Cost Estimation Predictive Modeling: Regression versus Neural Network

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Abstract: Cost estimation generally involves predicting labor, material, utilities or other costs over time given a small subset of factual data on "cost drivers." Statistical models, usually of the regression form, have assisted with this projection. Artificial neural networks are non-parametric statistical estimators, and thus have potential for use in cost estimation modeling. This research examined the performance, stability and ease of cost estimation modeling using regression versus neural networks to develop cost estimating relationships (CERs). Results show that neural networks have advantages when dealing with data that does not adhere to the generally chosen low order polynomial forms, or data for which there is little *a priori* knowledge of the appropriate CER to select for regression modeling. However, in cases where an appropriate CER can be identified, regression models have significant advantages in terms of accuracy, variability, model creation and model examination. Both simulated and actual data sets are used for comparison.

1. Introduction

Cost estimation is a fundamental activity of many engineering and business decisions, and normally involves estimating the quantity of labor, materials, utilities, floor space, sales, overhead, time and other costs for a set series of time periods. These estimates are used typically as inputs to deterministic analysis methods, such as net present value or internal rate of return calculations, or as inputs to stochastic analysis methods, such as Monte Carlo simulation or decision tree analysis. They may also be used in less quantitative analysis, such as the analytic hierarchy process or ranking schemes. Unfortunately, as critical as this activity is, cost estimating must frequently be done without the benefit of perfectly sampled cost driver data or adequate sample sizes. Moreover, cost estimating is often performed for new products or processes, for which good quality historical data does not exist. Thus, the cost model must make the most of sparse, noisy and approximate information.

Least squares regression has been used to support many cost estimating decisions and recent citations from the literature include the following diverse applications: capital and operating cost equations in southwestern U.S. mining operations [3], software development costs [22], roads in rural parts of developing countries [14], equipment and tooling configurations in plastic molding [23, 24], query costs in data bases [39], maintenance scheduling in power plants [5], urban water supply projects [34], and design for manufacturability [13]. Undoubtedly, there are many more unpublished instances and a recent survey by Mason, et al. [20] showed that professional cost estimators regularly use regression to build their cost models.

There has also been some interest in applying newer computational techniques, such as fuzzy logic and artificial neural networks, to the field of cost estimation. Applying fuzzy techniques to cash flow analysis has been used successfully. Ward discussed using fuzzy composition to estimate NPV after specifying the membership functions for future cash flows [36], and Choobineh and Behrens compared interval mathematics and fuzzy approaches in cost estimation [4]. A drawback of the fuzzy approach is that the relationships are developed from qualitative information of the cost estimating problem, usually elicited from a knowledgeable person. Fuzzy relationships are not primarily empirical models like regression and neural networks.

Artificial neural networks are purely data driven models which through training iteratively transition from a random state to a final model. They do not depend on assumptions about functional form, probability distribution or smoothness, and have been proven to be universal approximators [8, 12]. While theoretically universal approximators, there are practical problems in neural network model construction and validation when dealing with stochastic relationships, or noisy, sparse or biased data. It is these practical, not theoretical, drawbacks that this paper investigates.

There has been work done on neural networks for prediction of time series [11, 19, 28, 31], as well as studies of using neural networks for predicting financial phenomena, such as currency exchange [26, 38], bond ratings [7, 30] and stock prices [15, 16, 27, 37]. This body of research in mainly centered on sequential prediction using indicator data, usually in known and large amounts. More pertinent to the use of neural networks for cost estimation is the research directed at neural networks as surrogates for regression. Probably the most fundamental work on this aspect is by Geman et al., which extensively discusses the bias / variance dilemma of any estimation model [9], a subject also discussed in [18]. The trade-off of any model development is that of bias, or assumption of model form, and variance, or the dependence of the model on the data set used to construct the model, termed here the "construction sample." A model that is under-parameterized (or incorrectly parameterized), results in a biased model. A model that is over-parameterized has high variance which fits the construction sample well, but generalizes poorly to the model population, as estimated by the "validation sample." This bias / variance trade off becomes particularly evident when working with small data sets where a smooth form is hardly, if at all, discernible from the variability of the data. For a simple linear regression model, the bias is the assumed linear functional form, while the variance is the determination of the slope and intercept parameters using the construction data set. For neural network models, the choices between the bias and variance are less well defined. Neural networks have many more free parameters (each trainable weight) than corresponding statistical models, but are tolerant of redundancy.

There have been several citations from the literature on the use of neural network models to assist with cost estimation decisions. Recent published general works include reducing the dependence on contingency factors in civil engineering costing by supplementing the procedure with neural networks [1], software cost estimation [17, 35], a self organizing network within an expert system [25], and some miscellaneous financial applications [32]. Work that specifically compares neural network to regression models for cost estimation includes costing of a pressure vessel by Brass, Gerrard and Peel [2, 10] and material cost estimation of carbon steel pipes by de la Garza and Rouhana [6]. While the paper by Brass, et al. claimed a 50% improvement when using a neural network instead of a regression model, their results are almost certainly biased since no separate validation sample was used. This is known as the "resubstitution" method of model validation and is biased downwards (sometimes severely) [33]. The latter paper compared linear regression, nonlinear regression and neural networks for estimating the material cost of 16 pipes, however this comparison also seems flawed. The regressions were constructed using the entire set of 16 observations while the neural network was constructed using a training set of 10 observations. The remaining 6 observations were used as a validation set, however the results reported were mean squared errors over the entire training and testing sets. Despite these apparent faults, the authors reported substantial improvements when using neural networks over both of the regression approaches. Shtub and Zimmerman compared costing six product assembly strategies and found the neural network approach was generally superior to regression [29]. Another paper found, however, that when estimating a simple linear function with sparse data, that regression could be better than neural networks for both average and maximum error metrics [21].

