

MECH 4420 Homework #6
Due Wednesday 10/22/2025 in class
(Note: checkoff due 10/17/2025)

Steady State Handling

The steady-state handling results that we developed in class give a lot of insight into what happens in the linear region of the tire curve. In racing, however, the vehicle attempts to operate at the limits of handling. Therefore, it is important to include the saturation of tire force and the dependence of this saturation on load when examining the handling characteristics over the full range from parking lot maneuvers to the friction limits. To do this, there are a number of available tire models that handle load dependence, saturation and a wealth of other things. One model that produces surprisingly good correlation with experiments is the “Magic Formula” tire model, so named since the formulation has no underlying physical rationale (this is often called the Pacejka tire model after one of its developers, Hans Pacejka). Pacejka parameters are a common means of communicating tire data between tire suppliers and vehicle manufacturers.

The Magic Formula for side force as a function of slip angle is:

$$F_y = D \sin(C \tan^{-1}(B\Phi))$$

$$\Phi = (1 - E)\alpha + (E/B) \tan^{-1}(B\alpha)$$

each of the coefficients in the model is described as a function of vertical load (and can be modified to also become a function of camber angle):

$$C = 1.30$$

$$D = a_0 F_z^3 + a_1 F_z^2 + a_2 F_z$$

$$BCD = a_3 \sin(a_4 \tan^{-1}(a_5 F_z))$$

$$E = a_6 F_z^2 + a_7 F_z + a_8$$

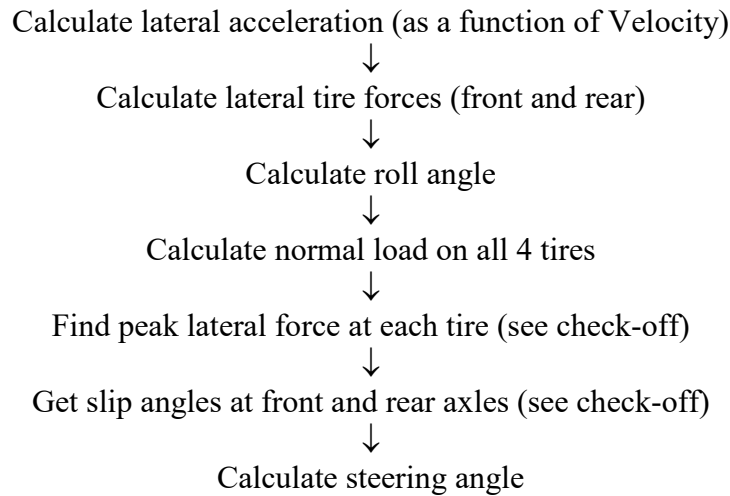
The coefficient D represents the peak side force while the product of coefficients BCD represents the cornering stiffness. One set of real data (**which requires vertical force in kN and slip angle in degrees and gives side force in N**) from SAE Paper 870421 is:

$a_0 = 0$	$a_3 = 1078$	$a_6 = 0$
$a_1 = -22.1$	$a_4 = 1.82$	$a_7 = -0.354$
$a_2 = 1011$	$a_5 = 0.208$	$a_8 = 0.707$

1) Once we put empirical data into a formula like this it is easy to lose sight of what is going on. To prevent this, plot the following to get an idea of the tire behavior predicted by this model:

- Plot side force as a function of slip angle for normal loads of 2, 4, 6 and 8kN.
- Plot the cornering stiffness (BCD) as a function of normal load from 0 to 8kN. Explain using this graph how lateral load transfer could be used to change the tire cornering properties on one of the axles.
- Plot the peak side force (D) as a function of normal load from 0 to 8kN. Explain how the maximum traction should differ between cases of minimal and large lateral load transfer.

2) Using MATLAB, develop a program that calculates the steer angle required for any level of lateral acceleration. The flow of the program should parallel the development in Gillespie and in class but substitute this tire model. The structure should look like:



The MATLAB script should solve for the slip angle given inner and outer normal loads and the combined lateral force.

3) Inspired by a particularly thrilling lecture in vehicle dynamics, you decide to devote your weekends to racing and want to adapt your car to the purpose. You decide to get a handle on the characteristics of your car by running it through the simulator developed above. Plot the steer angle as a function of lateral acceleration up to the limits (watch out for tire lift-off as you change parameters – a tire that lifts off the ground should not be producing force though the existing script does not check for this). Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor? Assume that your car has the following parameter set and it initially has the front roll bar installed. Assume also that your test track has a radius of 50m.

$$m = 1475 \text{ kg}$$

$$L = 2.58 \text{ m}$$

$$a = 1.16 \text{ m}$$

$$b = 1.42 \text{ m}$$

$$h_{cg} = 0.6 \text{ m}$$

$$h_f = h_r = 0.1 \text{ m}$$

$$t_f = t_r = 1.44 \text{ m}$$

$$K_{\phi_f} = 232 \text{ Nm/deg (springs only)}$$

$$K_{\phi} = 480 \text{ Nm/deg (springs and anti - roll bar)}$$

$$K_{\phi_r} = 250 \text{ Nm/deg (springs only)}$$

4) You're bumming. This doesn't seem like it is going to win you many autocrosses. Figuring that the front anti-roll bar might be causing you problems, you decide to remove it. Plot the new steering angle required as a function of the lateral acceleration. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor?

