The Selective Traveling Salesman Problem with Release Dates and Drone Resupply

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To reduce distribution costs and increase the number of orders that logistics companies can deliver, this presentation puts forth a new and innovative use of drones for resupplying dispatch vehicles while they are en route. We consider a setting where a single truck has a limited period of time to deliver orders that become available throughout the day. Drone resupply would avoid the need for the truck to return to the depot every time new orders become ready for dispatch, as these orders could be sent to the truck via drone.

To the best of our knowledge, the use of drones for resupply has been studied only twice. Dayarian, Savelsbergh, and Clarke (2020) concentrated on showing the benefits of this particular use through heuristic approaches without presenting a mathematical formalization of the problem. McCunney and Van Cauwenberghe (2019) presented simulation tests to analyze the impact of a parcel delivery system where drones are used to transport new orders to transshipment points, from where they are picked up by the dispatch vehicles. Hence, the mathematical formulation presented in the next section is the first mathematical programming model for this new class of routing problems. Unlike the two papers listed above, we assume that full order information, including release dates, is available at the time of planning.

1. Mathematical Formulation

Considering a tandem between a truck and a drone, we present the problem of determining a delivery route for the truck, synchronized with drone resupply, to minimize distribution costs and penalties incurred for not delivering parcels within a limited time frame. We assume that release dates of customer orders are known at the beginning of the planning horizon (see Archetti, Feillet, & Speranza, 2015). Orders can then be loaded on the truck either at the depot or via the drone while en route. Additionally, we assume that the drone can rendezvous with the truck only at customer locations, and we consider an unloading time when this meeting takes place. Finally, we assume that the drone has a given capacity and flight endurance, whereas the truck is uncapacitated.

Let $N = \{1, ..., n\}$ be the set of customers and $D \subset N$ be the subset that could be visited by the drone according to its endurance. We represent the depot with the subindices 0 and $n+1$ and we define $N_0 = N \cup \{0\}$, $N_{n+1} = N \cup \{n+1\}$, $D_0 = D \cup \{0\}$, and $D_{n+1} = D \cup \{n+1\}$.

1.1. Parameters

- p: Penalty cost for not delivering an order.
- h: Cost for elapsed time between arrival and departure of the truck at each node.
- c_{ij}: Truck travel cost associated with the arc (i,j).
- f: Fixed cost per use of the drone.
- w_i: Release date of the order of customer i.
- t_{ij}: Truck travel time associated with the arc (i,j).
- d_j: Drone flying time between the depot and the node j.
- δ: Time for receiving and unloading orders from the drone.
- A: Drone maximum load capacity.
- W: Limited time frame for delivering parcels.

1.2. Decision Variables

- x_{ij}: 1, if node j is visited immediately after node i by the truck. 0, otherwise.
- z_i: 1, if node i is visited by the truck. 0, otherwise.
- r_{ij}: 1, if node j is visited after node i by the drone. 0, otherwise.
- u_i: 1, if the drone flies to node i for resupplying the truck with new orders. 0, otherwise.
- y_{ij}: 1, if the order of customer j is loaded onto the truck at node i. 0, otherwise.
- T_i: Time when the truck departs node i.
- s_i: Time when the drone is launched for node i.
- ϵ_i: Elapsing time between arrival and departure of the truck at node i.

1.3. Mixed Integer Programming Formulation

The Mixed Integer Programming (MIP) formulation is presented in (1)-(22). The objective function (1) minimizes the total truck traveling and waiting costs, the cost of using the drone, and the penalty cost incurred for not delivering parcels. Non-negativity and integrality constraints are excluded given the limitation of space.

$$\min \sum_{i \in N_0} \sum_{j \in N_{n+1} \setminus \{i\}} c_{ij} x_{ij} + \sum_{i \in D} h \cdot \epsilon_i + \sum_{i \in D} f \cdot u_i + \sum_{i \in N} p(1 - z_i) \tag{1}$$

$$\sum_{j\in N} x_{0j} = 1 \tag{2}$$

$$\sum_{i\in N} x_{i,n+1} = 1 \tag{3}$$

$$\sum_{j\in N_{n+1}\setminus\{i\}} x_{ij} = z_i \qquad \forall i \in N \tag{4}$$