This paper is distinct from those just cited by the completeness and probity of the investigation that systematically includes the aspects of data set size, data set imperfections in the form of white noise and sampling bias, and the impact of model commitment in regression. The trade-offs of using neural networks for cost estimation under a variety of simulated environments are investigated to test the practical ramifications of the bias / variance dilemma. Then, a real problem in cost estimating that has been the subject of prior published research [2, 9] is considered and a detailed comparison is made using the cross validation method. Finally, the paper concludes with observations on the usability, accuracy and sensitivity of neural networks versus regression CERs for cost estimation.

2. The Simulated Problem and the Design of Experiments

A function in two variables using a simulated data set was selected so that sampling bias, sample noise and sample sizes could be controlled. However, the primary reason for using a simulated data set was the identification of the correct, or true, CER. The function:

$$z = 20x + y^3 + xy + 400 \tag{1}$$

included nonlinear and cross terms, and represents the input of two independent cost driver variables, x and y, such as number and kind of parts or raw materials or labor to determine the output z, the amount of resource required. The nominal range of x was 0 to 100 and y was 0 to 50.

The design of experiments tested four factors: the modeling method of developing the CER, the sample size available for CER construction, the magnitude and distribution of data imperfections (noise), and the bias of the sample. For each CER method, a full factorial experiment with five levels of construction sample size, three levels of noise and three levels of bias was created resulting in a total of 45 separate prediction models for each CER. The experimental design is summarized in Table 1. The bias of the construction sample deserves more explanation. One level was unbiased, that is selected with uniform probability across the nominal range. The second level was biased towards the mean, that is selected with Gaussian probability with μ = mean of the nominal range, and coefficient of variation (c.v. = σ/μ) = 0.30. The third level was biased towards the ends of the nominal range, that is selected equally from two Gaussians, each with μ = one extreme of the nominal range and c.v. = 0.15. The experiments simulated conditions of varying data sparseness, data imperfections (deviations from a smooth function), and sampling imperfections (sample bias). The best case would be a large sample size with perfect sampling and perfect adherence to the CER. The worst case would be the smallest sample with biased sampling and significant noise in the relationship between x and y, and z.

INSERT TABLE 1 HERE

A total of 45 neural network models were built for the experiments detailed above. Each neural network consisted of two input neurons, one output neuron, and two intermediate hidden layers with two neurons each. This architecture was determined after brief experimentation as adequate for the problem but not overly parameterized. See Figure 1 for the network structure. Each network was trained using a classical backpropagation algorithm with a smoothing term added which allows current weight changes to be based in part on past weight changes:

$$D_p W_{ij} = \eta \left(\alpha D_{p-1} W_{ij} + (1 - \alpha) \delta_{pi} O_{pi} \right) \quad (2)$$

where $D_p W_{ij}$ is the change in weight connecting neuron *j* to neuron *i* for input vector p, O_{pi} is the output of neuron *i* for input vector p, δ_{pi} is the error of the output of neuron *i* for input vector p times the derivative of the sigmoidal transfer function, η is the training rate, and α is the smoothing factor. Networks were trained to a maximum error of 0.1 for each construction data point, or failing that, a maximum number of iterations through the construction set (epochs) of 10000.

INSERT FIGURE 1 HERE

To compare to regression modeling, there was one important aspect that had to be added. An initial requirement of regression modeling is the *a priori* selection of the functional form, known as model commitment. Model commitment may be done on the raw data, or on transformed data, where the transformation decision is another prerequisite to the actual calculation of the regression model. Functional form selection is usually accomplished by assuming a low order polynomial or providing a variety of terms and using a stepwise regression approach. Note that although a stepwise regression approach can prevent over-specified models, a commitment *a priori* to some set of functional forms must still be made.

To allow for different possibilities during model commitment, three regression formulations were chosen. The first assumed that the exact CER was known ($z = \beta_o + \beta_1 x + \beta_2 y^3 + \beta_3 xy$), though coefficients (including the intercept) were to be determined by the data. This is a best case for the regression. A second CER was obtained by stepwise regression at $\alpha = 0.05$ using all possible terms of a third order polynomial, including cross terms. This would be a typical approach by a knowledgeable analyst. The third CER was a reasonable assumption on the nonlinearity of the *y* term. This CER used a functional form of $z = \beta_o + \beta_1 x + \beta_2 y^2$. The third

CER was a worst case for the regression (although one might assume an even gloomier regression that uses only first order terms). In summary, 45 regression models for each of the three CERs using the same data sets as used for the neural network models were built, for a total of 135 regression models.

