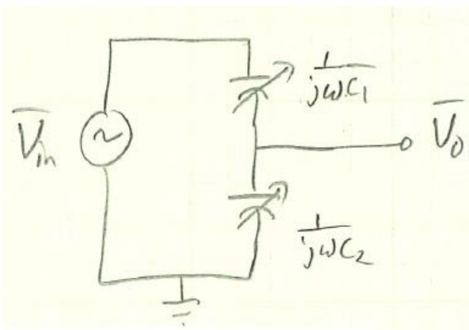
Lecture 9/20/21

Capacitive Sensing

1. Capacitor Interface Circuity: AC Voltage Division: Techniques to recover the amplitude

From last time:

$C_1 = \frac{\varepsilon_o \varepsilon_r A}{d_1} = \frac{\varepsilon_o \varepsilon_r A}{d_o + x(t)}$ and $C_2 = \frac{\varepsilon_o \varepsilon_r A}{d_2} = \frac{\varepsilon_o \varepsilon_r A}{d_o - x(t)}$



$$\frac{\bar{V}_o}{\bar{V}_{in}} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1}{C_2 + C_1}$$ → An AC voltage divider

Let $\bar{V}_{in} = V_1 \sin(\omega t)$

Therefore: $\bar{V}_o = V_1 \sin(\omega t) \left[\frac{C_1}{C_2 + C_1}\right]$ → select $\omega >> \omega_{MEMS}$

Which could have this form: $\bar{V}_o(t) = V_1 \sin(\omega t) \left[0.5\left(1 - \frac{x(t)}{d_o}\right)\right]$

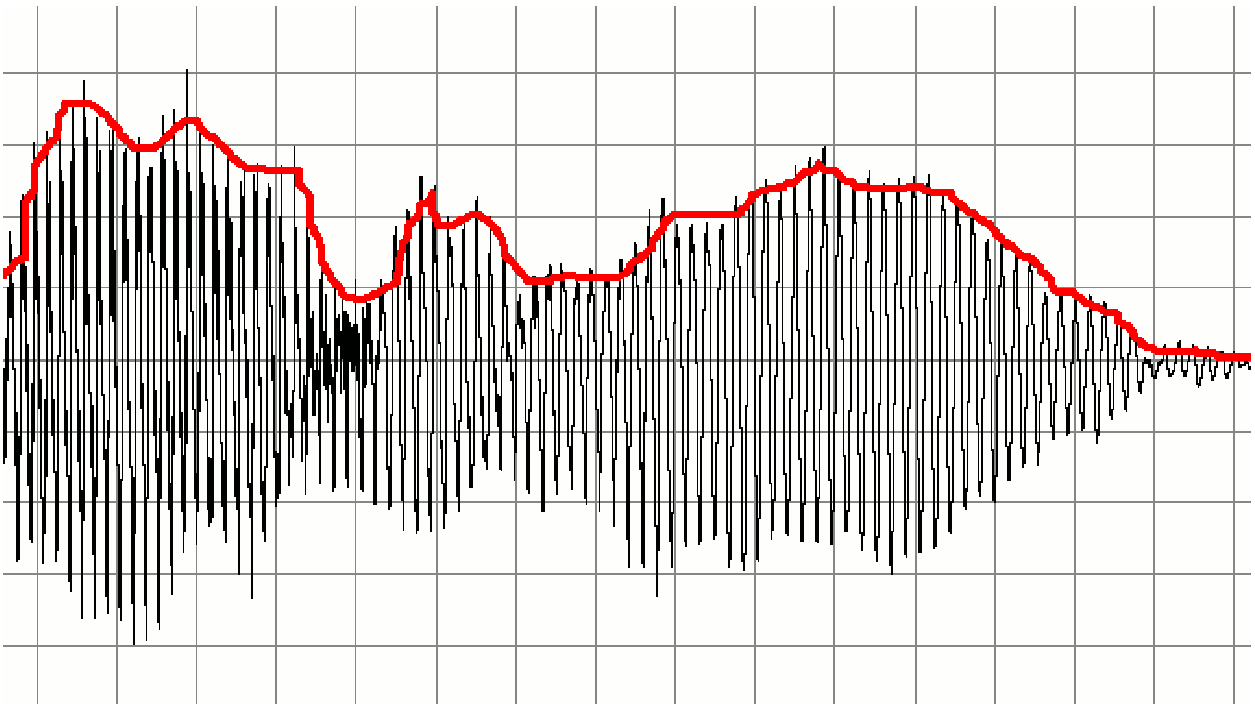
The amplitude of $\bar{V}_o(t)$ is a linear function of x(t).

