale being analyzed [15]. The wavelet transform is based on the same premise as the Fourier transform. However instead of representing the signal as a superposition of sines and cosines it represents the signal as a superposition of a function called a mother wavelet. There are several mother wavelets which can be used to perform an analysis. This work used the Daubechies (db10) mother wavelet to perform the wavelet transform in Matlab. There are several functions in Matlab which will perform various versions of the wavelet transform. For this work the 'cwt' command was used which computes the continuous wavelet transform. It returns an amplitude coefficient of each scale of the wavelet being analyzed. This coefficient represents the amplitude scaling of the mother wavelet at that frequency level. This coefficient is analogous to the coefficient which is returned when taking the FFT of a signal.

There are two specific cases which will be examined in this work. The first is a terrain profile with a high amplitude step increase. To generate this profile, the W-M function was again used, and then a 0.25 m increase was added to the profile after a certain point. This increase could represent a large rock, log, or in an urban environment a curb. The second case is one in which the general roughness of the terrain changes. To generate the terrain profile with a changing roughness, the W-M function was used to generate two profiles each with different degrees of roughness. The two profiles were then combined end to end.

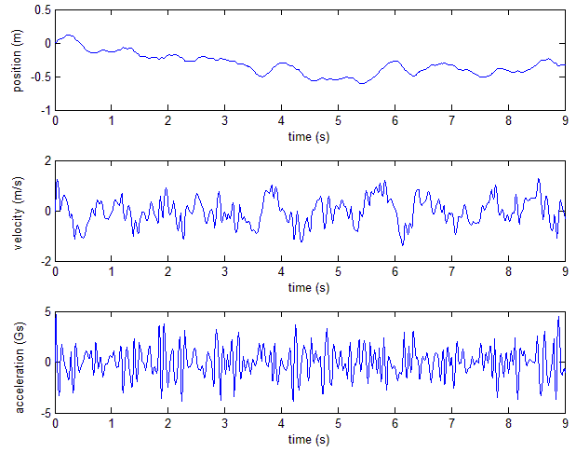
DISCUSSION AND RESULTS

Roughness Comparison

To compare and validate our vehicle model as means for evaluating the terrain profiles being generated, the quarter car model was simulated on each of the terrains shown in Fig. 1. The vehicle response of these simulations is shown in Fig. 5. As expected the vehicle which travels over the rougher terrain experiences more severe motions (higher magnitude velocities and accelerations). Note that according to the simulation, the body will experience negative accelerations above 1 G. This is not physically realizable, since the vehicle will lift off of the ground long before this magnitude of negative acceleration is reached. This does reveal a flaw in our simplified vehicle model for use in this study. The tire spring used in the model can carry both tension and compression, which is not true of an actual tire which can only carry a compressive load. This is a key effect that our vehicle model does not include that will affect the accelerations of the vehicle body. In the future, this issue will be overcome with more sophisticated vehicle models. However, the current model should suffice in allowing us to make some comparisons for our terrain characterization methodologies.



(a)



(b)

FIGURE 5: VEHICLE RESPONSE OF SPRUNG MASS (z_s) IN SIMULATION FOR (a) MILD TERRAIN (b) ROUGH TERRAIN

Fourier Transform Characterization

To analyze the Fourier transform characterization the vehicle was simulated traveling on a terrain profile to establish truth data for the position and velocity of the sprung mass. The FFT of the terrain profile data was taken. The terrain was then re-generated using the inverse FFT with a varying number of coefficients. Figure 6 shows the position of the sprung mass of the vehicle in the truth simulation compared with the regenerated profile simulation. The error between the motion of the vehicle on the regenerated surface was then determined based on the truth data. Figure 7 shows a plot of the error as a function of the number of Fourier coefficients used to regenerate the terrain. It can be seen that when only a few coefficients are used the error is highest. The error decreases rapidly as the first several coefficients are used. However, at a certain point the error begins to flatten out. Most of the detail of the terrain is determined by the first coefficients considered, not much additional information that is gained in adding the upper coefficients. It should be noted that each of these coefficients is directly related to a frequency level in the profile. Thus, this can also be interpreted as the highest frequency level considered in the terrain regeneration. In terms of the vehicle motions of the body, the terrain profile is characterized primarily by the long wavelength high amplitude terms. The short wavelength low amplitude frequency components of the profile do not heavily affect the motion of the vehicle. However, these frequency components may play a large role in other vehicle characteristics such as traction.

