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On Cooperative Relay in Cognitive Radio Networks

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Abstract

Cognitive radios (CR) and cooperative communications represent new paradigms that both can effectively improve the spectrum efficiency of future wireless networks. In this paper, we investigate the problem of cooperative relay in CR networks for further enhanced network performance. We investigate how to effectively integrate these two advanced wireless communications technologies. In particular, we focus on the two representative cooperative relay strategies, *decode-and-forward* (DF) and *amplify-and-forward* (AF), and develop optimal spectrum sensing and p-Persistent CSMA for spectrum access. We develop an analysis for the comparison of these two relay strategies in the context of CR networks, and derive closed-form expressions for network-wide throughput achieved by DF, AF and direct link transmissions. Our analysis is validated by simulations. We find each of the strategies performs better in a certain parameter range; there is no case of dominance for the two strategies. The significant gaps between the cooperative relay results and the direct link results exemplify the diversity gain achieved by cooperative relays in CR networks.

Keywords: Cognitive radio networks, cooperative communications, relay, cooperative diversity, spectrum sensing, performance modeling.

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1. Introduction

According to Cisco's recent study, wireless data traffic is expected to increase by a factor of 66 times by 2013. Much of this future wireless data traffic will be video based services driven by the need for ubiquitous access to wireless multimedia content. Such drastic increase in traffic demand will significantly stress the capacity of future wireless networks.

Cognitive radios (CR) provide an effective solution to meeting this critical demand by exploiting co-deployed networks and aggregating underutilized spectrum for future wireless networks [2–6]. CR was motivated by the spectrum measurements by the FCC, where a significant amount of the assigned spectrum is found to remain underutilized. CR represents a paradigm change in spectrum regulation and access, from exclusive use by primary users to shared spectrum for secondary users, which can enhance spectrum utilization and achieve high throughput capacity. *Cooperative wireless communications* represents another new paradigm for wireless communications [7–9]. It allows wireless nodes to assist each other in data delivery, with the objective of achieving greater reliability and efficiency than each of them could attain individually (i.e., to achieve the so-called *cooperative diversity*). Cooperation among wireless nodes enables opportunistic use of energy and bandwidth resources in wireless networks, and can deliver many salient advantages over conventional point-to-point wireless communications.

Recently, there has been some interesting work on cooperative relay in CR networks [10, 11]. In [10], the authors considered the case of two single-user links, one primary and one secondary. The secondary transmitter is allowed to act as a "transparent" relay for the primary link, motivated by the rationale that helping primary users will lead to more transmission opportunities for CR nodes. In [11], the authors presented an excellent overview of several cooperative relay scenarios and various related issues. A new MAC protocol was proposed and implemented in a testbed to select a spectrum-rich CR node as relay for a CR transmitter/receiver pair.

★Part of this work was presented at IEEE GLOBECOM 2010, Miami, FL, USA, December 2010 [1].

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In this paper, we investigate the problem of cooperative relay in CR networks. We assume a primary network with multiple licensed bands and a CR network consisting of multiple cooperative relay links. Each cooperative relay link consists of a CR transmitter, a CR relay, and a CR receiver. The objective is to develop effective mechanisms to integrate these two wireless communication technologies, and to provide an analysis for the comparison of two representative cooperative relay strategies, i.e., *decode-and-forward* (DF) and *amplify-and-forward* (AF), in the context of CR networks. We first consider cooperative spectrum sensing by the CR nodes. We model both types of sensing errors, i.e., miss detection and false alarm, and derive the optimal value for the sensing threshold. Next, we incorporate DF and AF into the p-Persistent Carrier Sense Multiple Access (CSMA) protocol for channel access for the CR nodes. We develop closed-form expressions for the network-wide capacities achieved by DF and AF, respectively, as well as that for the case of direct link transmission for comparison purpose.

Through analytical and simulation evaluations of DF and AF-based cooperative relay strategies, we find the analysis provides upper bounds for the simulated results, which are reasonably tight. We also find cross-point with the AF and DF curves when some system parameter is varied, indicating that each of them performs better in a certain parameter range. There is no case that one completely dominates the other for the two strategies. The considerable gaps between the cooperative relay results and the direct link results exemplify the diversity gain achieved by cooperative relays in CR networks.

The remainder of this paper is organized as follows. The system model is described in section 2. We analyze the two CR cooperative relay strategies in Section 3. Our simulation evaluations are presented in Section 4. Related work is discussed in Section 5 and Section 6 concludes the paper. The notation used in the paper is summarized in Table 1.