So, how do we recover the amplitude of $\bar{V}_o(t)$, V_1?

It is desirable to produce a DC voltage proportional to V_1.

a. Envelope Detection

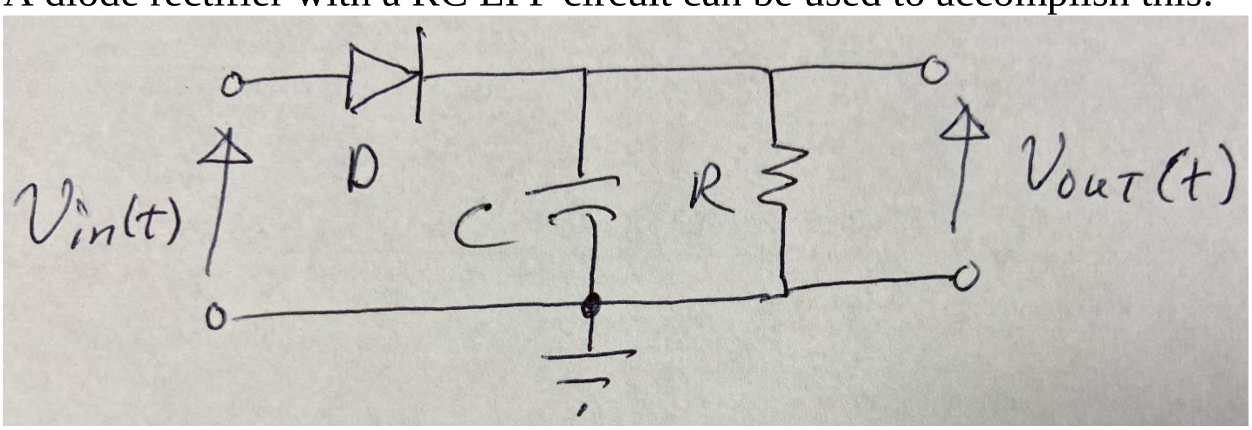
An envelope detector is a diode circuit that recovers the demodulated envelope of an AM modulated signal.



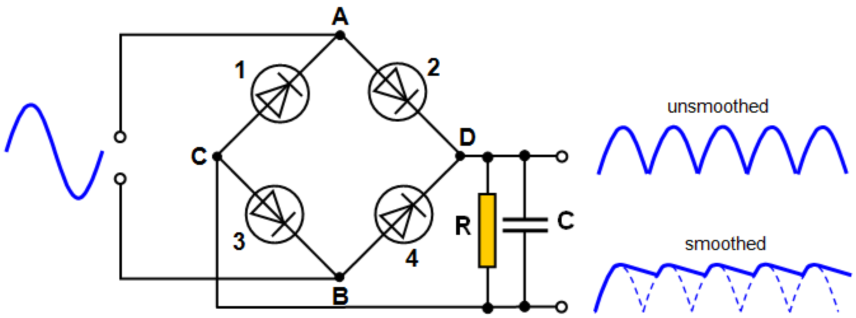
Curtesy: https://en.wikipedia.org/wiki/Envelope_detector#/media/File:C_Envelope_follower.png

The black waveform is the AM signal. The red waveform is the envelope or message we wish to recover.