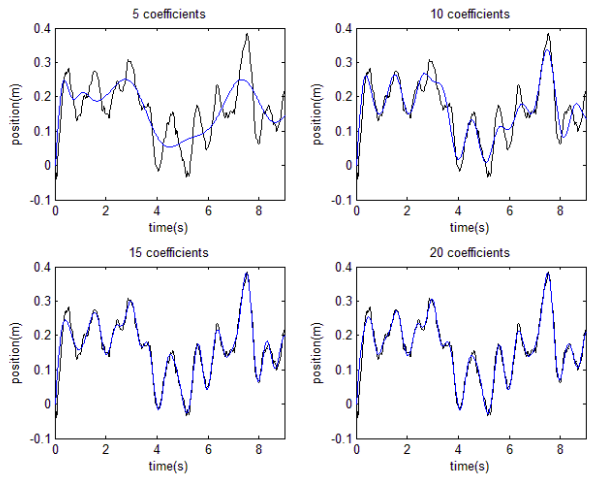
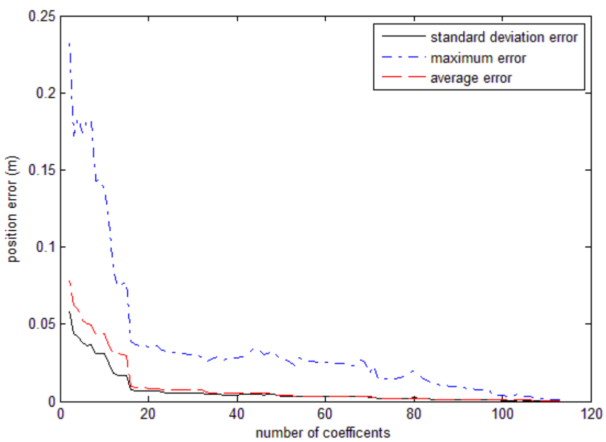
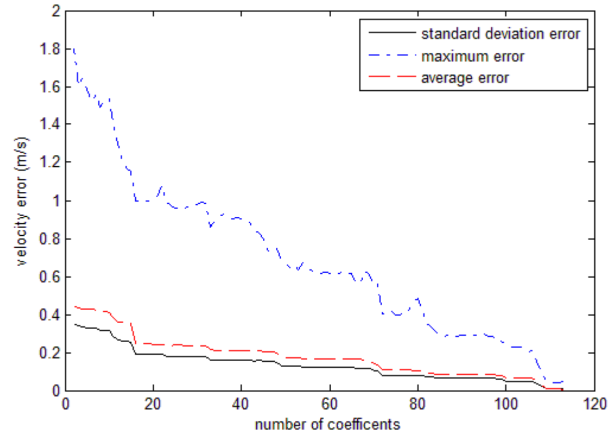


FIGURE 6. POSITION OF SPRUNG MASS FOR SIMULATIONS OVER REGENERATED SURFACES WITH DIFFERENT NUMBERS OF FOURIER COEFFICIENTS.



(a)



(b)

FIGURE 7. ERROR BETWEEN FULL TERRAIN SIMULATION AND REGENERATED TERRAIN SIMULATION AS A FUNCTION OF FOURIER TERMS USED FOR (a) POSITION (b) VELOCITY

Wavelet Transform Characterization

Figure 8 shows a terrain profile to which a step increase has been introduced. This could represent a log or rock ledge. Deconstructing this profile with the wavelet transform breaks the profile into its fundamental parts, making the extraction of these types of features easier. Looking at the high frequency component of the profile at the bottom of Fig. 8 it is easy to see the peak where the step increase occurs. Figure 9 shows the vehicle response of the sprung mass when traveling over the profile with the step increase. The step increase can be seen as point of high magnitude in the velocity and acceleration plots and as a high magnitude change in height in the position plot.

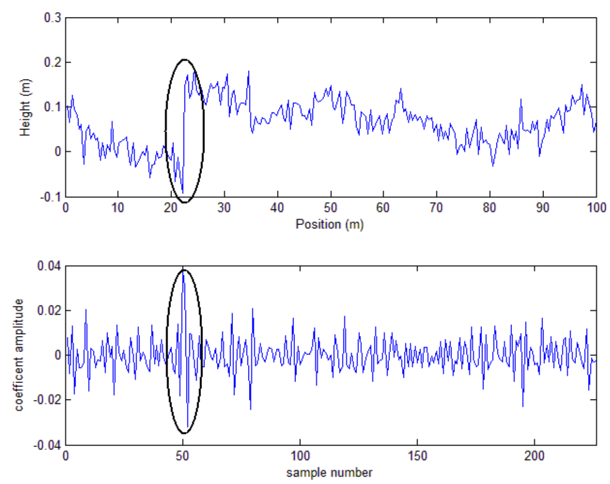


FIGURE 8. PROFILE WITH A DISCONTINUITY WHICH HAS BEEN DECOMPOSED INTO ITS HIGH FREQUENCY COMPONENT USING THE WAVELET TRANSFORM

