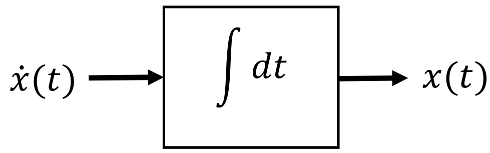
Thursday, 1/14/21

Simulating Dynamic Systems in MATLAB Simulink

Example: given $A\ddot{x} + B\dot{x} + Cx = f(t)$ (1) modelling a second order linear dynamic system

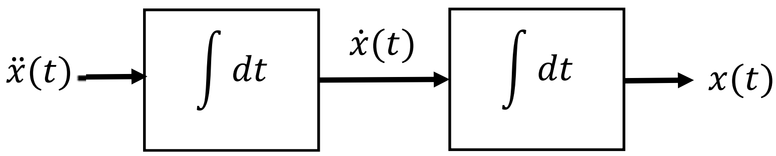
Realize that $x = \int \dot{x}\,dt$, which can be represented pictorially as:



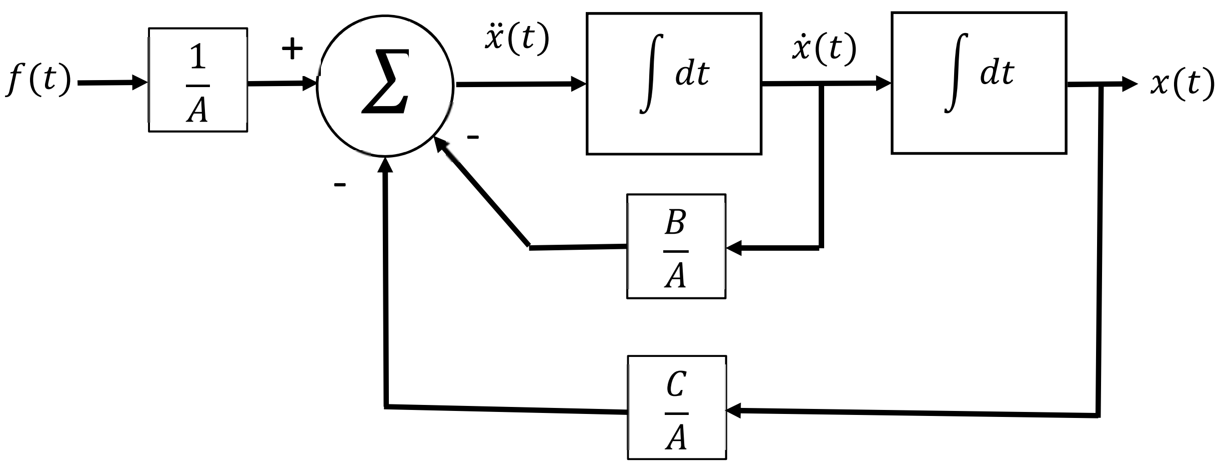
Rearrange (1) so that: $\ddot{x} = \frac{1}{A}f(t) - \frac{B}{A}\dot{x} - \frac{C}{A}x$ (2)

This form is very easy to implement in a simulation diagram using integrators

Begin with a chain or integrators representing all states:

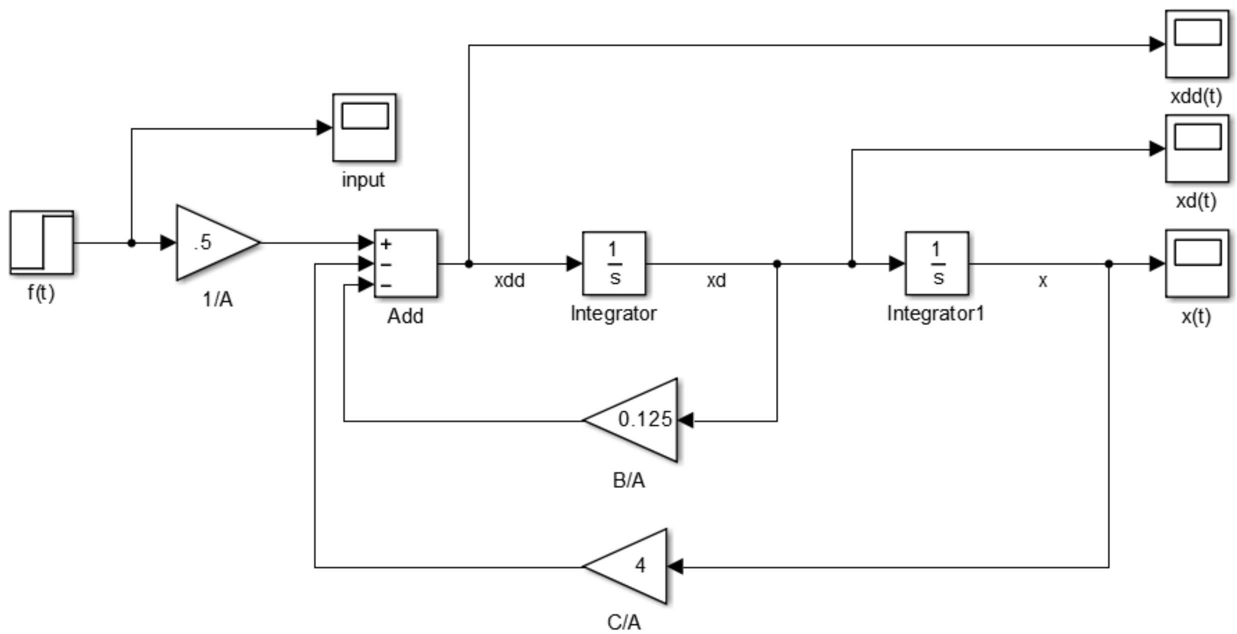


Then add a summing junction and feedback terms:

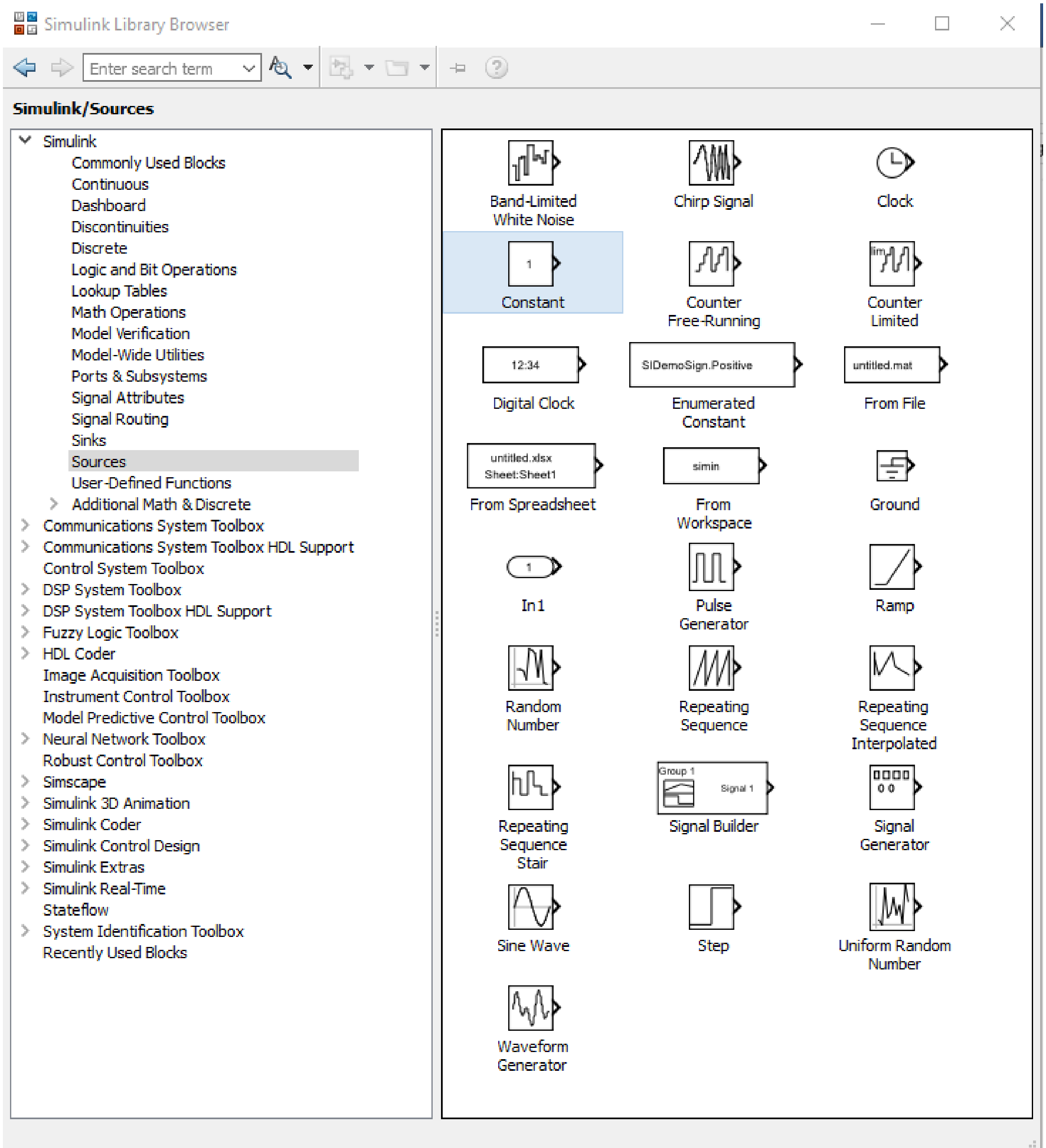


This can be built in MATLAB Simulink where the 1/s block is used for the integral block