3. Results from the Simulated Problem

Performance of interpolative predictions over the validation sample is reported in this section; interpolation is used here to mean that the validation sample is drawn from the same nominal range as was the construction sample. Four validation sets were used, each consisting of 100 uniform randomly drawn values of x and y over the specified nominal ranges of x and y. Each of the four sets was subjected to different noise (or error) distributions. The first set had no noise, i.e., z was the exact function calculation. The last three had Gaussian distributed errors with $\mu = 0$ and c.v. of 0.05, 0.10 and 0.20, respectively. The addition of noise was designed to test if interpolation ability was influenced by the similarity of the noise level in the data used to construct the model and the noise level of the general population.

An Analysis of Variance (ANOVA) was performed on the five factors (CER method, sample size, noise in the construction sample, noise in the validation sample, and sample bias), and all main effect factors were significant at $\alpha = 0.05$ except for sample size (n), which was found to be insignificant at any reasonable α. This insensitivity to n is rather surprising, although it will be shown below that the interaction between sample size and method is significant. Furthermore, the largest sample size, $n = 80$, did consistently result in better predicting models than the smaller sample sizes. The factor of CER had the most contribution to the sum of squares, and was the most significant factor by a large margin. The second most significant factor was the bias in the construction sample, and while noise in construction and validation samples were significant, they did not contribute largely to the sum of squares. A Tukey's test for mean differences at $\alpha = 0.05$ resulted as shown in Table 2. For method, the regression models that were a result of successful model commitment (exact CER and stepwise third order) were grouped together. The neural network and the second order regression CERs were grouped together, and both had significantly greater root mean square error levels than the exact and stepwise regressions across all experiments. Noise in the construction set was divided into two groups - low noise (c.v. = 5% or 10%) and high noise (c.v. = 20%) - where the low noise resulted in better performing prediction models. The noise in the validation set did not contribute much to the sum of squared errors, but formed two significant groups with the noiseless validation set in both groups. It is difficult to draw any consistent conclusions from this factor. Finally, bias in the construction set is important with sets that are unbiased or biased towards the middle resulting in better performing CERs, while the construction sets concentrated at the extremes formed significantly poorer performing CERs.

INSERT TABLE 2 HERE

Two way interactions with method were also examined, and all were significant at $\alpha = 0.05$, except for the interaction between method and bias, as shown in Table 3. Additionally, the interaction between noise in the construction set and noise in the validation set was unexpectedly insignificant. It was hypothesized that CERs constructed for one level of noise would perform best when predicting under that level of noise. This was not found to be the case, and indicates that all CERs were relatively robust to the consistency of the noise level from construction sample to validation sample.

INSERT TABLE 3 HERE

To scrutinize the relative performance of the neural network and the second order regression, results of the parametric paired t-test and the non-parametric Wilcoxen Signed Rank test are shown in Table 3. The paired t-test showed no difference between the mean root mean squared error of the two methods, however this was primarily due to the high and dissimilar variance of both methods, invalidating the test. The rank based Wilcoxen Signed Rank showed that the regression was significantly more accurate than the neural network with a p-value of 0.0231 and is a more appropriate result. An F test also showed that the variance of error for the neural network approach is significantly lower than for the regression approach, which indicates more stability of the neural network approach relative to a poorly formulated regression model. Another look at comparative performance is provided by Figure 2 that shows the relative error as a function of absolute distance of validation point from the center of the *x* / *y* plane for the exact functional form regression, the second order regression and the neural network. The larger scatter of the second order polynomial can be easily seen while the neural network errors generally increase as a function of the distance from the center.

INSERT FIGURE 2 HERE

To summarize the results of the detailed performance experiments, when the all important model commitment phase of regression is successful, the neural network approach is a poor choice. However, when an *a priori* CER is unknown and an inferior, but still reasonable choice is made (*viz*. the second order regression), the neural network approach is of nearly comparable precision. Additionally, the neural network may be less dependent on the sample data used and more robust to the conditions of the problem, as evidenced by significantly lower variance across all factors. All modeling approaches are better when the construction set has less noise and is unbiased, both of which are consistent with what would be expected.

4. A Real World Cost Estimation Data Set

Gerrard, et al. [2, 10] reported 20 samples of pressure vessel costs as a function of the height, diameter and wall thickness obtained from a manufacturer who had recently priced such vessels for new chemical production. Using a linear CER of these three independent variables, $y = b_o + b_1 x_1 + b_2 x_2 + b_3 x_3$, where the independent variables refer to vessel design parameters, the authors claimed that the neural network approach outperformed the regression approach.[1] However, this conclusion as to the superiority of the neural network approach is based on the resubstitution method where the construction sample is identical to the validation sample; this is known to be biased downwards (see [33] for a description of this validation method). Therefore, the results of Gerrard, et al. must be viewed with suspicion concerning the neural network, whose many free parameters could allow the error on data used in constructing the model to go to zero (this is the error measured by resubstitution), but gives no information on the expected error on the population in general, as estimated by performance on an independent validation sample.