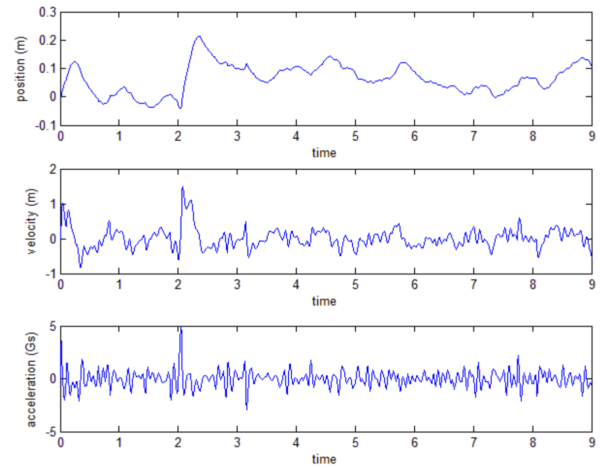


FIGURE 9 VEHICLE RESPONSE OF SPRUNG MASS (z_s) WHEN DRIVING OVER PROFILE WITH A STEP INCREASE

In order to determine a change in the roughness, the high frequency component of the Wavelet transform of the terrain profile shown in Fig. 10 can again be studied. There is a very distinct difference in the high frequency content of the Wavelet transform after the roughness of the terrain profile changes. Figure 11 shows the vehicle response corresponding to this same profile with a change in roughness. The velocity and acceleration plots both exhibit an increase in amplitude after the roughness changes.

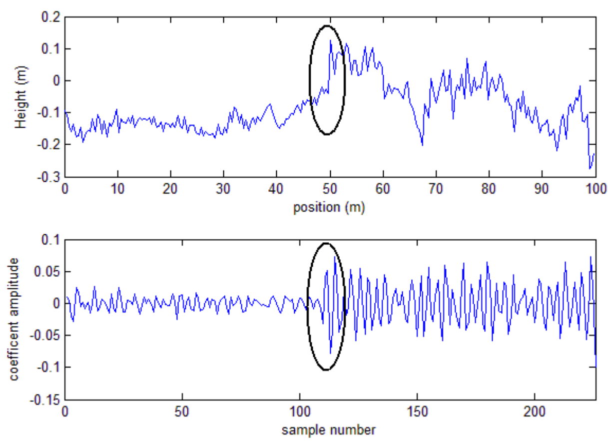


FIGURE 10. PROFILE WITH A CHANGE IN ROUGHNESS AND THE HIGH FREQUENCY WAVELET DECOMPOSITON DETECTING CHANGE

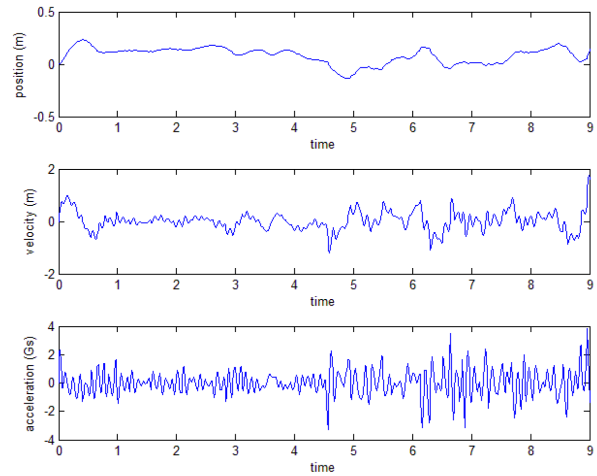


FIGURE 11. VEHICLE RESPONSE OF SPRUNG (z_s) MASS WHEN DRIVING OVER SURFACE WITH A CHANGE IN ROUGHNESS

Implications for UGV control

From the information presented in the previous section it is important to note that both the high amplitude step and change in roughness can be detected by the vehicle response. However, when controlling a UGV with this information, waiting for the vehicle response would be too late. A better method would use the wavelet to detect these features and use the vehicle response to validate and correct for wavelet calculations. Using an external sensor such as a Lidar, the terrain ahead of the UGV can be scanned and these key features can be detected. This information can then be fed back into the vehicle controller to make adjustments based on future input. As previously mentioned, to yield the most robust and efficient terrain characterization methodology these various techniques should be combined. For example if the RMS roughness of the surface is low enough (paved road) it may not be necessary to perform any of the other more complex and computationally intensive techniques. Using a more synergistic approach to characterizing the terrain allows for checks within the algorithm which will yield a more robust solution.

There are several challenges which must be addressed in using these techniques for vehicle control. All of the methods presented in this work consider a 2D profile, yet the difference of the terrain between the right and left sides of the vehicle is a very important effect that must be considered. Using 3D formulations may make extraction of key features more difficult. Determining the potential effect of the identified terrain features on the vehicle is not a trivial task. Additionally, once the effect is determined the appropriate action must be conveyed to the vehicle controller, which again is not trivial. Sometimes speeding up over a certain terrain is easier on the vehicle than slowing down. Ultimately the terrain characterization must be combined with all of the other UGV control considerations such as path planning, obstacle avoidance, and tracking other vehicles. To effectively control the vehicle all of these need to be implemented in real time which will present a completely different set of challenges.

