Lab 8. Speed Control of a DC motor

The Motor Drive
Motor Speed Control Project

1. Generate PWM waveform
2. Amplify the waveform to drive the motor
3. Measure tachometer signal (motor speed)
4. Find parameters of a motor model
5. Control motor speed with a computer algorithm
Buehler 12 volt permanent-magnet dc motor with tachometer output

**Electrical Connections**

yellow/green -- tachometer output

blue/red -- motor winding

*Note: Tachometer wires may not have two colors on some units.*
Exploded view
Some questions

- Required power? \[ P = V_{\text{motor}} \times I_{\text{motor}} \]
- Ac tachometer signal behavior?
Set up an experiment

- Measure $V_{\text{motor}}$, $V_R$, and $V_{\text{tach}}$
- $I_{\text{motor}} = V_R$ (because $R = 1\ \Omega$)
Experimental results

Current reaches 1 amp during startup!
Some observations

- $V_{tach}$ amplitude grows with motor speed
- $V_{tach}$ frequency also grows with speed
- Initial current $I_{motor}$ peaks around 1 A
- Steady state $I_{motor}$ is approx. 250 mA

Why does the process behave this way?

Some analytical modeling...
Motor electro-mechanical models

- $R_a$ – armature winding resistance
- $L_a$ – armature winding inductance
- $i_a$ – armature current
- $V_t$ – motor terminal voltage
- $e_a$ – back emf
- $T_m$ – developed torque
- $T_L$ – torque needed for load
- $\omega$ – rotational speed
- $B$ – friction coefficient
- $J$ – moment of inertia
Motor electrical dynamics

\[ v_t = R_a \cdot i_a + L \frac{di_a}{dt} + e_a \]

\[ e_a = K\omega_m \]

\( e_a \) = “back emf” (electromotive force) generated within armature windings

Note: Emf \( e_a \) = 0 at standstill, and increases linearly with motor speed. Current \( i_a \) is high at low speed.
Mechanical dynamics analogous to electrical circuits!

Equations for these systems have similar form.
**Motor mechanical dynamics**

\[ T_m = J \cdot \frac{d\omega}{dt} + B \cdot \omega + T_L \]

\[ T_m = K \cdot i_a \]

- \( T_m \) = developed torque increases with current
- \( J \) = motor moment of inertia
- \( B \) = motor friction coefficient
- \( \omega \) = angular velocity of the motor
- \( T_L \) = torque required to drive the load
Laplace transformed equations

- **Electrical**

\[ V_t(s) = R_a \cdot I_a(s) + L_a \cdot sI_a(s) + K \cdot \Omega(s) \]

- **Mechanical**

\[ K \cdot I_a(s) = J \cdot s\Omega(s) + B \cdot \Omega(s) + T_L(s) \]
Steady state analysis (s=0)

- Electrical steady state
  \[ V_t = R_a \cdot I_a + K \cdot \Omega \]

- Mechanical steady state
  \[ K \cdot I_a = B \cdot \Omega + T_L \]

- Solve for speed
  \[ \Omega = -\frac{R_a}{R_a B_m + K^2} \cdot T_L + \frac{K}{R_a B_m + K^2} \cdot V_t \]
Motor speed vs. load torque

- Speed is related to load torque and terminal voltage

\[ \Omega = -\frac{R_a}{R_a B_m + K^2} \cdot T_L + \frac{K}{R_a B_m + K^2} \cdot V_t \]

- Increasing \( V_t \)
- Operating points
- Speed 1
- Speed 2
- Load 1
- Load 2
What we now know:

- For a given load, motor speed is proportional to voltage applied to its terminals.
- Use of a PWM signal allows the *average* voltage of the signal to be varied by varying duty cycle.

\[ V_{avg} = V_{digital} \left( \frac{T1}{T1 + T2} \right) \]

- T1 = “ON” time
- T2 = “OFF” time

- We have a 12 Vdc motor (max. terminal voltage is 12 Vdc)
  - A 3 volt signal will be insufficient to produce full speed, PLUS …
  - Motor may draw 1 A of current, whereas microcontroller output pins can typically supply only milliamperes

  Idea: Use a single transistor switch to amplify the digital PWM signal to drive the motor.
Basic transistor switch

(ideal models)
Switching an inductive load (motor winding)

- Inductor voltage-current law:
  \[ V_L(t) = L \frac{di_C}{dt} \]

- As current \( i_C \) is switching **off**,
  - \( \frac{di_C}{dt} \) is large and **negative**
  - Inductor voltage \( V_L \) is large and **negative**
  - Collector voltage > \( V_{cc} \)

- \( Q \) may be destroyed!
Switching an inductive load
(need to protect switch Q)

- Use anti-parallel diode $D$!!!
  - reverse biased when $Q$ is ON
  - gives alternate current path when $Q$ switches OFF (when inductor voltage becomes negative)
  - protects $Q$
    - Collector voltage is clamped to $V_{cc} + V_{diode}$