To overcome the questionable results of [2, 10], the analysis was replicated using the cross validation method (also called the jackknife method) [33] in which the 20 samples were assigned to 20 groups, each containing one of the samples. Nineteen of these groups were then used to predict the remaining one-sample group. Thus, each of the 20 sample costs was predicted with the 19 remaining samples serving as the construction set. The validation and construction data, predicted costs, prediction error, prediction error squared, and absolute relative error results are shown in Table 4.

INSERT TABLE 4 HERE

Table 5 reports error statistics. The *Mean Absolute Relative Error* is calculated by subtracting the predicted value from the actual, taking the absolute value, and then dividing by the actual. Accordingly, mean absolute relative error can be interpreted as the average absolute

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[1] Gerrard et al. also reported that an exponential CER, *viz*. $y = ax_1^{b1} x_2^{b2} x_3^{b3}$, gave somewhat better results than the linear CER, but that the neural network still outperformed the regression. This is reasonable since nonlinear transformations of the independent variables might be expected to improve the predictive performance of regression given the nature of the product. Since the neural network still outperformed the regression, and since there are a variety of nonlinear models that could be rationally proposed, the original linear CER has been used for comparison purposes. Clearly, regression would be expected to outperform the neural network if the analyst does indeed know or can closely guess the underlying analytic relationship between cost and the cost drivers. Thus, the

percentage deviation from the actual cost over all the samples. The maximum and minimum errors are also shown. Samples 1, 6, 19 and 20 contained values in either their independent or dependent variables, such that when the cross validation method was used, the prediction constituted an *extrapolation* outside the data set. In Sample 1, both the height and actual cost were outside the range of the data used to construct the models. In Sample 6, the diameter was outside the data set. In Sample 19, the vessel diameter was outside the range of the data set. In Sample 20, the height, thickness, and cost were all outside the data set. Because of the unreliability of extrapolation with both regression and neural networks, the measures of error were recalculated excluding these four predicted costs. These are referred to as the "16 point" error measures.

INSERT TABLE 5 HERE

The significance of the differences for the RMS errors is based on the square of the errors for the 20 samples and is not, *per se*, the significance of the RMS error. This was done by first subtracting the square of the neural network error from the regression error and then using the t distribution to test the null hypothesis that the mean of the differences was equal to zero. In Table 5, p-values for a one-sided paired t-test are shown. In the case of relative absolute error, the statistic was the mean of the difference in absolute relative errors. A one-sided paired t-test was also used. It can be seen that the neural network dominated the regression CER on all error metrics, regardless of whether extrapolation was considered. These were statistically significant at a confidence of 95%, or better.

A scattergram of the regression and neural predicted costs vs. a line of perfect prediction is shown in Figure 3. The graph confirms the tendency of the neural network's predictions to be

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issue is not whether regression *can* outperform neural networks in estimating costs, but is one of the relative closer to the line of perfect prediction than those of regression. Figure 4 shows vessel cost as a function of the three design parameters. Assuming that there are not large measurement errors in the cost and design parameter data, nonlinear and/or discontinuous relationships are suggested in each graph. Therefore, other product attributes may be needed to accurately predict costs. The neural network's superior performance can be explained on the basis that it was able to capture these nonlinearities and discontinuities, along with their interactions, to better compensate for missing product attributes that drive cost. Product attribute interactions are unknown, but might yield to investigation. For example, cost might be accurately predicted in part by some function of the volume of the tank, where the volume would be proportional to one-half the diameter squared times height. Numerous regression models can be constructed along these lines, and it is possible that with enough knowledge of the fabrication process that a superior regression model could eventually be obtained. This, however, defeats a main purpose of the parametric cost estimating approach which is to overcome a lack of insightful knowledge of the fabrication process and materials, and their interactions. The pressure vessel cost data illustrates one situation in which the neural network approach provided superior results in relation to a simple, but credible, regression CER.

INSERT FIGURES 3 AND 4 HERE

5. Conclusions

These results suggest that an artificial neural network may be an attractive substitute for regression if the model commitment step (functional form selection, interaction selection and data transformation) of regression cannot be accomplished successfully. By this, it is meant that the cost data does not enable fitting a commonly chosen model, or does not allow the analyst to

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performance of the two models in the absence of known analytic relationships.

discern the appropriate CER. The problem of model commitment becomes more complex as the dimensionality of the independent variable set grows. Visualizing functional shape is extremely difficult in more than three dimensions. While neural networks alleviate this issue, there is the considerable danger of choosing an overdetermined neural model, especially when dealing with small samples. Conclusions as to model accuracy from the resubstitution method can be misleading, and care must be taken to achieve unbiased estimates of neural network performance. The laborious procedure of cross validation, which entailed the construction and validation of twenty neural networks in the pressure vessel example, can provide a reliable empirical estimation of accuracy over the target population.

Below are listed some important issues other than model accuracy to be considered when using regression versus neural networks to estimate cost functions.