Another challenge that will arise in the implementation of these techniques in performing a real time terrain characterization and control of a vehicle is the collecting, storing, and processing of the data. It is logical to assume that the quality of the terrain characterization is dependent on the resolution of the scans. This resolution will be directly related to the sample rate of the sensors being used. Therefore as the vehicle's speed increases the resolution of the data being collected will decrease. To counter this effect, higher sample rate sensors can be used, but this will result in higher computational demand in the data processing. Also in general the motion the body of the vehicle experiences will also be dependent on the vehicle's speed. In order to accurately scan the ground, one must compensate for the motion of the vehicle to which the sensor is mounted. Thus the quality of the terrain characterization will be related to the speed at which the vehicle is traveling.

In addition to using these characterization techniques to control a UGV, they can also be used to generate more realistic terrains for vehicle simulation. If an artificial representative terrain can be generated from smaller set of parameters, the vehicle response can be approximated for a terrain without having the deterministic data, which could become cumbersome. This opens up the possibility of running real time vehicle simulations to determine possible vehicle responses to a terrain over which the vehicle is traveling.

CONCLUSIONS

This paper has shown that both the Fourier transform and Wavelet transform are viable tools which can be added to the more traditional methods of terrain characterization. The Fourier transform can be used to determine a reduced set of parameters which will effectively describe the terrain profile with a vehicle response error relative to the number of coefficients used. The Wavelet transform can detect events such as high amplitude step increases and changes in the roughness of the terrain. Combining several methods of terrain characterization will yield a more robust solution.

ACKNOWLEDGMENTS

The authors would like to thank TARDEC for their support in this work.

REFERENCES

[1] Howe, J.G., Lee, D., Chrstos, J.P., Balling, O., Myers, T.T., Allen, R.W., Gorsich, D.J. and Reid, A., 2005," Further Analysis of Potential Road/Terrain Characterization Rating Metrics," SAE 2005-01-3562.

[2] Howe, J.G., Lee, D., Chrstos, J.P., Myers, T.T., Allen, R.W., Gorsich, D.J. and Reid, A., 2004," Analysis of Potential Road/Terrain Characterization Rating Metrics," SAE 2004-01-2640.

[3] Chaika, M. and Gorsich, D., 2004," Some Statisitical Tests in the Study of Terrain Modeling," International Journal of Vehicle Design, **36**(2/3), 132-148.

[4] Kern, J.V. and Ferris, J.B., 2006," Preliminary Results for Model Identification in Characterizing 2-D Topographic Road Profiles," Proceedings of SPIE- the International Society for Optical Engineering, **6228**.

[5] Kern, J.V. and Ferris, J.B., 2006," Characterizing 2-D Topographic Road Profiles," ASME International Mechanical Engineering Congress and Exposition.

[6] Andren, P., 2006," Power Spectral Density Approximations of Longitudinal Road Profiles " International Journal of Vehicle Design, **40**(1/2/3), 2-14.

[7] Gorisch, D., 2003," Terrain Roughness Standards for Mobility and Ultra-Reliability Predictions," Proceedings of the SAE World Congress.

[8] Sayers, M.W., 1989," Two Quarter-Car Models for Defining Road Rougness: IRI and HRI," Transportation Research Record, **1215**, 165-172.

[9] Sayers, M.W., 1995," On the Calculation of International Roughness Index from Longitudinal Road Profile," Transportation Research Record, **1501**, 1-12.

[10] Malmedahl, G.A., Dembski, N., Rizzoni, G., Soliman, A. and Disaro, L., 2006," A Method for the Characterization of Off-Road Terrain Severity," 2006-01-3498.

[11] Wagner, S.M. and Ferris, J.B., 2007," Reduced Order ARIMA Models of 2-D Terrain Proifles Using Singular Value Decomposition," ASME International Mechanical Engineering Congress and Exposition.

[12] Wang, Q., Guo, J. and Chen, Y., 2006," Fractal Modeling of Off-Road Terrain Oriented to Vehicle Virtual Test," Journal of Zhejiang University SCIENCE A, **7**, 287-292.

[13] Yang, J. and Komvopoulos, K., 2005," A Mechanics Apporach to Static Friction of Elastic-Plastic Fractal Surfaces," Journal of Tribology, **127**(2).

[14] Ganti, S. and Bhushan, B., 1995," Generalized fractal analysis and its applications to engineering surfaces," Wear, **180**, 17-34.

[15] Graps, A., 1995," An introduction to wavelets," Computational Science & Engineering, IEEE, **2**(2), 50-61.