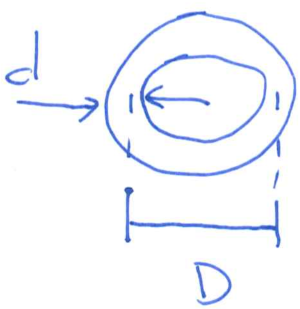
Spring Rates

$$K_s = \frac{Gd^4}{8D^3N}$$



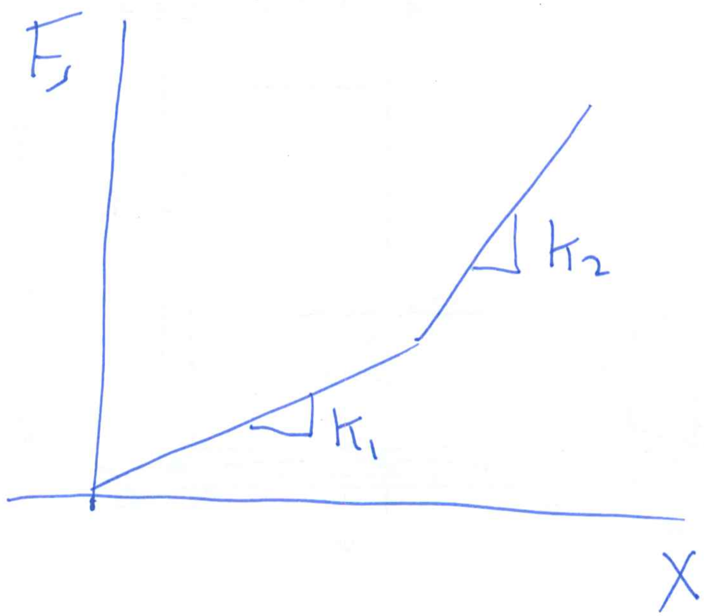
$N \Rightarrow$ # of "active" coils

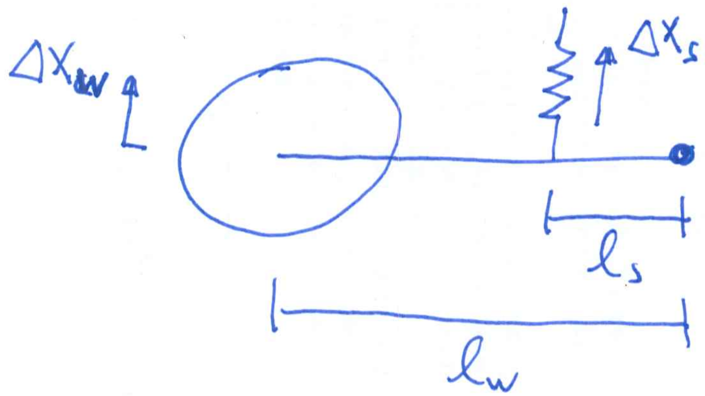
$G \Rightarrow$ shear modulus $(11.0 \times 10^6 \text{ psi})$

Varyin pitch will change # of active coils when the spring compresses

$\Longrightarrow$ Progressive Rate Springs

(Same thing NASCAR does w/ spring Rubbers)





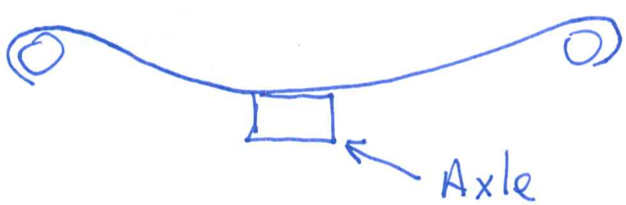
$$K_{susp}\,\Delta X\, l_w = K_{spring}\,\Delta X_s$$

$$\Delta X_w\, l_w = \Delta X_s\, l_s$$

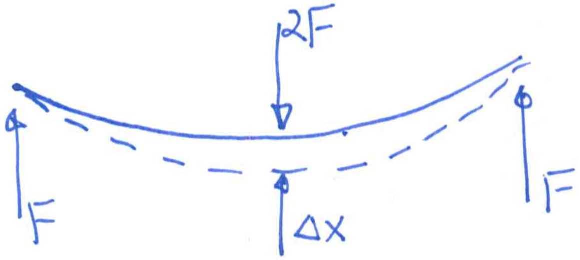
$$K_{susp} = K_{spring}\left(\frac{l_s}{l_w}\right)^2$$

☆ The amount the wheel (or car body) deflects is not the same as the amount of deflection in the spring. Therefore, the "apparent" stiffness of the suspension is different than the stiffness of the ~~spr~~ actual spring.