Table 1. Notation

Symbol	*Definition*
M	number of licensed channels
$S_m(t)$	status of channel m at time slot t
λ_m	transition probability of channel m from idle to idle
μ_m	transition probability of channel m from busy to idle
η_m	utilization of channel m
γ_m	maximum allowable collision probability on channel m
N	number of CR cooperative relay links
N_m	number of CR nodes sensing channel m
H_0^m	hypothesis that channel m is idle
H_1^m	hypothesis that channel m is busy
ϵ_m	false alarm probability on channel m
δ_m	miss detection probability on channel m
Θ_i^m	sensing result of channel m at CR node i
D_m	decision variable for channel m
τ_m	sensing threshold for channel m
$a_m(\vec{\Theta}_m)$	probability that channel m is idle
$a_m^{(j)}$	the jth largest value of $a_m(\vec{\Theta}_m)$
$\vec{\theta}_m^{(j)}$	argument of $a_m(\vec{\Theta}_m)$ achieving the jth largest value $a_m^{(j)}$
P_s	source transmit power
P_r	relay transmit power
G_0^k	path gain of kth relay link from transmitter to receiver
G_1^k	path gain of kth relay link from transmitter to relay
G_2^k	path gain of kth relay link from relay to receiver
$\sigma_{r,k}^2$	noise at the kth relay
$\sigma_{d,k}^2$	noise at the kth receiver
κ	decoding threshold for received SNR
$\bar{F}_{G_0^k}(x)$	CCDF of G_0^k
$\bar{F}_{G_1^k}(x)$	CCDF of G_1^k
$\bar{F}_{G_2^k}(x)$	CCDF of G_2^k
L	packet length
P_{DF}^k	decoding probability of DF
P_{AF}^k	decoding probability of AF
P_{DL}^k	decoding probability of DL
N_{DF}	no. received frames in two consecutive time slots with DF
N_{AF}	no. received frames in two consecutive time slots with AF
N_{DL}	no. received frames in two consecutive time slots with DL
C_{DF}	network-wide capacity with DF
C_{AF}	network-wide capacity with AF
C_{DL}	network-wide capacity with DL

2. System Model

We assume a primary network and a spectrum band that is divided into M licensed channels, each modeled as a time slotted, block-fading channel. The state of each channel evolves independently following a discrete time Markov process [3]. The status of channel m in time slot t is denoted as $S_m(t)$, for $m = 1, 2, \cdots, M$. When $S_m(t) = 0$, the channel is in the idle state; when $S_m(t) = 1$, the channel is in the busy state (i.e., being used by primary users). Let λ_m and μ_m be the transition probability to remain in state 0 and the transition probability from state 1 to 0 for channel m, respectively. The channel model is illustrated in Fig. 1. The utilization of channel m with respect to primary

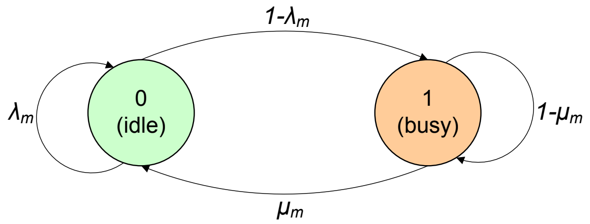

Figure 1. The discrete-time two-state Markov model for channel m, $m = 0, 1, \ldots, M-1$.

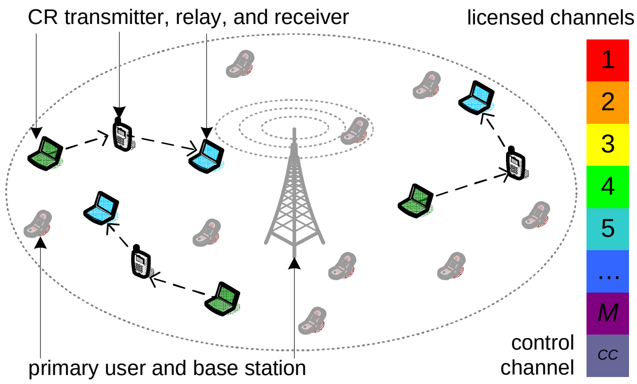

Figure 2. Illustration of colocated primary and CR networks. The CR network consists of a number cooperative relay links, each consisting of a CR transmitter, a CR relay and a CR receiver.

user transmissions, denoted by $\eta_m = \Pr\{S_m(t) = 1\}$, can be written as:

$$\eta_m = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} S_m(t) = \frac{1-\lambda_m}{1-\lambda_m+\mu_m}. \quad (1)$$

As illustrated in Fig. 2, there is a CR network colocated with the primary network. The CR network consists of N sets of cooperative relay links, each including a CR transmitter, a CR relay, and a CR receiver. Each CR node (or, secondary user) is equipped with two transceivers, each incorporating a software defined radio (SDR) that is able to tune to any of the M licensed channels and a control channel and operate from there.

We assume CR nodes access the licensed channels following the same time slot structure [3]. Each time slot is divided into two phases, the *sensing phase* and the *transmission phase*, as shown in Fig. 3. In the sensing phase, a CR node chooses one of the M channels to sense using one of its transceivers, and then exchanges sensed channel information with other CR nodes using the other transceiver over the control channel. During the transmission phase, the CR transmitter and/or relay transmit data frames on licensed channels that are believed to be idle based on sensing results, using one or

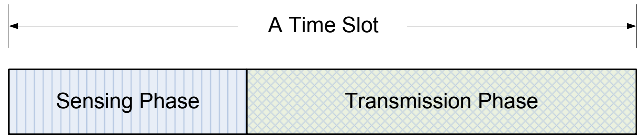

Figure 3. Time slot structure: a time slot consists of a sensing phase and a transmission phase.

both of the transceivers. We consider cooperative relay strategies AF and DF, and compare their performance in the following sections.