- a.k.a. freewheeling diode
Drive design practical model

\[ V_{\text{in}} = V_{OH} \]

\[ I_B \]

\[ V_{BE(sat)} \]

\[ V_{CE(sat)} \]

\[ I_{load} \]
Drive design considerations

- Maximum load current, $I_{LOAD}$
- Transistor characteristics
  - current gain, $h_{FE}$
  - voltage $V_{BE\text{(sat)}}$ in saturation mode
- Microcontroller limitations
  - digital pin output voltage (high), $V_{OH}$
  - digital pin output current, $I_{IO} \approx 20$ mA (max)
Design equations

- Constraints for base current in the ON state
  
  \[ I_{IO} > I_B >> \frac{I_{LOAD}}{h_{FE}} \]

- Calculate base series resistance, \( R \)
  
  \[ R = \frac{V_{OH} - V_{BE(sat)}}{I_B} \]
EE Board variable power supply

Positive Supply
VP+ output voltage & current limit

VP+ ON

Actual VP+ Current

Waveforms Power Supply Window
Connect grounds of multiple power supplies
Lab procedure

- Verify proper PWM signal generation
- Study amplifier behavior
  - Measure $V_{in}$, $V_{BE}$, $V_{CE}$
  - Compare to theoretical assumptions
- Study motor behavior
  - Measure tachometer output (yellow/green leads)
  - Plot motor speed vs. PWM signal duty cycle
  - Repeat for several PWM signal frequencies
  - Analyze data and discuss results
Choice of devices

- Transistor (Q)
  - 2N3904 is cheap but under-rated for current
  - 2N2222 has higher current rating
  - Both may be destroyed if motor is stalled

- Diode (D)
  - 1N4001 is a rectifier diode: a bit slow, has large diameter leads
  - 1N4148 (or 1N914) is a switching diode: faster, but has low current rating (but is not expensive)
### Absolute Maximum Ratings

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CEO}$</td>
<td>Collector-emitter voltage (base open)</td>
<td>40</td>
<td>V</td>
</tr>
<tr>
<td>$V_{CBO}$</td>
<td>Collector-base voltage (emitter open)</td>
<td>75</td>
<td>V</td>
</tr>
<tr>
<td>$V_{EBO}$</td>
<td>Emitter-base voltage (collector open)</td>
<td>6</td>
<td>V</td>
</tr>
<tr>
<td>$I_{C}$</td>
<td>Collector current</td>
<td>1</td>
<td>A</td>
</tr>
</tbody>
</table>

### Electrical Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Conditions</th>
<th>min</th>
<th>max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{FE}$</td>
<td>Dc current gain</td>
<td>$I_{C} = 150$ mA, $V_{CE} = 1$ V</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{CE(sat)}$</td>
<td>Collector-emitter saturation voltage</td>
<td>$I_{C} = 150$ mA, $I_{B} = 15$ mA</td>
<td>0.3</td>
<td>1.2</td>
<td>V</td>
</tr>
<tr>
<td>$V_{BE(sat)}$</td>
<td>Base-emitter saturation voltage</td>
<td>$I_{C} = 150$ mA, $I_{B} = 15$ mA</td>
<td>0.6</td>
<td>1.2</td>
<td>V</td>
</tr>
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# 2N3904 NPN transistor data

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<td>$V_{CBO}$</td>
<td>Collector-base voltage (emitter open)</td>
<td>60</td>
<td>V</td>
</tr>
<tr>
<td>$V_{EBO}$</td>
<td>Emitter-base voltage (collector open)</td>
<td>6</td>
<td>V</td>
</tr>
<tr>
<td>$I_C$</td>
<td>Collector current</td>
<td>200</td>
<td>mA</td>
</tr>
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<tbody>
<tr>
<td>$h_{FE}$</td>
<td>Dc current gain</td>
<td>$I_C = 100$ mA, $V_{CE} = 1$ V</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{CE(sat)}$</td>
<td>Collector-emitter saturation voltage</td>
<td>$I_C = 50$ mA, $I_B = 5$ mA</td>
<td>0.3</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>$V_{BE(sat)}$</td>
<td>Base-emitter saturation voltage</td>
<td>$I_C = 150$ mA, $I_B = 5$ mA</td>
<td>0.95</td>
<td></td>
<td>V</td>
</tr>
</tbody>
</table>

Source: Fairchild Semiconductor
### 1N4148 switching diode data

Source: Fairchild Semiconductor

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<tr>
<td>$V_{RRM}$</td>
<td>Maximum repetitive reverse voltage</td>
<td>100</td>
<td>V</td>
</tr>
<tr>
<td>$I_O$</td>
<td>Average rectified forward current</td>
<td>200</td>
<td>mA</td>
</tr>
<tr>
<td>$I_F$</td>
<td>Dc forward current</td>
<td>300</td>
<td>mA</td>
</tr>
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<td>$I_C$</td>
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<tr>
<td>$V_F$</td>
<td>Forward voltage</td>
<td>$I_F = 100$ mA</td>
<td>1</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>$I_R$</td>
<td>Reverse leakage</td>
<td>$V_R = 20$ V</td>
<td>0.025</td>
<td></td>
<td>µA</td>
</tr>
<tr>
<td>$t_{rr}$</td>
<td>Reverse recovery time</td>
<td>$I_F = 10$ mA, $V_R = 6$ V, $I_{rr} = 1$ mA, $R_L = 100$ ohm</td>
<td>4</td>
<td></td>
<td>ns</td>
</tr>
</tbody>
</table>