Can use "Cantilever" Model for leaf springs

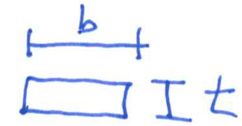


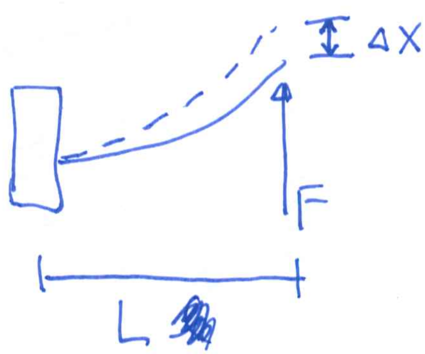
$$F = \frac{3EI}{L^3}\Delta X$$



$I \Rightarrow$ Area Moment of Inertia

$$\left(\frac{bt^3}{12}\right)$$

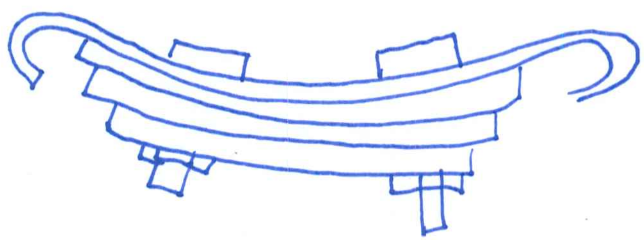




$E \Rightarrow$ Youngs Modulus

$(30 \times 10^6 \text{ psi})$

$$K_s = \frac{Ebt^3}{4L^3}$$

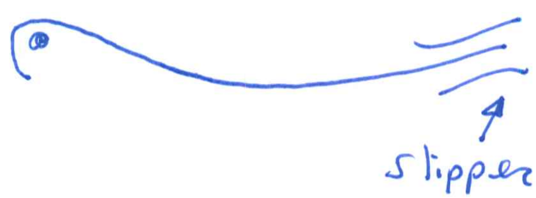


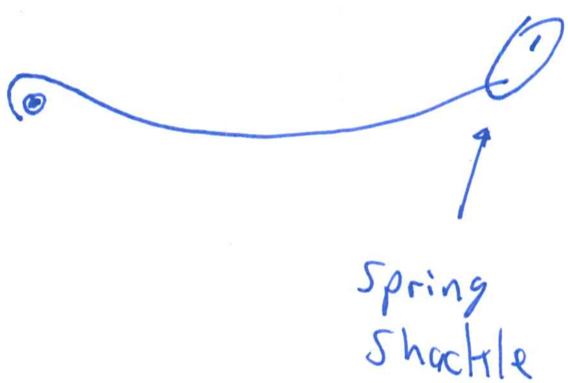
Bosch Handbook:

$$K_s = \frac{\left(2 + \frac{n'}{n}\right) E n b t^3}{6L^3}$$

$n \Rightarrow$ total # of leafs

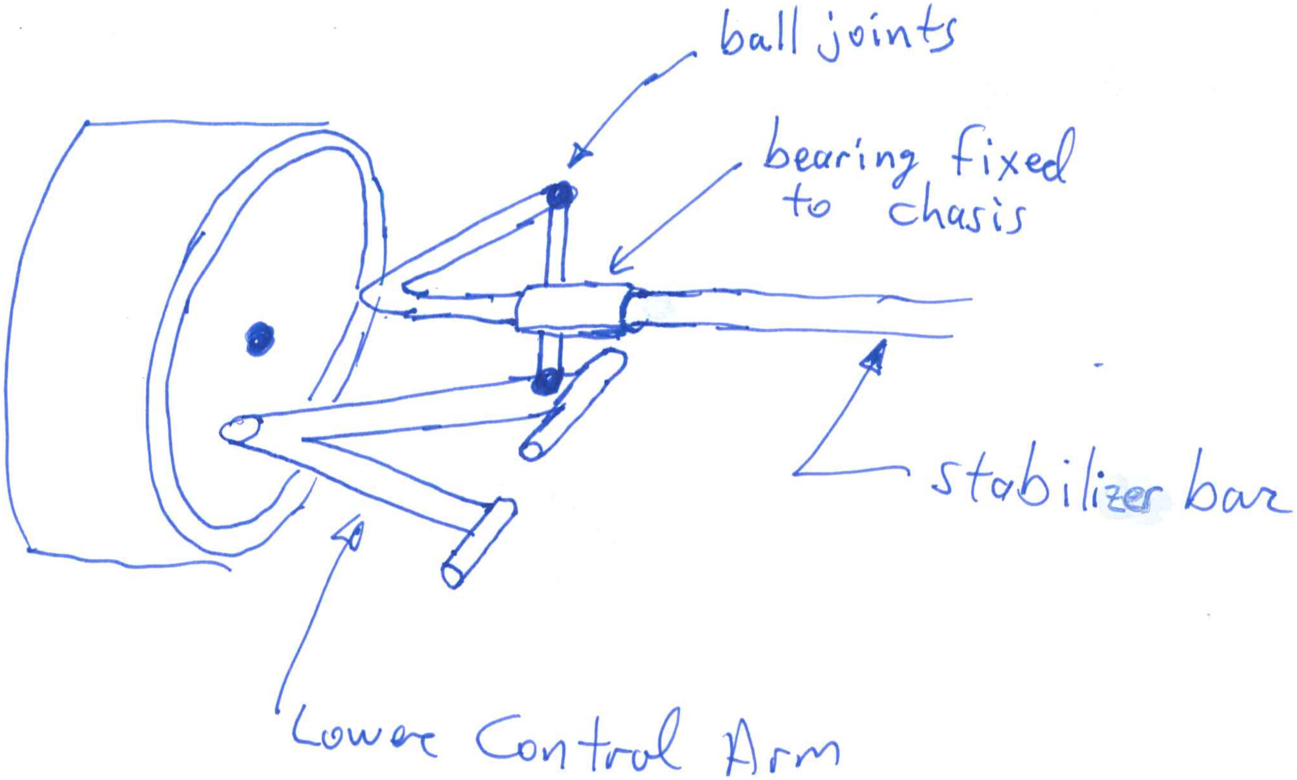
$n' \Rightarrow$ # of leafs at the end of the spring



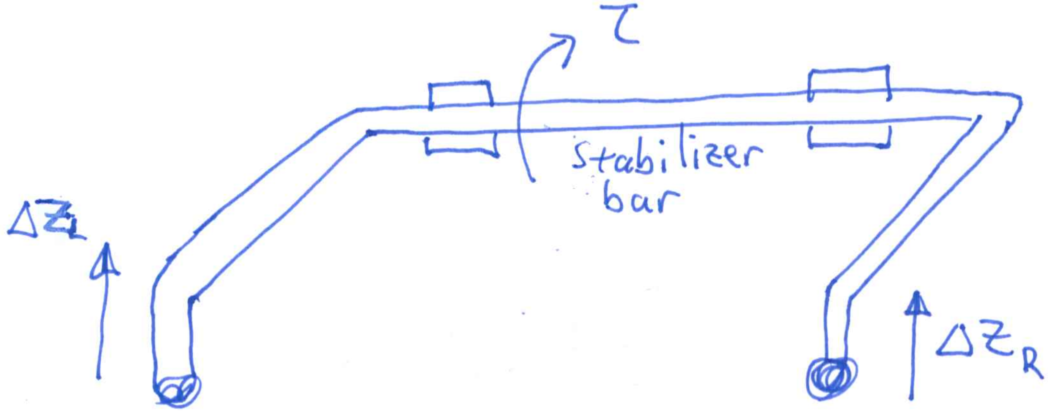


✱ Some "fudge" from experience in the above formula (doesn't match cantilever calculation)