3. Cooperative Relay in CR Networks

In this section, we investigate how to effectively integrate the two advance wireless communication technologies, and present an analysis of the cooperative relay strategies in CR networks. We first examine cooperative spectrum sensing and derive the optimal sensing threshold. We then consider cooperative relay and spectrum access, and derive the network-wide throughput performance achievable when these two technologies are integrated.

3.1. Spectrum Sensing

Although precise and timely channel state information is desirable for spectrum access and primary user protection, continuous, full-spectrum sensing is both energy inefficient and hardware demanding [3]. Recall that each CR node is equipped with two transceivers and one has to be used for exchanging spectrum sensing results over the control channel. Thus each CR node can sense only one of the licensed channels at a time, using the remaining transceiver. Without loss of generality, we assume that each CR node senses a fixed channel throughout the time slots and receives from the control channel sensing results on other channels, i.e., distributed by other CR nodes.

During the sensing process, two kinds of detection errors may occur. A *false alarm* refers to the case when an idle channel is considered busy. Consequently, the CR nodes will not attempt to access that channel and a spectrum opportunity will be wasted. A *miss detection* refers to the case when a busy channel is considered idle. Since CR nodes will attempt to access this channel in the transmission phase, collisions with primary user transmissions will occur subsequently. Such spectrum sensing errors have been considered in the design of MAC protocols for CR networks [12, 13].

In this paper, we adopt hypothesis test to detect the availability of channel m. The null hypothesis H_0^m is "channel m is idle." The alternative hypothesis H_1^m is "channel m is busy." Let ϵ_i^m and δ_i^m be the probabilities of false alarm and miss detection, respectively, when CR node i senses channel m. We have

$$\epsilon_i^m = \Pr\left\{\Theta_i^m = 1 | H_0^m\right\} \quad \text{and} \tag{2}$$

$$\delta_i^m = \Pr\left\{\Theta_i^m = 0 | H_1^m\right\}, \tag{3}$$

where $\Theta_i^m \in \{0, 1\}$ is the channel m sensing result of channel m at node i.

Assume there are N_m CR nodes sensing channel m. After the sensing phase, each CR node obtains a *sensing result vector* $\vec{\Theta}_m = [\Theta_1^m, \Theta_2^m, \cdots, \Theta_{N_m}^m]$ for channel m. The conditional probability $a_m(\vec{\Theta}_m)$ on channel m availability is

$$\begin{aligned}
& a_m\left(\Theta_1^m, \Theta_2^m, \cdots, \Theta_{N_m}^m\right) \\
\cong\ & \Pr\left\{H_0^m | \Theta_1^m, \Theta_2^m, \cdots, \Theta_{N_m}^m\right\} \\
=\ & \frac{\Pr\left\{\Theta_1^m, \Theta_2^m, \cdots, \Theta_{N_m}^m | H_0^m\right\} \Pr\left\{H_0^m\right\}}{\sum_{j\in\{0,1\}} \Pr\left\{\Theta_1^m, \Theta_2^m, \cdots, \Theta_{N_m}^m | H_j^m\right\} \Pr\left\{H_j^m\right\}} \\
=\ & \frac{\prod_{i=1}^{N_m} \Pr\left\{\Theta_i^m | H_0^m\right\} \Pr\left\{H_0^m\right\}}{\sum_{j\in\{0,1\}} \prod_{i=1}^{N_m} \Pr\left\{\Theta_i^m | H_j^m\right\} \Pr\left\{H_j^m\right\}} \\
=\ & \left[1 + \frac{\Pr\left\{H_1^m\right\}}{\Pr\left\{H_0^m\right\}} \prod_{i=1}^{N_m} \frac{\Pr\left\{\Theta_i^m | H_1^m\right\}}{\Pr\left\{\Theta_i^m | H_0^m\right\}}\right]^{-1} \\
=\ & \left[1 + \frac{\eta_m}{1-\eta_m} \prod_{i=1}^{N_m} \frac{(\delta_i^m)^{1-\Theta_i^m}(1-\delta_i^m)^{\Theta_i^m}}{(\epsilon_i^m)^{\Theta_i^m}(1-\epsilon_i^m)^{1-\Theta_i^m}}\right]^{-1}.
\end{aligned} \tag{4}$$

If $a_m(\vec{\Theta}_m)$ is greater than a *sensing threshold* τ_m, channel m is believed to be idle; otherwise, channel m is believed to be busy. The decision variable D_m is defined as follows.

$$D_m = \begin{cases} 0, & \text{if } a_m(\vec{\Theta}_m) > \tau_m \\ 1, & \text{if } a_m(\vec{\Theta}_m) \le \tau_m. \end{cases} \tag{5}$$

CR nodes only attempt to access channel m where D_m is 0. Since function $a_m(\vec{\Theta}_m)$ in (4) has N_m binary variables, there can be 2^{N_m} different combinations corresponding to 2^{N_m} values for $a_m(\vec{\Theta}_m)$. We sort the 2^{N_m} combinations according to their $a_m(\vec{\Theta}_m)$ values in the non-increasing order. Let $a_m^{(j)}$ be the jth largest function value and $\vec{\theta}_m^{(j)}$ the argument that achieves the jth largest function value $a_m^{(j)}$, where

$$\vec{\theta}_m^{(j)} = [\theta_1^m(j), \theta_2^m(j), \cdots, \theta_{N_m}^m(j)].$$

In the design of CR networks, we consider two objectives:

1. how to avoid harmful interference to primary users, and
2. how to fully exploit spectrum opportunities for the CR nodes.

