Automatic Gain Control

As previously shown, $\ddot{x} + (x^2 - 1)\dot{x} + x = 0$ is a nonlinear system that produces a limit cycle with a distorted sinusoid for $x(t)$, which is not a very good sinusoidal oscillator. A better gain adjustment would be to adjust the damping to zero as a function of the amplitude of $x(t)$. The subsystem that accomplishes this is called Automatic Gain Control or AGC.

A simple AGC could consist of rectifying $x(t)$ using a diode circuit, then lowpass filtering the rectified signal, then comparing the filtered signal with a desired amplitude, and adjusting the gain using a four quadrant multiplier.

Here is a simplified version of that using Simulink with abs($x$) used as an ideal fullwave rectifier circuit (next page).

A plot of $x(t)$ vs. time is presented. The system is still a complex nonlinear system, but the output ($x(t)$) is a better sinewave than in the previous system. The process of reaching steady state is complex, though. A phase plot of $x_d$ vs. $x$ is presented for $x$ and $x_d$ during the time span of 150s to 200s. Observe that this phase plot of $x_d$ vs. $x$ is pretty close to a perfect circle even through there is still sinusoidal fluctuation in the feedback control signal, filt(t).
Phase Plot: $x_d$ vs. $x$

$\mathbf{f}(t)$ vs. time
Phase plot of $x_d$ vs $x$ for approximately 150s to 200s.
Constraints on \( A(j\omega) \)

The simplest electronic oscillators are op amp based, and \( A(j\omega) \) is typically a simple op amp fixed gain amplifier, such as the negative gain and positive gain amplifiers shown below.

![Negative Gain Configuration](image1)

Negative Gain Configuration

\[
V_o = -V_i \frac{R_2}{R_1}
\]

![Positive Gain Configuration](image2)

Positive Gain Configuration
\[ V_o = V_i \left( 1 + \frac{R_2}{R_1} \right) \]

The gain equations above are for ideal op amp performance. However, op amps are non-ideal, and these non-idealities limit their performance in oscillator and other applications:

(1) Op amp frequency response.

The op amp is a complex circuit comprised of nonlinear active elements (primarily transistors), as well as passive elements (primarily capacitors). Op amps possess many poles. However, quality op amps are compensated and can be modelled by:

\[ A(j\omega) = \frac{A_{OL}}{1 + j\omega/a}, \]

where \( A_{OL} \) is the maximum open loop gain of the op amp, and \( \omega_a \) is due to the dominant pole of the compensated op amp and is the 3dB frequency. At \( \omega_a \), the op amp open loop gain has dropped by 3dB and has a -45° phase shift. At \( \omega_a \) and higher, the open loop gain decrease by approximately 20dB per decade. The unity gain bandwidth, \( \omega_t = A_{OL} x \omega_a \). Consider an op amp modelled with \( A_{OL} = 1 \times 10^8 \) V/V and \( \omega_a = 2\pi(10 \times 10^6) \) rad/s:

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As such, the gain bandwidth (GBW) product is necessary in selecting an appropriate op amp for a given oscillator circuit. Although varying somewhat with the actual amplifier configuration, the 3dB bandwidth (and the -45° phase shift) for a particular amplifier design is approximately proportional to the open loop 3dB bandwidth divided by the closed loop gain. The higher the required gain to achieve oscillation for a particular oscillator circuit, the lower the achievable oscillation frequency with that circuit. Since op amps will have at least a 45° phase shift at the BW limit for a particular gain, and probably a greater phase shift due to higher frequency poles, the useful frequency range for a particular gain is usually no more than 1/10 of the BW at that gain.
Magnitude of $A(j\omega)$ Vs. Frequency

Phase of $A(j\omega)$ Vs. Frequency
(2) Slew rate limit

Most op amps have optimal performance with small signals. Large amplitude signals can be affected by the slew rate limitation of the op amp at lower frequencies than small amplitude signals. Consider a sinusoid of the form:

\[ V(t) = V_p \sin(\omega t) \]

The maximum value of the derivative of \( V(t) \) will give us the minimum slew rate needed for the op amp. Therefore

\[ \dot{V}(t) = \omega V_p \cos(\omega t) \]

Which is maximum at \( \omega t = 0 \).

