Simplified Circuit Analysis of BJT Colpitts and Hartley Oscillators

a. Colpitts Oscillator

Notice that the biasing resistors are not shown in this simplistic circuit.

Simplistic Small signal model:

Let’s further simplify by assuming that $r_\pi$ is so large that we can approximate $r_\pi$ as an open circuit:
Here, $V_{\pi} = V_o$

To evaluate the conditions for which the circuit will oscillate, one approach is to break the circuit at the $v_{\pi}$ negative terminal, and evaluate $T(s)$ for the condition that $T(j\omega) = 1|0^\circ$, modelled as a positive feedback oscillator.

Therefore define the circuit as:

\[
T(s) = \frac{V_{out}}{V_{in}}(s) = \frac{V_o}{V_s}(s)
\]
\[ V_o \left( sC_1 + \frac{1}{sL} \right) - V_1 \left( \frac{1}{sL} \right) = 0 \]

\[ V_o (s^2LC_1 + 1) = V_1 \quad (1) \]

\[ V_1 \left( \frac{1}{R} + sC_2 + \frac{1}{sL} \right) - V_o \left( \frac{1}{sL} \right) = -g_mV_\pi = -g_mV_s \]

\[ V_1 \left( \frac{sL}{R} + s^2LC_2 + 1 \right) - V_o = -g_m sLV_s \quad (2) \]

(1)\(\rightarrow\)(2)

\[ V_o (s^2LC_1 + 1) \left( \frac{sL}{R} + s^2LC_2 + 1 \right) - V_o = -g_m sLV_s \]

Then calculate \(T(s)\) and \(T(j\omega)\)

For oscillation: find the constraints so that \(T(j\omega) = 1|0^\circ\).

Obviously, there are \(s^4\) terms, making this a \(4^{th}\) order system.

Much arithmetic will be required to obtain \(T(j\omega) = 1|0^\circ\).

Another approach may be simpler.

From this tank circuit configuration, we know that:

\[ \omega = \frac{\sqrt{C_1 + C_2}}{\sqrt{LC_1 C_2}} \]
For oscillation, the energy supplied by $g_m V_o$ must be equal to or greater than the energy dissipated by $R$:

$$E|_{g_m V_o} \geq E|_R$$

$$V_o \left( s C_1 + \frac{1}{s L} \right) - V_1 \left( \frac{1}{s L} \right) = 0$$

$$V_o (s^2 L C_1 + 1) = V_1$$

$$E|_{g_m V_o} = (-g_m V_o) V_1 t$$

$$E|_{g_m V_o} = -\frac{g_m V_1^2 t}{1 + s^2 L C_1}$$

$$E|_{g_m V_o} = -\frac{g_m V_1^2 t}{1 - \omega^2 L C_1}$$

$$E|_{g_m V_o} = -\frac{g_m V_1^2 t}{1 - \frac{C_1 + C_2}{C_2}}$$

$$E|_{g_m V_o} = -\frac{g_m V_1^2 t}{\frac{C_1}{C_2}}$$

$$E|_{g_m V_o} = \frac{g_m V_1^2 t C_2}{C_1}$$

$$E|_R = \frac{V_1^2}{R} t$$

Therefore:

$$\frac{g_m V_1^2 t C_2}{C_1} \geq \frac{V_1^2}{R} t$$
\[ \frac{g_m C_2}{C_1} \geq \frac{1}{R} \]
\[ g_m R \geq \frac{c_1}{c_2} \quad \text{This is the condition for oscillation.} \]

b. Hartley Oscillator

Simplified small signal model:

Here, the tank circuit has \( \omega = \frac{1}{\sqrt{C(L_1+L_2)}} \)

\[ V_o \left( \frac{1}{sL_1} + sC \right) - V_1(sC) = 0 \]
\[ V_o(1 + s^2LC_1) = V_1s^2LC_1 \]
Using the energy balance relationship:

\[ E|_{g_mV_o} \geq E|_R \]

\[ E|_{g_mV_o} = (-g_mV_o)V_1 t \]

\[ E|_{g_mV_o} = -\frac{g_mV_1^2 t s^2 C L_1}{1 + s^2 C L_1} \]

\[ E|_{g_mV_o} = \frac{g_mV_1^2 t \omega^2 C L_1}{1 - \omega^2 C L_1} \]

\[ E|_{g_mV_o} = \frac{(g_mV_1^2 t) L_1}{L_1 + L_2} \]

\[ E|_{g_mV_o} = \frac{(g_mV_1^2 t) L_1}{L_1 + L_2 - L_1} \]

\[ E|_{g_mV_o} = \frac{(g_mV_1^2 t) L_1}{L_2} \]

\[ E|_R = \frac{V_1^2}{R} t \]

\[ \frac{(g_mV_1^2 t) L_1}{L_2} \geq \frac{V_1^2}{R} t \]

\[ \frac{g_mL_1}{L_2} \geq \frac{1}{R} \]

\[ g_mR \geq \frac{L_2}{L_1} \]

This is the condition for oscillation.