For primary user protection, we limit the collision probability with primary user with a threshold. Let γ_m be the *tolerance threshold*, i.e., the maximum allowable interference probability with primary users on channel m. The probability of collision with primary users on channel m is given as $\Pr\{D_m = 0 \mid H_1^m\}$; the probability of detecting an available transmission opportunity is $\Pr\{D_m = 0 \mid H_0^m\}$. Our objective is to maximize the probability of detecting available channels, while keeping the collision probability below γ_m. Therefore, the optimal spectrum sensing problem can be formulated as follows.

$$\max_{\tau_m} \quad \Pr\{D_m = 0 | H_0^m\} \tag{6}$$

$$\text{subect to:} \quad \Pr\{D_m = 0 | H_1^m\} \le \gamma_m. \tag{7}$$

From their definitions, both $\Pr\{D_m = 0 \mid H_1^m\}$ and $\Pr\{D_m = 0 \mid H_0^m\}$ are decreasing functions of τ_m. As $\Pr\{D_m = 0 \mid H_1^m)\}$ approaches its maximum allowed value γ_m, $\Pr\{D_m = 0 \mid H_0^m\}$ also approaches its maximum. Therefore, solving the optimization problem (6) $\sim$ (7) is equivalent to solving

$$\Pr\{D_m = 0 \mid H_1^m\} = \gamma_m.$$

If $\tau_m = a_m^{(j)}$, we have

$$\begin{aligned}
& \Pr\{D_m = 0 | H_1^m\}\left(a_m^{(j)}\right) \\
=\ & \Pr\left\{a_m(\vec{\Theta}_m) > a_m^{(j)} | H_1^m\right\} \\
=\ & \sum_{l=1}^{j-1} \Pr\left\{a_m(\vec{\Theta}_m) = a_m^{(l)} | H_1^m\right\} \\
=\ & \sum_{l=1}^{j-1} (\delta_i^m)^{1-\theta_i^m(l)}(1-\delta_i^m)^{\theta_i^m(l)}.
\end{aligned} \tag{8}$$

Obviously, $\Pr\{D_m = 0 \mid H_1^m\}(a_m^{(j)})$ is an increasing function of j. The optimal sensing threshold τ_m^* can be set to $a_m^{(j)}$, such that

$$\Pr\{D_m = 0 \mid H_1^m\}\left(a_m^{(j)}\right) \le \gamma_m$$

and

$$\Pr\{D_m = 0 \mid H_1^m\}\left(a_m^{(j+1)}\right) > \gamma_m.$$

The algorithm for computing the optimal sensing threshold τ_m^* is presented in Algorithm 1.

Once the optimal sensing threshold τ_m^* is determined, $\Pr\{D_m = 0 \mid H_1^m\}$ can be computed as given in (8) and

Algorithm 1: Algorithm for Computing the Optimal Sensing Threshold

Compute $a_m^{(j)}$ and the corresponding $\vec{\theta}_m^{(j)}$, for all j ;
Initialize $p_c = \Pr\left\{a_m(\vec{\Theta}_m) = a_m^{(1)}|H_1^m\right\}$ and $\tau_m = a_m^{(1)}$;
Set $j = 1$;
while $p_c \le \gamma_m$ **do**
  $j = j + 1$;
  $\tau_m = a_m^{(j)}$;
  $p_c = p_c + \Pr\left\{a_m(\vec{\Theta}_m) = a_m^{(j)}|H_1^m\right\}$;
end

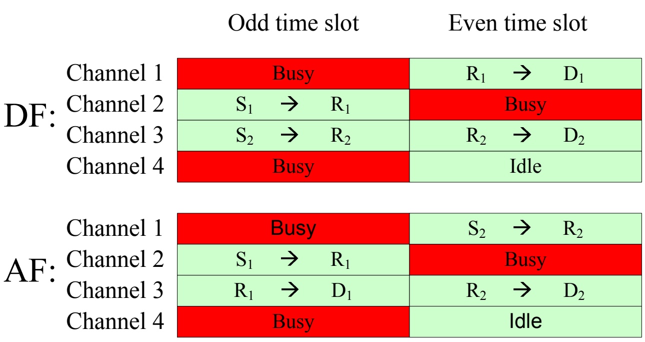

Figure 4. Illustration of the protocol operation of AF and DF, where $S_i \Rightarrow R_i$ represents the transmission from source to relay and $R_i \Rightarrow D_i$ represents the transmission from relay to destination, for the ith cooperative relay link.