5) Thinking this is going well, you decide to put the anti-roll bar you removed from the front on the rear axle. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor? Does this seem like a good modification?

6) You decide to buy an additional anti-roll bar of equal stiffness so that you have bars on both the front and the rear. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor?

7) From this configuration, assume that you decide to modify the suspension kinematics. Can you make a roll center height change that gets you around the corner faster? If so, what do you do?

8) Finally, assume you decide to make some major modifications to the car and can move the c.g. location (though not the height) anywhere you want and add total roll stiffness up to 600 Nm/deg divided as you want between front and rear. What should you do to get around the corner as fast as you can? What is your rationale for this? Plot the steering angle versus lateral acceleration. What is the peak lateral acceleration?

Just as a point of interest, if you were to get seriously into racing, you would want to be able to consider several other modifications as well that might come in handy on race day. One of these is changing the tire pressure, which alters the tire characteristics (cornering stiffness and peak). Another is to add wedge, cranking up the pre-load on a rear spring to create a diagonal imbalance in the tire normal force distribution. In many racing circuits, there are also suspension modifications that can be made. In NASCAR, the rear roll center height used to be adjustable by changing the track rod of the suspension (no more as NASCAR went to an independent rear suspension in 2022). Many of these vehicle settings are done prior to a race using simulation to test different setups. Then on race day, minor adjustments are made to account for errors or changes in track conditions. The key to understanding the challenge of racing is to see that the tire properties change as the tires heat up and wear. Since your front tires are steering while your rear tires carry all of the tractive forces in acceleration, they wear at different – and rather unpredictable – rates. Thus, the tricky balancing act you have done on this assignment only holds for one point in time. As race engineers, you need to make the call on how much to change each parameter so that the car will run as well as it can between pit stops. You also have rather imprecise data from which to work if you have any data at all. If you make the wrong call... well, that's racing, and you'll get 'em next week.

HW #6 Check Off

These functions will be needed to complete Problem #2.

- a) Write a matlab function to calculate $F_{y\max}$, given F_{zi} , F_{zo} (using the Pacejka model). This is fairly straightforward to write using basic logic. However you want to do it in a more sophisticated manner, therefore use the matlab function “fminsearch”. This requires that you write a matlab function (which “fminsearch” will call) that takes in α , F_{zi} , F_{zo} , and returns F_y such as:

```
function Fy=calc_max_fy(alpha,Fzi,Fzo)
```

insert code here

```
Fy=-(Fy_inner+Fy_outer)
```

Note: the negative sign because “fminsearch” is looking for a minimum – therefore, you must flip the Pacejka curve. Therefore “fminsearch” will actually return $-F_{y\max}$ (use “abs” after “fminsearch” to convert to max F_y).

- b) Write a matlab function to calculate α , given F_{zi} , F_{zo} , and F_y
For this use the Matlab function “fsolve” or “fzero”

This requires that you write a matlab function (which “fsolve” will call) that takes in α , F_{zi} , F_{zo} , $F_{y_desired}$ and returns F_y such as:

```
function Fy=calc_Fy(alpha,Fzi,Fzo,Fy_desired)
```

insert code here

```
Fy=Fy_inner+Fy_outer-Fy_desired
```

- c) Check the functions for parts (a) & (b) by writing a script that plots F_{ytot} (for a given F_{zi} and F_{zo}) vs. α and verifying $F_{y\max}$ and the α for $F_{y_desired}$! The script will use “calc_Fy” (with $F_{y_desired}=0$), “fminsearch” calling “calc_max_Fy” and “fsolve” calling “calc_Fy” (with the given $F_{y_desired}$). Show this to me in class for your check-off:

```
plot(alpha,Fy_tot)
hold on
plot(alpha_for_Fymax,Fymax,'*b')
plot(alpha_for_Fydes,Fydes,'*r');
hold off
```

Things to be careful about:

The minimum value for F_z is 0 (maximum ΔF_z is F_z)

If F_z is zero then F_y at that tire is 0

Have to take front and rear axles separately

If you are going to have several m-files and functions that use a large set of variables, it is often beneficial to place these variables in a separate m-file called “load_pacejka_parameters.m” for example. Then in any m-file or function that requires these parameters you simply include the line:

```
load_pacejka_parameters
```

Be careful that the variables in “load_pacejka_parameters” have unique names so as to not accidentally rename a variable in another m-file. Capital variable names usually help. This is not too necessary for this assignment, but a useful tool to keep in mind when running simulations with lots of variable definitions that are used in multiple m-files.