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Lateral Buckling of Stepped Beams

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Report

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Sponsored by

The Alabama Department of Transportation
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DISCLAIMER

ABSTRACT

Continuous multispan steel bridges are common along the state and interstate highway systems. The structural system is tied together by a reinforced concrete deck slab and transverse steel members, or diaphragms, which are connected to the girders. Many of these bridges were designed as noncomposite systems and constructed with no shear stud connectors between the top flange of the girders and the deck slab. Although there are no shear connectors, continuous lateral support of the top flange is provided by embedment in the slab. Buckling of the girders can only result from negative bending moments near the interior supports of these continuous span bridges. Often, the flange thickness and width at the interior supports in the continuous girders is increased to resist high negative moments so that the girder is a nonprismatic beam.

Design equations for lateral-torsional buckling (LTB) resistance in the American Institute of Steel Construction (AISC) *LRFD Specifications* (1998) account for only prismatic and web-tapered beams. A study of LTB capacity of stepped beams has been performed, and simple design equations are proposed. Finite element method (FEM) buckling analysis results for stepped beams under uniform moment have been used to develop new design equations that account for the change in cross section of stepped beams. Traditional moment gradient factors for prismatic beams were reviewed and were found to be sufficiently accurate for some stepped beam cases. A new moment gradient factor equation has been developed from the finite element analysis results and is proposed for the determination of the LTB resistance of stepped beams under general loading conditions.

Furthermore, FEM buckling analyses of a number of different beams with prismatic or stepped cross sections having continuous lateral top flange bracing have been conducted. The beams are subjected to a concentrated load or a uniformly distributed load applied at top flange, and end moments. New moment gradient factors for prismatic or stepped beams with

continuous lateral top flange bracing have been developed using the results of the FEM investigation. In all applicable areas, a significant review and comparison between existing design specifications and the proposed equations are presented.

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CHAPTER 1. INTRODUCTION

BACKGROUND

Continuous multi-girder steel bridges are common along the state and interstate highway systems. The girder cross section is sometimes increased suddenly, or stepped, near the interior supports of continuous girders to provide increased resistance to high negative moments. Steps in the cross section can be achieved by adding cover plates to the beam flanges, changing the size of the hot rolled cross section, or changing the flange thickness and/or width for built-up sections. Common practice for fabrication of built-up welded beams favors the use of a constant web depth and flange width with increases in flange thickness to provide increased moment of inertia. The multi-girder steel bridge is tied together by a reinforced concrete deck slab and transverse steel members, or diaphragms, which are connected to the girders. Many of these bridges were designed as noncomposite systems and constructed with no shear stud connectors between the top flange of the girders and the deck slab. Although there are no shear connectors, continuous lateral support of the top flange is provided by embedment in the slab as shown in Fig. 1.1. Buckling of the girders can only result from negative bending moments near the interior supports of these continuous span bridges.

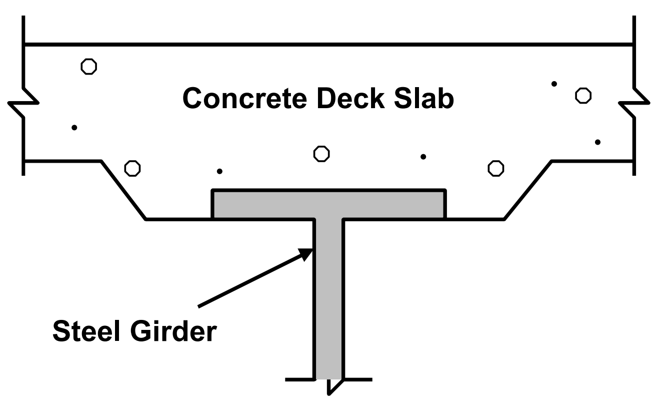


FIG. 1.1 Embedment of Girder Flange in Concrete Deck Slab

Design of steel beams for flexure requires consideration of limit states of yielding, local buckling, and lateral-torsional buckling (LTB). Design specifications typically provide rules for investigation of LTB that assume a constant cross section between braced points. Designers of stepped beams can use these rules by providing bracing at the locations where the cross section changes, or by calculating the LTB moment capacity based on the properties of the smaller cross section in the unbraced length. These approaches may lead to inefficient designs. Designers in the past sometimes considered inflection points in the deflected shape of continuous beams, points of zero moment, to be braced points even when no external bracing was attached to the beam at those points. Yura (1993) clearly illustrated the error in this assumption. There is no theoretical justification for assuming a zero moment point is a braced point.

There have been a number of previous investigations of the effects of elastic restraints on the elastic lateral-torsional buckling of beams. However, the majority of earliest studies have concentrated on beams with discrete lateral restraints (Powell and Klingner, 1970; Mutton, B. R. and Trahair 1973; Winter 1980). Trahair (1979) investigated the case of simply supported mono-symmetric I-shaped beam-columns with general uniform restraints. Trahair (1993) also provided a summary of the various theoretical studies that have been conducted on beams with continuous braces. The general solutions provided are in many instances to complicated for design purposes, or are in graphical forms that are difficult to incorporate into design codes. Many of the previous solutions in the literature are not sufficiently general for use in designing multi-span beams because the solutions do not reflect the effects of cross section change, continuous top flange bracing, or the moment gradient due to general loading.

This report describes the buckling behavior as determined from FEM analyses of prismatic and stepped beams with bracing at discrete locations or with continuous lateral top flange bracing. A calculation procedure is provided that is analogous to current lateral-torsional buckling solutions used in the design of girders.

OBJECTIVES

The overall goal of this report is to present a calculating procedure of new equations that can be used in beam design for determining the lateral-torsional buckling moment resistance of girders or beams. This report focuses primarily on finite element method studies of the buckling behavior of realistic structural systems. Traditional design procedures for calculating the lateral-torsional buckling resistances of beams are reviewed, and the accuracy of some of those methods is assessed.

Specific objectives are to consider the following detailed topics: lateral-torsional bucking of stepped beams with discrete bracing, and prismatic or stepped beams with continuous top flange bracing. A brief description of these topics is given below.

Stepped Beams

Most previous solutions for LTB of stepped beams consider simply supported beams with increased moment of inertia near midspan. None provides a method that can be readily used for design of stepped beams having end moments and increased moment of inertia at the ends of the span.

Goal of this study is to develop a cross section modifier, C_{st}, to account for the cross section changes of stepped beams. Equations for C_{st} are presented along with an expression for the moment gradient factor C_b so that both the effects of the stepped cross section and moment gradient can be accounted for. Results from FEM buckling analyses were used to develop the new equations and to study the accuracy of the weight average approach for calculating of the LTB moment resistance of stepped beams.

Stepped Beams with Continuous Top Flange Bracing

Early experts such as Newmark (1948) recognized that diaphragms had only a small effect on the performance of a bridge after the concrete deck was completed. The most recent AASHTO *LRFD Bridge Design Specifications* (1998) does not have strict requirement for diaphragms but allow the designer to use diaphragms as needed. The LTB resistance of beams with continuous bracing provided by concrete deck needs to be considered for potentially reducing the number of diaphragms in a bridge.

A goal of this study is to develop a new design equation for calculating the LTB moment resistance of prismatic or stepped beams with continuous lateral top bracing. With information compiled from existing stepped beams in a number of bridge structures, numerical studies are executed to obtain simple design equations for general loading conditions. This investigation provides valuable insight into lateral-torsional buckling behavior. A better understanding of stepped beams with continuous restraints should minimize the use of very conservative bracing in new structures as well as permit the replacement or removal of interior diaphragms that have been found to cause extensive fatigue cracking.

CHAPTER 2. LATERAL-TORSIONAL BUCKLING OF STEPPED BEAMS

INTRODUCTION

The analysis of frames having nonprismatic members is covered in many classical textbooks on structural analysis. Trahair and Kitipornchai (1971) studied the effects of steps in the minor axis flexural rigidity, the torsional rigidity, and the warping rigidity. The effects of the position of the steps on the elastic LTB resistance of simply supported I-beams with central concentrated loads were also studied. Fertis and Keene (1990) presented a method for the LTB analysis of nonprismatic members with arbitrary variation in moment of inertia, including cases where the material is stressed beyond the elastic limit, causing the modulus of elasticity to vary along the span. El-Mezaini et al. (1991) investigated the linear elastic behavior of frames with nonprismatic members by using isoparametric plane stress finite elements. Gupta et al. (1996) developed a finite-element formulation for LTB analysis of continuous or single-span, non-prismatic I-shaped beams subjected to static loadings. The formulation was applicable to continuous stepped or taper beams.

AISC *LRFD Specifications* (1998) define the elastic LTB moment capacity for doubly symmetric I-shaped members and channels as:

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \qquad (2\text{-}1)$$

where C_b is the moment gradient modifier; L_b is the laterally unbraced length; E is the modulus of elasticity of steel; G is the shear modulus of elasticity of steel; J is the torsional constant of the beam; I_y is the moment of inertia of the beam about the Y-axis; C_w is the warping constant of the beam. Eq. (2-1) with C_b of 1 is the elastic lateral-torsional buckling resistance (M_{ocr}) for an I-shaped prismatic section under the action of constant moment in the plane of the web over the laterally unbraced length (Timoshenko, 1961). C_b is a moment gradient modifier that accounts for the increased resistance to LTB when the applied loading does not produce constant, or uniform, moment over the entire unbraced length of beam.

AASHTO *LRFD Bridge Design Specifications* (1998) define the nominal flexural resistance based upon lateral-torsional buckling of I-shaped girders as:

$$M_n = 3.14EC_b(\frac{I_{yc}}{L_b})\sqrt{0.772\left(\frac{J}{I_{yc}}\right)+9.87\left(\frac{d}{L_b}\right)^2} \leq M_y \qquad (2\text{-}2)$$

where I_{yc} is the moment of inertia of the compression flange about an axis in the plane of the web; d is the depth of the girder; and M_y is the yield moment resistance. Equation (2-2) is a simplified expression of Eq. (2-1) for I-shaped girders.

The AASHTO *LRFD Bridge Design Specifications* (1998) and AISC *LRFD Specification* (1998) have incorporated the following expression for C_b, which is applicable for linear and nonlinear moment diagrams

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max}+3M_A+4M_B+3M_c} \qquad (2\text{-}3)$$

where M_{max} is the maximum moment along L_b; M_A, M_B, and M_C are the respective moments at $L_b/4$, $L_b/2$, and $3L_b/4$; L_b is the distance between braced points. There is no sign convention associated with Eq. (2-3); the absolute value is used for all moments. The loading is assumed to be applied at midheight of the cross section.

Helwig et al. (1997) presented the following simplified equation for C_b for singly symmetric I-shaped girders subjected to transverse loading applied at different heights on the cross section:

$$C_b^* = 1.4^{2y/h} C_b \tag{2-4}$$

where C_b can be calculated using Eq. (2-3), h is the beam depth, and y is the distance from the midheight to the point of load application. The distance y is negative for loading applied above midheight, and positive for loading applied below midheight. For the two basic simple span cases of loading at the top flange, Eq. (2-4) gives a C_b^* value of 0.94 for a point load at midspan and 0.81 for a load uniformly distributed over the entire span. Eq. (2-3) and (2-4) were developed for use with prismatic beams. Neither equation has been shown to be accurate for stepped beams.

Trahair and Kitipornchai (1971) suggested the following approximate solution for the elastic LTB resistance load of stepped beams of constant depth h:

$$Q = Q_{1,1} - 2\alpha(Q_{1,1} - Q_{\beta,\gamma}) \tag{2-5}$$

in which $Q_{1,1}$ is the buckling load for a uniform member having the larger cross section along entire beam span ($\beta = \gamma = 1$); $Q_{\beta,\gamma}$ is the buckling load for a uniform member having a reduced cross section corresponding to the values of β, and γ; α, β, and γ are ratios defining the relative length and relative width and thickness of the large and small cross sections, respectively. Eq. (2-5) was developed for beams with increased moment of inertia near midspan under a concentrated load.

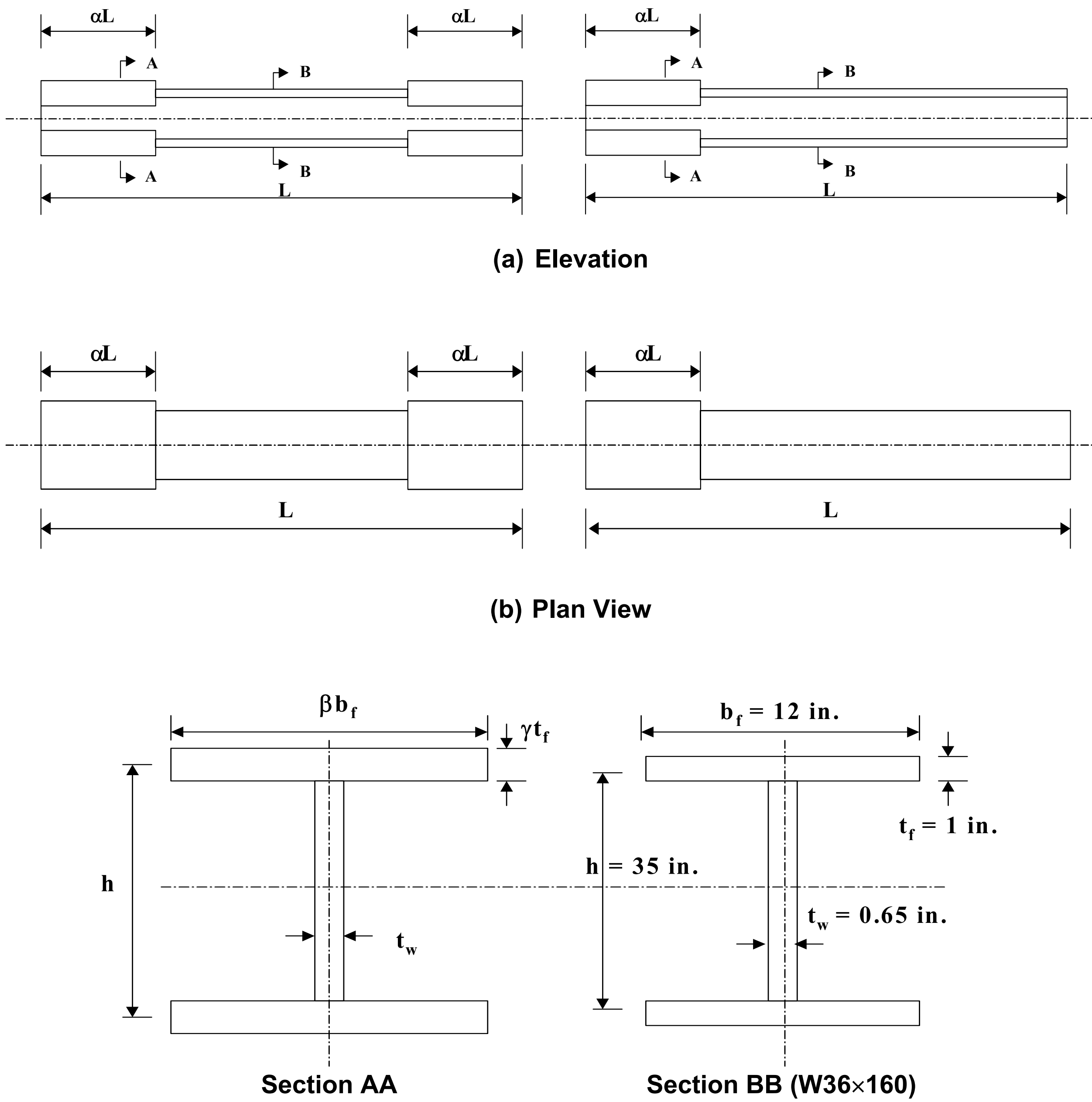
The additional capacity of stepped beams provided by increasing cross section properties (I_y, J, and C_w) at the interior supports of continuous beams can also be estimated using a weighted average approach (WAA) as suggested by Trahair (1993). For each cross section parameter, an average value is calculated by weighting the actual parameters in the individual segments of the span by the proportion of the total span in that segment. Comparisons between this WAA and FEM results are presented later.

OVERVIEW AND FINITE ELEMENT MODELING

The goal of the work presented here was to develop a modifier, C_{st}, that can be multiplied by the critical moment from Equation (2-1) to account for the cross section changes of stepped beams. Equations for C_{st} are presented later along with an expression for C_b so that both the effects of the stepped cross section and moment gradient can be accounted for. Equations for C_{st} were developed using FEM eigenvalue buckling analysis results for stepped beams under constant moment loading. FEM buckling analyses of stepped beams subjected to top flange loading were also performed. These results were used to make adjustments to the coefficients of Eq. (2-3) so that the C_b modifier could be used along with the C_{st} modifier.

Two basic types of stepped beams are considered here: Singly stepped beams with increased flange size at one end, and doubly stepped beams with increased flange size at both ends. Fig. 2.1 shows the doubly and singly stepped beams used in this study. As shown in Fig. 2.1, the flanges of the smaller cross section were fixed at 12 in. (304.8 mm) by 1 in. (25.4mm) while the width and/or thickness of the flanges of the larger cross section were varied. The web thickness and height of beam was kept at 0.65 in. (16.51mm) and 35 in. (889mm), respectively. The ratio of the flange thicknesses, γ, the ratio of the flange widths, β, and the ratio of stepped length of beam, α, are also defined in Fig. 2.1. For the special case of β = 1 and γ = 1, the beam is prismatic. From the geometry of typical stepped bridge girders, ranges of the three ratios for stepped beams were established, and these ranges are given in Table 2.1. The ranges given in Table 2.1 define the limits of the FEM investigation.