Therefore, the op amp slew rate should be greater than \( 2\pi V_p f \), where \( f \) is the oscillation frequency.

(3) Circuit considerations

Good circuit design techniques should be employed in all op amp circuits, including the use of decoupling capacitors, balancing the impedances between the two inputs, and restricting the size of feedback resistors to reasonable values. If resistances are too small, currents may be too large for the op amp to sufficiently source or sink. If resistances are too large, noise issues can result and/or the frequency response of the op amp circuit can be adversely affected. Use good grounding techniques: ground planes or star configurations for power and ground signals.

(4) Op amp phase response

The phase response of an op amp will certainly affect the oscillator circuit design. A nonlinear phase response is particularly problematic for chaotic oscillator circuits, which operate over a wide bandwidth compared to sinusoidal oscillator circuits.

These constraints limit the use of op amp based oscillators to low frequency operation, often below 1 MHz, although operation up to a few MHz may be
possible with high frequency op amps in low gain oscillator circuits. As a result, op amp based oscillator circuits requiring the lowest possible gain are desirable for the highest frequency operation. Also, in oscillator circuits that employ multiple op amps, it may be possible to distribute the required gain among several op amp stages, thereby reducing the gain that any one op amp subcircuit has to provide.

**Constraints on \( \beta(j\omega) \)**

The \( \beta(j\omega) \) subcircuit often consists of some passive elements (R, C and L) and occasionally op amps. The primary purpose of the \( \beta(j\omega) \) subcircuit is to provide a closed loop phase delay at a desired frequency to satisfy the Barkhausen stability criterion for oscillation. Therefore, both the magnitude response and the phase response for \( \beta(j\omega) \) subcircuits are important. Let’s consider some simple circuits that are sometimes used in \( \beta(j\omega) \) subcircuits:

(1) Passive one-pole RC lowpass filter

\[ G(j\omega) = \frac{V_o}{V_i} (j\omega) = \frac{1}{1 + j\omega RC} \]

\[ |G(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \]

\[ |G(j\omega)| = -\tan^{-1}(\omega RC) \]

Filter resonant frequency = \( \omega_r = \frac{1}{RC} \)

At \( \omega = \omega_r \):

\[ |G(j\omega_r)| = \frac{1}{\sqrt{2}} \]

And
\[ |G(j \omega_r) = -45^\circ | \]

(2) Passive one-pole RC highpass filter

\[
G(j \omega) = \frac{V_o}{V_i}(j \omega) = \frac{j \omega}{j \omega + \frac{1}{RC}}
\]

\[
|G(j \omega)| = \frac{\omega}{\sqrt{\left(\frac{1}{RC}\right)^2 + (\omega)^2}}
\]

\[ |G(j \omega) = 90^\circ - \tan^{-1}(\omega RC) \]

Filter resonant frequency = \( \omega_r = \frac{1}{RC} \)

At \( \omega = \omega_r \):

\[ |G(j \omega_r)| = \frac{1}{\sqrt{2}} \]

And

\[ |G(j \omega_r) = 45^\circ \]
(3) Simple RC BPF

\[ G(j\omega) = \frac{V_o}{V_i}(j\omega) = \frac{j\omega CR}{1 - (\omega CR)^2 + 2j\omega CR} \]

\[ |G(j\omega)| = \frac{\omega CR}{\sqrt{(1 - (\omega CR)^2)^2 + (2\omega CR)^2}} \]

\[ |G(j\omega)| = 90^\circ - \tan^{-1}\left(\frac{2\omega CR}{1 - (\omega CR)^2}\right) \]

Filter resonant frequency = \( \omega_r = \frac{1}{RC} \)

At \( \omega = \omega_r \):

\[ |G(j\omega_r)| = \frac{1}{2} \]

And

\[ |G(j\omega_r)| = 0^\circ \]