Stabilizer Bars

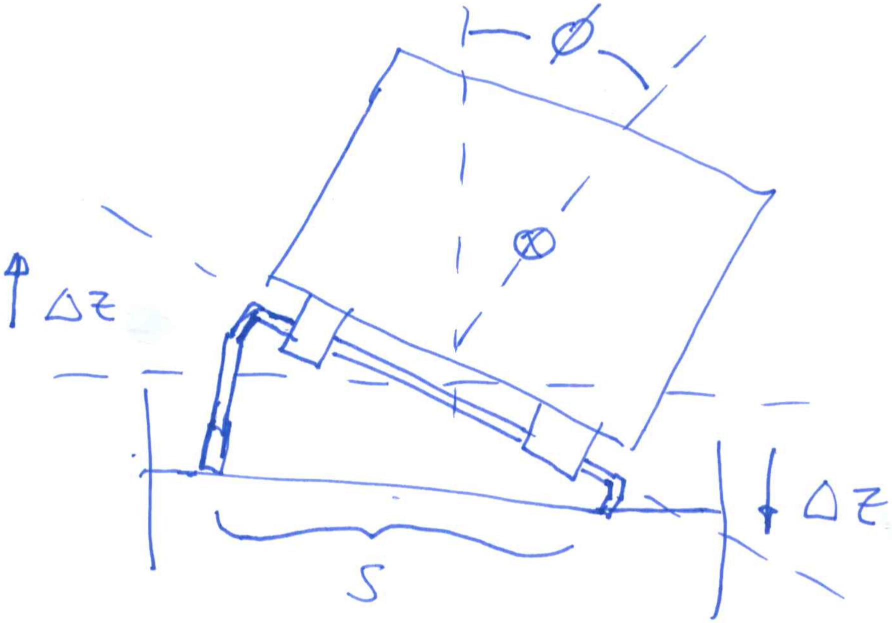


Stabilize bar Resists Vehicle Roll ~~while~~ but does not resist uniform vertical motion (i.e "bounce")



In Bounce: If $\Delta Z_L = \Delta Z_R \Rightarrow \tau = 0$

In Roll: If $\Delta Z_L \neq \Delta Z_R \Rightarrow \tau \neq 0$



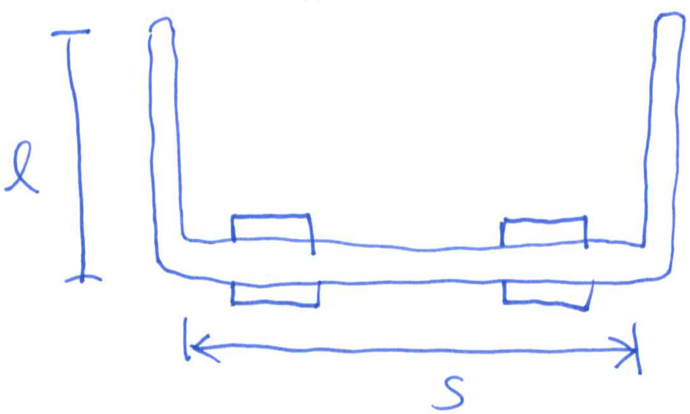
1) Relate Vehicle Roll to stabilizer Deflection

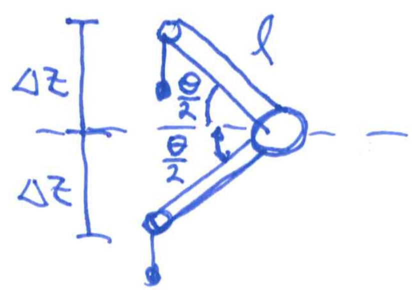
$$\Delta Z = \frac{S}{2}\phi \qquad \text{(SAA)}$$