The length, L, in Fig. 2.1 represents a typical unbraced length in a beam. At the ends of the unbraced length, the beam was free to warp. The ratio of the unbraced length to the height of the stepped beam models was varied from 15 to 25 in the investigation of beams subjected to constant moment. It was found that the value of C_{st} for stepped beams increases as the length-height ratio increases, and these results are presented later in this report. A length-height ratio



(c) Cross Section

FIG. 2.1 Definition of Parameters for Doubly and Singly Stepped Beams (a) Elevation (b) Plan View (c) Cross Sections (1 in. = 25.4 mm)

of 21 was used for beams subjected to applied transverse loading.

The 3D finite-element program MSC/NASTRAN (1998) and graphical package MSC/PATRAN (2000) was used to perform the eigenvalue buckling analyses. MSC/NASTRAN is a well-established general-purpose finite-element code that is extremely versatile and powerful, especially in modeling the behavior of metal structures. For the present investigation, MSC/NASTRAN was used to model the full three-dimensional configuration of the cross section using QUAD4 elements.

TABLE 2.1 Ranges of Ratios for Stepped Beams Investigated

Model Type	Length Ratio (α)	Flange Width Ratio (β)	Flange Thickness Ratio (γ)
Doubly Stepped Beam	0.167 - 0.333	1.0 - 1.4	1.0 - 1.8
Singly Stepped Beam	0.167 - 0.5	1.0 - 1.4	1.0 - 1.8

FINITE ELEMENT RESULTS AND DESIGN RECOMMENDATIONS

Stepped Beams under Constant Moment

81 models were analyzed for doubly stepped beams, and 108 models were analyzed for singly stepped beams. Span-to-height ratios of 15, 20, and 25 were used. At each span-to-height ratio, the 27 combinations of α, β, and γ shown in Table 2.2 were used for doubly stepped beams, and the 36 combinations shown in Table 2.3 were used for singly stepped beams.

Doubly Stepped Beams

From the results of the finite element investigation, it was found that the buckling moments varied approximately with the square of the length ratio (α), and a change of flange thickness is more significant than a change of flange width on the LTB capacity of a stepped beam. The proposed design equation for doubly stepped beams under uniform bending is:

$$M_{ost} = C_{st} M_{ocr} \qquad (2\text{-}6)$$

in which

$$C_{st} = C_o + 6\alpha^2(\beta\gamma^{1.3} - 1) \qquad (2\text{-}7)$$

where M_{ost} is the critical LTB moment; M_{ocr} is the LTB moment of an equal length prismatic beam having the smaller cross section along the entire span; C_o is a constant equal to 1 for constant moment loading and α, β, and γ are the ratios previously defined in Fig. 2.1.

Fig.2.2 (a), (b), and (c) provide comparisons between FEM results and the proposed solution for span-to-height ratios of 15, 20, and 25, respectively. C_{st} from Eq. (2-7) is plotted as a solid line. Individual FEM results were plotted by dividing the critical moment from the FEM

TABLE 2.2 Parameters Used in the FEM Analyses of Doubly Stepped Beams

α	β	γ
0.167	1.0	1.2
0.167	1.2	1.0; 1.4; 1.8
0.167	1.4	1.0; 1.4; 1.8
0.25	1.0	1.4; 1.8
0.25	1.2	1.0; 1.4; 1.8
0.25	1.4	1.0; 1.4; 1.8
0.333	1.0	1.2; 1.4; 1.6; 1.8
0.333	1.2	1.0; 1.4; 1.8
0.333	1.4	1.0; 1.2; 1.4; 1.6; 1.8

TABLE 2.3 Parameters Used in the FEM Analyses of Singly Stepped Beams

α	β	γ
0.167	1.0	1.2; 1.4; 1.8
0.167	1.2	1.0; 1.4; 1.8
0.167	1.4	1.0; 1.4; 1.8
0.25	1.0	1.2; 1.4; 1.8
0.25	1.2	1.0; 1.4; 1.8
0.25	1.4	1.0; 1.4; 1.8
0.333	1.0	1.2; 1.4; 1.8
0.333	1.2	1.0; 1.4; 1.8
0.333	1.4	1.0; 1.4; 1.8
0.5	1.0	1.2; 1.4; 1.8
0.5	1.2	1.0; 1.4; 1.8
0.5	1.4	1.0; 1.4; 1.8

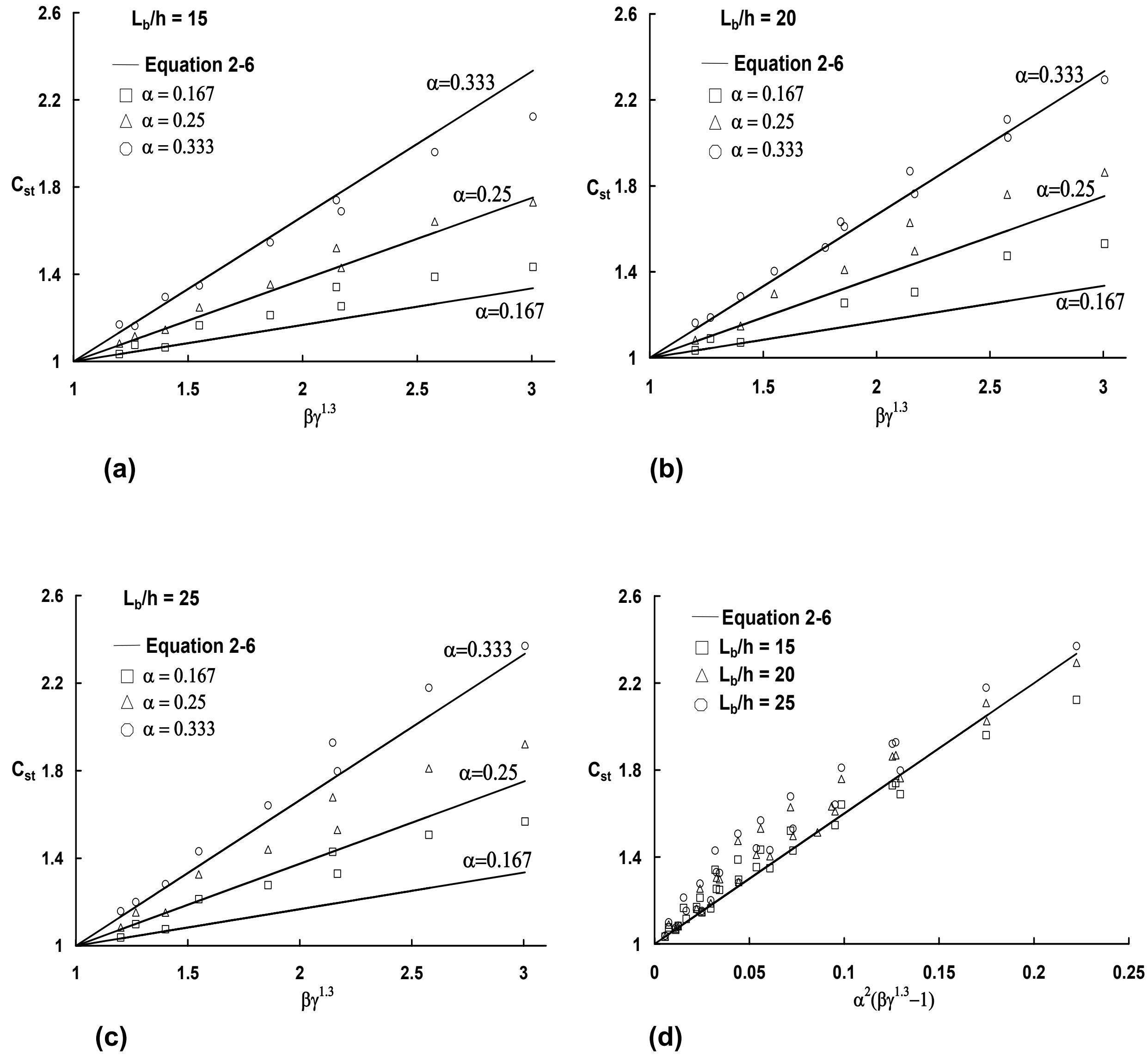


FIG. 2.2 FEM Results and Proposed Solution for Doubly Stepped Beams: (a) L_b/h=15; (b) L_b/h=20; (c) L_b/h=25; (d) All Models

analysis by M_{ocr} as defined for Eq. (2-6). The FEM results of Fig.2.2 show that C_{st} values increase as span-to-height ratios increase. Fig.2.2 (d) shows a comparison between the proposed solution and all FEM results from Fig. 2.2 (a), (b), and (c). The proposed solution is linear in the quantity $\alpha^2(\beta\gamma^{1.3}-1)$. Fig. 2.2 (d) shows that the proposed solution is conservative for most cases, but it is unconservative for some cases with L_b/h = 15 and 20 at α = 0.333. The maximum difference for an unconservative estimate is 10% at L_b/h = 15 with α = 0.333, β = 1.4, and γ = 1.8. The maximum difference for a conservative estimate is 16% at L_b/h = 25 with α = 0.167, β = 1.2, and γ = 1.8. The accuracy of the proposed solution is acceptable over the range of parameters investigated.

Singly Stepped Beam

The proposed design equation for singly stepped beams under uniform bending is:

$$M_{ost} = C_{st} M_{ocr} \tag{2-8}$$

in which

$$C_{st} = C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1) \tag{2-9}$$

where M_{ost} is the critical LTB moment; M_{ocr} is the LTB moment of an equal length prismatic beam having the smaller cross section; C_o is a constant equal to 1 for uniform bending; α, β, and γ are the ratios previously defined in Fig. 2.1.

A comparison between FEM results and the proposed solution for span-to-height ratios of 15, 20, and 25 is shown in Fig. 2.3. These graphs also show that the C_{st} values from the FEM results for singly stepped beams increase as span-to-height ratio increases. Fig. 2.3 shows that the proposed solution gives conservative values for most cases, but for some cases, it is

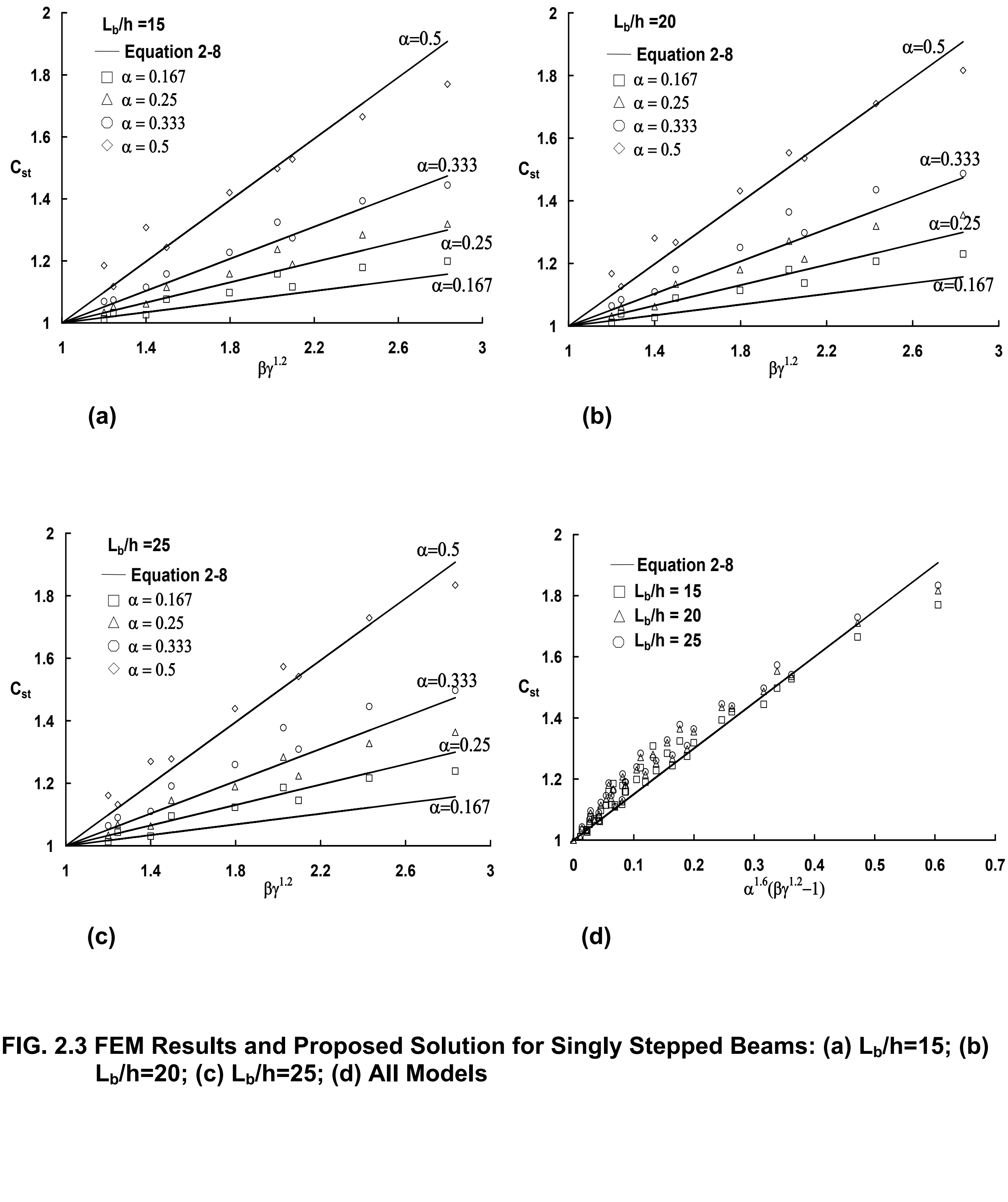


FIG. 2.3 FEM Results and Proposed Solution for Singly Stepped Beams: (a) L_b/h=15; (b) L_b/h=20; (c) L_b/h=25; (d) All Models

unconservative. The maximum difference for an unconservative estimate is 7% at L_b/h = 15 with α = 0.5, β = 1.4, and γ = 1.8. The maximum difference for a conservative estimate is 9% at L_b/h = 25 with α = 0.25, β = 1.0, and γ = 1.8. Fig. 2.3 (d) shows that the proposed solution is in good agreement with the FEM results.

Stepped Beams under General Loading Conditions

FEM eigenvalue analyses of 1134 beam models with applied loading were performed. For doubly stepped beams, the 27-parameter combinations of Table 2.2 were investigated for 18 applied load cases. For singly stepped beams, the 36-parameter combinations of Table 2.3 were investigated for 18 applied load cases. The load cases included top flange loading by a concentrated load at midspan or a uniformly distributed load, in combination with no end moments, one end moment, or moments at each end. All these 1134 models were for beams with L_b/h of 21.

The proposed design equation for stepped beams under general loading conditions is:

$$M_{st} = C_{bst} C_{st} M_{ocr} \tag{2-10}$$

in which C_{st} and M_{ocr} are as defined previously; and C_{bst} is a moment gradient modifier. C_o values for use in the C_{st} equation, and C_{bst} values are discussed below. Best estimates for the critical LTB moment are obtained by choosing C_o and C_{bst} values based on the number of zero moment points, inflection points in the deflected shape, within the unbraced length.

Cases investigated with no zero moment points include simply supported beams subjected to uniform moment, to only one end moment, or to applied transverse loading. For cases with no zero moment points, C_o for Equations (2-7) and (2-9) should be taken equal to 1. Uniform moment cases were discussed previously. Best results for non-uniform moment are obtained by using Eq. (2-3) for calculating the moment gradient modifier, C_{bst}, along with the modifier for load height given by Eq. (2-4). Comparisons of FEM results and Eq. (2-10) for singly and doubly stepped beams are provided in Fig. 2.4. In that figure, the ratios of FEM result to the result from Eq. (2-10) are plotted against the value of γ. The parameter γ was used to provide some separation in the results. Ratios are plotted for values of the combinations of parameters listed in Table 2.2 and 2.3. Fig. 2.4 shows that the proposed design equation produces reasonably accurate results that are conservative for almost all the cases investigated.

Cases investigated with one zero moment point in the unbraced length include propped cantilever beams with uniformly distributed loading or a concentrated load at midspan. For cases with one zero moment point, C_o for Eq. (2-7) and Eq. (2-9) for singly and doubly stepped beams should be taken as 1. The following moment gradient modifier should be used, and the effect of top flange loading is included:

$$C_{bst} = \frac{10M_{\max}}{4M_{\max} + M_A + 7M_B + M_C} \qquad (2\text{-}11)$$

Comparisons of FEM results and Eq. (2-10) for singly and doubly stepped beams are provided in Fig. 2.5. In that figure the ratio of the FEM result to the result from Eq. (2-10) are plotted against the value of γ similar to Fig 2.4. Ratios are plotted for values of the combinations of parameters listed in Table 2.2 and 2.3. Fig. 2.5 shows that the proposed design equation produced reasonably accurate results that are conservative for almost all the cases investigated.