A diode rectifier with a RC LPF circuit can be used to accomplish this:



More advanced rectifier circuits, such as using a four diode full wave bridge rectifier, or using an op amp based "super diode" could also be used for envelope detectors:



Curtesy: https://www.schoolphysics.co.uk/age16-19/Electronics/Semiconductors/text/Rectification_/index.html

b. Synchronous Demodulator (Lock-in Amplifier)

Consider this:

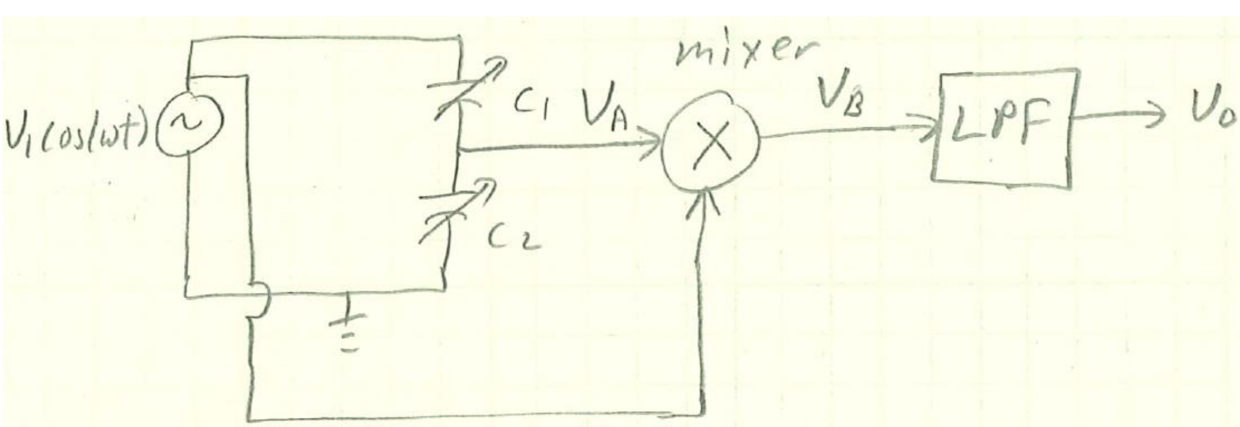
$$Acos(\omega t) \times Bcos(\omega t) = 0.5AB[\cos(\omega t - \omega t) + \cos(\omega t + \omega t)]$$

$$= 0.5AB[\cos(0) + \cos(2\omega t)]$$

$$= 0.5AB + 0.5ABcos(2\omega t)$$

↑ DC term ↑ AC term at 2ωt

So let's connect our differential capacitive sensor into a new circuit:



$V_A = V_1 \cos(\omega t)\left[0.5\left(1 - \frac{x(t)}{d_o}\right)\right]$ → same form as with the AC voltage divider

$V_B = V_A V_1 \cos(\omega t)$

$= 0.5\, V_1^2 \left[0.5\left(1 - \frac{x(t)}{d_o}\right)\right] + 0.5\, V_1^2 \cos(2\omega t)\left[0.5\left(1 - \frac{x(t)}{d_o}\right)\right]$

↑ DC term ↑ AC term at 2ωt

The LPF attenuates the AC term so that:

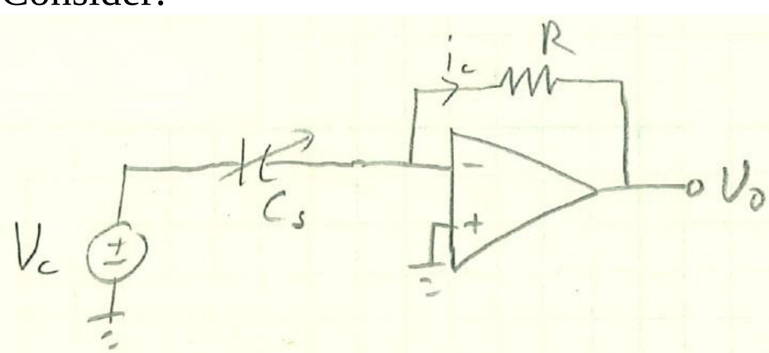
$$V_o \approx 0.5\, V_1^2 \left[0.5\left(1 - \frac{x(t)}{d_o}\right)\right] = V_c\left(1 - \frac{x(t)}{d_o}\right)$$

where V_c is a constant: $V_c = 0.5\, V_1^2 (0.5)$

Now, V_o is a DC voltage that is a linear function of x(t).