$\Pr\{D_m = 0 \mid H_0^m\}$ can be computed as:

$$\begin{aligned}
& \Pr\{D_m = 0|H_0^m\} \\
= \; & \Pr\left\{a_m(\vec{\Theta}_m) > \tau_m^*|H_0^m\right\} \\
= \; & \sum_{l=1}^{j-1} \Pr\left\{a_m(\vec{\Theta}_m) = a_m^{(l)}|H_0^m\right\} \\
= \; & \sum_{l=1}^{j-1} (\epsilon_i^m)^{\theta_i^m(l)}(1-\epsilon_i^m)^{1-\theta_i^m(l)}. \qquad (9)
\end{aligned}$$

3.2. Cooperative Relay Strategies

During the transmission phase, CR transmitters and relays attempt to send data through the channels that are believed to be idle. We assume fixed length for all the data frames. Let G_1^k and G_2^k denote the path gains from the transmitter to relay and from the relay to receiver, respectively, and let $\sigma_{r,k}^2$ and $\sigma_{d,k}^2$ denote the noise powers at the relay and receiver, respectively, for the kth cooperative relay link. We examine the two cooperation relay strategies DF and AF in the following. For comparison purpose, we also consider direct link transmission below.

Decode-and-Forward (DF). With DF, the CR transmitter and relay transmit separately on consecutive odd and event time slots: the CR transmitter sends data to the corresponding relay in an *odd* time slot; the relay node then decodes the data and forwards it to the receiver in the following *even* time slot, as shown in Fig. 4.

Without loss of generality, we assume a data frame can be successfully decoded if the received signal-to-noise ratio (SNR) is no less than a *decoding threshold* κ. That is, outage probability of the cooperative channel is used to approximate packet loss probability. We assume gains on different links are independent to each other. The receiver can successfully decode the frame if it is not lost or corrupted on both links. The *decoding rate* of DF at the kth receiver, denoted by P_{DF}^k, can be computed as,

$$\begin{aligned}
P_{DF}^k &= \Pr\left\{\left(\frac{P_s G_1^k}{\sigma_{r,k}^2} \ge \kappa\right) \text{ and } \left(\frac{P_r G_2^k}{\sigma_{d,k}^2} \ge \kappa\right)\right\} \\
&= \bar{F}_{G_1^k}\left(\frac{\sigma_{r,k}^2 \kappa}{P_s}\right) \bar{F}_{G_2^k}\left(\frac{\sigma_{d,k}^2 \kappa}{P_r}\right), \qquad (10)
\end{aligned}$$

where P_s and P_r are the transmit powers at the transmitter and relay, respectively, $\bar{F}_{G_1^k}(x)$ and $\bar{F}_{G_2^k}(x)$ are the complementary cumulative distribution functions (CCDF) of path gains G_1^k and G_2^k, respectively.

Amplify-and-Forward (AF). With AF, the CR transmitter and relay transmit simultaneously in the same time slot on different channels. A pipeline is formed connecting the CR transmitter to the relay and then to the receiver; the relay amplifies the received signal and immediately forwards it to the receiver in the same time slot, as shown in Fig. 4. Recall that the CR relay has two transceivers. The relay receives data from the transmitter using one transceiver operating on one or more idle channels; it forwards the data simultaneously to the receiver using the other transceiver operating on one or more *different* idle channels.

With this cooperative relay strategy, a data frame can be successfully decoded if the SNR at the receiver is no less than the decoding threshold κ. Then the decoding rate of AF at the kth receiver, denoted as P_{AF}^k, can be computed as,

$$\begin{aligned}
P_{AF}^k &= \Pr\left\{\frac{P_r}{G_1^k P_s + \sigma_{r,k}^2} \frac{P_s G_1^k G_2^k}{\sigma_{d,k}^2} \ge \kappa\right\} \\
&= \int_0^{+\infty} \bar{F}_{G_2^k}\left(\frac{(P_s x + \sigma_{r,k}^2)\sigma_{d,k}^2 \kappa}{P_s P_r x}\right) dF_{G_1^k}(x). \qquad (11)
\end{aligned}$$

Direct Link Transmission. For comparison purpose, we also consider the case of direct link transmission (DL). That is, the CR transmitter transmits to the receiver via the direct link; the CR relay is not used in this case. Let the path gain be G_0^k with CCDF $\bar{F}_{G_0^k}(x)$, and recall that the noise power is $\sigma_{d,k}^2$ at the receiver, for the kth direct link transmission.

Following similar analysis, the decoding rate of DL at the kth receiver, denoted as P_{DL}^k, can be computed as

$$P_{DL}^k = \Pr\left\{\frac{P_s G_0^k}{\sigma_{d,k}^2} \geq \kappa\right\} = \bar{F}_{G_0^k}\left(\frac{\sigma_{d,k}^2 \kappa}{P_s}\right). \quad (12)$$

3.3. Opportunistic Channel Access

We assume greedy transmitters that always have data to send. The CR nodes use p-Persistent CSMA for channel access. At the beginning of the transmission phase of an odd time slot, CR transmitters send Request-to-Send (RTS) with probability p over the control channel. Since there are N CR transmitters, the transmission probability p is set to $1/N$ to maximize the throughput (i.e., to maximize P_1 in (13) given below).