Cases investigated with two zero moment points in the unbraced length include cases with negative end moments, and uniformly distributed loading or a concentrated load at midspan. For cases with two zero moment points, C_o for Eq. (2-7) and (2-9) for singly and

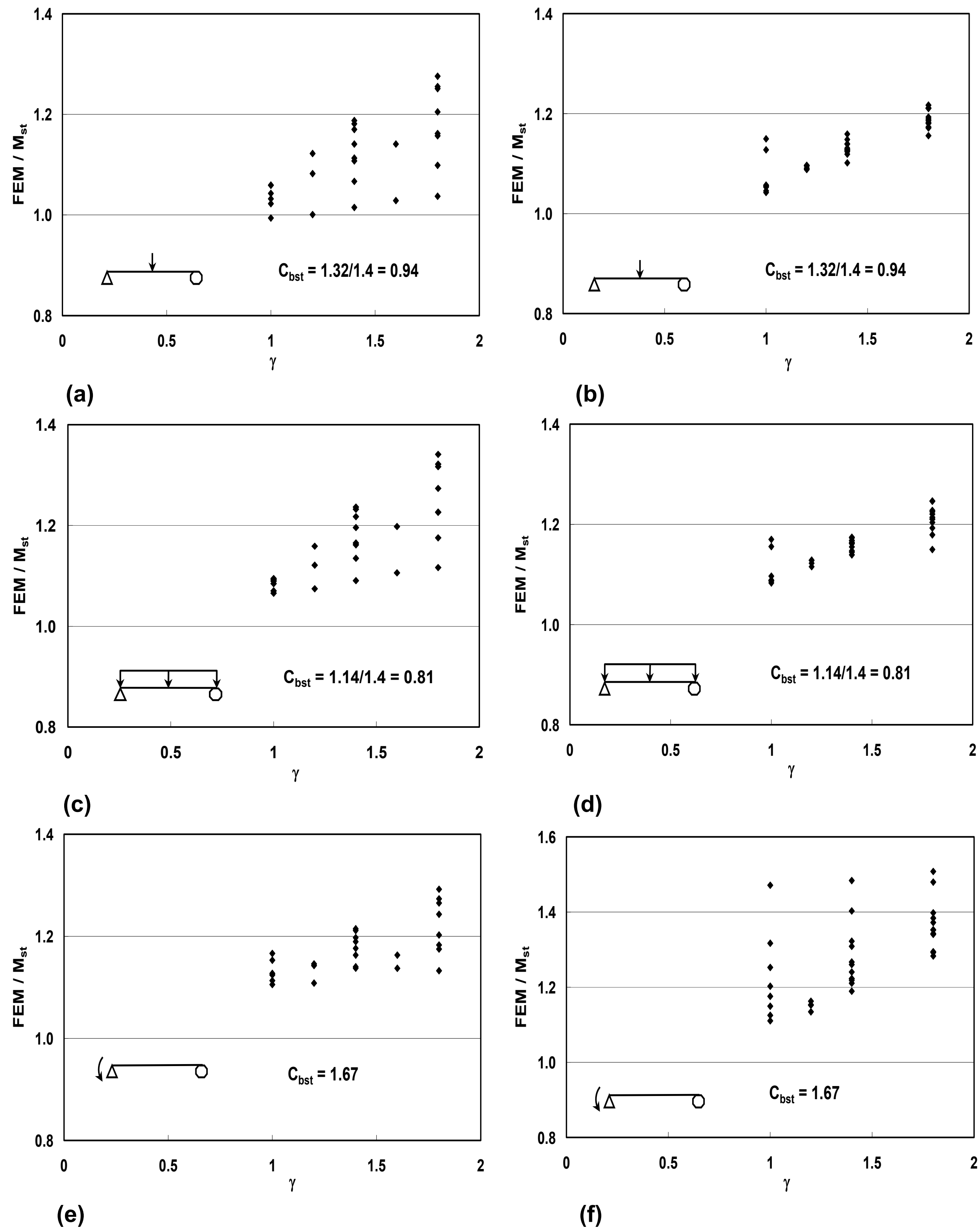


FIG. 2.4 Cases with No Zero Moment Points: (a), (c) and (e) Doubly Stepped Beams; (b), (d) and (f) Singly Stepped Beams

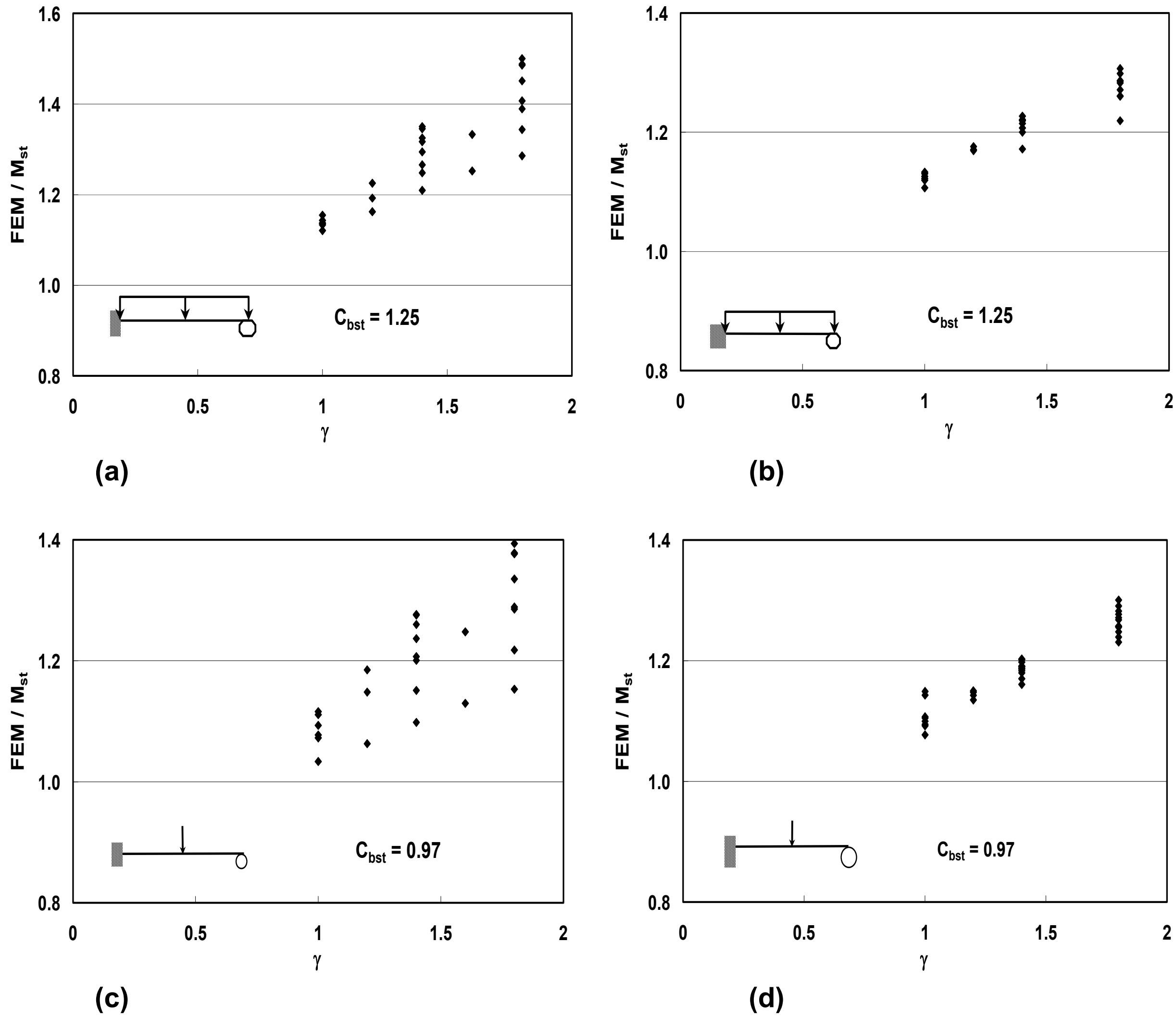


FIG. 2.5 Cases with One Zero Moment Point: (a) and (c) Doubly Stepped Beams; (b) and (d) Singly Stepped Beams

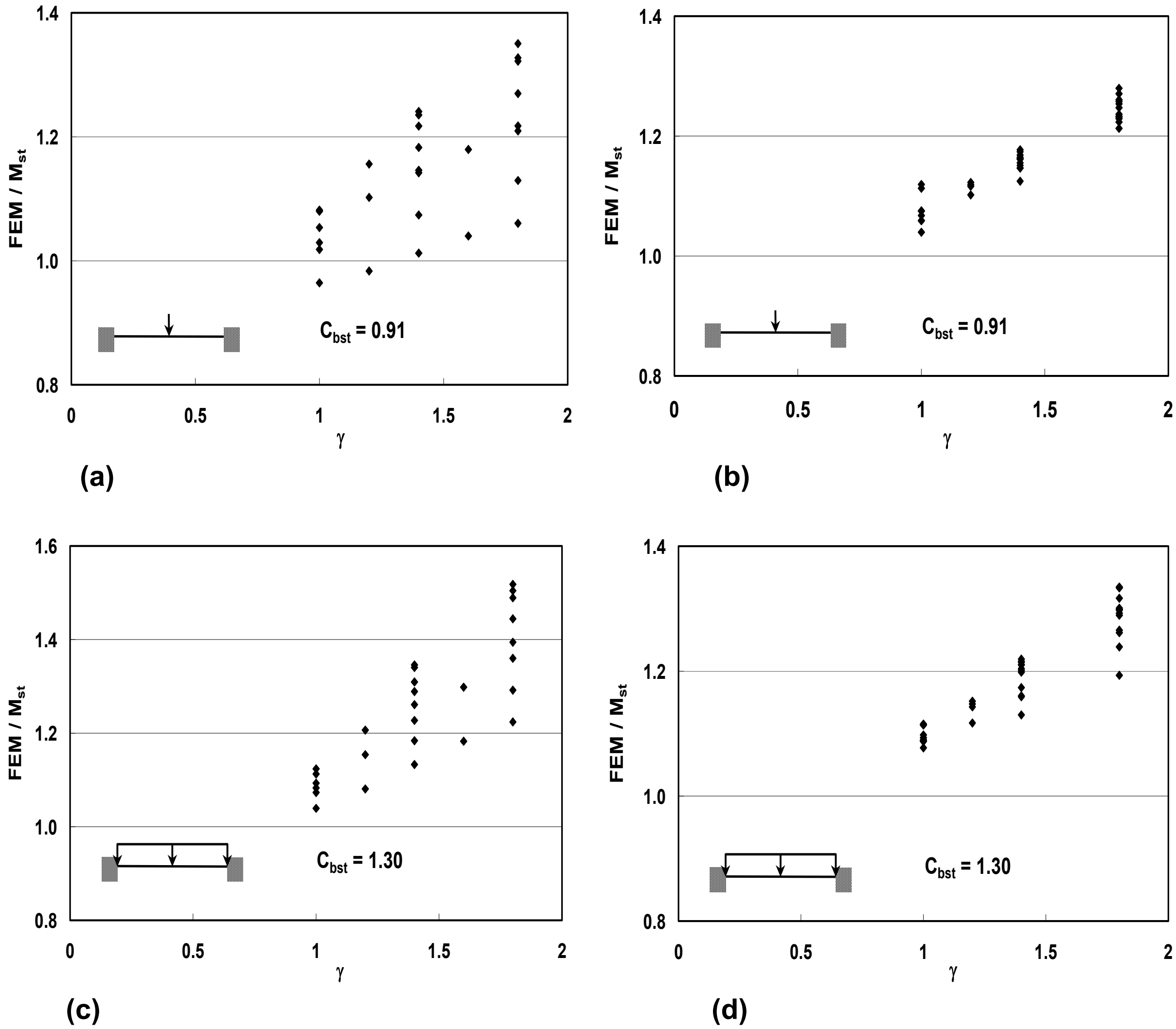


FIG. 2.6 Cases with Two Zero Moment Points: (a) and (c) Doubly Stepped Beams; (b) and (d) Singly Stepped Beams

doubly stepped beams should be taken as 0.85. The moment gradient modifier of Eq. (2-11) should be used, and the effect of top flange loading is included. Comparisons of FEM results and Eq. (2-10) for singly and doubly stepped beams are provided in Fig. 2.6. In Figure 2.6 the ratio of the FEM result to the result from Eq. (2-10) are plotted against the value of γ similar to above. Ratios are plotted for values of the combinations of parameters listed in Table 2.2 and 2.3. Fig. 2.6 shows that the proposed design equation produced reasonably accurate results that are conservative for almost all the cases investigated.

SUMMARY AND APPLICATONS

Studies on how to determine the LTB capacity of continuous beam spans with increased cross sections at the interior supports were executed. Results from FEM analyses for a number of different stepped beams were used to develop design equations. The critical LTB moments from Eq. (2-10) are calculated by choosing C_o in the C_{st} equation and C_{bst} values based on the number of inflection point within the unbraced length. Table 2.4 summarizes the application of C_o, C_{st}, and C_{bst} values for design of stepped beams.

Table 2.5 and Table 2.6 show comparisons between the proposed equations and FEM results for doubly and singly stepped beam spans of existing highway bridges. Each model of Table 2.5 is an interior span of a continuous beam as described in Fig. 2.7. Each model of Table 2.5 was loaded by a uniformly distributed load on the top flange and had a negative bending moment at each end. Models A, B and C of Table 2.6 are end spans of continuous beams. These models were loaded by a uniformly distributed load on the top flange and one negative end moment, the other end moment was equal to zero. Case D of Table 2.6 is an interior span and was loaded by a uniformly distributed load on the top flange and negative end moments.

Percentages in the last column of the Table 2.5 and 2.6 indicate the difference between the FEM result and the proposed solution of Eq. (2-10). These tables also include an estimate of

LTB resistance by using weighted average cross-section parameters (WAA) along with Eq. (2-3), and Eq. (2-4). Table 2.5 and 2.6 indicate that the LTB estimates from the weighted average approach are unconservative. The proposed solution produced conservative estimates of the LTB resistance for all cases. Note that most of the models of Table 2.5 and 2.6 have higher length-to-height ratios than the value of 21 used for the FEM models used in developing the proposed solution. The details of the process of applying the proposed solution, Eq. (2-10), are illustrated in the Appendix.

TABLE 2.4 Summary of Modifier for LTB Moment Resistance of Stepped Beams

Number of Inflection Points	C_o	C_{st}		C_{bst}
		Double	Single	
0	**1**	$C_o + 6\alpha^2(\beta\gamma^{1.3} - 1)$	$C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$	$1.4^{2y/h}\left(\frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_c}\right)$
1	**1**	$C_o + 6\alpha^2(\beta\gamma^{1.3} - 1)$	$C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$	$\frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$
2	**0.85**	$C_o + 6\alpha^2(\beta\gamma^{1.3} - 1)$	$C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$	$\frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$

TABLE 2.5 Comparisons between Proposed Solution and FEM Results of Real Structural System for Doubly Stepped Beams

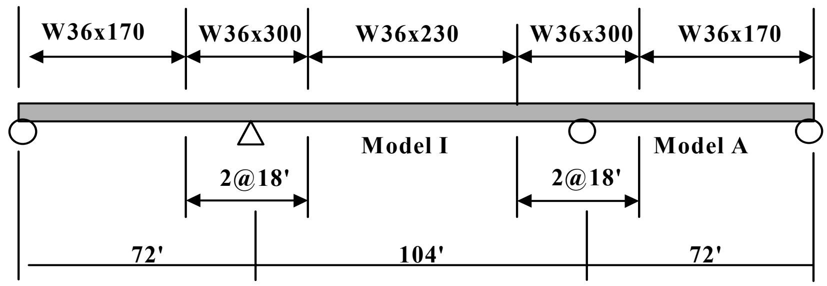
Model	Stepped Ratio			L_b/h	Applied Load Moments					LTB Moments (kip-ft)						Diff. (%)
	α	β	γ		M_0	M_A	M_B	M_C	M_1	M_{ocr} (Eq. 2-1)	C_{bst} (Eq. 2-11)	C_{st} (Eq. 2-7)	M_{st} (Eq. 2-10)	M_{st} (FEM)	M_{st} (WAA)	
I	0.17	1.01	1.33	36.0	-1	0.27	0.69	0.27	-1	651	1.07	0.93	648	1020	1066	36.4
II	0.22	1.36	1.07	28.3	-1	0.23	0.64	0.23	-1	399	1.12	0.99	442	606	833	27.1
III	0.24	1.13	1.67	30.0	-1	0.13	0.51	0.14	-0.99	179	1.28	1.27	290	372	562	22.0
IV	0.13	1.0	1.80	27.5	-1	0.19	0.59	0.19	-1	277	1.18	0.96	313	501	596	37.5

Note: 1 kip = 4.45 KN; 1 ft = 0.3048 m; 1 kip-ft = 1.356 KN-m

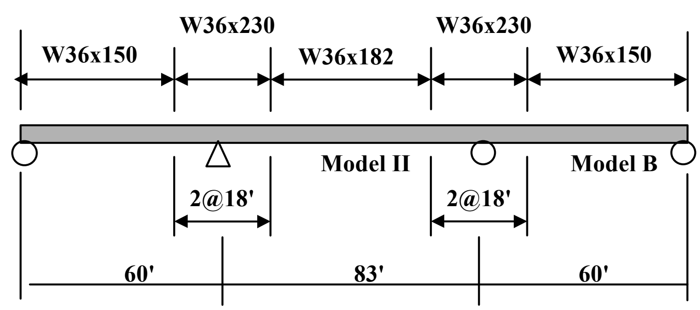
TABLE 2.6 Comparisons between Proposed Solution and FEM Results of Real Structural System for Singly Stepped Beams

Model	Stepped Ratio			L_b/h	Applied Load Moments					LTB Moments (kip-ft)						Diff. (%)
	α	β	γ		M_0	M_A	M_B	M_C	M_1	M_{ocr} (Eq. 2-1)	C_{bst} (Eq. 2-11)	C_{st} (Eq. 2-9)	M_{st} (Eq. 2-10)	M_{st} (FEM)	M_{st} (WAA)	
A	0.25	1.38	1.53	24.6	-1	-0.14	0.31	0.36	0	411	1.50	1.21	746	1190	1253	37.3
B	0.30	1.37	1.34	20.6	-1	-0.11	0.36	0.39	0	397	1.43	1.21	686	911	1092	24.7
C	0.17	1.0	1.80	20.6	-1	-0.08	0.39	0.42	0	397	1.38	1.09	597	812	812	26.5
D	0.26	1.01	1.16	22.8	-1	0.04	0.31	0.08	-0.75	395	1.58	0.89	555	766	924	27.5

