Graph Terminology

- vertex, node, point
- edge, line, arc
- \( G = (V, E) \)
  - \( V \) is set of vertices
  - \( E \) is set of edges
- Each edge joins two different vertices

Undirected Graph

- edges do not have a direction
- The edge from 1 to 2 is also an edge from 2 to 1
- Edge (1, 2) implies that there is also an edge (2, 1) [The same edge]

Directed Graph

- edges have a direction
- Edge (2, 1) means only that there is an edge from 2 to 1
- In this example, there is no edge from 1 to 2

Weighted Graph

- weights (values) are associated with the edges in the graph
- may be directed for undirected
- Weighted graphs are also referred to as networks

Complete Graph

- For each pair of vertices, there is one edge
- If \( G = (V, E) \) is a complete graph, and \( |V| = n \), then can you calculate \( |E| \)?
**Subgraph**
- A subgraph $G'$ of graph $G = (V, E)$ is a graph $(V', E')$ that $V' \subseteq V$ and $E' \subseteq E$.

**Path**
- the sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$.
- Denoted as path $i_1, i_2, \ldots, i_k$.
- **Simple path** – all vertices (except possibly first and last) are different.
- Length of path is sum of the lengths of the edges.

**Representation of Graphs**
- Adjacency matrix
- Incidence matrix
- Adjacency lists: Table, Linked List
- Space/time trade-offs depending on which operation are most frequent as well as properties of the graph.

**Can we use tree traversal algorithms to traverse graphs?**

**Why?**

**Depth first search**
- Starting from vertex $v$
- Mark $v$ as marked
- Select $u$ as an unmarked node adjacent to $v$
- If no $u$, quit
- If $u$, begin depth first search from $u$
- When search from $u$ quits, select another node from $v$
- Similar to preorder tree traversal

**Breadth first search**
- Starting from node $v$
- Identify all nodes adjacent to $v$
- Add these to the set
- Determine set of unvisited nodes which are adjacent to this set
- Add these to the set
- Continue until no new nodes are encountered
An Example

What would the visit orders for DFS(1), DFS(5), BFS(1), BFS(5) look like?