2. Capacitor Interface Circuity: Transimpedance amplifier (TIA)

Consider:



Note: some op amps are not stable with the input tied to a capacitor, and the output will break into high frequency oscillation.

But assuming the op amp configuration is stable: $v_o = -i_c R$

Note: for this inverting amplifier circuit, since the input is a current and the output is a voltage, the gain is a resistance with units of Ω.

For a fixed capacitor: $i_c = C\frac{dv_c}{dt}$. Note: as a differentiator of v_c, the TIA is noisy: it amplifies high frequency noise.

However, the more general case is: $i_c = C\frac{dv_c}{dt} + v_c\frac{dC}{dt} = C\frac{dv_c}{dt} + v_c\frac{\partial C}{\partial x}\frac{dx}{dt}$

a. If v_c is a constant, V_c, then $i_c = V_c\frac{\partial C}{\partial x}\frac{dx}{dt}$

If the capacitor's electrodes are is relative motion, then $\frac{dx}{dt}$ is a velocity term. In steady state, time varying $C_s(t)$ pumps i_c into the circuit.

b. If $V_c = V_A\sin(\omega t)$ and $\omega \gg \omega_{MEMS}$,

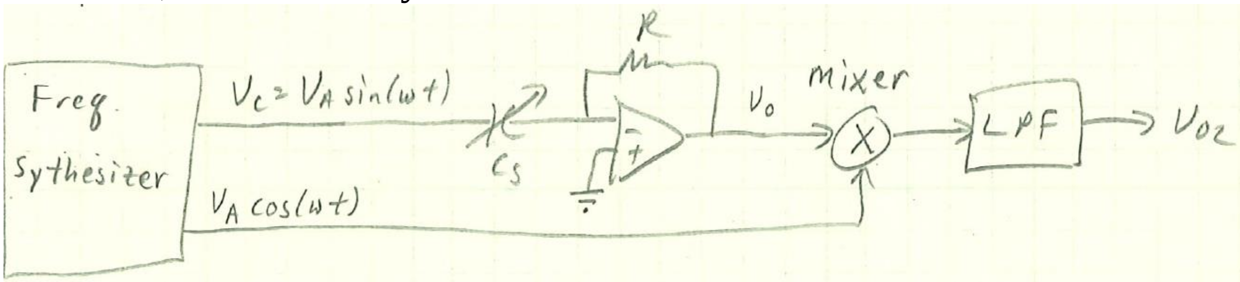
then for short time periods of several V_c cycles, C_s is nearly constant and

$v_c\frac{dC_s}{dt} \approx 0$ (i.e. very small change during the measurement time)

So: $i_c \approx C_s\frac{dV_c}{dt} = C_s V_A \omega\cos(\omega t)$

And finally: $v_o \approx -C_s R V_A \omega\cos(\omega t)$ for quick measurements of C_s.

However, we can add synchronous demodulation here too:



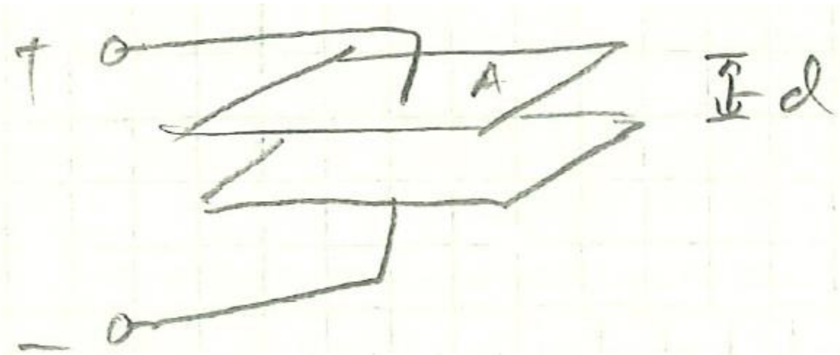
$\therefore V_{o2} = LPF[V_o \times V_A \cos(\omega t)] = -0.5 C_s R V_A^2 \omega = k C_s$

where k is a constant: $k = -0.5 R V_A^2 \omega$

So once again, V_{o2} is a DC voltage proportional to C_s, our sensor's capacitance.