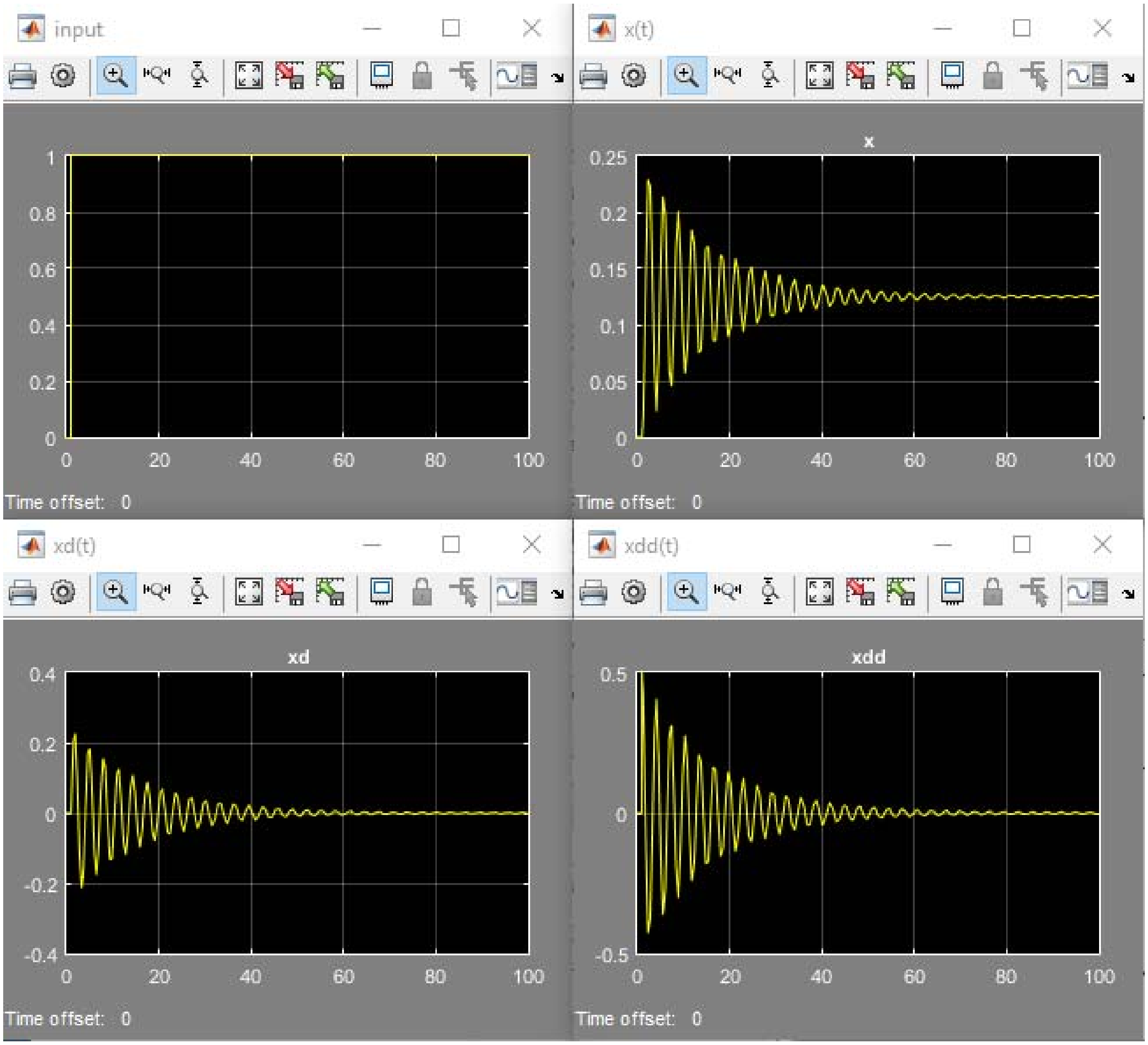
Example: $2\ddot{x} + 0.25\dot{x} + 8x = u(t-1)$