The following three cases may occur:

- *Case 1*: none of the CR transmitters sends RTS for channel access. The idle licensed channels will be wasted.
- *Case 2*: only one CR transmitter sends RTS, and it successfully receives Clear-to-Send (CTS) from the receiver over the control channel. It then accesses some of or all the licensed channels that are believed to be idle for data transmission in the transmission phase.
- *Case 3*: more than one CR transmitters send RTS and collision occurs on the control channel. No CR node can access the licensed channels, and the idle licensed channels will be wasted.

Let P_0, P_1 and P_2 denote the probability corresponding to the three cases enumerated above, respectively. We then have

$$P_0 = (1-p)^N = \left(1 - \frac{1}{N}\right)^N \quad (13)$$

$$P_1 = Np(1-p)^{N-1} = \left(1 - \frac{1}{N}\right)^{N-1} \quad (14)$$

$$P_2 = 1 - P_0 - P_1. \quad (15)$$

The CR cooperative relay link that wins the channels in the odd time slot will continue to use the channels in the following even time slot. A new round of channel competition will start in the next odd time slot following these two time slots.

Since a licensed channel is accessed with probability P_1 in the odd time slot, we modify the tolerance threshold γ_m as $\gamma_m' = \gamma_m/P_1$, such that the maximum allowable collision requirement can still be satisfied. In the even time slot, the channels will continue to be used by the winning cooperative relay link, i.e., to be accessed with probability 1. Therefore, the tolerance threshold is still γ_m for the even time slots.

3.4. Capacity Analysis

Once the CR transmitter wins the competition, as indicated by a received CTS, it begins to send data over the licensed channels that are inferred to be idle (i.e., $D_m = 0$) in the transmission phase. We assume the *channel bonding and aggregation* technique is used, such that multiple channels can be used collectively by a CR node for data transmission [14, 15].

With DF, the winning CR transmitter uses all the available channels to transmit to the relay in the odd time slot. In the following even time slot, the CR transmitter stops transmission, while the relay uses the available channels in the even time slot to forward data to the receiver. If the number of available channels in the even time slot is equal to or greater than that in the odd time slot, the relay uses the same number of channels to forward all the received data. Otherwise, the relay uses all the available channels to forward part of the received data; the excess data will be dropped due to limited channel resource in the even time slot. The dropped data will be retransmitted in some future odd time slot by the transmitter.

With AF, no matter it is an odd or even time slot, the CR transmitter always uses half of the available licensed channels to transmit to the relay. The relay uses one of its transceivers to receive from the chosen half of the available channels. Simultaneously, it uses the other transceiver to forward the received data to the receiver using the remaining half of the available channels.

Let D_m^{od} and D_m^{ev} be the decision variables of channel m in the odd and even time slot, respectively (see (5)). Let S_m^{od} and S_m^{ev} be the status of channel m in the odd and even time slot, respectively. We have,

$$\begin{aligned} &\Pr\left\{D_m^{od} = i, S_m^{od} = j, D_m^{ev} = k, S_m^{ev} = l\right\} \quad (16) \\ &= \Pr\left\{D_m^{ev} = k | S_m^{ev} = l\right\} \Pr\left\{D_m^{od} = i | S_m^{od} = j\right\} \times \\ &\quad \Pr\left\{S_m^{ev} = l | S_m^{od} = j\right\} \Pr\left\{S_m^{od} = j\right\}, \text{ for } i, j, k, l \in \{0, 1\}. \end{aligned}$$

where $\Pr\{S_m^{od} = j\}$ are the probabilities that channel m is busy or idle, $\Pr\{S_m^{ev} = l \mid S_m^{od} = j\}$ are the channel m transition probabilities. $\Pr\{D_m^{ev} = k \mid S_m^{ev} = l\}$ and $\Pr\{D_m^{od} = i \mid S_m^{od} = j\}$ can be computed as in (8) and (9).

Let N_{DF}, N_{AF} and N_{DL} be the number of frames successfully delivered to the receiver in the two consecutive time slots using DF, AF and DL, respectively. Define $\bar{S}_m^{od} = 1 - S_m^{od}$, $\bar{S}_m^{ev} = 1 - S_m^{ev}$, $\bar{D}_m^{od} = 1 - D_m^{od}$ and

$\bar{D}_m^{ev} = 1 - D_m^{ev}$. We have

$$N_{DF} = \left(\sum_{m=1}^{M} \bar{S}_m^{od} \bar{D}_m^{od}\right) \wedge \left(\sum_{m=1}^{M} \bar{S}_m^{ev} \bar{D}_m^{ev}\right) \quad (17)$$

$$N_{AF} = \left\lfloor \frac{1}{2} \sum_{m=1}^{M} \bar{S}_m^{od} \bar{D}_m^{od} \right\rfloor + \left\lfloor \frac{1}{2} \sum_{m=1}^{M} \bar{S}_m^{ev} \bar{D}_m^{ev} \right\rfloor \quad (18)$$

$$N_{DL} = \left(\sum_{m=1}^{M} \bar{S}_m^{od} \bar{D}_m^{od}\right) + \left(\sum_{m=1}^{M} \bar{S}_m^{ev} \bar{D}_m^{ev}\right), \quad (19)$$

where $x \wedge y$ represents the minimum of x and y, and $\lfloor x \rfloor$ means the maximum integer that is not larger than x.