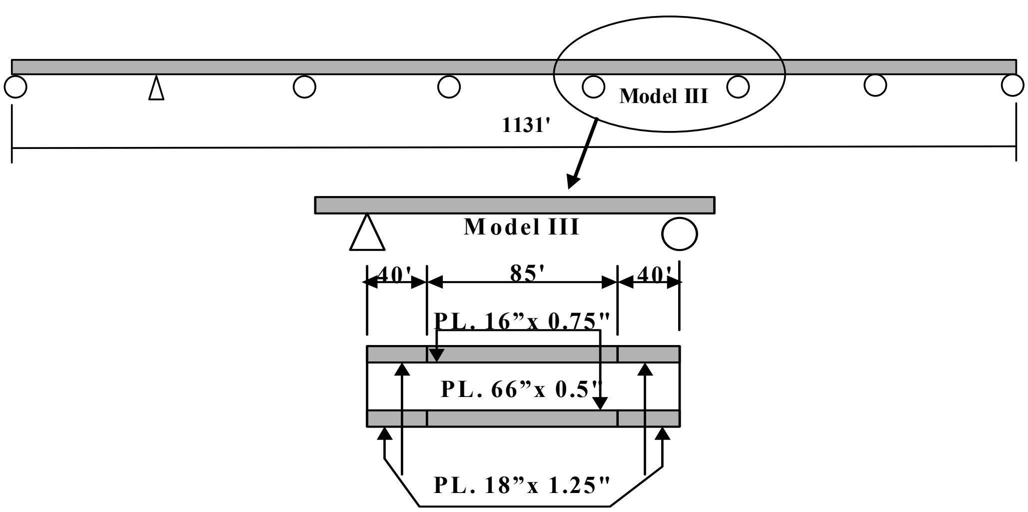
Note: 1 kip = 4.45 KN; 1 ft = 0.3048 m; 1 kip-ft = 1.356 KN-m



(a) Three-Span Continuous Bridge on I-59/20 in Birmingham, Alabama

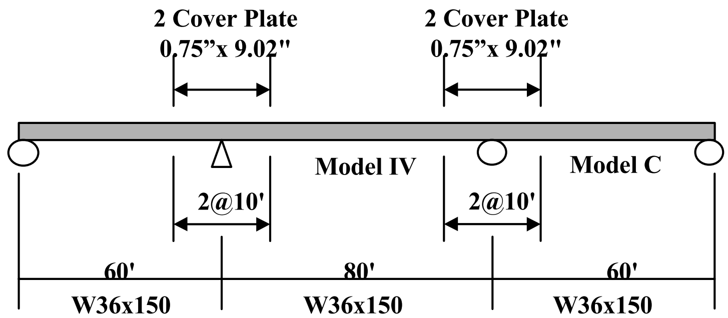


(b) Three-span Continuous Bridge on I-59 in Birmingham, Alabama

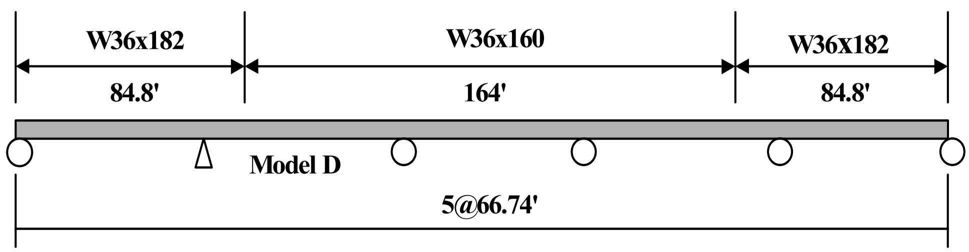


(c) Seven-Span Continuous Bridge in Phoenix, Arizona

FIG. 2.7 Beam Details for Existing Bridges with Stepped Cross Sections



(d) Three-Span Continuous Bridge at Tuscaloosa, Alabama



(e) Five-Span Continuous Bridge for a Railroad Viaduct on I-65 in Birmingham, Alabama

FIG. 2.7 Beam Details for Existing Bridges with Stepped Cross Sections (Continued)

CHAPTER 3. BUCKLING OF STEPPED BEAMS WITH CONTINUOUS BRACING

INTRODUCTION

Current American Association of State Highway and Transportation Officials (AASHTO) *Specifications* (1996) require transverse diaphragms to be placed in multi-girder steel bridges at a maximum spacing of 25 ft (7.6 m) between lines of diaphragms. A theoretical explanation for the required maximum spacing is not available. The AASHTO *LRFD Bridge Design Specifications* (1998) do not have a strict requirement for diaphragms but allow the designer to use diaphragms as needed.

The unbraced length L_b in Eq. (2-1) is typically interpreted by designer to be the distance between diaphragm lines when the resistance to LTB is calculated. Using this interpretation, the M_{cr} from Eq. (2-1) does not account for any additional capacity provided by lateral bracing between the ends of the segment such as provided by the slab.

Yura (1993) studied various loading and bracing combinations to develop improved moment gradient factors, C_b, for cases involving lateral-torsional buckling. A solution of Yura's that is particularly pertinent to this research is for beams with bracing at the ends and with continuous lateral bracing of the top flange (SSRC *Guide*, 1998). If the moments at either or both ends produce a compressive stress on the bottom flange, the moment gradient factor is given as:

$$C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) + \frac{8}{3}\frac{M_{CL}}{(M_0 + M_1)} \qquad (3\text{-}1)$$

where, M_0 is the end moment that produces the largest compressive stress on the bottom flange; M_1 is the other end moment; and M_{CL} is the moment at the centerline of the segment. Positive values should be substituted into above equation for M_0 and M_1 when these moments produce compressive stress in the bottom flange. A positive value should be substituted for M_{CL} when this moment produces tensile stress in the bottom flange. For the quantity ($M_0 + M_1$) in the equation, M_1 should be taken as zero when term M_1 is negative. Eq. (3-1) is easy to use in beam design, but its accuracy is not well documented.

A topic of importance in the general area of beam design is the LTB behavior of non-prismatic beams. Girders or beams with stepped cross section are used frequently because of low cost and ease of application. Steps in the cross section can be achieved by adding cover plates to the beam flanges, changing the size of the hot rolled cross section, or changing the flange thickness and/or width for built-up sections. Most studies performed on non-prismatic beams consider beams without intermediate bracing. None provides a method that can be readily used for design of stepped beams having continuous top flange bracing. Therefore, study of stepped beams with continuous lateral top flange bracing as presented here is necessary to be consistent with many real structural systems common in bridges.

OVERVIEW AND FINITE ELEMENT MODELING

Goals of the work presented here were to evaluate the accuracy of Eq. (3-1) for beam design and to develop a modifier, C_{st}, that can be applied to the moment gradient modifier to account for the cross section changes of stepped beams. Equations for C_{st} are presented later along with Eq. (3-1) so that the effects of the stepped cross section, moment gradient, and continuous top flange bracing can be accounted for. FEM eigenvalue buckling analyses of beams were performed to provide buckling data that allowed the goals to be accomplished.

Helwig et al. (1997) showed that the moment gradient factor, C_b, increased as the ratio of the unbraced length to the height of beams, L_b/h, increase if a load is applied at top flange. The correlation between the ratio of the unbraced length to the height and LTB capacity of beams was also investigated in the research here. It was found that the value of C_b for stepped beams increases as the length-height ratio, L_b/h, increases. The results are presented later to develop the length-height ratio factor for stepped beams.

As in Chapter 2, two basic types of stepped beams are considered : Singly stepped beams with increased flange size at one end, and doubly stepped beams with increased flange size at both ends. Fig. 3.1 shows the doubly and singly stepped beams with continuous lateral top flange bracing used in this study. As shown in Fig. 3.1, the model configurations of the flanges and the web are the same as shown in Fig. 2.1, only the bracing conditions are different.

The 3D finite-element program MSC/NASTRAN (1998) and a graphical package MSC/PATRAN (2000) was used to investigate analytically the lateral-torsional buckling behavior. The full three-dimensional configuration of the cross-section using QUAD4 elements was considered. Fig. 3.2 shows loading conditions having three types of moment diagrams from combinations of end moments and uniform loading that were considered initially in the investigation. Additional loadings were investigated as illustrated later in this report.

FINITE-ELEMENT METHOD RESULTS AND DESIGN RECOMMENDATIONS

LTB of Prismatic Beams with Continuous Bracing

Fig.3.3 (a) and (b) are graphs of M_{cr} /M_{ocr} versus L_{cb}/L_b. M_{cr} is the FEM result for the critical moment of a prismatic beam with continuous lateral top flange bracing. M_{ocr} is the elastic LTB resistance for an I-shaped prismatic section under the action of constant moment in the plane of the web over the laterally unbraced length (Timoshenko, 1961).

Fig.3.3 (a), and (b) are graphs for load type II, and type III, respectively. Buckling of beams with continuous top flange lateral bracing results from compression in the bottom flange produced by negative end moments. As shown in Fig.3.3, the ratio M_{cr}/M_{ocr} increases significantly when less than approximately 30% of the bottom flange is in compression (L_{cb}/L_b <

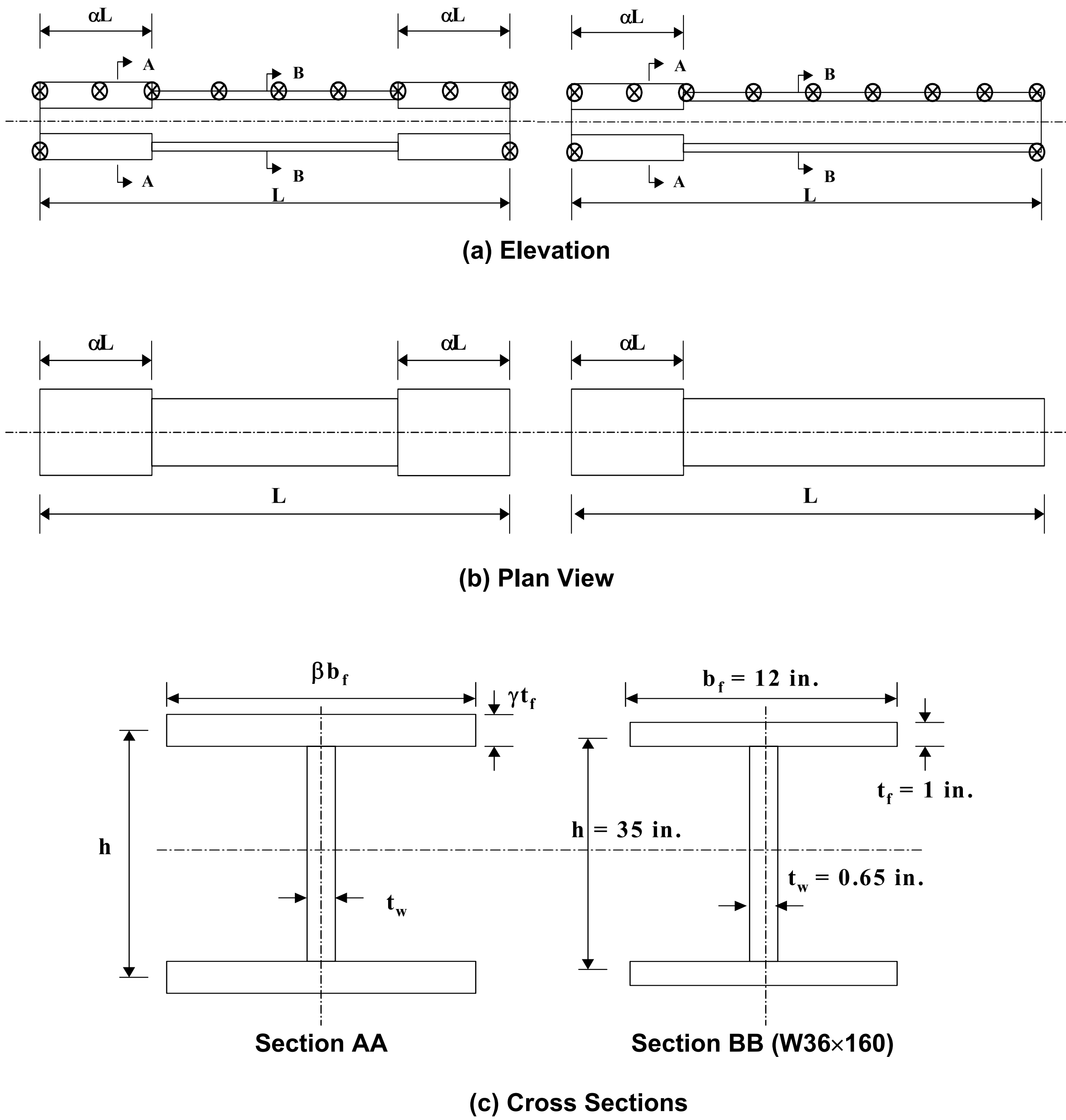


FIG. 3.1 Parameters Defining Doubly and Singly Stepped Beams: (a) Elevation; (b) Plan View; (c) Cross Sections (1 in. = 25.4 mm)

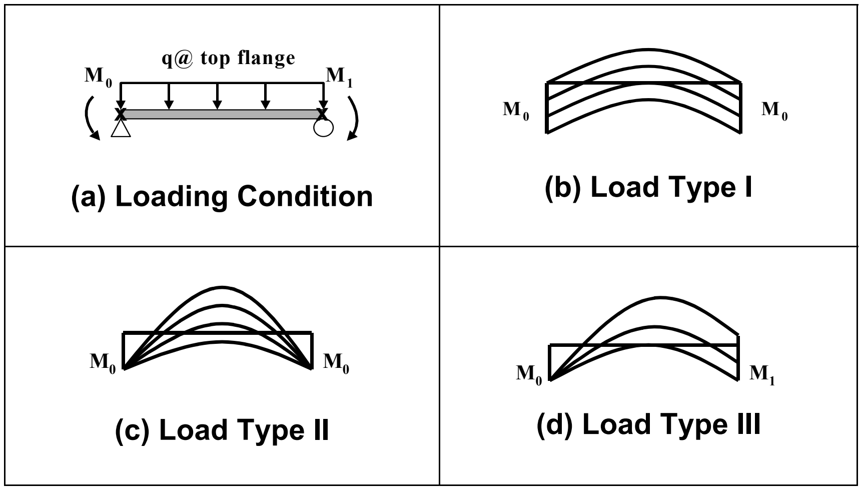
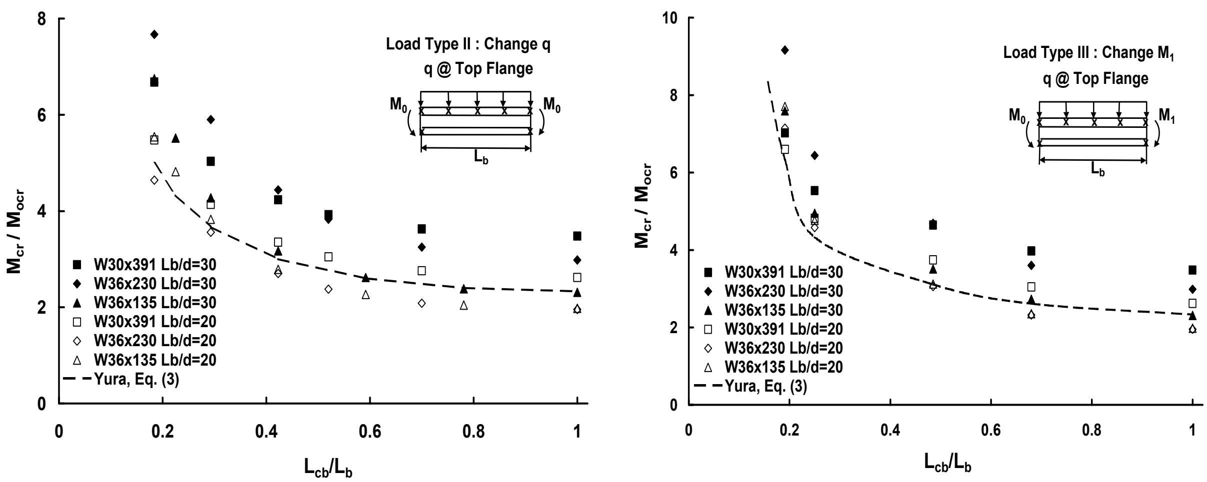


FIG. 3.2 Moment Diagrams of Three Load Types



(a) Load Type II **(b) Load Type III**

FIG. 3.3 Results from FEM and Eq. (3-1)

0.3). Fig.3.3 also shows that the ratio M_{cr}/M_{ocr} for beams with top flange loading increases as the length-depth ratio increases.

The dashed line in Fig. 3.3 (a) and (b) is the result from Eq. (3-1). Eq. (3-1) predicts C_b values that are conservative relative with the FEM results for most cases, particularly for the W30x391 (relatively stocky beam) with L_b/d of 20 or 30. For the W36x230 and W36x135 with L_b/d of 20, the C_b values obtained from Eq. (3-1) are a little unconservative with respect to FEM results. The maximum difference between an unconservative estimate from Eq. (3-1) and the FEM results is 14% for the W36x135 with L_b/d of 20 at $L_{cb}/L_b = 1$ for load type II.