Consider a “real circuit” for a Colpits oscillator
Imagine the complexity in analyzing this circuit…
Other Transistor Oscillators

a. Phase Shift Oscillator

This circuit uses a common emitter (large negative gain) amplifier. The oscillator uses negative feedback to satisfy the BSC: $1|-180^\circ$.

$C_{BP}$ is a large capacitance bypass capacitor. $C_C$ is a large capacitance coupling capacitor. $R_{B1}$, $R_{B2}$, $R_C$ and $R_E$ bias the transistor and determine the amplifier’s gain.

b. Wien Bridge Oscillator

This circuit uses a two stage amplifier (two negative gain common emitter amplifiers) to achieve a positive gain for the Wein bridge oscillator.
c. Armstrong (1912 – U.S) or Meissner (1913 - Austria) Oscillator

Without biasing resistors shown. The transformer provides feedback from the output of the common emitter amplifier back around to the base. A transformer can easily provide a 180° phase shift. Transformer’s inductance with C determines the oscillation frequency.

Same oscillator with possible biasing resistors shown.
d. Oscillators Using Crystal and Ceramic Resonators

Some limiting factors in LC oscillators include:

1. The size and weight of inductors (particularly at low frequencies)
2. Resistive losses in inductors (low inductor Q)
3. Inductive accuracy of inductors
4. Long term stability

Piezoelectric crystal and ceramic resonators are therefore used to improve these issues.

Quartz is a piezoelectric material. Piezoelectric materials experience a small dimensional change in response to an applied voltage. Also, they generate an electric charge in response to an applied mechanical stress. Some ceramics also exhibit piezoelectric properties.

Schematic symbol for piezoelectric crystal and ceramic resonators:

![Schematic symbol](image)

Overall equivalent circuit:

![Overall equivalent circuit](image)

\( C_1 \): equivalent capacitance, motional capacitance

\( L_1 \): equivalent inductance, motional inductance

\( R_1 \): equivalent resistance, series resistance
Co: inner electrode capacitance, holder or shunt capacitance

The resonator has an impedance $Z = R + jX$

- $f_r$ is the resonant frequency: $f_r = \frac{1}{2\pi\sqrt{L_1C_1}}$

- $f_a$ is the anti-resonant frequency: $f_a = f_r\sqrt{1 + C_1 + C_0}$

- $Q_m$ is the mechanical quality factor: $Q_m = \frac{1}{2\pi F_r C_1 R_1}$

In between $f_r$ and $f_a$, $Z$ is normally inductive. See example on next page.

In between $f_r$ and $f_a$, the resonator can be modelled as:

\[
\begin{align*}
R_E & \quad \text{is the effective resistance over this frequency range.} \\
L_E & \quad \text{is the effective inductance over this frequency range.} \\
Z & = R_E + j\omega L_E
\end{align*}
\]

Example $L_1$, $C_1$, $R_1$, $C_0$ and $Q_m$ values for some ceramic and crystal resonators <below>.

Note: higher oscillation modes (overtones) do exist due to the mechanical resonance. They only occur at odd multiples of the fundamental frequency ($3^{rd}$, $5^{th}$, etc.). Crystals specified for frequencies over 30 MHz often use overtones. Your oscillator circuit must therefore be designed appropriately if using an overtone mode.
Figure 1.5) Impedance and Phase Characteristics for Ceramic Resonators

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>CERAMIC RESONATOR</th>
<th>CRYSTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>455kHz</td>
<td>2.50MHz</td>
</tr>
<tr>
<td>$L_1$ ($\mu F$)</td>
<td>$8.8 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
</tr>
<tr>
<td>$C_1$ (pF)</td>
<td>14.5</td>
<td>4.7</td>
</tr>
<tr>
<td>$C_0$ (pF)</td>
<td>256.3</td>
<td>33.3</td>
</tr>
<tr>
<td>$R_1$ (Ω)</td>
<td>9.0</td>
<td>17.6</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>2734</td>
<td>912</td>
</tr>
<tr>
<td>$\Delta f$ (kHz)</td>
<td>12</td>
<td>147</td>
</tr>
</tbody>
</table>

Courtesy ECE International, Inc.
Crystal and ceramic resonators can be used in linear or square wave oscillators. In a linear oscillator, they are used to replace the discrete inductor. Example linear oscillator circuits:

Pierce oscillator circuit.

The Pierce oscillator is a derivative of the Colpitts oscillator.

Clapp oscillator circuit with collector grounded.

The Clapp oscillator is also a derivative of the Colpitts oscillator, except that a 3rd capacitor, $C_3$, is used in series with the inductor (inherent in the resonator).

\[ \omega_o = \sqrt{\frac{1}{L \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)}} \]
Clapp oscillator circuit with base grounded.

With each of these circuits, the crystal should be modelled with $C_1$, $L_1$, $R_1$ and $C_0$ included.

These oscillator circuits are referred to as aperiodic oscillators, because they will occasionally operate in an overtone (higher frequency) mode. $C_2$ can be replaced with a resonant circuit to prevent this condition from occurring.

Note, these oscillators do not have AGC circuits. If the gain is too high, you may get a square wave output instead of a sinusoidal output.
Example crystal oscillator circuit using a common emitter amplifier in a Colpitts configuration.