⇒ **Credibility**: Management and customer confidence in parametric methods is a widely recognized problem regardless of what parametric approach is used. This is particularly true in the case of firm business proposals which must always satisfy management and sometimes customer criteria as to what constitutes a proper methodology. In the bottom-up approach to cost estimating, there is a credible audit trail of detailed work procedures and methods, materials, and schedules. This allows assumptions to be examined and produces an aura, if not the reality, of accuracy. Parametric methods in general and regression in particular are employed because it is either (i) not feasible, or (ii) not cost effective to develop this micro-level specification.

However, with regression one at least can argue logically why the model of cost behavior is reasonable. This is because the analyst creates an CER equation which checks with common sense. It is credible on a term-by-term basis. Few cost estimators are heroic enough to publish a CER that contains an intercept or term that defies common sense even if the equation does a remarkable job of predicting costs.

Now consider neural networks. In this case, the "equation" will not check with common sense even if one were to extract it by examining the weights, architecture, and nodal transfer functions that were associated with the final trained model. The artificial neural network truly becomes a "black box" CER. Explaining to a customer how it arrived at its answer could be much like explaining how one plays tennis by doing a dissection of the tennis player's brain tissue. Moreover, the analyst may wish to fit the data to a particular parametric form. This is possible with regression but not practical with neural networks.

⇒ **Tactical Issues**: The neural network approach does not mitigate any of the difficulties associated with preliminary activities when using statistical parametric methods, nor does it create any new ones. The analyst is still left with a choice of cost drivers and frequently must make a one-time commitment to collecting specific cost data before analysis begins. As a practical matter, neural networks are capable of accepting a larger number of potential cost drivers than regression, and will accommodate multicollinearity readily. For both approaches, software has been developed to ferret out inputs that appear to contribute little to prediction and thereby simplify the application. Regression produces a CER that may be easily imbedded in computer-aided cost estimating systems. This is not the case with neural networks although many commercial systems generate high-level source code, C for example, that reproduces the behavior of the trained network.

⇒ **Replicating the Results**: Training a neural network is an algorithmic procedure and the results can most certainly be replicated as long as one uses the identical computer code, the same initial weights, the same training data, and the same deterministic method of presenting the data during training. However, if even one of these parameters is altered, the resulting neural network would almost certainly be different from the original one. This difference is apt to be extremely minor, however it is not inconceivable that major differences could occur. This is one of aspects of the "art" of neural network construction and validation. Moreover, producing near optimal neural network models involves iteratively identifying good combinations of network architecture, training methods and stopping criteria. Currently, the learning curve in building and interpreting neural network models is more imposing than that of statistical models, where decisions are fewer and guidance is readily available from texts and software.

By way of conclusion, it is expected that neural networks will be used with increasing frequency as a substitute for regression by the parametric cost estimating community because analysts will find that in particular situations neural networks provide a superior cost estimate. They will be considered a viable alternative to regression if one has a poor idea of the underlying cost behavior or suspects that there are functional discontinuities and significant nonlinearities, especially in data sets of large independent variable dimensionality. However, the concerns of neural network modeling apart from model accuracy should not be ignored and represent formidable hurdles to widespread use and acceptance of neural CERs.

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Table 1. Design of Experiments.

Factor	Number of Levels	Levels
Sample Size	5	5, 10, 20, 40, 80
Noise - Construction Sample	3	Gaussian with $\mu = 0$ and c.v. = 0.05, 0.10 and 0.20
Bias - Construction Sample	3	No bias (uniform random), Mid-value bias (Gaussian about mean), End bias (Gaussian about extremes)
CER Method	4	Neural network and three regressions: exact form, stepwise of third order polynomial, second order polynomial

Table 2. ANOVA for Main Effects.

Factor	F Value	P Value	Homogeneous Groups*
CER Method	174.57	0.0000	(exact regression, stepwise regression), (neural network, second order regression)
Sample Size	0.10	0.9780	None
Noise in Construction Sample	10.25	0.0001	(0.05, 0.10), (0.20)
Noise in Validation Sample	6.43	0.0003	(0, 0.05, 0.20), (0, 0.10)
Bias in Construction Sample	37.46	0.0000	(uniform, mid-value), (extremes)

* Using Tukey's Procedure at $\alpha = 0.05$.

Table 3. ANOVA for Interactions and Two Sample Test Results.
(All Two Sample Tests are Neural Network versus Second Order Regression.)

Factor	F Value	p-Value
CER Method	235.98	0.0000
Sample Size	0.14	0.9644
Noise in Construction Sample	13.85	0.0000
Noise in Validation Sample	8.69	0.0000
Bias in Construction Sample	50.64	0.0000
Method * Sample Size	10.14	0.0000
Method * Noise/Construction	6.70	0.0000
Method * Noise/Validation	2.10	0.0275
Method * Bias/Construction	17.51	0.0000
Noise/Construction * Noise/Validation	0.20	0.9762
Method/Paired t Test - Mean[#]	0.30*	0.7636
Method/Two Sample F Test - Variance	3.36	0.0000
Method/Paired Wilcoxen Signed Rank	2.272[+]	0.0231

* t statistic.
[#] Inappropriate test.
[+] Wilcoxen Signed Rank statistic.

Table 4. Data and Prediction Errors for Pressure Vessel Problem.

