Testing Course Homework 2
Solution

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Problem 3.1: Economic decision

We start with the following formula for the price of the car derived by John (Equation 3.2 on page 38 of book):

\[ P = 20,000 + \frac{20,000}{n} \text{ dollars} \]

where \( n \) is the number of breakdowns per 15,000 miles since John’s car is driven 15,000 miles in a year. Because Laura drives only 5,000 miles per year, her car is expected to have \( n/3 \) breakdowns per year. Assuming a linear depreciation to zero value over 20 years and an average repair cost of $250 per breakdown, the annual cost of driving is

\[ C = \frac{P}{20} + K + \frac{250n}{3} \text{ dollars} \]

\[ = 1,000 + \frac{1,000}{n} + K + \frac{250n}{3} \text{ dollars} \]

where \( K \) is the cost of gasoline and regular maintenance, assumed to be the same for all models. To minimize this cost, we write

\[ \frac{dC}{dn} = -\frac{1,000}{n^2} + \frac{250}{3} = 0 \quad \text{or} \quad n = \sqrt{12} \]

This is a minimum because \( \frac{d^2C}{dn^2} > 0 \). The price of car for minimum transportation cost is,

\[ P = 20,000 + \frac{20,000}{\sqrt{12}} = 25,774 \text{ dollars} \]

Laura should invest in a car priced around 25,774 dollars.
Problem 3.7: Defect level

Defect level, $DL(T)$, given by Equation 3.20 (p. 50 of the book), can be written as:

$$DL(T) = 1 - \frac{(1 + TAf / \beta)^\beta}{(1 + Af / \beta)^\beta}$$

$$= 1 - \frac{e^{TAf}}{e^{Af}} = 1 - e^{-Af(1-T)}, \text{ as } \beta \to \infty$$

Also, as $\beta \to \infty$, Equation 3.19 (p. 50 of the book) gives the yield,

$$Y = \left(1 + \frac{Af}{\beta}\right)^{-\beta} = e^{-Af}$$

Substituting this expression for yield in the defect level, we get

$$DL(T) = 1 - (e^{-Af})^{1-T} = 1 - Y^{1-T}$$

which is the required result.

Problem 4.7: Fault indistinguishability

Without loss of information we will write a function $f(V)$ as $f$. Thus, the left hand side of Equation 4.3 is:

$$[f_0 \oplus f_1] \oplus [f_0 \oplus f_2]$$

$$= [f_0f_1 + \overline{f_0}f_1] \oplus [f_0f_2 + \overline{f_0}f_2]$$

$$= (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2) + (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2)$$

$$= (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2) + (f_0f_1)(f_0f_2 + \overline{f_0}f_2)$$

$$= (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2) + (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2)$$

$$= (f_0f_1 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2) + (f_0 + \overline{f_0}f_1)(f_0f_2 + \overline{f_0}f_2)$$

$$= f_0f_1f_2 + \overline{f_0}f_1f_2 + f_0f_1 + f_0f_1\overline{f_2}$$

$$= (f_0f_1f_2 + f_0f_1\overline{f_2} + f_0f_1 + f_0f_1\overline{f_2})$$

$$= f_1f_2 + f_1\overline{f_2}$$

$$= f_1 \oplus f_2$$

Left hand side of Equation 4.4

This completes the derivation of Equation 4.4 from Equation 4.3.

Problem 4.8: Functional equivalence

Faulty functions for the circuit of Figure 4.12 corresponding to the two faults are:

$$i(c \ s - a - 0) = b(\overline{a}b) = \overline{a}b$$

$$i(f \ s - a - 1) = (a + b)\overline{a} = \overline{a}b$$

The two faulty functions are indistinguishable and hence the two faults are equivalent.

Solution: Testing course homework 2: Economics, yield and fault modeling
Problem 4.10: Fault collapsing for test generation

The circuit of Figure 4.9 has 18 single stuck-at faults. Gate-level fault equivalence, as shown in the following figure reduces the number to 12. The faults in shaded boxes have been collapsed as shown by arrows. Many ATPG and fault simulation programs will collapse faults as shown above. However, functional fault collapsing can further reduce the number of faults to 10. As shown in Example 4.11 (see page 75 of the book), the s-a-1 faults on A1 and B1 are equivalent, and so are the s-a-1 faults on A2 and B2.

Whether we take the set of 12 faults or the set of 10 faults, their detection requires all four input vectors.