As discussed, the probability that a frame can be successfully delivered is P_{DF}^k, P_{AF}^k, or P_{DL}^k for the three schemes, respectively. Recall that spectrum resources are allocated distributedly for every pair of two consecutive time slots. We derive the capacity for the three cooperative relay strategies as

$$C_{DF} = \mathrm{E}[N_{DF}] \times \sum_{k=1}^{N} \frac{P_{DF}^k P_1 L}{2N T_s} \quad (20)$$

$$C_{AF} = \mathrm{E}[N_{AF}] \times \sum_{k=1}^{N} \frac{P_{AF}^k P_1 L}{2N T_s} \quad (21)$$

$$C_{DL} = \mathrm{E}[N_{DL}] \times \sum_{k=1}^{N} \frac{P_{DL}^k P_1 L}{2N T_s}, \quad (22)$$

where L is the packet length and T_s is the duration of a time slot. The expectations are computed using the results derived in (16) ∼ (19).

4. Performance Evaluation

We evaluate the performance of the cooperative relay strategies with analysis and simulations. The analytical capacities of the schemes are obtained with the analysis presented in Section 3. The actual throughput is obtained using MATLAB simulations. The simulation parameters and their values are listed in Table 2, unless specified otherwise. We consider $M = 5$ licensed channels and a CR network with seven cooperative relay links. The channels have identical parameters for the Markov chain models. Each point in the simulation curves is the average of 10 simulation runs with different random seeds. We plot 95% confidence intervals for the simulation results, which are negligible in all the cases.

We first examine the impact of the number of licensed channels. To illustrate the effect of spectrum sensing, we let the decoding rate P_{AF}^k be equal to P_{DF}^k. In Fig. 5, we plot the throughput of AF, DF, and DL under increased number of licensed channels. The analytical curves are upper bounds for the simulation curves in all the cases, and the gap between the two is reasonably small. Furthermore, as the number of license channels is increased, the throughput of both AF and DF are increased. The slope of the AF curves is larger than that of the DF curves. There is a cross point between five and six, as predicted by both simulation and analysis curves. This indicates that AF outperforms DF when the number of channels is large. This is because AF is more flexible than DF in exploiting the idle channels in the two consecutive time slots. The DL analysis and simulation curves also increases with the number of channels, but with the lowest slope and the lowest throughput values.

In Fig. 6, we demonstrate the impact of channel utilization on the throughput of the schemes. The channel utilization η is increased from 0.3 to 0.9, when primary users get more active. As η is increased, the

Table 2. Simulation Parameters and Values

Symbol	*Value*	*Definition*
M	5	number of licensed channels
λ	0.7	channel transition probability from idle to idle
μ	0.2	channel transition probability from busy to idle
η	0.6	channel utilization
γ	0.08	maximum allowable collision probability
N	7	number of CR cooperative relay links
P_s	10 dBm	transmit power of the CR transmitters
P_r	10 dBm	transmit power of CR relays
L	1 kb	packet length
T_s	1 ms	duration of a time slot

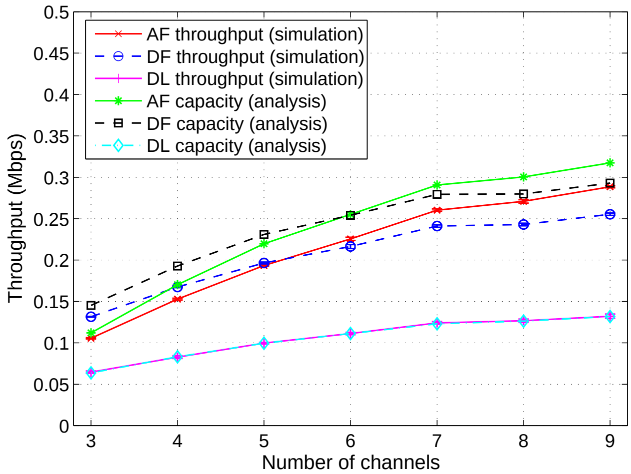

Figure 5. Throughput performance versus number of licensed channels.

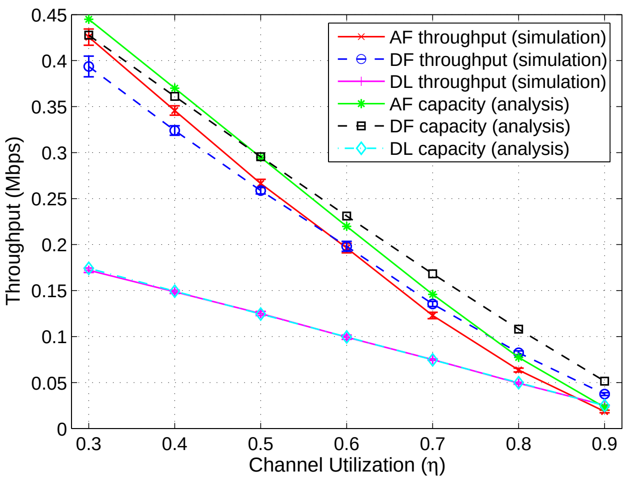

Figure 6. Throughput performance versus primary user channel utilization.