Sample	Vessel Height	Vessel Diameter	Vessel Thickness	Actual Cost	Predicted Cost		Error (Act. - Predicted)		Error Squared		Absolute Rel. Error	
					MLR	NN	MLR	NN	MLR	NN	MLR	NN
1	1200	1066	10	$10,754	$30,608	$10,904	($41,362)	$150	1.7 E+ 09	22500	384.62%	1.39%
2	4500	1526	15	$18,172	$33,008	$22,691	$14,836	$4,519	2.2 E+ 08	2 E+ 07	81.64%	24.87%
3	6500	1500	16	$23,605	$42,543	$23,725	$18,938	$120	3.6 E+ 08	14400	80.23%	0.51%
4	12250	1200	12	$23,956	$9,059	$22,941	($14,867)	($985)	2.2 E+ 08	970225	62.14%	4.12%
5	21800	1050	12	$28,400	$17,671	$29,665	($10,729)	$1,265	1.2 E+ 08	1600225	37.18%	4.45%
6	23300	900	14	$31,400	$27,913	$33,913	($3,487)	$2,513	1.2 E+ 07	6315169	11.11%	8.00%
7	26700	1500	15	$42,200	$60,239	$52,673	$18,039	$10,473	3.3 E+ 08	1.1 E+ 08	42.75%	24.82%
8	12100	3000	11	$47,970	$65,920	$53,942	$17,950	$5,972	3.2 E+ 08	3.6 E+ 07	37.42%	12.45%
9	17500	2400	12	$48,000	$57,477	$49,440	$9,477	$1,440	9 E+ 07	2073600	19.74%	3.00%
10	26500	1348	14	$51,000	$46,899	$47,959	($4,101)	($3,041)	1.7 E+ 07	9247681	8.04%	5.96%
11	28300	1800	14	$53,900	$65,797	$60,063	$11,897	$6,163	1.4 E+ 08	3.8 E+ 07	22.07%	11.43%
12	14700	2400	10	$54,600	$38,866	$40,081	($15,734)	($14,519)	2.5 E+ 08	2.1 E+ 08	28.82%	26.59%
13	26600	1500	15	$58,040	$58,394	$53,263	$354	($4,777)	125316	2.3 E+ 07	0.61%	8.23%
14	24800	2500	13	$61,790	$77,577	$72,069	$15,787	$10,279	2.5 E+ 08	1.1 E+ 08	25.55%	16.64%
15	25000	2100	14	$61,800	$70,022	$64,632	$8,222	$2,832	6.8 E+ 07	8020224	13.30%	4.58%
16	24700	2000	16	$67,460	$78,380	$63,756	$10,920	($3,704)	1.2 E+ 08	1.4 E+ 07	16.19%	5.49%
17	29500	2250	13	$80,400	$73,871	$69,240	($6,529)	($11,160)	4.3 E+ 07	1.2 E+ 08	8.12%	13.88%
18	21900	3150	12	$85,750	$87,376	$81,911	$1,626	($3,839)	2643876	1.5 E+ 07	1.90%	4.48%
19	32300	5100	17	$207,800	$177,548	$208,266	($30,252)	$466	9.2 E+ 08	217156	14.56%	0.22%
20	53500	3000	29	$240,000	$185,358	$217,750	($54,642)	($22,250)	3 E+ 09	5 E+ 08	22.77%	9.27%

Highlighted cells indicate extrapolations appearing in examples 1, 6, 19 and 20

Table 5. Prediction Errors for Full and Interpolation Only Pressure Vessel Data Set.

	RMS Error		Mean Absolute Relative Error		Max Error		Min Error	
	20 points	16 points	20 points	16 points	20 points	16 points	20 points	16 points
MLR	20,203	12,599	45.97%	30.39%	($54,642)	$18,938	$354	$354
NN	7,809	6,699	9.52%	10.72%	($22,250)	($14,519)	$120	$120
Significance	$p < 0.05$	$p < 0.001$	$p < 0.05$	$p < 0.005$				

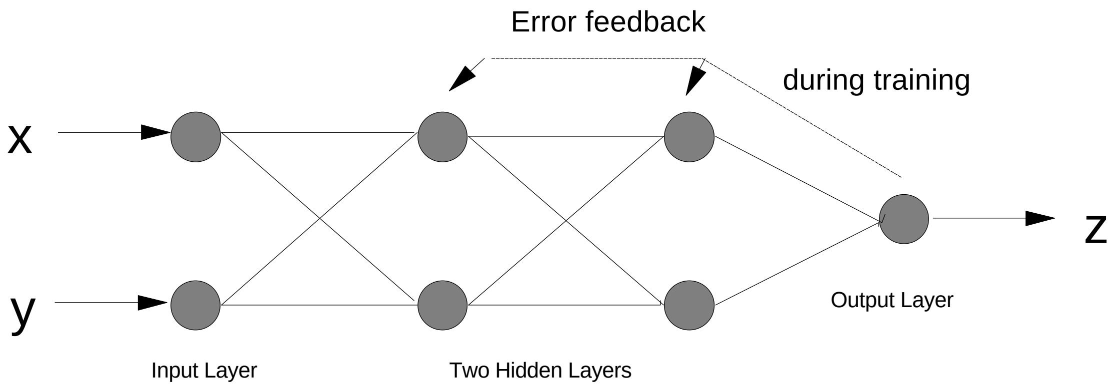

Figure 1. Neural Network Architecture.