Simulink model of the simulation diagram

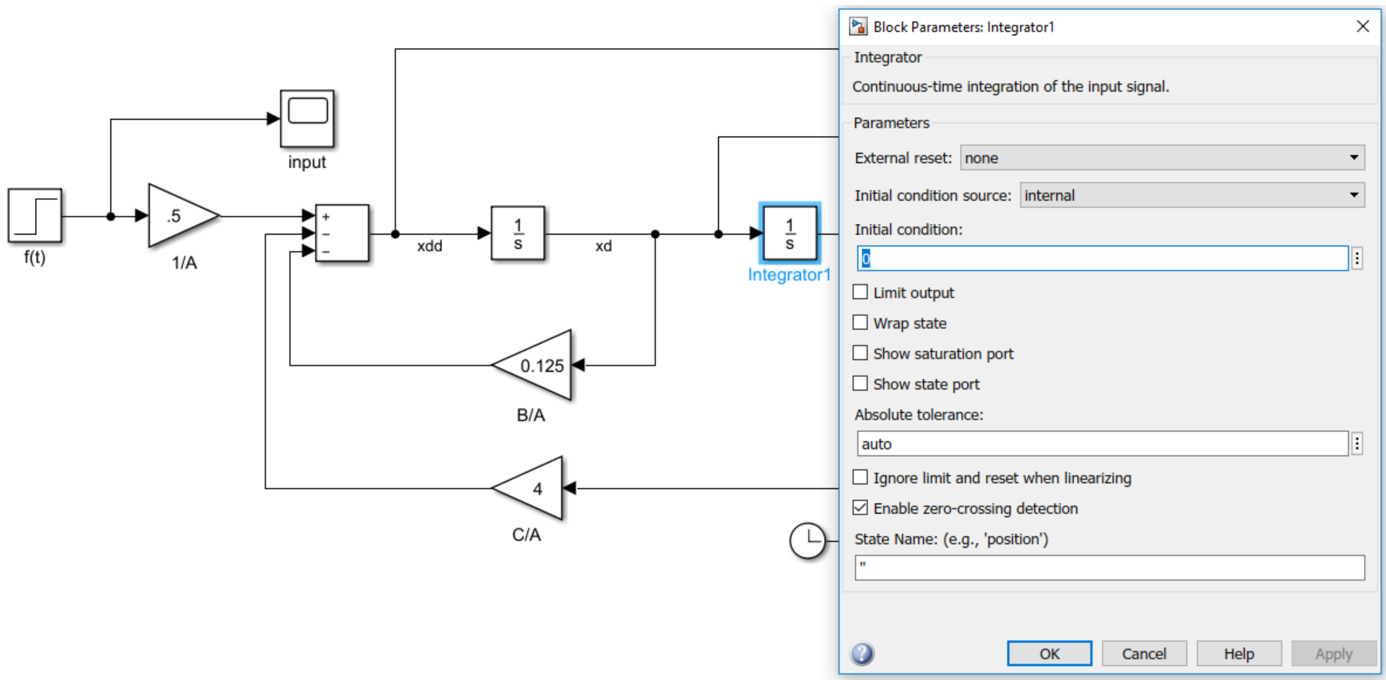


Simulink Library Browser

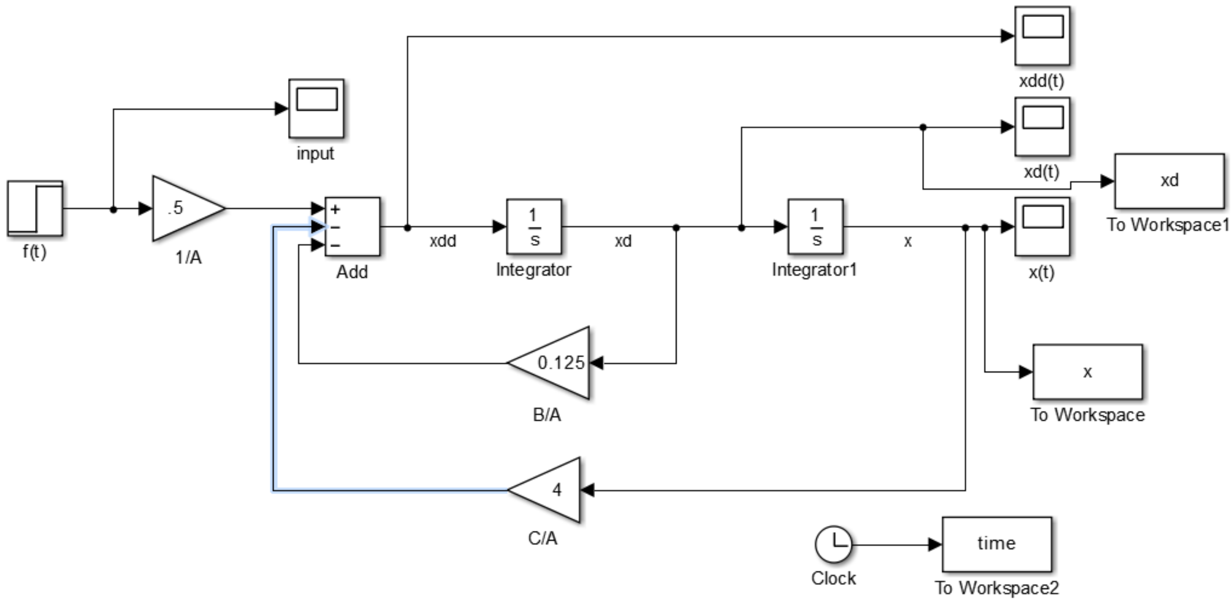

input
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
Time offset: 0
x(t)
x
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
Time offset: 0
xd(t)
xd
0.4
0.2
0
-0.2
-0.4
0
20
40
60
80
100
Time offset: 0
xdd(t)
xdd
0.5
0
-0.5
0
20
40
60
80
100
Time offset: 0

Setting Initial Conditions:

Click on the integrator block and select the initial condition for the signal output by that integrator:

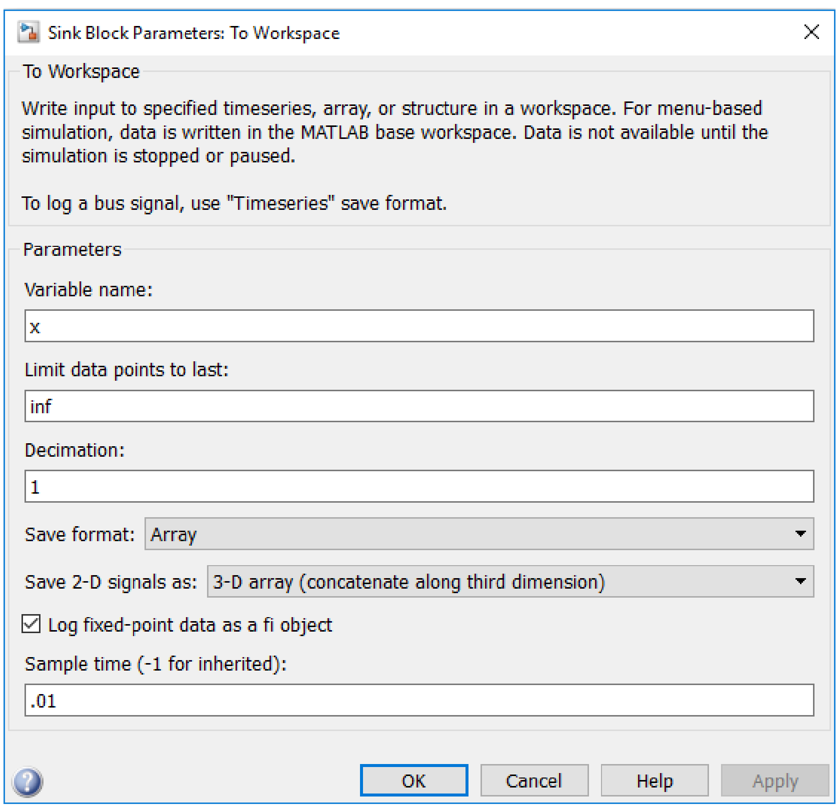


Bringing Simulink Data into the MATLAB Workspace



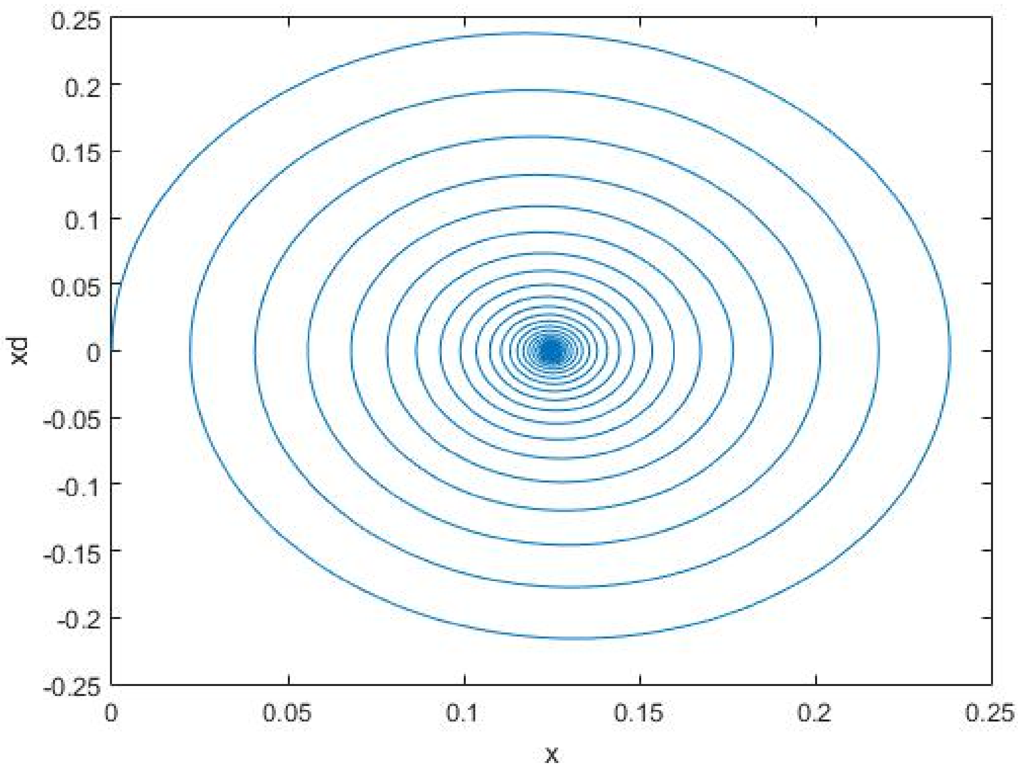
“To Workspace” blocks added to get data into Matlab Workspace

Setting the parameters for each “To Workspace” block:



Then after running the simulation, you can process the Simulink data in the workspace or via an m-file.

Example: plot(x,xd)



This is a plot of x-dot vs. x. It is called a phase plot and is very useful in analyzing dynamical systems.

<u>Hand out Homework 1</u>:

Review of Second Order Dynamic Systems

Consider systems of the form: $A\ddot{x} + B\dot{x} + Cx = f(t)$

Example: mechanical spring-mass-damper system: $m\ddot{x} + c\dot{x} + kx = f(t)$

Often convenient to analyze using Laplace Transforms: $ms^2X(s) + csX(s) + kX(s) = F(s)$

Then $X(s) = \frac{F(s)/m}{s^2+s\frac{c}{m}+\frac{k}{m}} = \frac{F(s)/m}{s^2+s2\zeta\omega_o+\omega_o^2} = \frac{F(s)/m}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$, where:

ω_o is the natural frequency

Q is the quality factor

ζ is the damping ratio

finally, $x(t) = L^{-1}\left[\frac{F(s)/m}{s^2+s\frac{c}{m}+\frac{k}{m}}\right]$

$\zeta = 0$ or $Q \to \infty$: undamped system

$0 < \zeta < 1$ or $Q \to \infty > Q > ½$: underdamped system

$\zeta = 1$ or $Q = ½$: critically damped system

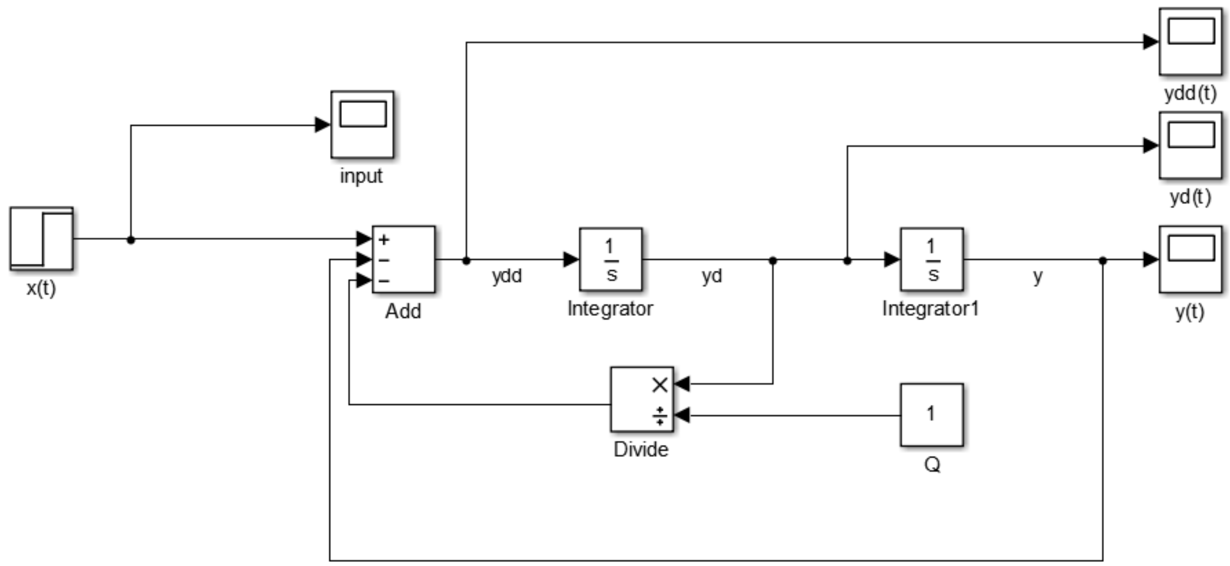
$\zeta > 1$ or $Q < ½$: overdamped system