Capacitive Fringing Field Sensors

Consider:

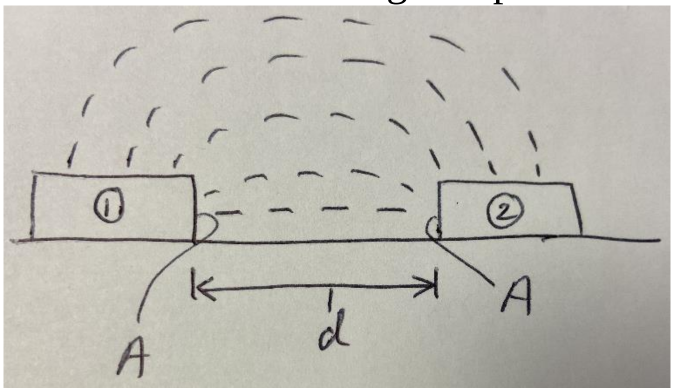


For $A \gg d^2$: $C \approx \frac{\varepsilon_o \varepsilon_r A}{d}$, and fringing effects are small,

But A ≈ d^2 or if $d^2 > A$, $C \neq \frac{\varepsilon_o \varepsilon_r A}{d}$.

Actually now, $C > \frac{\varepsilon_o \varepsilon_r A}{d}$ due to fringing effects.

Consider the case using two planar electrodes:

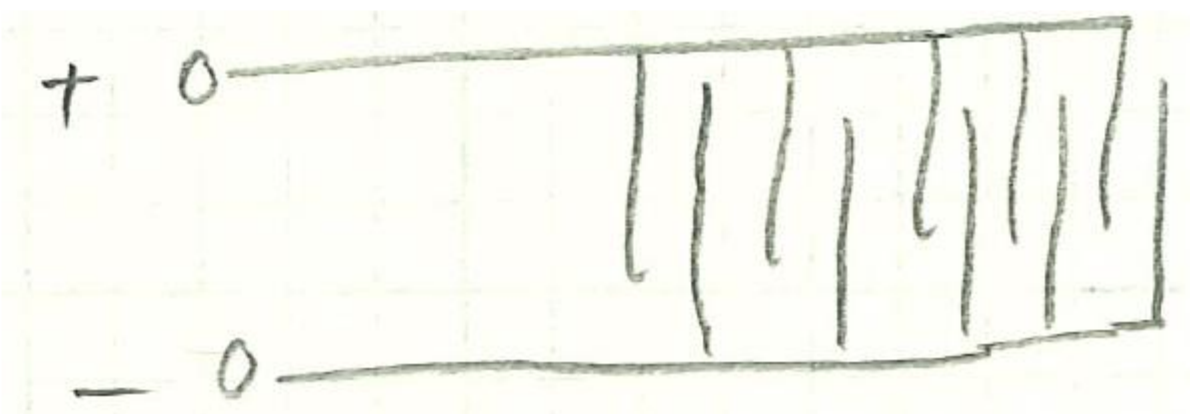


1 and 2 are the electrodes, d is the distance between them, and A is the area of each electrode facing each other. The dashed lines represent electric flux lines.

The capacitance between 1 and 2 can be modeled by:

$C \approx \frac{\varepsilon_o \varepsilon_r A \gamma}{d}$, where γ is a fringing scale factor and γ > 1.

Often, the two electrodes are arranged in an interdigitated electrode (IDE) layout on a planar surface, realizing capacitive fringing field sensor:



Let n = number of interdigitated fingers.

$$C \approx \frac{(n-1)\varepsilon_o \varepsilon_r A \gamma}{d}$$

Sometimes, the IDE is coated with a thin insulating layer, such as polyimide (PCB technology) or silicon dioxide (MEMS technology).