Fig. 3.4 shows a comparison between Eq. (3-1) and FEM results for beams with a concentrated load and either one or two negative end moments. As can be seen in Fig. 3.4, Eq. (3-1) gives very unconservative values for these loading cases. The reason for this can be explained by inspection of the bending moment diagrams shown in Fig. 3.5. A beam subjected only to a concentrated load and negative end moments has a more negative moment area, thus a larger percentage of the bottom flange in compression than a beam subjected to uniformly distributed load and negative end moments if these beams have same values of M_1, M_0 and M_{CL}.

The following equation for C_b similar to Eq. (3-1) is proposed for I-shaped beams subjected to a concentrated load on the top flange and negative moments at either or both ends:

$$C_b = 2.5 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) + \frac{5}{3}\frac{M_{CL}}{(M_o + M_1)} \qquad (3\text{-}2)$$

where M_0 is the end moment that produces the largest compressive stress on the bottom flange; M_1 is the other end moment; and M_{CL} is the moment at the centerline of the segment. The positive and negative definitions of M_0, M_1 and M_{CL} are same as the Eq. (3-1). Fig.3.4 also

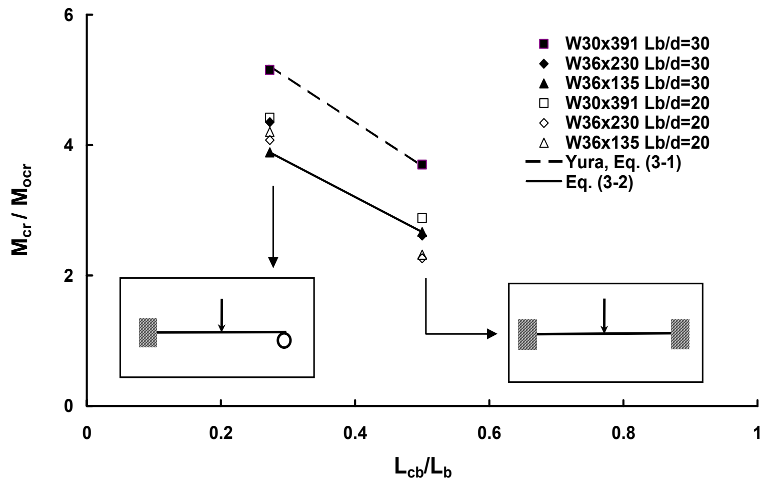


FIG. 3.4 Results from FEM, Eq. (3-1), and Eq. (3-2) for Beams with a Point Load

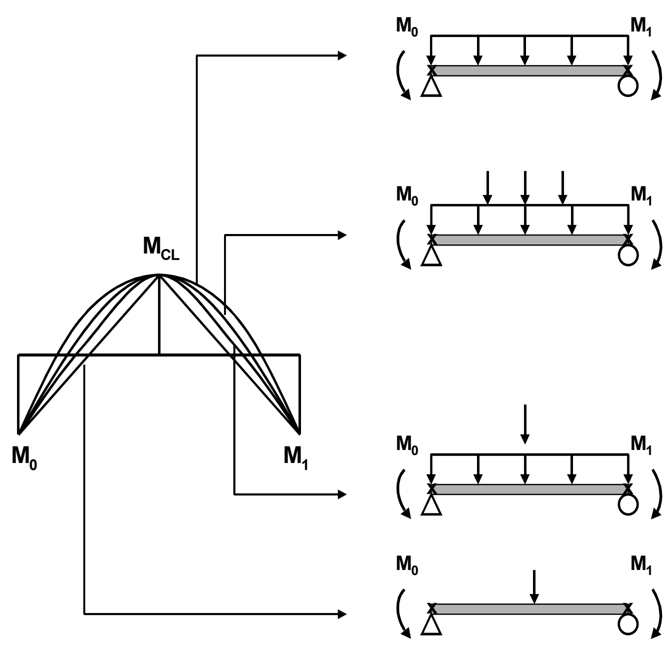


FIG. 3.5 Moment Diagrams for Several Loading Conditions

includes a comparison between Eq. (3-2) and FEM results for propped cantilevers and fixed beams with a concentrated load. In this figure, the maximum difference for an unconservative estimate is 18% for the W36x230 beam with L_b/d of 20. The maximum difference for a conservative estimate is 25% for a propped cantilever W30x391 beam with L_b/d of 30.

Fig.3.5 shows the moment diagrams for beams subjected to a concentrated load, beams subjected to uniformly distributed load, beams subjected to both uniformly distributed load and a concentrated load or a series of concentrated loads with end negative moments. All moment diagrams have the same values of M_{CL}, M_0, and M_1. Fig. 3.5 shows that the shape of moment diagram for a beam subjected to both uniformly distributed load and a series of concentrated loads, which is a typical design loading condition on a bridge, is more similar to the shape of the moment diagram for a beam subjected to uniformly distributed load than the shape of the moment diagram for a beam subjected to a concentrated load. For beams having both uniformly distributed load and a concentrated load or a series of concentrated loads, Eq. (3-1) is recommended for obtaining LTB moment resistances of the beams.

LTB of Stepped beams with Continuous Bracing

FEM eigenvalue analyses of 1071 beam models were performed. For doubly stepped beams, the 27-parameter combinations of Table 2.2 were investigated for 17 applied load cases. For singly stepped beams, the 36-parameter combinations of Table 2.3 were investigated for 17 applied load cases. The load cases included top flange loading by a concentrated load at midspan or a uniformly distributed load, in combination with one end moment, or one moment at each end. All these 1071 models were for beams with L_b/h of 21. Additional 48 beam models were analyzed to develop a new design factor for considering the variety of the critical moment resistance due to change of L_b/h.

The proposed design equation for stepped beams is:

$$M_{st} = FC_{bst}C_{st}M_{ocr} \tag{3-3}$$

in which C_{bst} should be calculated using Eq. (3-2) for stepped beams under a concentrated load and Eq. (3-1) for stepped beams under uniformly distributed load or both a concentrated load and uniformly distributed load; $C_{st} = C_o + 6\alpha^2(\beta\gamma^{1.3}-1)$ for doubly stepped beams and $C_{st} = C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2}-1)$ for singly stepped beams; M_{ocr} is the LTB moment of an equal length prismatic beam having the smaller cross section along the entire span. F is the length-height ratio factor. For prismatic beams, F and C_{st} are taken as 1. Best estimates for critical buckling moments of stepped beams are obtained by choosing C_o based on the number of negative end moments, and F based on the type of stepped beams, singly or doubly stepped beams. C_o values and F factors are discussed below.

Cases investigated with a negative moment at each end include beams under uniform bending, and beams with uniformly distributed loading or a concentrated load at midspan with two end moments. For cases with two negative end moments, C_o in the equation C_{st} for singly and doubly stepped beams should be taken as 0.9 as illustrated by the beam sketches in Fig. 3.6. Comparisons of FEM results and Eq. (3-3) with F of 1 for singly and doubly stepped beams are shown in Fig. 3.6. In that figure, the ratio of FEM results to the result from Eq. (3-3) are plotted against the value of γ. The parameter γ was used to provide some separation in the results. Ratios are plotted for the parameter value combinations listed in Table 2.2 and 2.3. Fig. 3.6 shows that the proposed design equation produces reasonably accurate results that are conservative for almost all the cases investigated.

Cases investigated with a negative moment at one end include beams with one end moment with no transverse load, and propped cantilever beams with a uniformly distributed load or a concentrated load. For cases with a negative moment at one end, C_o in the equation C_{st} for singly and doubly stepped beams should be taken as 1.25. Comparison of FEM results and Eq.

(3-3) with F of 1 are provided in Fig. 3.7. In Figure 3.7 the ratio of the FEM result to the results from Eq. (3-3) are plotted against the value of γ similar to Fig. 3.6. Results in Fig.3.7 show that the proposed design equation produced reasonably accurate results that are conservative for almost all cases investigated.

Results presented in Fig. 3.8 illustrate that the critical moment resistance of stepped beams with continuous lateral top flange bracing increases as the length-height ratio increases. FEM results are shown in Fig. 3.8 for doubly and singly stepped beams with length-height ratios from 15 to 40 with α = 0.167, β = 1.2, and γ = 1.4. Results are presented as ratios of the critical moment from the FEM results, M_{cr}, to the critical moment for an equal length prismatic beam with the same cross section as the smaller portion of the stepped beam, M_{ocr}. L_{cb} is the length of bottom flange in compression.

From the results of the FEM investigation, the following expressions for F were developed. For doubly stepped beams, $F = \frac{L_b}{20h}$, and for singly stepped beams, $F = \frac{L_b}{40h} + 0.5$.

The h is the distance between centroid of top and bottom flange. These expressions for F are valid for values of L_b/h from 15 to 40.

Fig. 3.9 (a) and (b) are graphs of L_b/h versus F. The values of F are obtained using the ratio of M_{st} from FEM results to $C_{bst}C_{st}M_{ocr}$ from the proposed solution. These graphs include the FEM results of stepped beam models having α = 0.167 and combination of β=1.0, 1.2, or 1.4 and γ = 1.0, 1.4, or 1.8. These graphs indicate that the proposed solutions give somewhat unconservative values for some stepped beams with α = 0.167, β = 1.2, or 1.4 and γ = 1.0 with L_b/h = 30 or 40. However, the proposed equations give reasonably accurate results that are conservative for almost all the cases investigated.

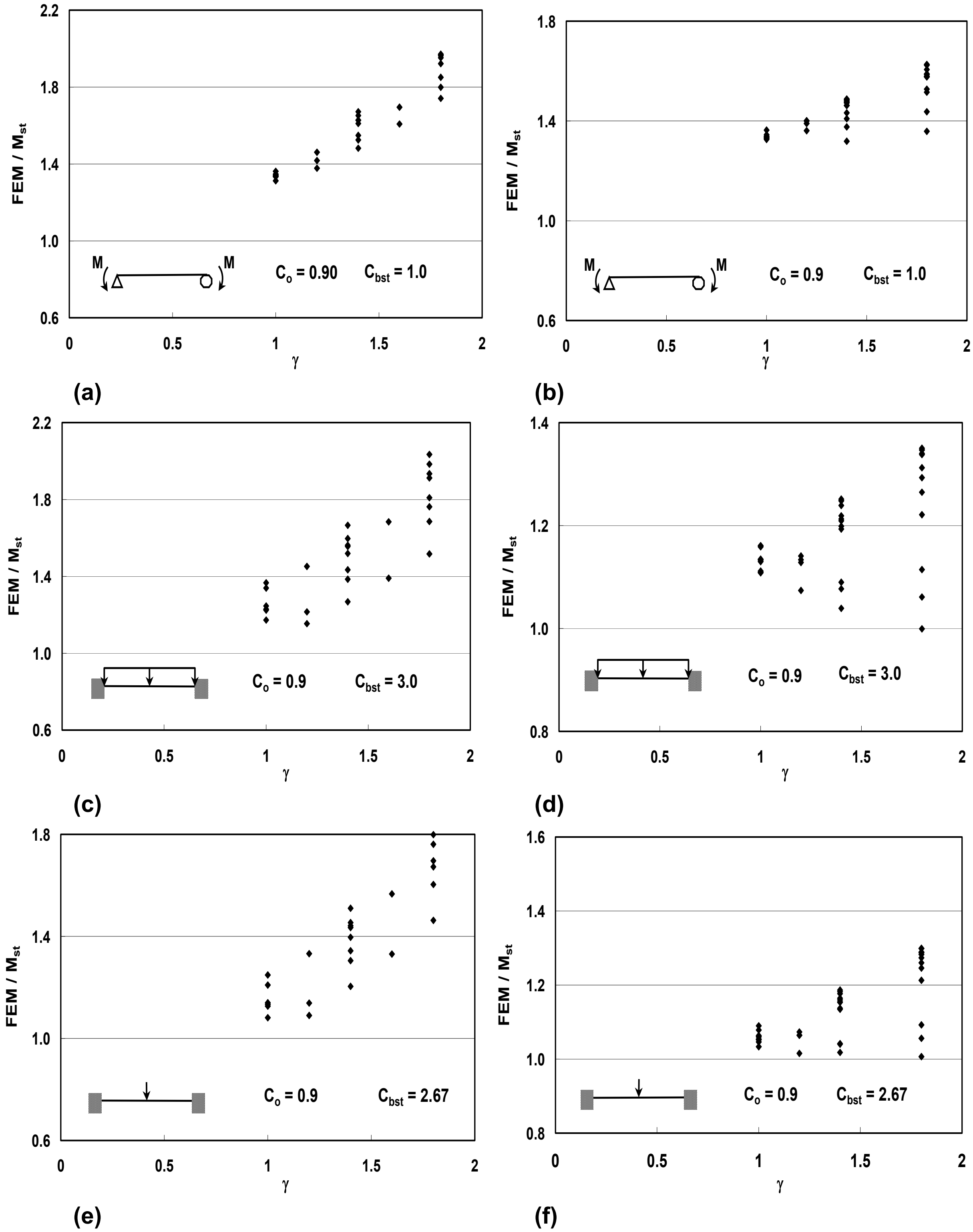


FIG. 3.6 Cases with Two Negative End Moment: (a), (c) and (e) Doubly Stepped Beams; (b), (d) and (f) Singly Stepped Beams

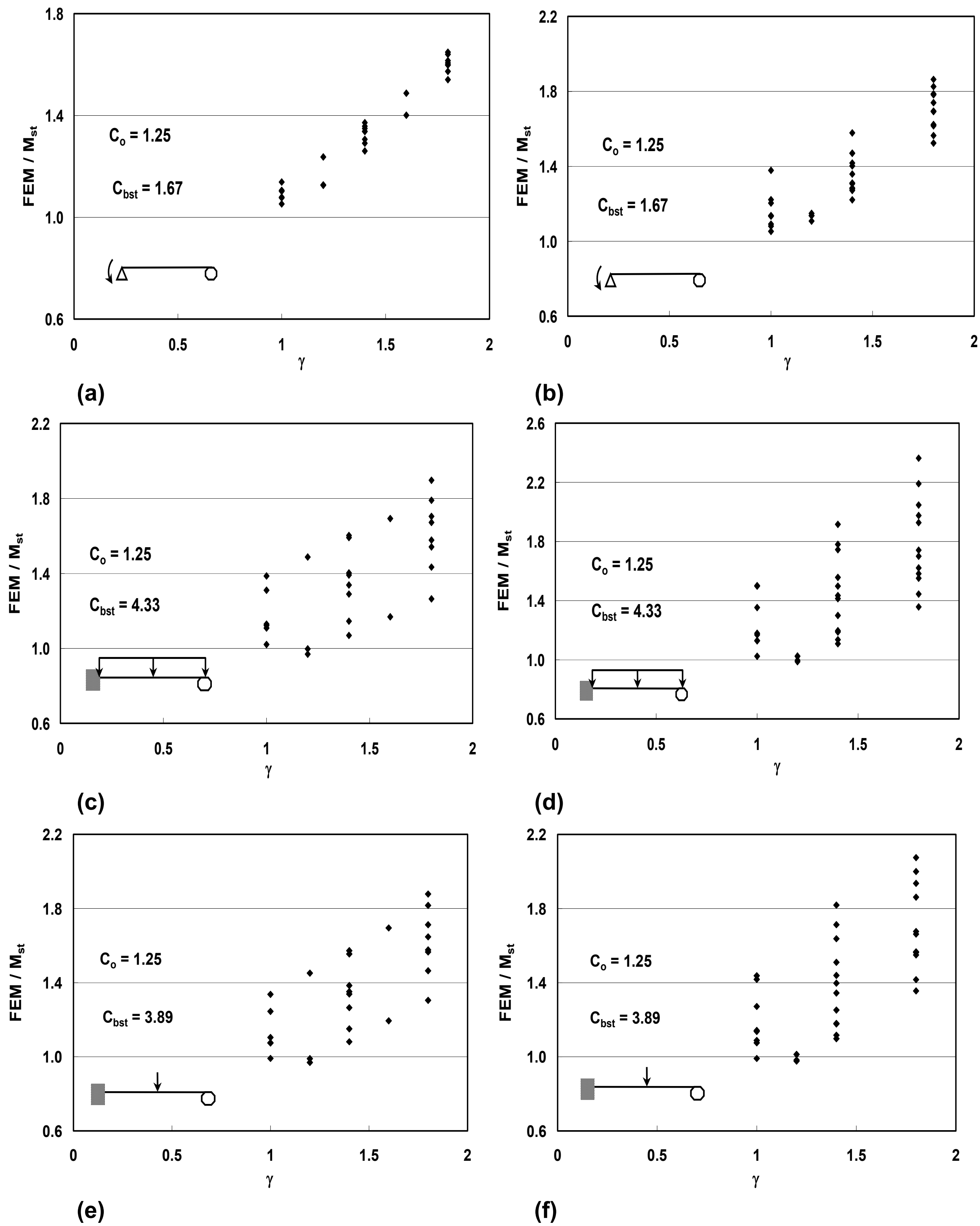
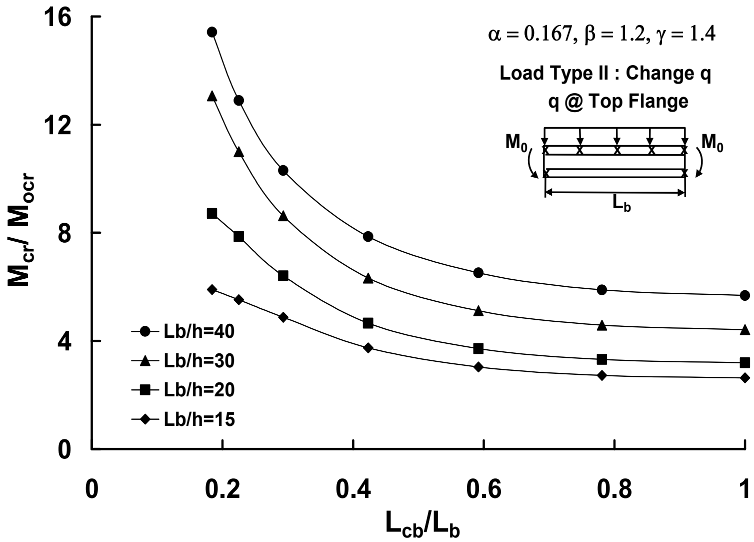
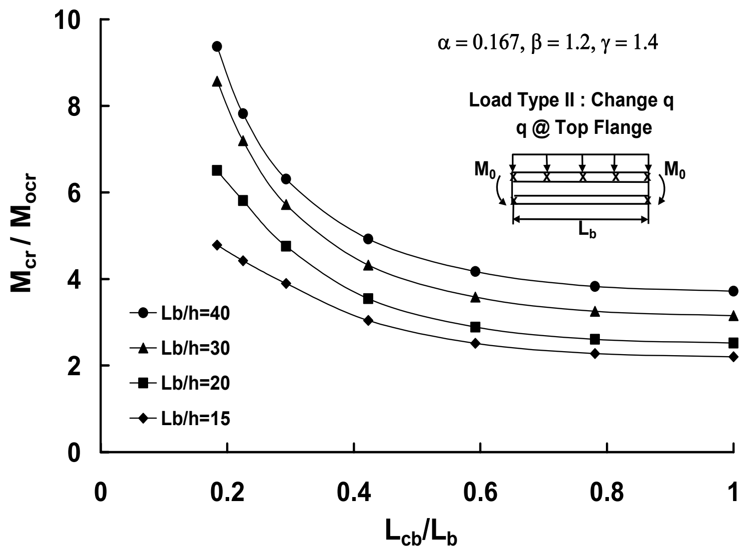