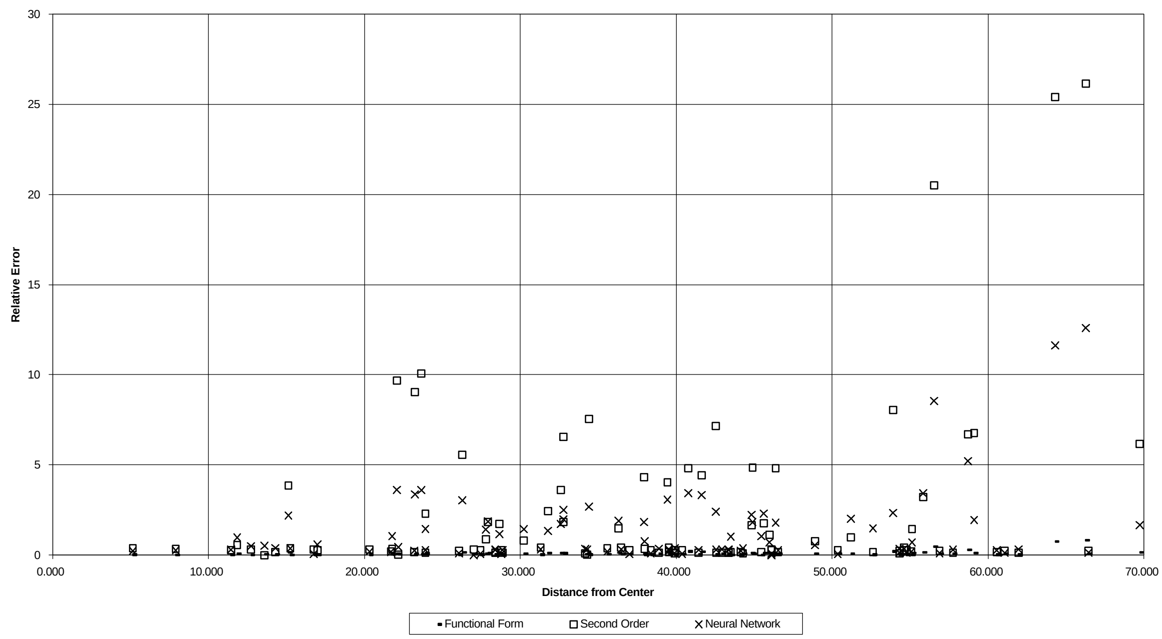

Figure 2. Normalized RMS Error by Absolute Distance from Center of *xy* Plane.