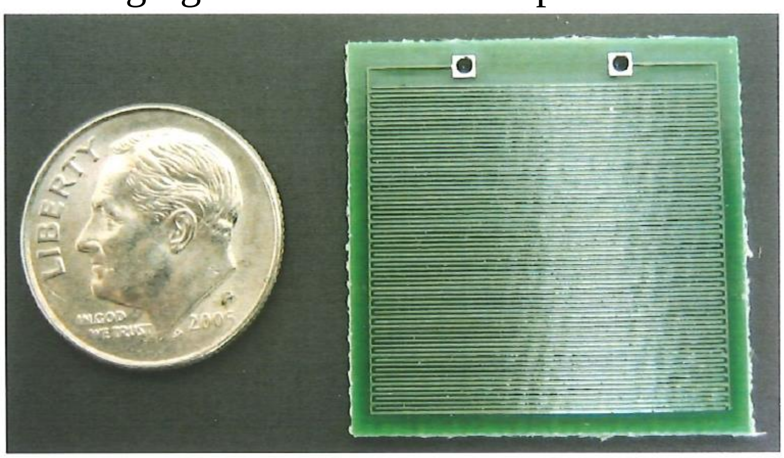
When the electrode width is equal to d, the sensing range above the sensor is approximately 1.25d to 1.5d.

Applications for capacitive fringing field sensors:

1. Detecting the presence of liquid water ($\varepsilon_r|_{air} \approx 1$ and $\varepsilon_r|_{water} \approx 80$ at room temperature)
2. Measuring the moisture content of many materials
3. Measuring the level of water and other liquids
4. Detecting ice: above ~ 10 KHz, $\varepsilon_r|_{water} >> \varepsilon_r|_{ice}$
5. Measuring relative humidity

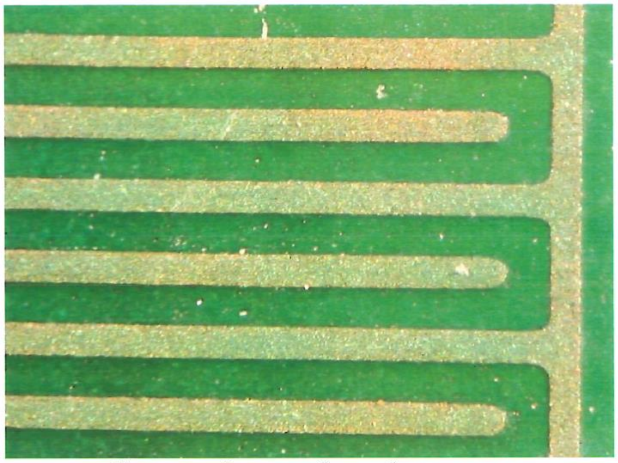
Meso scale versions, such as in PCB technology, can have relatively large capacitances → 100s of pF.

PCB Capacitive Fringing Field Sensor Example:

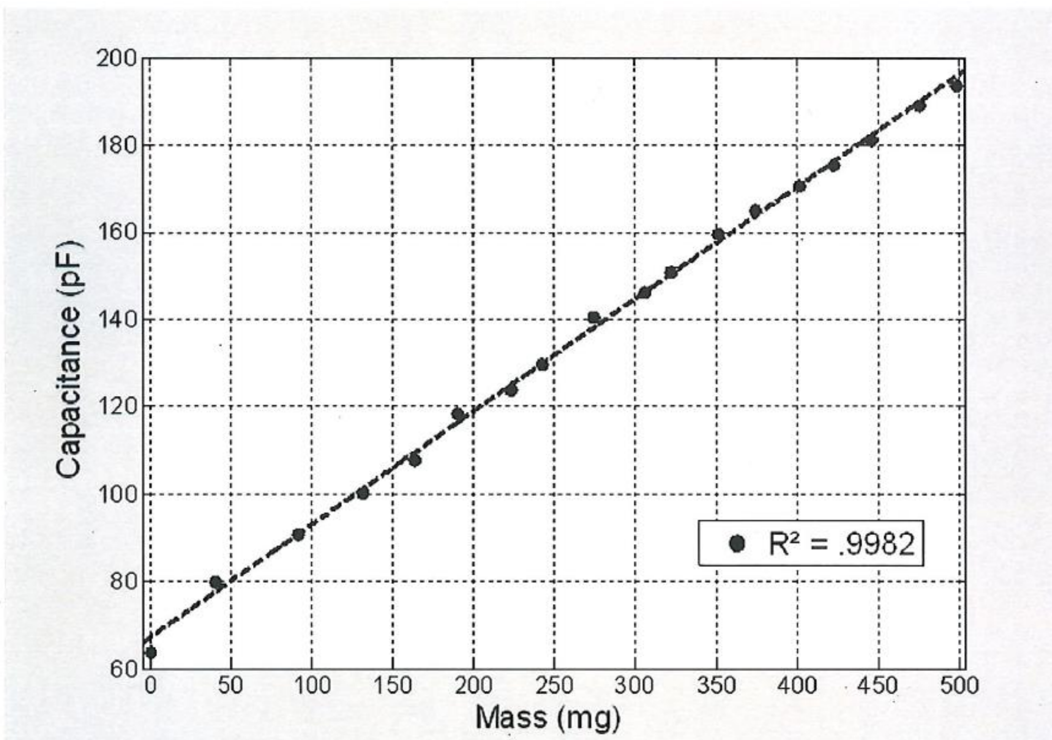


Photo

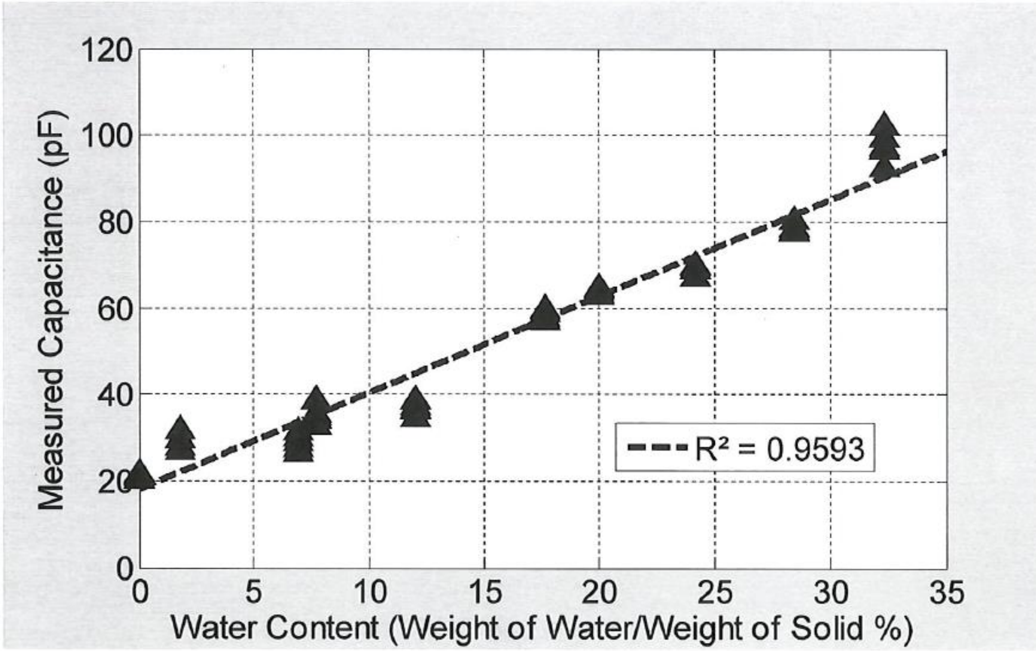
Device: 25.4 mm x 25.4 mm
70 interdigitated fingers (~150µm wide)
22.4 mm electrode overlap
63.9pF capacitance in air
321.3pF capacitance when submerged in water



Close up photo or electrode structure



Mass of water drop sensor response



Measured moisture content of soil