FIG. 3.7 Cases with One Negative End Moment: (a), (c) and (e) Doubly Stepped Beams; (b), (d) and (f) Singly Stepped Beams

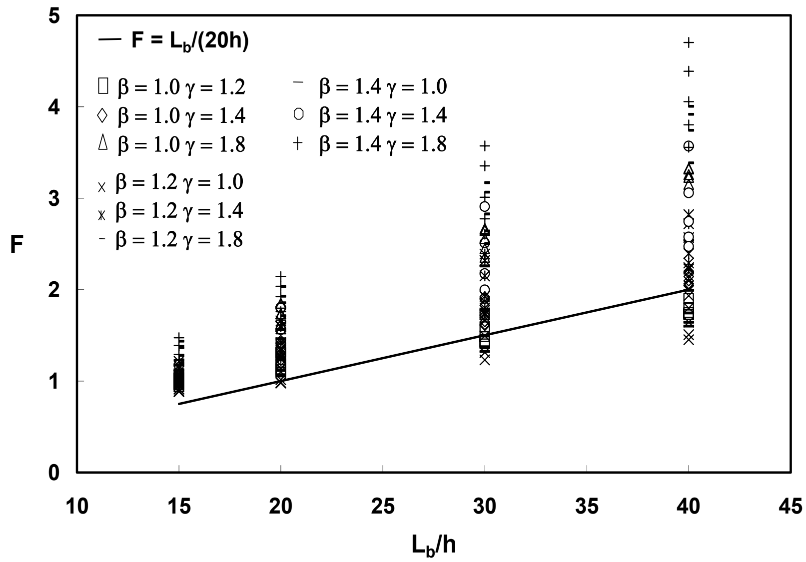


(a) Doubly Stepped Beams

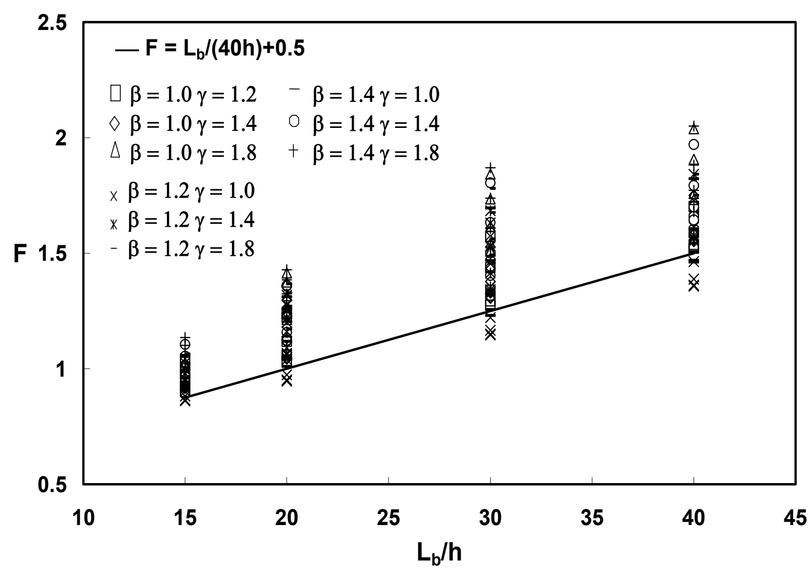


(b) Singly Stepped Beams

FIG. 3.8 FEM Results of a Stepped Beam with Different Length-height Ratio



(a) Doubly Stepped Beams



(b) Singly Stepped Beams

FIG. 3.9 FEM Results and Factor F with $\alpha = 0.167$

SUMMARY AND APPLICATONS

This chapter presents the results from computational studies using FEM for a number of different prismatic and stepped beams with continuous top flange bracing. The results of LTB analyses presented herein are compared with the proposed solutions. The critical LTB moment resistance from Eq. (3-3) is calculated by choosing C_o values based on the number of negative end moments that produce compressive stress at the bottom flange. Factor F in Eq. (3-3) accounts for the effect of L_b/h on LTB resistance. F and C_{st} equations are chosen based whether the beam is a doubly or singly stepped beam. Moment gradient factor C_{bst} can be chosen based on applied top flange loading condition, uniformly distributed load or a concentrated load. Table 3.1 summarizes the application of C_o, C_{st}, F, and C_{bst} values for design of stepped beams with continuous top flange bracing.

Five existing bridges beams shown in Fig. 2.7 are used to apply the proposed equations and to check the accuracy of LTB resistances from Eq. (3-3). Table 3.2 and Table 3.3 show the cross-section properties of the stepped beams in the example structural systems. Table 3.4 and Table 3.5 show comparisons between the results from the proposed equations and the FEM for doubly and singly stepped beam spans. Each model of Table 3.4 is an interior span of a continuous beam as described in Fig. 2.7. Each model of Table 3.4 was loaded by a uniformly distributed load on the top flange and a negative bending moment at each end. Models A, B and C of Table 3.5 are end spans of continuous beams. These models were loaded by a uniformly distributed load on the top flange and one negative moment, the other end moment was equal to zero. Case D of Table 3.5 is an interior span and was loaded by a uniformly distributed load on the top flange and negative end moments. The magnitudes of the negative end moments at all beam models were obtained from structural analyses for continuous span beams subjected to the uniformly distributed load applied to each model. Equation (3-3) produced conservative estimates of the LTB resistance for all cases. The details of the process of applying the proposed solution Eq. (3-3) are illustrated in the Appendix.

TABLE 3.1 Summary for LTB Moment Resistances of Stepped Beams with Continuous Top flange Bracing

C_{st} and F	
Double	Single
$C_{st} = C_o + 6\alpha^2(\beta\gamma^{1.3} - 1)$ $F = \frac{L_b}{20h}$	$C_{st} = C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$ $F = \frac{L_b}{40h} + 0.5$
C_{bst}	
Uniform Load, Or Combined Uniform And Concentrated Loads	A Concentrated Load
$3 - \frac{2}{3}(\frac{M_1}{M_0}) + \frac{8}{3}\frac{M_{CL}}{(M_o + M_1)}$	$2.5 - \frac{2}{3}(\frac{M_1}{M_0}) + \frac{5}{3}\frac{M_{CL}}{(M_o + M_1)}$

Note: For cases with one negative end moment C_o=1.25;
for cases with two negative end moments, C_o=0.9.

TABLE 3.2 Cross-Section properties of Doubly Stepped Beams in Existing Structural Systems

Model No.	Stepped Ratios			Cross Section			
	Length (α)	Flange Width (β)	Flange Thickness (γ)	Support	Center	L_b(ft)	h(in.)
I	0.173	1.011	1.333	W36x300	W36x230	104	34.64
II	0.217	1.364	1.067	W36x230	W36x182	83	35.15
III	0.243	1.125	1.667	Plate Girder	Plate Girder	165	66.0
IV	0.125	1.000	1.798	Cover plates on the flanges	W36x150	80	34.91

Note : 1 ft = 0.3048 m; 1 in. = 25.4 mm

TABLE 3.3 Cross-section properties of Singly Stepped Beams in Existing Structural Systems

Model Case	Stepped Ratios			Cross Section			
	Length (α)	Flange Width (β)	Flange Thickness (γ)	Larger	Smaller	L_b(ft)	h(in.)
A	0.250	1.384	1.527	W36×300	W36×170	72	35.07
B	0.300	1.372	1.340	W36×230	W36×150	60	34.91
C	0.167	1.000	1.800	Cover plates on the flanges	W36×150	60	34.91
D	0.263	1.010	1.160	W36×182	W36×160	66.7	35.0

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm

TABLE 3.4 Comparison between Proposed Solution and FEM Results for Doubly Stepped Beam Spans of Existing Bridges

	Stepped Ratio				Applied Load Moments			LTB Moments (kip-ft)						
Model	α	β	γ	L_b/h	M_0	M_{CL}	M_1	M_{ocr} (Eq. 2-1)	F	C_{bst} (Eq.3-1)	C_{st}	M_{st} (Eq.3-3)	M_{st} (FEM)	Diff. (%)
I	0.17	1.01	1.33	36.0	-1	0.69	-1	651	1.80	3.26	0.98	3744	4687	20
II	0.22	1.36	1.07	28.3	-1	0.64	-1	399	1.42	3.18	1.04	1874	3219	42
III	0.24	1.13	1.67	30.0	-1	0.51	-0.99	179	1.50	3.02	1.32	1070	1592	33
IV	0.13	1.0	1.80	27.5	-1	0.59	-1	277	1.38	3.11	1.01	1201	2230	46

Note: 1 kip = 4.45 KN; 1 ft = 0.3048 m; 1 kip-ft = 1.356 KN-m

TABLE 3.5 Comparison between Proposed Solution and FEM Results for Singly Stepped Beam Spans of Existing Bridges

	Stepped Ratio				Applied Load Moments			LTB Moments (kip-ft)						
Model	α	β	γ	L_b/h	M_0	M_{CL}	M_1	M_{ocr} (Eq. 2-1)	F	C_{bst} (Eq.3-1)	C_{st}	M_{st} (Eq.3-3)	M_{st} (FEM)	Diff. (%)
A	0.25	1.38	1.53	24.6	-1	0.31	0	411	1.12	3.83	1.46	2574	4650	45
B	0.30	1.37	1.34	20.6	-1	0.36	0	397	1.02	3.95	1.46	2335	4211	45
C	0.17	1.0	1.80	20.6	-1	0.39	0	397	1.02	4.04	1.34	2192	3479	37
D	0.26	1.01	1.16	22.8	-1	0.31	-0.75	395	1.07	2.98	0.94	1184	1445	18

Note: 1 kip = 4.45 KN; 1 ft = 0.3048 m; 1 kip-ft = 1.356 KN-m

CHAPTER 4. SUMMARY AND CONCLUSION

A number of finite-element analyses were executed to study the elastic buckling capacity of beams having cross-section change between bracing points and/or having continuous bracing at the top flange. From the results of finite element investigation and a review of previous research, new design equations were developed for use by practicing engineers to determine the LTB capacity of beams during the construction and the maintenance of bridges. The predicted values from these equations were compared to the results of finite-element analysis with regard to existing bridge situations, and the applicability and accuracy of the new equations were assessed.

Lateral-torsional buckling resistance of prismatic beams was found to be dramatically increased by providing continuous bracing along the length. It was shown that the LTB resistance of beams with continuous bracing depended upon the ratio of the length of bottom flange in compression to the unbraced length, L_{cb}/L_b. The accuracy of the equation for C_b proposed by Yura (SSRC *Guide,* 1998) was investigated. Yura's Equation applies to beam cases with continuous top flange lateral bracing, constant cross section, and uniformly distributed load at top flange. This equation can be used conservatively to calculate the LTB moment resistance of a beam having the L_b/d of 30. In the range of L_b/d less than 30, the critical moments of beams having stocky flanges and low web slenderness are also conservatively estimated by Yura Equation, but for some cases such as W36x230, it was somewhat less conservative. For beam design, Yura's Equation can be reasonably used to calculate the critical moment of prismatic beams with uniformly distributed load and end moments.

For beam cases with a concentrated load on the top flange, Yura's Equation gives unconservative values with respect to FEM results. The reason is that a beam subjected to a concentrated load and negative end moments has a larger proportion of the bottom flange in compression than a beam subjected to uniformly distributed load and negative end moments if these beams have same magnitude of end moments and moment at midspan. A new design equation for these cases was proposed by adjusting the coefficients of Yura's Equation. This proposed equation can be conservatively used for all prismatic beam spans with continuous bracing, but it would be uneconomical for some cases. It was recommended that the Yura's Equation be used for beams subjected to uniformly distributed load and the new equation be used for beams subjected only to a concentrated load. Yura's Equation is also recommended for beam cases having both uniformly distributed load and a concentrated load or a series of concentrated loads within the laterally unbraced length.

The study of LTB capacity of stepped beams, which are used to resist high negative moments at interior supports in continuous bridges, is important because of the significant increase in the critical moment resistance of this type beam. An investigation of the elastic LTB behavior of I-shaped stepped beams was conducted using finite element method and resulted in the development of design equations for beams having singly or doubly stepped cross sections within a laterally unbraced length. A number of loading conditions were considered. First, an equation for a C_{st} factor was developed to account for the changes in cross section when the beam is subjected to constant moment loading. It was found that the buckling moments varied approximately with the square or the 1.6th power of the length ratio (α) for doubly or singly stepped beams, that a change of flange thickness is more significant than a change of flange width, and that C_{st} values increase as span-to-height ratios increase. Two proposed C_{st} equations produced good agreement with the FEM results. Second, an equation for a C_{bst} for the moment gradient effect was considered along with the C_{st} equation. The proposed design equation for stepped beams under general loading conditions provided best estimates by choosing C_o in the C_{st} equations and C_{bst} values based on the number of zero moment points, inflection points in the deflected shape, within the unbraced length. Results from the design equations were demonstrated with comparisons between the proposed equations, the weighted average approach (WAA), and FEM results for doubly and singly stepped beam spans of existing highway bridges. The comparisons indicated that the LTB estimates from the WAA were unconservative and the proposed solution produced reasonable estimates.

The increase in LTB capacity of stepped beams due to continuous bracing provided by a concrete slab is significantly greater than the critical moment increase for prismatic beams. From FEM analyses of typical stepped beams with continuous lateral top flange bracing, it was confirmed that the present equations for beams with continuous lateral top bracing in the AASHTO *LRFD specifications* (1998) give very conservative values and that the critical moment greatly increases as the length-to-height ratio increases. Therefore, a new design equation was proposed using C_{st} and C_{bst} equations similar to the equation for stepped beams without continuous lateral bracing. The length-to-height ratio factor F was included in the new equation. Best estimates for the critical LTB moment were obtained by choosing C_o, C_{bst}, and F based on the number of negative end moment at ends, on the type of transverse loading at top flange, and on the category of stepped beam, respectively. Comparisons of results from the proposed equation with the critical moments of five existing bridges illustrated that the new simplified equation produced reasonable values for all cases with respect to FEM results. It is concluded from the comparative studies that the proposed equation gives the most accurate solutions for the LTB resistance moment of steel stepped beams with continuous top flange bracing of any design approach.

In conclusion, the new equations improve current design methods for calculating the lateral-torsional buckling resistance and can result to increase efficiency in building and bridge design. In all cases, the proposed solutions are sufficiently simple and accurate for use by designers to determine the LTB resistance of beams. The process for applying the proposed solution using AASHTO *LRFD Specifications* (1998) is illustrated in the Appendix.

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APPENDIX

EXAMPLE APPLICATIONS

INTRODUCTION

Three example applications of equations developed for the calculation of the lateral-torsional buckling resistance of stepped beams are presented in this appendix. The first example is a 3-span continuous beam with a uniformly distributed load and with bracing only at discrete locations. The cross section and geometry of the beams are the same as an existing 3-span continuous bridge on I-59 in Birmingham, Alabama. The second example is the same beam geometry and loading, but continuous top flange bracing is assumed. Example 3 is an application of the proposed equations to a non-composite bridge beam using AASHTO *LRFD Specifications* (1998). The beam geometry is similar to that of an existing bridge span of the CBD Interchange in Birmingham, Alabama. Cross section properties for the rolled steel W-shapes used in all the examples are summarized in Table A.1.