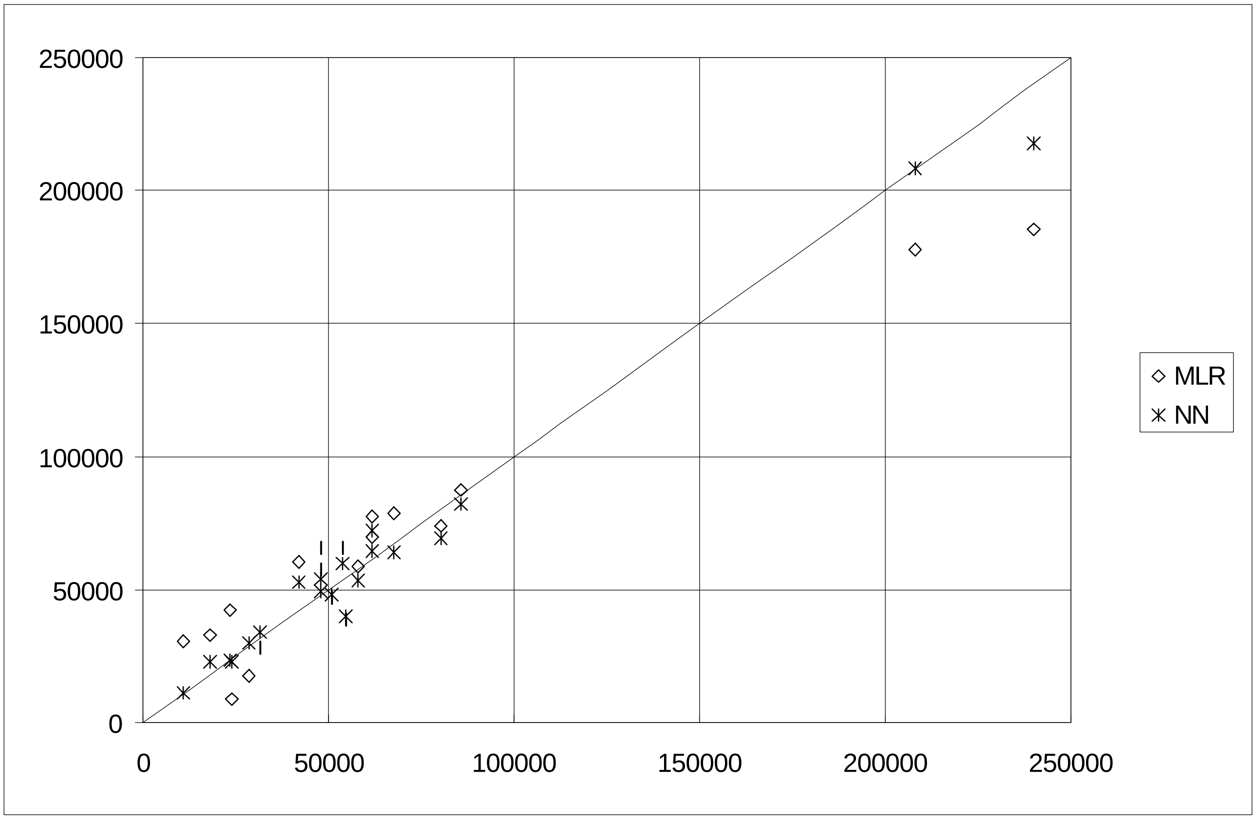

Figure 3. Predicted versus Actual Cost for Neural Network and Regression Model.

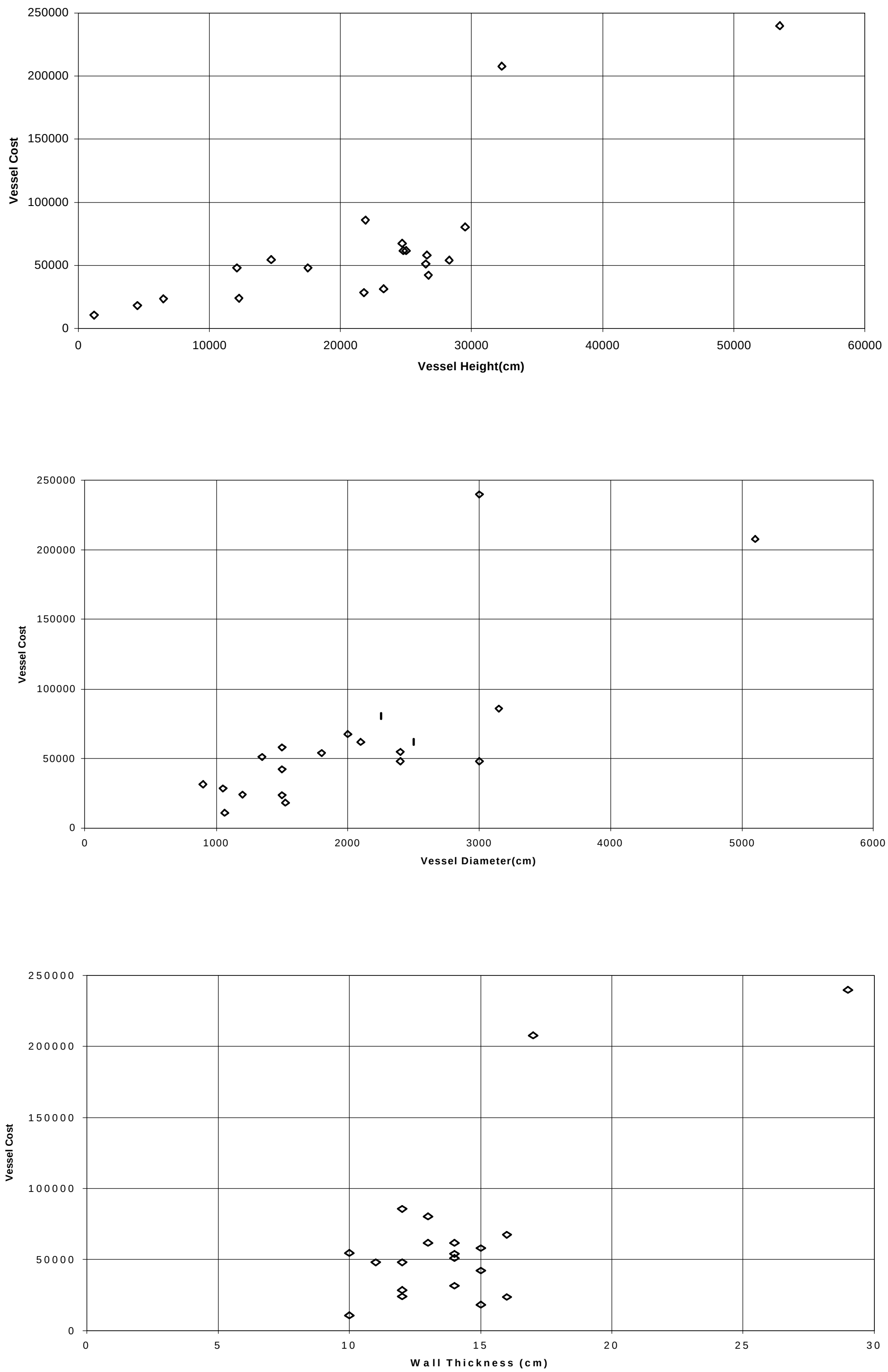

Figure 4. Pressure Vessel Cost versus Height, Diameter and Thickness.