$$\sum_{i\in N_0\setminus\{j\}} x_{ij} = z_j \qquad \forall j \in N \tag{5}$$

$$u_i \leq u_0 \qquad \forall i \in D \tag{6}$$

$$\sum_{j\in D} r_{0j} = u_0 \tag{7}$$

$$\sum_{i\in D} r_{i,n+1} = u_0 \tag{8}$$

$$\sum_{j\in D_{n+1}\setminus\{i\}} r_{ij} = u_i \qquad \forall i \in D \tag{9}$$

$$\sum_{i\in D_0\setminus\{j\}} r_{ij} = u_j \qquad \forall j \in D \tag{10}$$

$$u_i \leq z_i \qquad \forall i \in D \tag{11}$$

$$\sum_{j\in N} y_{ij} \leq A\cdot u_i \qquad \forall i \in D \tag{12}$$

$$\sum_{i\in D_0} y_{ij} = z_j \qquad \forall j \in N \tag{13}$$

$$T_j \geq T_i - W(1-y_{ij}) \qquad \forall i \in D_0, j \in N\setminus\{i\} \tag{14}$$

$$T_j \geq T_i + t_{ij} - W(1-x_{ij}) \qquad \forall i \in N_0, j \in N_{n+1}\setminus(D\cup\{i\}) \tag{15}$$

$$T_j \geq T_i + t_{ij} + \delta\cdot u_j - W(1-x_{ij}) \qquad \forall i \in N_0, j \in D\setminus\{i\} \tag{16}$$

$$T_j \geq s_j + d_j + \delta - W(1-u_j) \qquad \forall j \in D \tag{17}$$

$$\epsilon_j \geq T_j - (T_i + t_{ij}) - W(1-x_{ij}) \qquad \forall i \in N_0, j \in D\setminus\{i\} \tag{18}$$

$$s_j \geq T_i + d_i - W(1-r_{ij}) \qquad \forall i \in D, j \in D\setminus\{i\} \tag{19}$$

$$s_j \geq w_i\cdot y_{ji} \qquad \forall i \in N, j \in D \tag{20}$$

$$T_0 \geq w_j\cdot y_{0j} \qquad \forall j \in N \tag{21}$$

$$T_{n+1} \leq W \tag{22}$$

2. Results and Conclusions

We developed a set of 90 randomly generated instances of 10, 20, and 30 customers (30 per each size of the problem). All customers were uniformly random located over a 15 × 15 km^2 area. Deliveries can be made between 08:00 and 18:00 hours, however, orders are only accepted until 16:00 hours, and their release dates follow a uniform distribution during this time horizon. We alternated between locating the depot at the corner (15,0) and at the right (15,7.5) of the area. We considered a truck speed of 30 km/hr and a drone speed of 60 km/hr. We set the flight endurance of the drone as 30 minutes, and we considered a drone load capacity of 4 orders.

Considering a maximum run-time of 30 minutes, the performance of the MIP model is presented in Table 1, using an Intel(R) Xeon(R) Gold 5118 CPU @2.30 GHz (12 cores) with 64 GB RAM and CPLEX 12.8. In this table, f_r represents the fill-rate (i.e. the percentage of customers served), n_{drone} the number of drone trips, and o_r the number of orders resupplied. CPU times and total costs increase significantly as the size of the instances grows, being slightly higher when the depot is located at the corner. When 30 customers are considered, CPLEX is not able to reach optimal solutions in all instances, and the fill-rate is less than 100%. Finally, for both layouts and $n = 30$, the drone performed one flight to resupply the truck with more than three orders on average.

Table 1 **Average results reached by CPLEX considering a maximum run-time of 30 minutes.**

Layout	n	CPU Time(s)	Cost	GAP (%)	f_r (%)	n_{drone}	o_r
Right	10	0.15	60.58	0.00	100.00	0.03	0.03
	20	99.82	82.65	0.01	100.00	0.70	1.53
	30	904.44	151.79	2.24	98.22	1.23	3.63
Corner	10	0.17	64.13	0.00	100.00	0.07	0.10
	20	115.54	88.04	0.00	100.00	0.83	1.87
	30	1226.00	241.26	4.53	96.33	1.13	3.23

In contrast with a traditional parcel delivery system using a truck only, experiments shown that the median percentage of cost savings reaches up to 70% in instances with 30 customers. Additionally, the fill-rate increases from 85% to more than 95% on average for that size of the instances.

To conclude this presentation, we will discuss future work, which includes considering a fleet of vehicles and drones, where the former can perform several dispatch routes during the time horizon.

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