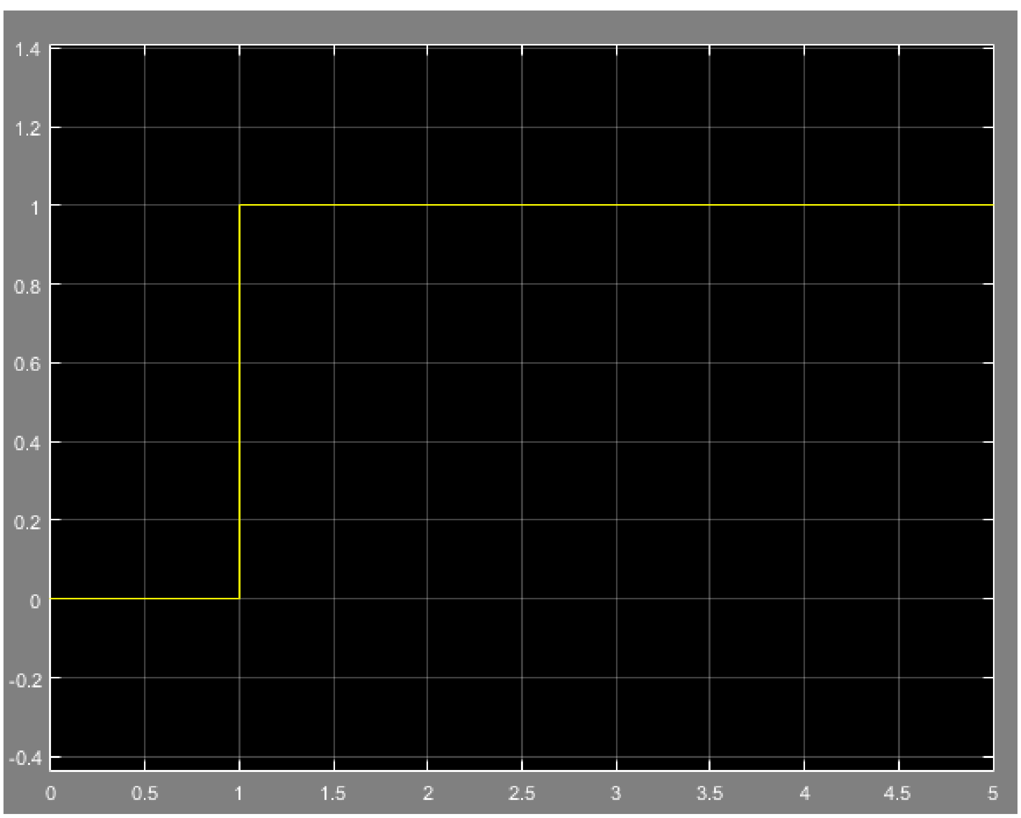
$\zeta = Q = 0.707$: maximally flat response (no resonant peak in the frequency domain)

Example. Consider this system with ω_o = 1 rad/s:

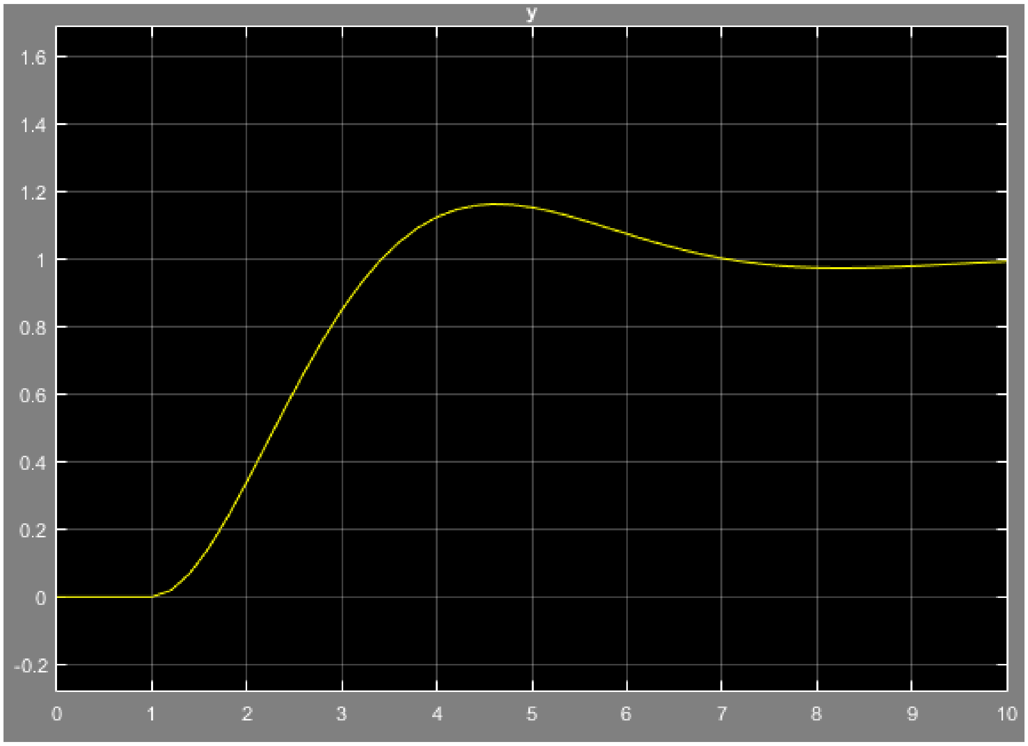
$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + s\frac{1}{Q} + 1}$$

Simulink model shown on next page with a step response:

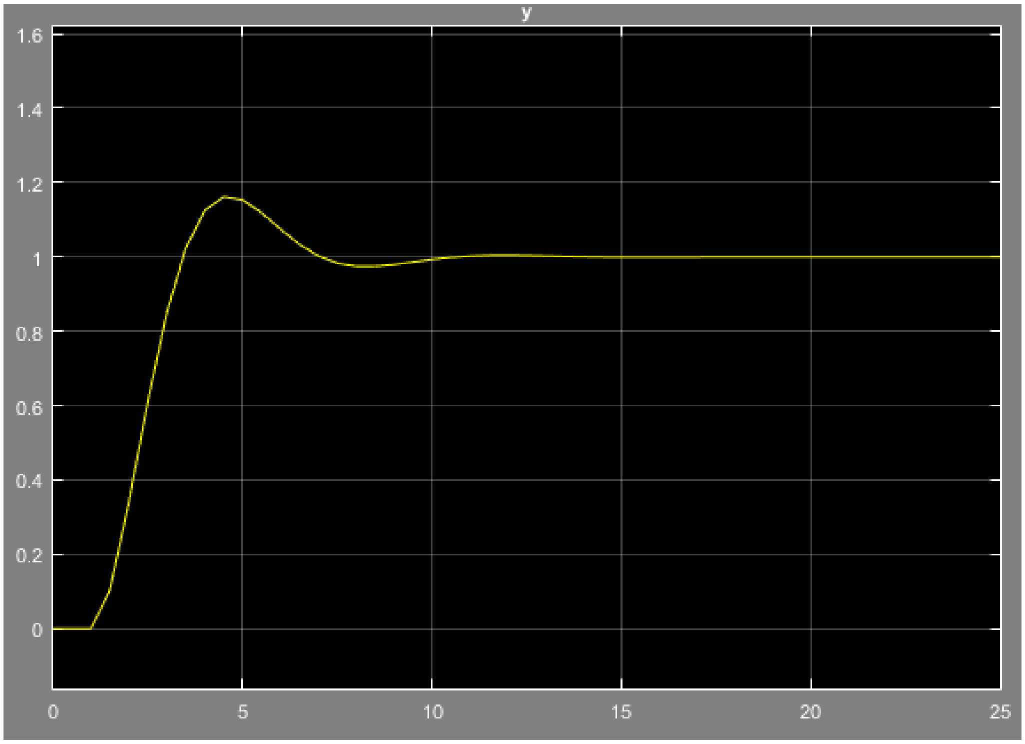

input
ydd(t)
yd(t)
y(t)
x(t)
+
−
−
Add
ydd
1
s
Integrator
yd
1
s
Integrator1
y
×
÷
Divide
1
Q


1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

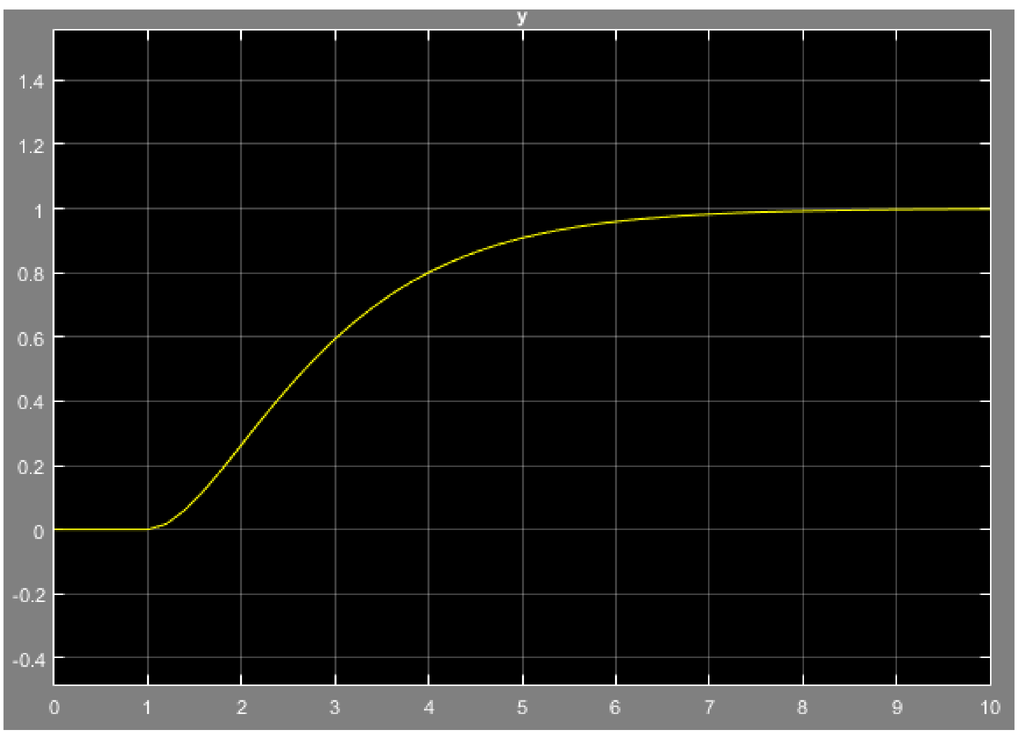
Input: x(t)