2) Relate Stabilizer deflection to stabilizer bar twist

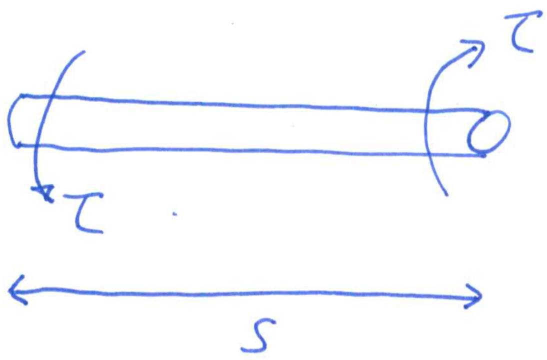
$$\Delta Z = \ell \frac{\theta}{2}$$

$$\theta_{bar} = \frac{2\Delta Z}{\ell} = \frac{S}{\ell}\phi$$





3) Relate twist in the bar to the Torque across the bar



$$\theta_{bar} = \frac{\tau s}{JG}$$

(Solid Mechanics formulation)

$J \Rightarrow$ Area Moment of Inertia
$G \Rightarrow$ shear Modulus of bar

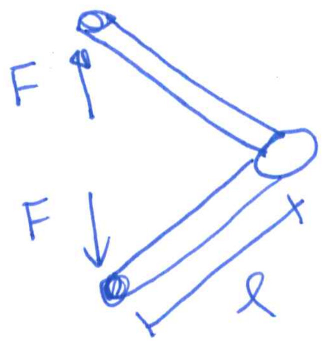
$$\tau = K_{bar}\,\theta_{bar}$$

$$K_{bar} = \frac{JG}{s}$$

$$J = \frac{\pi d^4}{32}$$

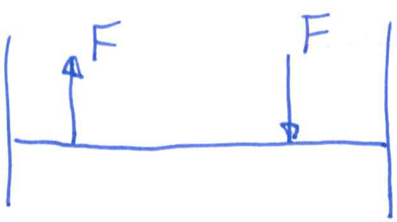
(If solid)

* Some vehicles use hollow pipe
$\Rightarrow$ why ??



4) Relate Moment (Torque) in the stabilizer bar to the Forces at the endpoint

$$F = \frac{\tau}{\ell}$$



5) Relate Force on the chasis to the Moment on the chasis

$$M = 2\left(\frac{s}{2}\right)F$$

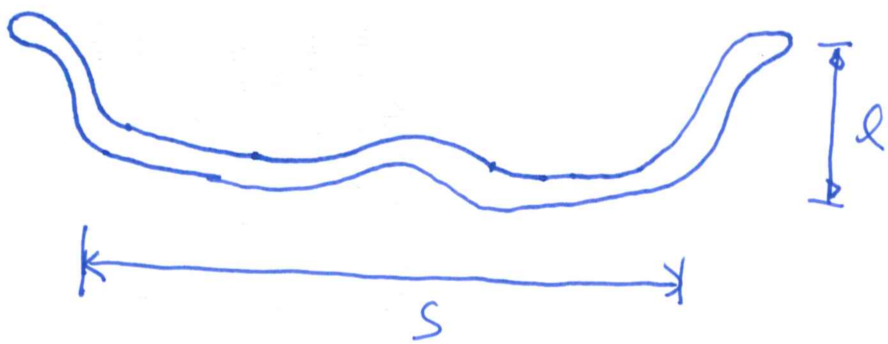
6) Putting it all together

$$M = 2\left(\frac{s}{2}\right)F = 2\left(\frac{s}{2}\right)\left(\frac{T}{l}\right) = s\left(\frac{K_{bar}\,\theta_{bar}}{l}\right)$$

$$M = s\left(\frac{K_{bar}}{l}\right)\left(\frac{s}{l}\right)\phi = \frac{K_{bar}\,s^2}{l^2}\,\phi$$

$$M = K_\phi^{stab}\,\phi$$

$$K_\phi^{stab} = \frac{K_{bar}\,s^2}{l^2} = \frac{JGs}{l^2}$$



- Bar is not straight like our analysis (due to "packaging")
- We assumed only deflection was in the component that runs laterally under the car (i.e. the s-length)
- Therefore the true stiffness ~~may~~ will be less

$$K_\phi^{true} \approx 0.9 K_\phi^{calc}$$

- Use FE to get stiffness of complex shape