Lab 11. Speed Control of a D.C. motor

Motor Characterization
Motor Speed Control Project

1. Generate PWM waveform
2. Amplify the waveform to drive the motor
3. Measure motor speed
4. Estimate motor parameters from measured data
5. Regulate speed with a controller
Goals of this lab

- Experimentally determine the control system model of the motor/hardware setup
  - Measure response to a step input
    (determine time constant, gain, etc.)

- This model will be used in the design of a speed controller
Motor control system modeled as a feedback system

(Frequency domain model)

- Setpoint $R(s)$
- Error $E(s)$
- Controller $C(s)$
- Amplifier $A(s)$
- Motor $M(s)$
- Sensor $H(s)$
- Disturbance $D(s)$

- Software
- PWM signal

- Tachometer + comparator/counter (period)
  or envelope detector (amplitude)

User entry (system input)
Simplified system model

Duty cycle of PWM signal

Switch setting

Setpoint \( R(s) \)

Error \( E(S) \)

Controller \( C(S) \)

Control Action \( X(S) \)

The Plant \( G(S) \)

Determine experimentally

Motor and Electronics

Measured Signal \( Y(S) \)

Computer Software

period or amplitude
What goes into the plant $G(s)$?

- Amplifier dynamics
- Electrical dynamics (motor winding has inductance and resistance)
- Mechanical dynamics (motor rotor has inertia and experiences friction)
- Sensor dynamics (filter has capacitance and resistance)

OVERALL: A 3rd order model (or higher)
An Empirical Modeling Approach

- Experimentally determine “plant” model, $G(s)$
  1. Apply a “step input” to the Plant
     - step change in the duty cycle of the PWM signal driving the motor
  2. Measure the motor system “response” to this step input
     - measure speed change over time
  3. Derive parameters of $G(s)$ from the measured response
Response $y(t)$ of a 1st-order system to a step input $x(t)$

Motor speed (ADC reading)

Plant input = change in PWM duty cycle (at $t = 0$)
First-order system model

System equation:

\[ Kx(t) = \tau \frac{dy}{dt} + y(t) \]

- \( x(t) = \) system input
- \( y(t) = \) system output
- \( K = \) gain
- \( \tau = \) time constant

Solution if step input applied at \( t=0 \) (step response):

\[ \Delta y(t) = K\Delta x(t)(1 - e^{-t/\tau}) \]

- \( \Delta x = \) input change at time \( t=0 \)

Laplace transform (plant transfer function):

\[ G(s) = \frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1} \]
Experimentally determining $G(s)$ for the first-order system

- After the transient period ($t$ large), study output $y$:
  \[ \Delta y = K \Delta x \]

  \[ K = \frac{\Delta y}{\Delta x} \]

  Experimentally measure change in $y$ (after large $t$) to compute gain, $K$.

- At $t=\tau$, step response is:
  \[ y(\tau) = K \Delta x (1 - e^{-\tau/\tau}) \]
  \[ y(\tau) = K \Delta x (0.632) \]

  Experimentally measure time at which $y(t) = 63.2\%$ of final value to determine time constant, $\tau$. 
Finding gain $K$

$$K = \frac{\Delta y}{\Delta x}$$
Finding time constant $\tau$

$$t = 0$$

settling time $\approx 4\tau$ or $5\tau$

$0.632 \Delta y$

$\Delta y$

$\Delta x$

$\tau$
Verify model in MATLAB/Simulink

(Controller to be added to this to compute the controller parameters.)
Later: Design a controller in Matlab/Simulink

One method: Proportional-Integral-Derivative (PID) controller

\[ a(t) = K_P e(t) + K_I \int_0^t e(t) \, dt + K_D \frac{de(t)}{dt} \]

Select P-I-D constants to produce the desired response.

- Start with P value to improve response time
- Use I term to eliminate steady-state error
- Use D term to further improve response
First-order response with delay

\[ y(t) \]

\[ y(t - \Delta t) \]
First-order system with delay

\[ G(s) = \frac{K}{\tau s + 1} e^{-\Delta ts} \]

\[ e^{-\Delta ts} \] represents time delay \( \Delta t \)
Second-order step response

- Overdamped (real, unequal poles)
- Underdamped
- Critically damped
Underdamped 2nd-order model

\[ G(s) = \frac{Y(s)}{X(s)} = K \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

- **Gain**
- **Damping factor**
- **Undamped natural frequency**
2nd-order model character (a)

- Underdamped (0 < \(\zeta\) < 1) model has complex conjugate poles:

\[
S_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}
\]

- Time constant: inverse of the |Re| part

\[
\tau = \frac{1}{\zeta \omega_n}
\]
Underdamped step response

overshoot

damped oscillation

frequency \( \omega_d = \frac{2\pi}{\text{period}} \)

\( K = \frac{\Delta y}{\Delta x} \)

settling time \( \approx 4\tau \)
2nd-order model character (b)

- oscillation frequency (rad/s): Im part
  \[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

- overshoot (% of final value)
  \[ \text{\% overshoot} = e^{-\left(\frac{\text{Re}}{\text{Im}}\right)\pi} \times 100 \]
  - a function only of damping factor
Other 2nd-order forms

- Critically damped model has 2 equal poles

\[ G(s) = \frac{K}{(\tau s + 1)^2} \]

- Overdamped model has unequal poles

\[ G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \]
Lab Procedure

- Re-verify hardware/software from previous labs
- Modify software to measure the period (or voltage) of the tachometer signal following a step input
  - “Step input” = change in selected speed
  - Save values in an array that can be transferred to the host PC after the motor is stopped
- Plot measured speed vs. time
- Choose a model (1st-order? 2nd-order?)
- Determine model parameters and write the transfer function $G(s)$
- Compare step response of $G(s)$ to the experimental response (suggested tool: MATLAB/Simulink)