Table A.1 Cross-Section Properties Used in Example Applications

<table>
<tr><td colspan="2">Cross Section</td><td>W36x170</td><td>W36x230</td><td>W36x280</td><td>W36x300</td></tr>
<tr><td rowspan="2">Flange</td><td>Width (b_f, inch)</td><td>12.03</td><td>16.47</td><td>16.595</td><td>16.655</td></tr>
<tr><td>Thickness (t_f, inch)</td><td>1.10</td><td>1.26</td><td>1.57</td><td>1.68</td></tr>
<tr><td rowspan="2">Web</td><td>Depth (D)</td><td>33.97</td><td>33.38</td><td>33.38</td><td>33.38</td></tr>
<tr><td>Thickness (t_w, inch)</td><td>0.68</td><td>0.76</td><td>0.885</td><td>0.945</td></tr>
<tr><td colspan="2">Depth (d, inch)</td><td>36.17</td><td>35.9</td><td>36.52</td><td>36.74</td></tr>
<tr><td colspan="2">Height (h, inch)
($d - t_f$)</td><td>35.07</td><td>34.64</td><td>34.95</td><td>35.06</td></tr>
</table>

EXAMPLE 1. Three-span continuous beam with discrete bracing

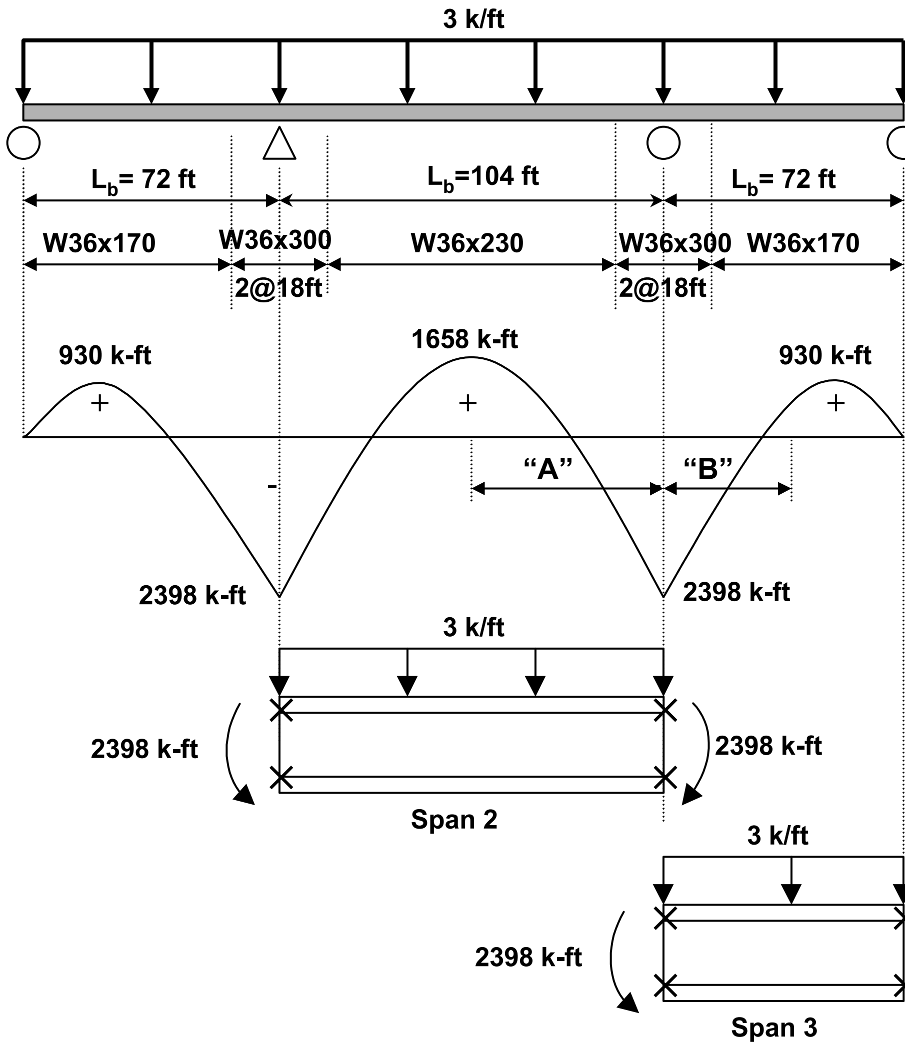

FIG. A.1 Beam Details for Example 1

1. Span 2

(1) Assume bracing at supports only.

M_{max} = 2398 k-ft, M_A = 644 k-ft, M_B = 1658 k-ft, M_C = 644 k-ft,

α = 18/104 = 0.173, β = 16.655/16.47 = 1.011, γ = 1.68/1.26 = 1.333,

E = 29000 ksi, ν = 0.3, G = E/2(1+ν) = 11154 ksi,

Unbraced length of Span 2, L_b = 104 ft, L_b/h = (104)(12)/(34.64) = 36

$$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$ = 651 k-ft [for W36×230 with L_b = 104 ft]

$$C_{st} = 0.85 + 6\alpha^2(\beta\gamma^{1.3} - 1)$$ = 0.934

$$C_{bst} = \frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$$ = 1.066

$M_{st} = C_{bst}\ C_{st}\ M_{ocr}$ = (1.066) (0.934) (651) = 648 k-ft < M_{max} = 2398 k-ft

∴LTB resistance is less than the maximum applied moment.

(2) Assume bracing at the supports and midspan (L_b = "A").

M_{max} = 2398 k-ft, M_A = 1405 k-ft, M_B = 644 k-ft, M_C = 623 k-ft,

α = 18/52 = 0.346, β = 16.655/16.47 = 1.011, γ = 1.68/1.26 = 1.333,

Unbraced length of half of Span 2, L_b = 52 ft, L_b/h = (52)(12)/(34.64) = 18

$$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$ = 1561 k-ft [for W36×230 with L_b = 52 ft]

$$C_{st} = 1 + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$$ = 1.117

$$C_{bst} = \frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$$ = 1.487

$M_{st} = C_{bst}\ C_{st}\ M_{ocr}$ = (1.487) (1.117) (1561) = 2593 k-ft > M_{max} = 2398 k-ft

∴LTB resistance is greater than the maximum applied moment.

Alternately, using AASHTO LRFD (1998),

$$C_b^{LRFD} = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$ = 2.045

$M_{cr} = C_b\ M_{ocr}$ = (2.045) (1561) = 3195 k-ft > M_{max} = 2398 k-ft

∴LTB resistance by LRFD is also greater than the maximum applied moment.

2. Span 3

(1) Assume bracing at supports only.

M_{max} = 2398 k-ft, M_A = 340 k-ft, M_B = 745 k-ft, M_C = 859 k-ft,

α = 18/72 = 0.25, β = 16.655/12.03 = 1.384, γ = 1.68/1.1 = 1.527,

E = 29000 ksi, ν = 0.3, G = E/2(1+ν) = 11154 ksi,

Unbraced length of Span 3, L_b = 72 ft, L_b/h = (72)(12)/(35.07) = 24.63

$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$ = 411 k-ft [for W36×170 with L_b = 72 ft]

$C_{st} = 1 + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$ = 1.212

$C_{bst} = \frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$ = 1.498

$M_{st} = C_{bst}\, C_{st}\, M_{ocr}$ = (1.212) (1.498) (411) = 746 k-ft < M_{max} = 2444 k-ft

∴LTB resistance is less than the maximum applied moment.

(2) Assume bracing at the supports and midspan (L_b = "B").

M_{max} = 2398 k-ft, M_A = 1248 k-ft, M_B = 340 k-ft, M_C = 324 k-ft,

α = 18/36 = 0.5, β = 16.655/12.03 = 1.384, γ = 1.68/1.1 = 1.527,

Unbraced length of half of Span 3, L_b = 36 ft, L_b/h = (36)(12)/(35.07) = 12.32

$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$ = 1029 k-ft [for W36×170 with L_b = 36 ft]

$C_{st} = 1 + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$ = 1.643

$C_{bst} = \frac{10M_{max}}{4M_{max} + M_A + 7M_B + M_C}$ = 1.771

$M_{st} = C_{bst}\, C_{st}\, M_{ocr}$ = (1.643) (1.771) (1029) = 2994 k-ft > M_{max} = 2398 k-ft

$\therefore$ LTB resistance is greater than the maximum applied moment.

Alternately, using AASHTO LRFD (1998),

$$C_b^{LRFD} = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} = 2.483$$

$M_{cr} = C_b M_{ocr}$ = (2.483) (1029) = 2555 k-ft > M_{max} = 2398 k-ft

$\therefore$ LTB resistance by LRFD is also greater than the maximum applied moment.

EXAMPLE 2. Three-span continuous beam with continuous top flange lateral bracing

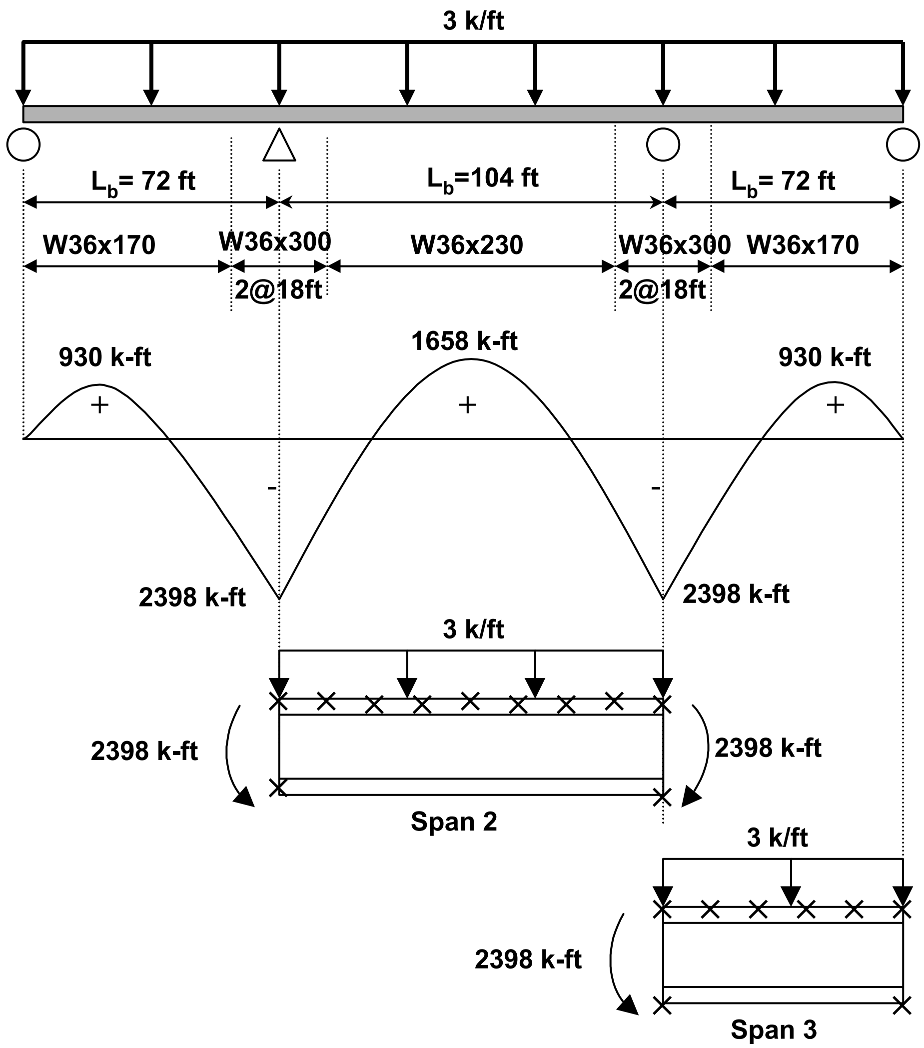

FIG. A.2 Beam Details for Example 2

1. Span 2

M_0 = 2398 k-ft, M_{CL} = 1658 k-ft, M_1 = 2398 k-ft,

α = 18/104 = 0.173, β = 16.655/16.47 = 1.011, γ = 1.68/1.26 = 1.333,

E = 29000 ksi, ν = 0.3, G = E/2(1+ν) = 11154 ksi,

Unbraced length of Span 2, L_b = 104 ft, L_b/h = (104)(12)/(34.64) = 36

$$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$ = 651 k-ft [for W36×230 with L_b = 104 ft]

$$C_{st} = 0.9 + 6\alpha^2(\beta\gamma^{1.3} - 1)$$ = 0.980

$$C_{bst} = 3 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) + \frac{8}{3}\frac{M_{CL}}{(M_o + M_1)}$$ = 3.260

$$F = \frac{L_b}{20h}$$ = 1.80

M_{st} = F C_{bst} C_{st} M_{ocr} = (1.80) (3.260) (0.980) (651) = 3744 k-ft > M_{max} = 2398 k-ft

∴ Buckling resistance is greater than the maximum applied moment.

2. Span 3

M_0 = 2398 k-ft, M_{CL} = 745 k-ft, M_1 = 0 k-ft,

α = 18/72 = 0.25, β = 16.655/12.03 = 1.384, γ = 1.68/1.1 = 1.527,

E = 29000 ksi, ν = 0.3, G = E/2(1+ν) = 11154 ksi,

Unbraced length of Span 3, L_b = 72 ft, L_b/h = (72)(12)/(35.07) = 24.63

$$M_{ocr} = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$ = 411 k-ft [for W36×170 with L_b = 72 ft]

$$C_{st} = 1.25 + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$$ = 1.462

$$C_{bst} = 3 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) + \frac{8}{3}\frac{M_{CL}}{(M_o + M_1)}$$ = 3.830

$$F = \frac{L_b}{40h} + 0.5$$ = 1.12

M_{st} = F C_{bst} C_{st} M_{ocr} = (1.12) (3.83) (1.462) (411) = 2577 k-ft > M_{max} = 2398 k-ft

∴ Buckling resistance is greater than the maximum applied moment.

EXAMPLE 3. Non-composite bridge analysis using proposed equation with AASHTO *LRFD specification* (1998)

STEP 1: Identify bridge geometry. An interior beam is investigated in this example.

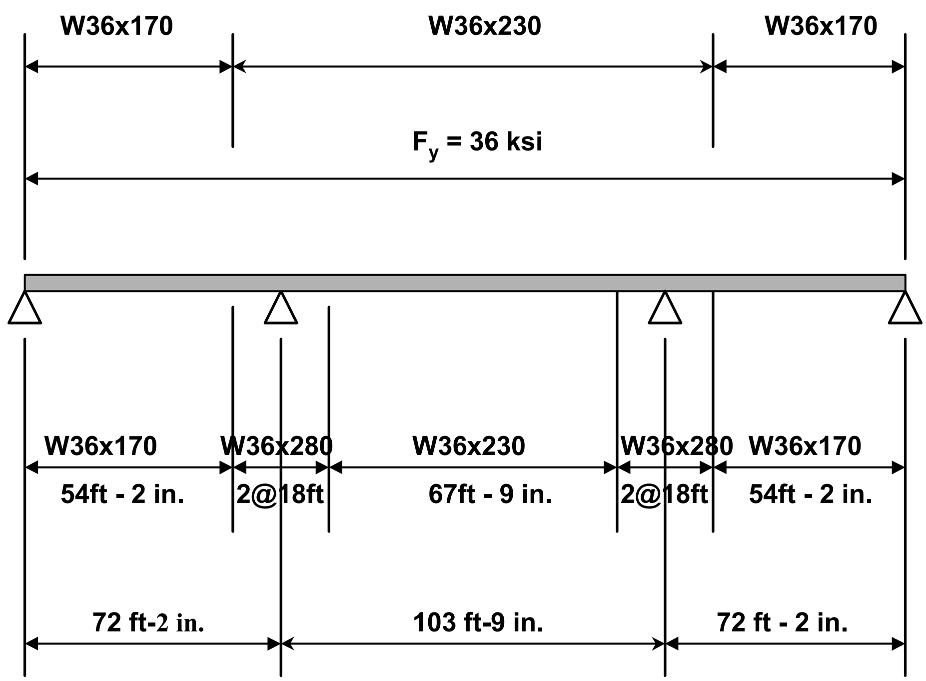

FIG. A.3 Beam Details for 3-Span Continuous Bridge

<u>Exterior Span</u>

S = 7.57 ft

Section = W36x170

S_x = 580 in.^{3}

Slab thickness = 5 in.

n_g = 8 girders

<u>Interior Span</u>

S = 7.57 ft

Section = W36x230

S_x = 837 in.^{3}

Slab thickness = 5 in.

n_g = 8 girders

f'_c = 4 ksi

Diaphragms = C18x42.7

4 Traffic Lanes

STEP 2: Determine the maximum factored moments for the governing load cases.