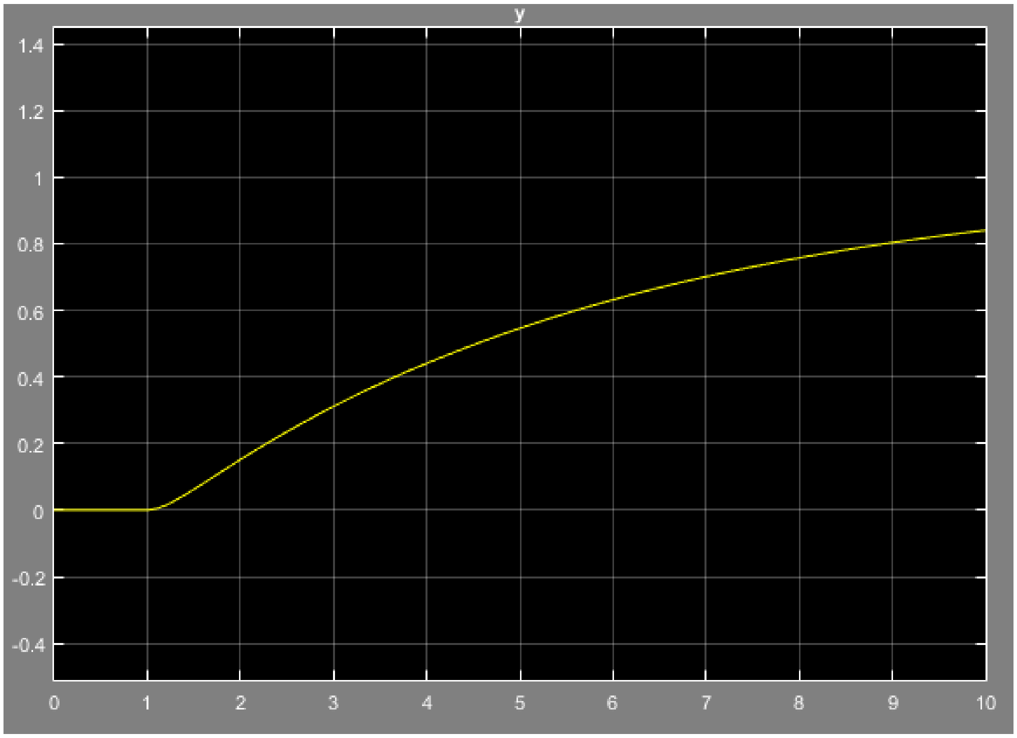
Output y(t) for Q = 1, underdamped. Observe the "ringing" at the resonant frequency.



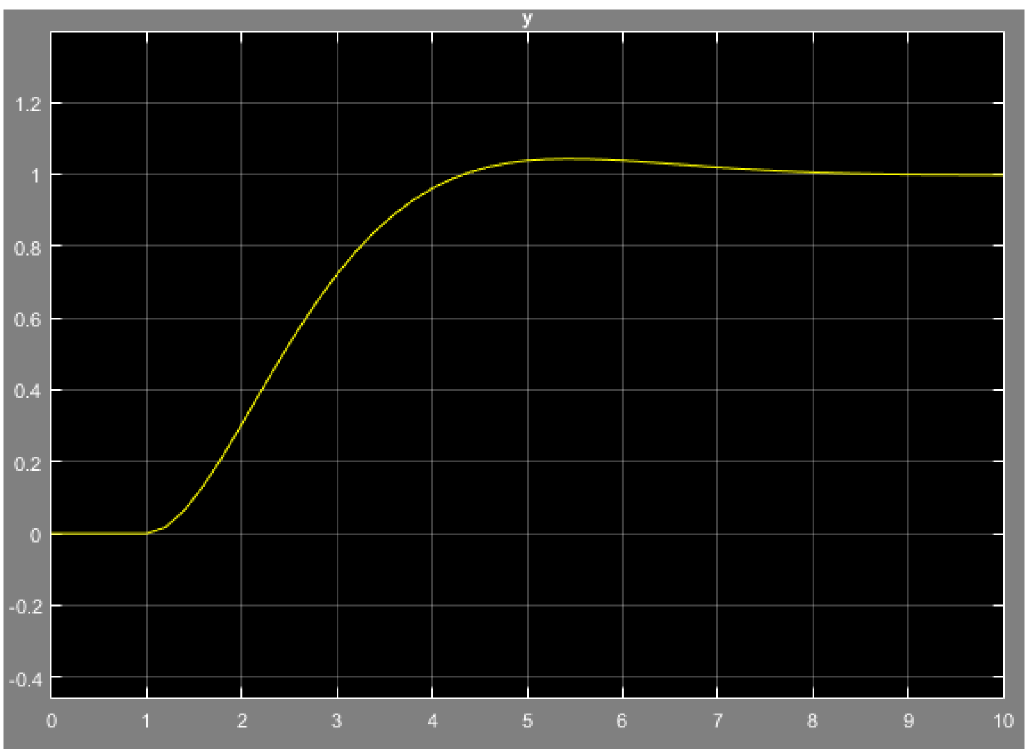
y(t) for Q = 1, but response run for 25s to observe decaying ringing.



y(t) for Q = 0.5, critically damped response.



y(t) for Q = 0.2, overdamped response.



y(t) for Q = 0.707, maximally flat response. Observe a slight overshoot with a reasonably fast response time.

State Variable Modelling

Example: $G(s) = \frac{Y(s)}{F(s)} = \frac{1}{ms^2+cs+k}$, a second order system

Obviously: $\ddot{y}(t) = \frac{1}{m}f(t) - \frac{k}{m}y(t) - \frac{c}{m}\dot{y}(t)$ (1)

Define the state variables:

Let $x_1(t) = y(t)$ and $x_2(t) = \dot{y}(t)$

Then:

$\dot{x}_1(t) = x_2(t)$ and $\dot{x}_2(t) = \ddot{y}(t)$

Therefore (1) becomes: $\dot{x}_2(t) = \frac{1}{m}f(t) - \frac{k}{m}x_1(t) - \frac{c}{m}x_2(\mathrm{t})$

Now the dynamical system can be represented in matrix form:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{c}{m}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}f(t)$$

$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

This is a very useful way for representing dynamical systems, and it is very applicable to numerical processing techniques. The general matrix form is:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

This is also applicable to higher order systems, systems described by multiple differential equations, and even nonlinear systems.

The state-space representation is a mathematic model of a physical system consisting of the input **u(t)**, output **y(t)** and state variables **x(t)** related by first order differential equations. The term “state space” refers to a dimensional space where the axes are the state variables. Therefore, the state of the modelled physical system can be represented as a vector within that space.