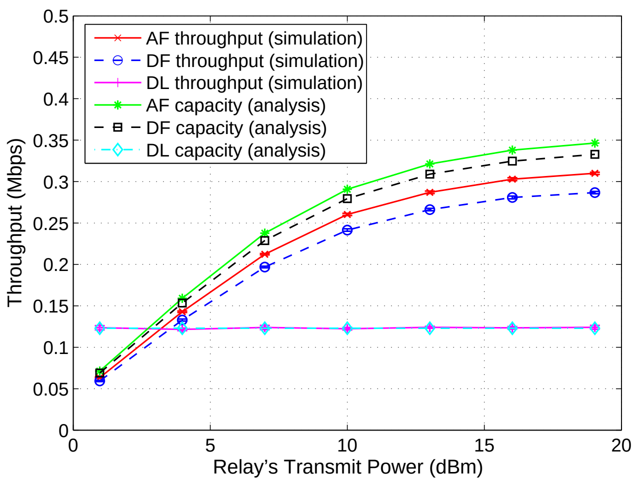

Figure 7. Throughput performance versus transmit power of relay nodes.

transmission opportunities for CR nodes are reduced and all the throughputs are degraded. We find the throughputs of AF and DF are close to each other when the channel utilization is high. AF outperforms DF in the low channel utilization region, but is inferior to DF in the high channel utilization region. There is a cross point between the AF and DF curves between $\eta = 0.5$ and $\eta = 0.6$. When the channel utilization is low, there is a big gap between the cooperative relay curves and and the DL curves.

In Fig. 7, we examine the channel fading factor. We consider Rayleigh block fading channels, where the received power is exponentially distributed with a distance-dependent mean. We fix the transmitter power at 10 dBm, and increase the relay power from one dBm to 18 dBm. As the relay power is increased, the throughput is also increased since the SNR at the receiver is improved. We can see the increasing speed of AF is larger than that of DF, indicating that AF has superior performance than DF when the relay transmit power is large. The capacity analysis also demonstrate the same trend. The throughput of DL does not depend on the relay node. Its throughput is better than that of AF and DF when the relay transmit power is low, since both AF and DF are limited by the relay-to-receiver link in this low power region. However, the throughputs of AF and DF quickly exceed that of DL and grow fast as the relay-to-receiver link is improved with the increased relay transmit power. The considerable gaps between the cooperative relay link curves and the DL curves in Figs. 5, 6 and 7 exemplify the diversity gain achieved by cooperative relays in CR networks.

5. Related Work

The theoretical foundation of relay channels was laid by the seminal work [16]. The capacities of the Gaussian relay channel and certain discrete relay channels are evaluated, and the achievable lower bound to the capacity of the general relay channel is established in this work. In [7, 8], the authors described the concept of cooperative diversity, where diversity gains are achieved via the cooperation of mobile users. In [9], the authors developed and analyzed low-complexity cooperative diversity protocols. Several cooperative strategies, including AF and DF, were described and their performance characterizations were derived in terms of outage probabilities.

In practice, there is a restriction that each node cannot transmit and receive simultaneously in the same frequency band. The "cheap" relay channel concept was introduced in [17], where the authors derived the capacity of the Gaussian degraded "cheap" relay channel. Multiple relay nodes for a transmitter-receiver pair are investigated in [18] and [19]. The authors showed that, when compared with complex protocols that involve all relays, the simplified protocol with no more than one relay chosen can achieve the same performance. This is the reason why we consider single relay in this paper.

In [20], Ng and Yu proposed a utility maximization framework for joint optimization of node, relay strategy selection, and power, bandwidth and rate allocation in a cellular network. Cai et al. [21] presented a semi-distributed algorithm for AF relay networks. A heuristic was adopted to select relay and allocate power. Both AF and DF were considered in [22], where a polynomial time algorithm for optimal relay selection was developed and proved to be optimal. In [23], a protocol is proposed for joint routing, relay selection, and dynamic spectrum allocation for multi-hop CR networks, and its performance is evaluated through simulations.

In [24], we investigate cooperative geographical routing in the context of wireless sensor networks. In a recent work [25], we investigate the problem of combing cooperative relay with CR for multiuser downlink video streaming, where interference alignment is incorporated to facilitate concurrent transmissions of multiple video packets.

6. Conclusion

In this paper, we studied the problem of cooperative relay in CR networks. We modeled the two cooperative relay strategies, i.e., DF and AF, which are integrated with p-Persistent CSMA. We analyzed their throughput performance and compared them under various parameter ranges. Cross-point with the AF and DF curves are found when some parameter is varied, indicating that each of them performs better in a certain parameter range; there is no case of dominance for the two strategies. Considerable gains were observed over conventional DL transmissions, as achieved by exploiting cooperative diversity with the cooperative relays in CR networks.

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