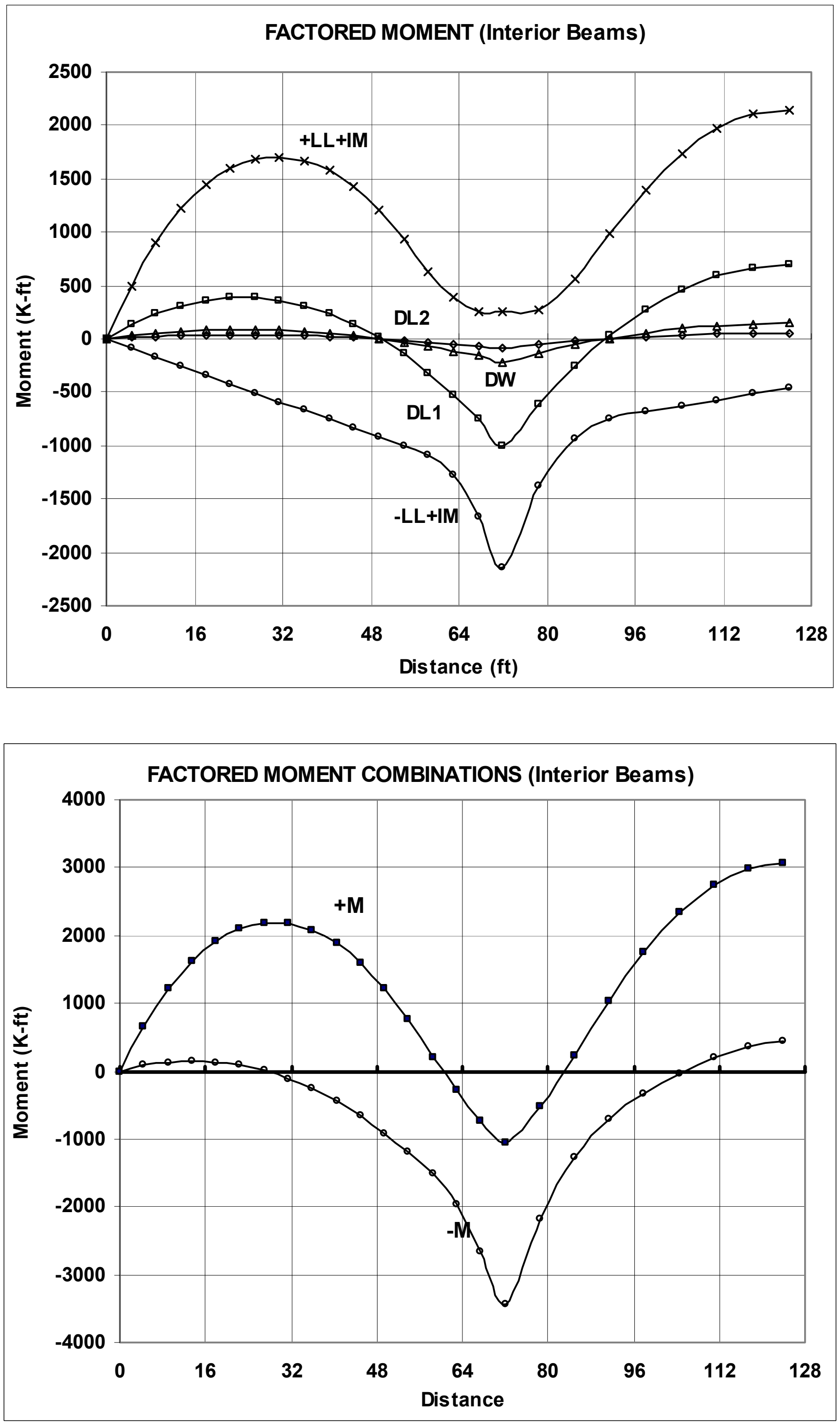

FIG. A.4 Factored Moments and Combinations

STEP 3: Determine the lateral-torsional buckling (LTB) moment capacity for the Exterior Span.

The noncomposite section flexural resistance based upon lateral torsional buckling from AASHTO LRFD Eq. 6.10.4.2.6a-1.

$$M_n = 3.14EC_b(\frac{I_{yc}}{L_b})\sqrt{0.772\left(\frac{J}{I_{yc}}\right)+9.87\left(\frac{d}{L_b}\right)^2} \le M_y$$

A) Calculate cross sectional parameters I_{yc}, J, D, D_c.

For this example, the smaller cross section is a W36x170.

$I_{yc} = t_f\, b_f^3/12 = (1.10)(12.03)^3/12 = 160 \text{ in.}^4$

$D = d\text{-}2t_f = 36.17\text{-}2(1.10) = 33.97$ in.

$D_{cp} = D/2 = 33.97/2 = 16.99$ in.

$$J = \frac{(2b_f t_f^3 + Dt_w^3)}{3} = \frac{[(2)(12.03)(1.10)^3 + (33.97)(0.68)^3]}{3} = 14.24 \text{ in.}^4$$

B) Verify the conditional limits.

Section proportion limit of flexural component

$0.1 \le \frac{I_{yc}}{I_y} \le 0.9$ [AASHTO LRFD Article 6.10.2.1]

$$\frac{I_{yc}}{I_y} = \frac{160}{320} = 0.5$$

$\therefore$ The formulas for LTB used in the Specification are valid for this section.

Compact-section web slenderness

$\frac{2D_{cp}}{t_w} \le 3.76\sqrt{\frac{E}{F_{yc}}}$ [AASHTO LRFD Article 6.10.4.1.2]

$$\frac{2D_{cp}}{t_w} = \frac{2(16.99)}{(0.68)} = 50.0$$

$$3.76\sqrt{\frac{E}{F_{yc}}} = 3.76\sqrt{\frac{29000}{36}} = 106.7$$ > 50.0

$\therefore$ Local buckling of web is prevented.

Compact-section compression-flange slenderness

$$\frac{b_f}{2t_f} \leq 0.382\sqrt{\frac{E}{F_{yc}}}$$ [AASHTO LRFD Article 6.10.4.1.3]

$$\frac{b_f}{2t_f} = \frac{(12.03)}{2(1.1)} = 5.47$$

$$0.382\sqrt{\frac{E}{F_{yc}}} = 0.382\sqrt{\frac{29000}{36}} = 10.84$$ > 5.47

$\therefore$ Local buckling of flange is prevented.

C) Calculate C_{st}, C_{bst}, F, and M_n.

Calculate M_n using proposed equations for singly stepped beam with continuous lateral bracing provided by deck. The Exterior Span is a singly stepped beam having one negative end moment.

$M_n = FC_{bst}C_{st}M_{ocr}$

$$F = \frac{L_b}{40h} + 0.5,$$

$$C_{bst} = 3 - \frac{2}{3}(\frac{M_1}{M_0}) + \frac{8}{3}\frac{M_{CL}}{(M_o + M_1)}, \quad C_{st} = C_o + 1.5\alpha^{1.6}(\beta\gamma^{1.2} - 1)$$

The moments used for the calculation of C_{bst} should be those corresponding to the critical load case. Here the critical loading is the combination of dead load plus live lane load plus truck-train loading. This combination produces the largest negative moment at the support, and is shown in Fig. A.5.

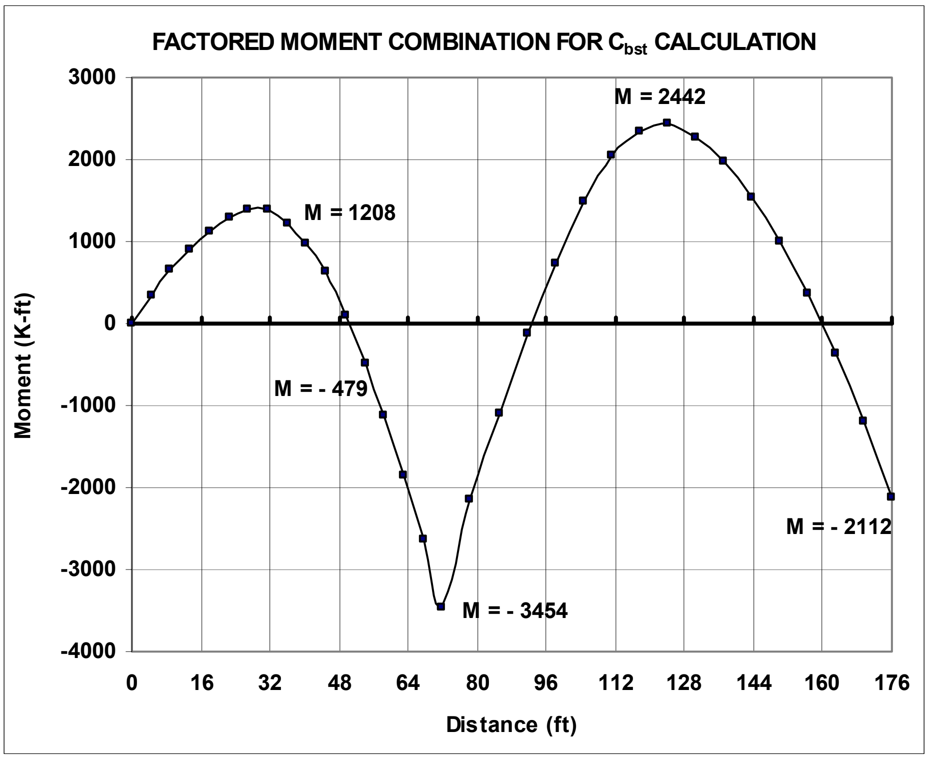

FIG. A.5 Factored Moment Combination for Calculating LTB

<u>L_b = 72.17 ft or 866 in., full length of exterior span</u>

M_1 = 0 kip-ft., M_0 = -3454 kip-ft., M_{CL} = 1208 kip-ft.

α = 18/72 = 0.25, β = 16.595/12.03 = 1.38, γ = 1.57/1.1 = 1.43

C_o = 1.25 [for one negative end moment]

h = 35.07 in. [for w36x170]

$M_y = F_yS_x$ = (36ksi) (580 in.^{3}) = 20880 kip-in = 1740 kip-ft [for w36x170]

M_p = 2000 kip-ft [for w36x170 with F_y = 36 ksi]

$M_y = F_yS_x$ = (36ksi) (1030 in.^{3}) = 37080 kip-in = 3090 kip-ft [for w36x280]

M_p = 3510 kip-ft [for w36x280 with F_y = 36 ksi]

$$F = \frac{864}{40(35.07)} + 0.5 = 1.12$$

$$C_{bst} = 3 - \frac{2}{3}(\frac{0}{3454}) + \frac{8}{3}\frac{1208}{(3454 + 0)} = 3.93$$

$$C_{st} = 1.25 + 1.5(0.25)^{1.6}[(1.38)(1.43)^{1.2} - 1] = 1.43$$

$$M_{ocr} = 3.14(29000)(\frac{160}{866})\sqrt{0.772\left(\frac{14.24}{160}\right) + 9.87\left(\frac{36.17}{866}\right)^2}$$ = 4932 kip-in = 411 kip-ft

$M_n = FC_{bst}C_{st}M_{ocr}$ = (1.12) (3.93) (1.43) (411) = 2587 kip-ft

M_n (2587 k-ft) < M_y (3090 k-ft) < M_0 (3454 k-ft) < M_p (3510 k-ft)

The resistances M_n = 2587 k-ft and M_y = 3090 k-ft are less than the maximum applied moment M_0 = 3454 k-ft. An interior diaphragm can be used in the exterior span to provide bracing and increase the LTB resistance. But, the flexural resistance can not exceed M_y = 3090 k-ft. The moment capacity with bracing provided by only one line of diaphragms at the center of the Exterior Span is evaluated below.

D) Re-evaluate assuming the Exterior Span is braced at midspan.

L_b = 36.08 ft or 433 in., from center of the exterior span to interior support

M_1 = 1208 kip-ft., M_0 = -3454 kip-ft., M_{CL} = -479 kip-ft.

α = 18/36 = 0.50, β = 16.595/12.03 = 1.38, γ = 1.57/1.1 = 1.43

C_o = 1.25 [For one negative end moment]

h = 35.07 in. [For w36x170]

$$F = \frac{433}{40(35.07)} + 0.5 = 0.81$$

$$C_{bst} = 3 - \frac{2}{3}(\frac{-1208}{3454}) + \frac{8}{3}\frac{-479}{(3454 + 0)} = 2.86$$

$$C_{st} = 1.25 + 1.5(0.5)^{1.6}[(1.38)(1.427)^{1.2} - 1] = 1.80$$

$$M_{ocr} = 3.14(29000)(\frac{160}{433})\sqrt{0.772\left(\frac{14.24}{160}\right) + 9.87\left(\frac{36.17}{433}\right)^2}$$ = 12480 kip-in = 1040 kip-ft

$M_n = FC_{bst}C_{st}M_{ocr}$ = (0.81)(2.86)(1.80)(1040) = 4337 kip-ft

M_y (3090 k-ft) < M_0 (3454 k-ft) < M_p (3510 k-ft) < M_n (4337 k-ft)

The resistance to LTB M_n = 4337 kip-ft is larger than the yield moment resistance M_y = 3090 k-ft. The yield moment, M_y, controls the resistance of the beam. This means that using the line of diaphragms at midspan to provide bracing allows the full cross section capacity to be utilized. Addition of more lines of diaphragms will not increase the flexural capacity beyond the yield moment. Hence, this beam cross section is not adequate to resist the applied moment of 3454 kip-ft.

L_b = 36.08 ft or 433 in., from center of the exterior span to exterior support

M_1 = 1208 kip-ft. (Creates tension in bottom flange.)

M_0 = 0 kip-ft.

M_{CL} = 1121 kip-ft. (Creates tension in bottom flange)

Neither end moment for this unbraced length creates compression in the bottom flange, so LTB will not occur.

STEP 4: Determine the LTB moment capacity for the Interior Span.

A) Calculate cross sectional parameters I_{yc}, J, D, D_c.

For this example, the smaller cross section is the W36x230.

$I_{yc} = t_f\, b_f^3/12 = (1.26)(16.47)^3/12 = 469$ in.^{4}

$D = d-2t_f = 35.90-2(1.26) = 33.38$ in.

$D_c = D/2 = 33.38/2 = 16.69$ in.

$$J = \frac{(2b_f t_f^3 + Dt_w^3)}{3} = \frac{[(2)(16.47)(1.26)^3 + (33.38)(0.76)^3]}{3} = 26.85 \text{ in.}^4$$

B) Verify the conditional limits.

<u>Section proportion limit of flexural component</u>

$$0.1 \leq \frac{I_{yc}}{I_y} \leq 0.9$$ [AASHTO LRFD Article 6.10.2.1]

$$\frac{I_{yc}}{I_y} = \frac{469}{940} = 0.5$$

$\therefore$ The formulas for LTB used in the Specification are valid for this section.

<u>Compact-section web slenderness</u>

$$\frac{2D_{cp}}{t_w} \leq 3.76\sqrt{\frac{E}{F_{yc}}}$$ [AASHTO LRFD Article 6.10.4.1.2]

$$\frac{2D_{cp}}{t_w} = \frac{2(16.99)}{(0.76)} = 43.9$$

$$3.76\sqrt{\frac{E}{F_{yc}}} = 3.76\sqrt{\frac{29000}{36}} = 106.7 > 43.9$$

$\therefore$ Local buckling of web is prevented.

<u>Compact-section compression-flange slenderness</u>

$$\frac{b_f}{2t_f} \leq 0.382\sqrt{\frac{E}{F_{yc}}}$$ [AASHTO LRFD Article 6.10.4.1.3]

$$\frac{b_f}{2t_f} = \frac{(16.47)}{2(1.26)} = 6.54$$

$$0.382\sqrt{\frac{E}{F_{yc}}} = 0.382\sqrt{\frac{29000}{36}} = 10.84 > 6.54$$

$\therefore$ Local buckling of flange is prevented.

C) Calculate C_{st}, C_{bst}, F, and M_n.

The Interior Span is a doubly stepped beam having two negative end moments. Calculate M_n using the proposed equations for a doubly stepped beam with continuous lateral bracing provided by deck.

$M_n = FC_{bst}C_{st}M_{ocr}$

$$F = \frac{L_b}{20h},$$

$$C_{bst} = 3 - \frac{2}{3}(\frac{M_1}{M_0}) + \frac{8}{3}\frac{M_{CL}}{(M_o + M_1)}, \quad C_{st} = C_o + 6\alpha^2(\beta\gamma^{1.3} - 1)$$

The moments used for the calculation of C_{bst} should be those corresponding to the critical load case. Here the critical loading is the combination of dead load plus live lane load plus truck-train loading. This combination produces the largest negative moment at the support, and is shown in Fig. A.5.

<u>L_b = 103.75 ft or 1245 in., full length of interior span</u>

M_1 = -2112 kip-ft., M_0 = -3454 kip-ft., M_{CL} = 2442 kip-ft.

α = 18/104 = 0.17, β = 16.595/16.47 = 1.01, γ = 1.57/1.1 = 1.25

C_o = 0.9 [For two negative end moments]

h = 34.64 in. [For w36x230]

$M_y = F_yS_x$ = (36ksi) (837 in.^{3}) = 30132 kip-in = 2511 kip-ft [for w36x230]

M_p = 2833 kip-ft [for w36x230 with F_y = 36 ksi]

$M_y = F_yS_x$ = (36ksi) (1030 in.^{3}) = 37080 kip-in = 3090 kip-ft [for w36x280]

M_p = 3510 kip-ft [for w36x280 with F_y = 36 ksi]

$$F = \frac{1245}{20(34.64)} = 1.80$$

$$C_{bst} = 3 - \frac{2}{3}(\frac{2112}{3454}) + \frac{8}{3}\frac{2442}{(3454 + 2112)} = 3.76$$

$$C_{st} = 0.9 + 6(0.17)^2[(1.01)(1.25)^{1.3} - 1] = 0.96$$

$$M_{ocr} = 3.14(29000)(\frac{469}{1245})\sqrt{0.772\left(\frac{26.85}{469}\right) + 9.87\left(\frac{35.90}{1245}\right)^2}$$ = 7887 kip-in = 657 kip-ft

$M_n = FC_{bst}C_{st}M_{ocr}$ = (1.80) (3.76) (0.96) (657) = 4270 kip-ft

M_y (3090 k-ft) < M_0 (3454 k-ft) < M_p (3510 k-ft) < M_n (4270 k-ft)

The LTB resistance M_n = 4270 kip-ft is larger than the yield moment M_y = 3090 k-ft, so the beam's resistance is limited to M_y. Since the yield moment controls, interior diaphragms are not required in the Interior Span for bending moment resistance. Resistance to wind loading and other considerations may make diaphragms or other bottom flange bracing necessary in the Interior Span.